

An Interactive Editor for Curve-Skeletons: SkeletonLab

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Abstract

Curve-skeletons are powerful shape descriptors able to provide higher level information on topology, structure and semantics of a given digital object. Their range of application is wide and encompasses computer animation, shape matching, modelling and remeshing. While a universally accepted definition of curve-skeleton is still lacking, there are currently many algorithms for the curve-skeleton computation (or *skeletonization*) as well as different techniques for building a mesh around a given curve-skeleton (*inverse skeletonization*). Despite their widespread use, automatically extracted skeletons usually need to be processed in order to be used in further stages of any pipeline, due to different requirements. We present here an advanced tool, named SkeletonLab, that provides simple interactive techniques to rapidly and automatically edit and repair curve skeletons generated using different techniques proposed in literature, as well as handcrafting them. The aim of the tool is to allow trained practitioners to manipulate the curve-skeletons obtained with skeletonization algorithms in order to fit their specific pipelines or to explore the requirements of newly developed techniques.

Keywords: Curve-Skeleton, 3D Meshes, Geometry Processing, Interactive Editing

1. Introduction

A curve-skeleton is a compact mono-dimensional representation able to provide meaningful information about both topology and the volume of a shape. While the skeleton of a two dimensional shape is defined as its *Medial Axis Transform* [1], which is the locus of centres of its maximal inscribed discs, this definition leads, for three dimensional shapes, to a collection of connected curves and sheets that are impractical to use in most real world applications.

Constraining the skeleton of a 3D shape to be a one-dimensional structure results in a simpler and more intuitive representation, that is also easy and natural to manipulate. As of today, however, there is no unique, precise and universally accepted definition of curve-skeletons, hence different approaches for its computation have been proposed, each obtaining results with different features, characteristics and defects [2].

The range of applications in which curve-skeletons are used is wide and comprises computer animation [3], shape matching [4], modelling [5], remeshing and quad layout extraction [6], polycube [7], and hexahedral mesh construction [8]. Since each of these applications requires skeletons with different properties, further processing is often required in order to obtain optimal results in the pipeline in which they will be used. Moreover, it is often difficult to define algorithms to process a skeletal structure in order to reflect the required features, due to the semantic nature of the information it conveys.

Tools for interactive creation of curve-skeletons already exist (e.g. in the the medical imaging field [9]). These tools usually allow the user to create a curve-skeleton from scratch so that it can be used as the input for an inverse skeletonization algorithm. The main difference with our tool is that, due to our different

goals, the basic operations they offer are not sufficient for editing an automatically extracted skeleton.

In [6] an algorithm for computing a coarse quad-layout starting from a shape and its curve skeleton was introduced. We discovered that using automatically extracted skeletons often brought to sub-optimal results even when using some simplification strategies. This led to the development of the tool we present in this paper, which has shown to be easy-to-use and practical for interactively editing curve-skeletons. It allowed us to obtain optimal results within our quad-layout computation method and we think it can allow other researchers or practitioners to obtain, even within minutes, curve-skeletons that are tailored for their specific tasks.

The rest of the paper is organized as follows: in Section 2 we will give a brief overview of the different algorithms for computing curve-skeletons at the state-of-the-art, interactive tools for curve-skeleton handcrafting will be discussed as well; in Section 3 we will discuss the limitation of using the current skeletonization approaches in real world applications and why the tool we are presenting in this paper could be a useful resource; in Section 4 we will give an overview of our tool, describing its functionalities; in Section 5 we will report the results of a complete user evaluation session conducted in our lab; in Section 6 we will present an example of a specific use case, to give an idea of the capabilities of our tool; in Section 7 we will draw our conclusions, consider the limitations, and explain what could be done in the future to improve the work.

The project page with binaries and link to the repository containing the entire source code of the presented tool is available at http://francescousai.info/skel_lab. This paper is a substantial extension and revision of the work “Skeleton Lab:

63 an Interactive Tool to Create, Edit, and Repair Curve-Skeletons”
 64 [10], presented at the 2015 EuroGraphics Italian Chapter Con-
 65 ference, held in Verona in October 2015. The work received the
 66 Best Paper Award at the end of the Conference.

67 2. Related work

68 Several automatic skeletonization methods have been pre-
 69 sented in the last two decades. These are commonly categorised
 70 based upon their input, which can be a surface mesh [11, 12], a
 71 point cloud [13, 14] or a voxel grid [15, 16]. Volumetric repre-
 72 sentations, while important, are not handled by our tool, hence
 73 they will be not discussed. We refer to [2] for a recent survey.
 74 Currently, a universally accepted definition of *curve-skeleton* is
 75 still lacking. The *medial geodesic skeleton* defined in [17] is
 76 the only meaningful attempt to fill this gap; however, while the
 77 theoretical definition is precise and sound, the implementation
 78 of the skeletonization algorithm provided by the authors presents
 79 some defects, such as long computational times and sensitivity
 80 to small perturbations of the surface. Both these issues arise due
 81 to the well known sensitivity of the medial axis itself, which
 82 is computed and processed to obtain the final skeleton. The
 83 main issue is the presence of noisy skeletal paths and spurious
 84 branches, especially between nodes where the branching occurs.

85 Since different skeletonization approaches exist, it is difficult
 86 to compare the quality of the results of each method. Important
 87 information about the desirable properties of a curve-skeleton
 88 can be found in [18].

89 The current trend in skeletonization is to use a contraction-
 90 based approach, which relies on contracting the 3D shape repre-
 91 sented as a triangle mesh accordingly to its mean curvature-flow
 92 until it collapses to a monodimensional object. We refer to
 93 [15, 19] for a qualitative comparison of the most representative
 94 methods based on this idea. A recent contraction-based method
 95 is [11] which is a robust skeletonization algorithm whose result-
 96 ing skeletons are topology-preserving, usually well centered and
 97 smoother than previous similar methods.

98 Notable exceptions to this trend are [20, 21, 22] which, rather
 99 than relying on the geometric properties of the input, operate
 100 emulating human perception, synthesising the curve-skeleton
 101 of a 3D object from those of its 2D silhouettes (obtained from
 102 different points of view).

103 This approach, although less robust than [17, 23, 11] presents
 104 heuristics for collapsing spurious branches and closing loops
 105 whenever two terminal nodes have intersecting maximal balls,
 106 which are beneficial for the resulting structure. A particularly
 107 notable example of point cloud based methods is the work of
 108 Huang et al. [14], which allows to obtain high quality skeletons
 109 from raw and potentially incomplete scans by using the L1-
 110 median operator.

111 2.1. Skeleton editing tools

112 The term *skeletonization* denotes the process of computing
 113 a curve-skeleton from a 3D shape. Similarly, *inverse skele-*
 114 *tonization* can be defined as the process of building a 3D shape
 115 around a curve-skeleton respecting its structural and volume
 116 information.

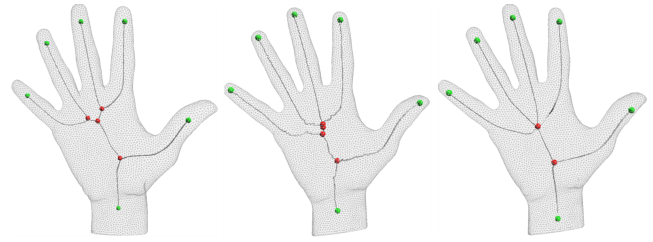


Figure 1: A direct comparison of three automatic skeletonization methods. [11] (left), [17] (center) and [21] (right) on the same hand model. One can notice that the terminal nodes are the same in all the three skeletons, while the internal nodes are different in number and position. It is difficult to tell which one is the most correct.

117 In recent years, inverse skeletonization became a quite active
 118 research topic, giving rise to different research works [24, 25,
 119 26] and commercial solutions [27, 28] for building 3D shapes,
 120 especially creature-like, from a user created skeleton.

121 The aim of these tools is to quickly create 3D models that
 122 will have to be further refined. They are designed to allow the
 123 user to handcraft a skeleton, providing a small set of simple
 124 skeleton editing operations such as adding new nodes, moving
 125 them around and changing the radii of their associated balls.
 126 These basic operations do not provide a sufficiently wide toolset
 127 for a user that needs to process the results of one of the presented
 128 skeletonization methods. Our tool addresses this need, allowing
 129 users to operate on curve-skeletons at different granularities with
 130 functionalities that are absent in other tools and have shown to
 131 be useful.

132 3. Motivation

133 From a practical point of view all the skeletonization methods
 134 presented in the previous section have specific properties and
 135 issues, and their resulting skeletons are typically difficult to use
 136 without processing. We noticed, from our own experience and
 137 the results of the experiments found in literature, that curve-
 138 skeletons are usually assumed to have some specific properties
 139 or are manually edited to have them (e.g. in [26, 7]).

140 Each of the methods presented in section 2 requires some
 141 input parameters to be set in order to fine-tune how the algorithm
 142 behaves, especially regulating its ability to catch small scale
 143 features. This brought us to make the following observations:

- 144 • The computation of a good curve-skeleton is not a fully
 145 automated operation. It requires a, sometimes complex,
 146 parameter setting that could be tightly coupled to both
 147 the specific application and the specific shape; this usually
 148 takes several attempts before obtaining the desired skeleton.
- 149 • Most of the parameters pertain to the detection of small
 150 scale features. On the one hand, more sensitivity usually
 151 means more noise and spurious branches; on the other
 152 hand, less noise could potentially mean missing meaningful
 153 features of the shape being processed.
- 154 • Given the semantic nature of the information encoded in a
 155 curve-skeleton it is difficult, from an application agnostic

156 point of view, to define what a meaningful feature is and
157 how much noise can be tolerated in order to capture it.

158 As can be seen in Fig. 1 different skeletonization methods
159 give very different results even on models of moderate complex-
160 ity. The three methods used in the experiment have different
161 approaches and lead to distinct results. Since we can not use a
162 single universal and formal definition of 3D curve-skeleton, it is
163 not possible to say which of the three skeletons is correct. Fur-
164 thermore we can say that rather than evaluating if an extracted
165 curve-skeleton is correct or not, it is worth to evaluate if it meets
166 certain desired properties and if it can bring optimal results or
167 not.

168 We have observed that it is usually not practical to rely on
169 a completely automatic pipeline, because the input parameters
170 of these algorithms must be fine tuned. Moreover, the user has
171 to decide the balance between meaningful features versus noise
172 and spurious branches. Taking this into consideration, we think
173 that working in a pipeline where an automatic extraction stage
174 (with a limited choice of standard parameter values) followed
175 by an interactive editing stage is more practical.

176 3.1. Applicative contexts

177 As mentioned in the introduction, in [6] we proposed an
178 algorithm for computing a coarse quad-layout starting from a
179 shape and its curve skeleton. We found that optimal skeletons for
180 this application are robust, connected, and reliable [18]. Strong
181 homotopy is not required but it is beneficial. Optimal results
182 have been obtained by using skeletons composed of a small
183 number of nodes that still approximate the shape accurately.
184 Another important requirement is for the skeletons to have the
185 associated balls of the endpoints not intersecting other balls.
186 In some cases, optimal results have been achieved with a little
187 interactive editing of the automatically extracted skeleton by
188 collapsing spurious branches, manually adjusting the missing
189 ones and sub-sampling the branches while retaining only the
190 most representative nodes.

191 Another applicative context which benefits from dealing with
192 a small number of nodes is the definition of kinematic skeletons.
193 They are in fact usually composed of few nodes for each branch,
194 requiring particular care on the nodes' position. In this scenario
195 an automatic simplification of a skeleton can potentially lead
196 to results that are not well suited for the purpose. This could
197 happen because the nodes associated with the shape's articula-
198 tions have to be preserved, but the presence of an articulation
199 itself is a semantic feature that is usually complex to capture
200 algorithmically.

201 4. Skeleton Lab

202 The presented tool, Skeleton Lab, allows the user to both
203 handcraft curve-skeletons and edit the existing ones, hence it
204 provides functionalities that are absent in the tools described
205 in Section 2.1, due to their different purposes. This section
206 will present its distinguishing features, their purpose and some
207 design choices.

208 We consider the skeleton as an attributed graph $G = (N, L)$
209 where N is the set of nodes and L the set of links between nodes.
210 Each node $N_i \in N$ has a few attributes:

- 211 • Position in space.
- 212 • Radius of the associated ball.
- 213 • Type of the node (the classification follows).
- 214 • List of its neighbouring nodes, since we do not explicitly
215 store the links.

216 We classify the nodes of the skeleton as: *Joint nodes (Jn)*
217 that have two incident arcs; *Leaf nodes (Ln)* that have only one
218 incident arc; and *Branching nodes (Bn)* that have more than two
219 incident arcs. We also allow the user to assign a higher semantic
220 value to a **Jn** marking it as an *Articulation (An)*, e.g. in case of
221 a human ankle or elbow.

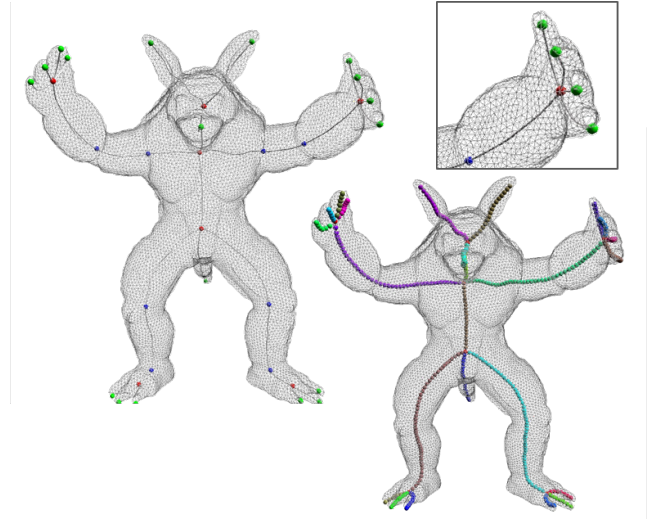


Figure 2: A visual explanation of the terminology we introduced. In the left image each **Bn** is red, each **Ln** is green, and each **An** is blue, while no **Jn** is shown. In the right image the nodes of **Branch** have a different color.

222 With the term **Branch** we refer to a sequence of linked nodes
223 starting and ending with either a **Bn** or a **Ln**. When a branch
224 ends with a **Ln** we call it *Ln-ending*, if both the starting and
225 ending nodes are **Bn** it is said to be *internal*, while when the
226 starting and the ending nodes are exactly the same, we have a
227 *Looping-Branch*.

228 With our tool the user is allowed to visualize and manipulate a
229 curve-skeleton and optionally view the triangle mesh it refers to.
230 We experimented our tool with skeletons obtained with different
231 methods, namely the ones presented in [17, 20, 11].

232 By design we have chosen to group functionalities into two
233 different modes:

234 **Node mode** in which each operation refers to the node or the
235 set of nodes currently selected.

236 **Branch mode** allows direct manipulation of a selected branch
237 with operations that are meaningful only when considering
238 an entire branch.

Moreover, there are several operations that affect the skeleton as a whole. From our experience manually editing a curve-skeleton is usually performed through a trial-and-error approach, hence undo/redo is available for both operations and selection.

4.1. Basic operations

All the tools presented in Section 2 allow the user to perform basic operations only, that are: adding a new node connected to an existing one; translating and rotate a selected set of nodes; changing the radii of their associated balls. These basic functionalities of a skeleton editor are available in our tool. They are trivial and, thus, will not be described further.

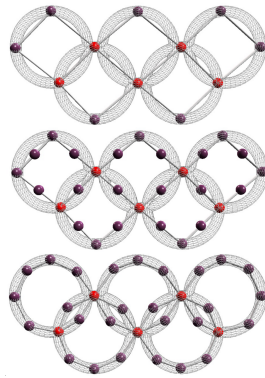
4.2. Node mode

Here we present all the possibilities offered in node mode.

4.2.1. Adding midpoint and constraining movement

Once two existing and directly connected nodes are selected, the user is allowed to create a new node connected to them and placed at the midpoint of the segment that connects them (see inset).

Strictly related to this operation, is the constrained movement of nodes. Our tool allows the user to move a selected **Jn** constraining its movement only to points in space that are linearly interpolated between the position of its two neighbours, in order to fine tune its position. These two operations have shown to be useful when creating a skeleton from scratch with a progressive refinement, where only **Bn**'s and **Ln**'s are created at first and branches can be progressively modelled adding **Jn**'s between them.



4.2.2. Node Removal

Removing one or more nodes is a simple operation with a number of different cases that have to be taken into account. In fact, when removing a single node there are three possible cases depending on what type of node is going to be removed:

- **Ln**: the node is removed without any further operations.
- **Jn**: the node is removed and the two nodes linked to it are then connected to each other.
- **Bn**: the user is required to choose between two possibilities. The first one is to delete all the node's links. Since we do not allow to create separate components, all the **Ln**-ending branches connected to the **Bn** will be removed. Other branches, the ones connected to other **Bn**, will be kept unchanged except for the fact that one of their endpoints will be removed, becoming a **Ln**-ending branch. Alternatively, the user can transfer all the links of the **Bn** that is going to be deleted to one of its neighbouring nodes, selected by the user.

The tool allows to handle isolated nodes or branches but does not allow to explicitly create them. This is a design choice: since a curve-skeleton is always a single connected component, SkeletonLab does not allow the user to create disconnected components, but it allows to load a disconnected skeleton in order to repair it interactively. When a user tries to remove a set of nodes in *Node Mode*, the operation is performed as a sequence of single node removals. When **Bn**'s are involved, all the **Ln**-ending branches that are connected to them are traversed and their nodes removed.

4.2.3. Copy and paste

Especially when creating a skeleton from scratch, the user may want to replicate a part of the skeleton that (s)he is editing. For this reason our tool allows to copy and paste a set of connected nodes. When copying, the user is required to choose a *Source* node, when pasting a *Destination* one. The *Source* node has to be chosen from the ones that are being copied, the *Destination* node can be any node of the skeleton. When pasting the copied nodes, *Source* will be merged with *Destination* in order to keep a single connected component. *Destination* will be modified only inheriting all *Source*'s copied neighbours. Copy and paste are disjoint operations, and, as such, one can perform them in different moments. The process is depicted in figure 3.

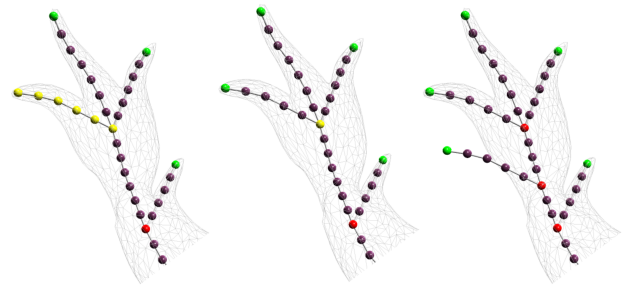


Figure 3: Copy&paste. The user selects a sequence of nodes (left), activates the copy (similarly to other softwares that supports this functionality), selects the *source* node (middle); then selects the destination node and pastes the copied nodes (right). The selected destination node has now become a **Bn** (right).

4.2.4. Creating and breaking links

A curve-skeleton is essentially an undirected graph. Skeleton Lab allows the user to explicitly manipulate the graph's connectivity with three operations:

- Merge two unconnected nodes creating a looping-branch.
- Create a link between two unconnected nodes.
- Break an existing link between two nodes. We use the Dijkstra's algorithm to check if breaking the link will create two separate components, if so we prevent the operation.

4.3. Branch mode

The operations explained so far operate on one or more selected nodes, while the operations that will be presented in this section operate on a higher level of granularity, allowing the users to work on an entire branch of the skeleton.

327 4.3.1. Pruning

328 The pruning operation allows the users to easily shorten the
329 selected **Ln**-ending branch, removing its current **Ln**.

330 4.3.2. Branch Removal

331 If the branch is a **Ln**-ending one, it will be removed without
332 further operations. In case of an internal branch the user will be
333 prompted to choose whether (s)he prefers to merge the two **Bn**'s
334 at their midpoint, or (s)he would rather retain the position of one
335 of the two. This operation is particularly useful when repairing
336 skeletons, since one of the most common defects is the presence
337 of short spurious branches.

338 4.3.3. Spurious branches removal

339 A spurious branch is a branch of the skeleton that breaks ho-
340 motopy, in the sense that it does not represent any meaningful
341 component of the shape. Identifying spurious branches would
342 require to understand the shape's features at a semantic level,
343 even if it can be approximated with the use of the volumetric
344 information provided by the radii of the associated balls. Skele-
345 ton Lab provides the capability of collapsing such branches
346 implementing the topological operations proposed in [20]. In
347 particular, every branch for which the Zones of Influence [29] of
348 its two endpoints intersect (i.e. their associated balls intersect)
349 will be removed.

350 4.3.4. Resampling

351 Skeletons computed with state-of-the-art skeletonization al-
352 gorithms are often composed of a large number of nodes, in
353 fact they can easily reach a thousand nodes, even for models of
354 moderate complexity. In usual applications, this level of detail
355 leads to redundancy and is not necessary, indeed it could be
356 preferable to deal with a smaller number of nodes.

357 In order to cope with this issue, we allow the users to change
358 the number of nodes of a branch with a *resampling* operation.

359 Resampling is done by arc-length parametrization adopting
360 the method presented in [30] with slight modifications in order to
361 take into account both position and radii of the involved nodes.
362 In this way, the three-dimensional structure of the branch is
363 maintained and the radii of the new nodes are approximated
364 from the original ones. Both sub-sampling and super-sampling
365 are allowed, but only super-sampling is a smoothness preserving
366 operation.

367 A special case for resampling occurs when the branch con-
368 tains some **An**'s, since they have been marked by the user as
369 semantically relevant **Jn**'s, their position and radii will be pre-
370 served.

371 Furthermore since subsampling must retain these nodes, the
372 minimum number of nodes a user can choose for resampling is
373 equal to $E + \#\mathbf{An}$, where E is the number of distinct endpoints
374 of the branch.

375 4.3.5. Approximation

376 The typical use case for resampling is skeleton simplification.
377 However, it could be sometimes required to maintain a subset
378 of the original nodes while simplifying. In these cases skeleton

379 simplification through resampling cannot be applied. A different
380 approach to lowering the number of nodes of a skeletal branch
381 is to approximate it. We implemented two different solutions.
382 The first one relies on the Douglas-Peucker algorithm [31]. The
383 second is a simple heuristic that operates traversing the branch
384 and removing all the nodes for which their distance to the last
385 node visited and not removed is smaller than a given threshold
386 (i.e., calculated as the sum of their associated ball radii multi-
387 plied by a scaling factor). These two approaches approximate
388 the branches keeping the original skeletal nodes, thus not guar-
389 anteeing to obtain a smooth curve with uniformly distributed
390 nodes.

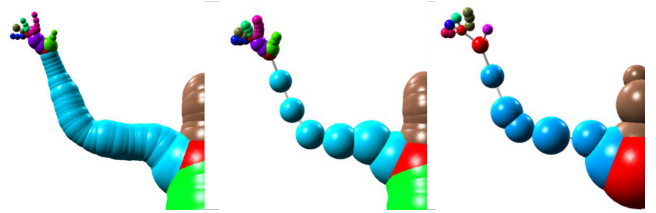


Figure 4: The original branch (left) composed by 50 nodes can be simplified by resampling (middle) or approximated with Douglas-Peucker algorithm (right). Both results are composed of 8 nodes.

391 4.4. Skeleton-wide operations

392 We here present a few operations that allow the user to manip-
393 ulate the skeleton position inside the shape described by a mesh
394 that has been loaded. These operations require an efficient im-
395 plementation of intersections and distance queries computation,
396 hence we decided to rely on the AABB tree implementation
397 found in [32].

398 4.4.1. Fix the external nodes

399 When a mesh is loaded along with a skeleton it is possible
400 to check if some of the skeleton's nodes are outside the mesh
401 and move them inside. This can be useful since most of the
402 skeletonization algorithms cannot guarantee that all the nodes
403 are internal to the mesh; even the more robust ones as [11] with
404 particular parameter settings can lead to branches crossing the
405 mesh boundaries instead of flowing inside them.

406 For each node N_i we find its closest face of the mesh F with
407 normal N_F . If N_i is outside the mesh we calculate its new posi-
408 tion as

$$N_i - [d(N_i, F) + \epsilon] \cdot N_F$$

409 where d is the Euclidean distance and ϵ a small positive num-
410 ber.

411 4.4.2. Skeleton recentering

412 Centeredness is one of the desired properties of a curve-
413 skeleton, which can be defined as the property of being medial
414 to the shape a skeleton describes. As stated in [18], a skeleton
415 is *perfectly centered* if it lies on the medial surface and it is
416 centered with respect to it. *Perfect centeredness* is difficult to
417 obtain, but it is also often undesirable due to the sensitivity of

418 the medial axis to small perturbations on the object’s surface.
 419 In order to achieve *perfect centeredness* a posteriori, one would
 420 need to have at hand both the curve-skeleton and the shape’s
 421 medial axis, while state-of-the-art algorithms do not provide
 422 such output. Moreover, several skeletonization algorithms do
 423 not compute the medial axis in their pipeline. Skeleton Lab
 424 implements three different algorithms for recovering the cen-
 425 teredness of the skeleton and estimating the radii of the maximal
 426 balls.

427 The presented methods rely on the definition of the local
 428 skeletal direction of each node N_i denoted with \vec{N}_i and computed
 429 as:

$$\vec{N}_i = \begin{cases} \vec{N_j N_i} & \text{if } N_i \text{ is Ln} \\ \vec{N_i N_k} & \text{if } N_i \text{ is Bn} \\ \vec{N_j N_i} + \vec{N_i N_k} & \text{if } N_i \text{ is Jn} \end{cases}$$

430 where N_j and N_k are, respectively, the predecessor and the suc-
 431 cessor of N_i in the branch they belong to, choosing an arbitrary
 432 traversal order. The position of N_i together with the direction \vec{N}_i
 433 defines a plane \mathcal{P}_i .

434 *Iterative recentering*

435 The first recentering algorithm relies on the description of the
 436 *approximate centeredness* proposed in [18]. For each skeletal
 437 node N_i we cast n uniformly distributed radial rays that lie on \mathcal{P}_i
 438 and have origin in N_i , computing their intersections with the ob-
 439 ject surface. For each ray R_h we consider only the intersections
 440 obtained pinching the object from the inside and we store only
 441 the intersection I_h that is closest to N_i . We also discard all the
 442 pairs of opposite rays for which at least one valid intersection is
 443 not found.

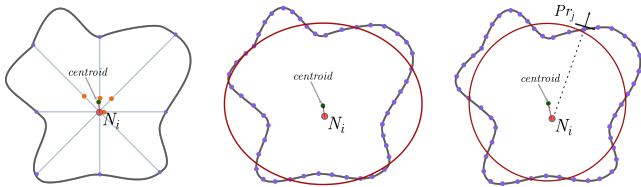


Figure 5: A section of the tubular branch where the node N_i is centered. On the left an application of iterative recentering; in the middle ellipse fitting; on the right SQEM for which only one of the tangent planes has been depicted.

444 For each pair of opposite rays (R_h, R'_h) we calculate the mid-
 445 point C_h of the intersection pair (I_h, I'_h) , then update the position
 446 of N_i as the centroid of all the computed midpoints C_h ’s and the
 447 new associated ball radius as the average semi-distance of all
 448 the intersection pairs. If N_i is a **Bn** we calculate its new position
 449 and radius, respectively, as the centroid of the positions and
 450 mean radius calculated separately for each incident branch. We
 451 observed that a single iteration of the procedure is usually not
 452 sufficient to obtain optimal results. Moreover, this procedure
 453 does not guarantee convergence, hence it is up to the user to
 454 iterate until the results are satisfactory.

455 *Recentering using ellipse fitting*

456 The great part of the shapes for which *makes sense* to compute
 457 a curve-skeleton, belong to the class of generalized cylinder
 458 assemblies. According to this assumption we can compute an
 459 approximated centered position for the skeletal nodes using an
 460 approach similar to the recentering procedure described in [14].
 461 Each branch is traversed and for each node N_i , the intersection
 462 between its associated plane \mathcal{P}_i and the object is computed.
 463 This intersection could generate different components, hence
 464 we choose the connected component that is closest to the node
 465 N_i . The selected component will be an arrangement of segments
 466 from which we compute a set of points R composed of all the
 467 segments’ endpoints and midpoints. We then use the approach
 468 proposed [33] to fit an ellipse on the set of points R , which lie on
 469 both the surface and \mathcal{P}_i , updating the position of N_i to the center
 470 of the ellipse thus obtained. The position of **Bn**’s is computed
 471 applying the described procedure for each outgoing branch and
 472 computing the centroid of the center of the ellipses. The radii
 473 of the associated balls is approximated as the distance of each
 474 node from its closest point on the object’s surface.

475 *Recentering using Spherical Quadric Error Metric*

476 The Spherical Quadric Error Metric (SQEM) has been intro-
 477 duced in [34] and allows to compute the sphere that optimally
 478 fits an input set of oriented planes. We use this metric to com-
 479 pute the new positions of the skeletal nodes and the radii of their
 480 associated balls. We follow an approach similar to the previous
 481 one. For each node N_i we compute the intersection of the object
 482 with \mathcal{P}_i and the point set R . The point set R is filtered from the
 483 outliers, removing all points for which the ZScore is greater than
 484 1.96 according to their distance from N_i . For each remaining
 485 point r_j in R we compute the direction $\vec{N_i r_j}$ which, together with
 486 r_j , will define a plane Pr_j . The set of all the planes Pr_j will be
 487 the input of the SQEM computation.

488 When N_i is a **Bn**, its new position is obtained computing the
 489 SQEM using as input the set of all the planes Pr_j obtained from
 490 the intersections computed for each outgoing direction of the
 491 **Bn** N_i .

492 4.4.3. *Comparison of the recentering algorithms*

493 All the three algorithms are able to improve the centeredness
 494 of the skeleton, however all of them have their own disadvan-
 495 tages. The iterative approach is particularly sensitive to the
 496 nodes position and local direction of the original skeleton, and
 497 the resulting paths are usually not smooth. Moreover, it is not
 498 able to deal with nodes that are outside the shape, hence their
 499 position needs to be fixed beforehand, as described in 4.4.1.
 500 Both ellipse fitting and SQEM based algorithms are more robust
 501 than the iterative procedure and they are able to produce good
 502 results even with nodes that are slightly outside the shape. By
 503 contrast they are not able to compute a reasonable position for
 504 those nodes lying at the branching parts of the shape. The ellipse
 505 fitting approach usually fails to reposition this kind of nodes,
 506 while SQEM provides reasonable results only for the shapes
 507 which are composed of only tubular parts (e.g., the *cactus*) and
 508 fails on the others (e.g., the *hand*). A comparison is shown in

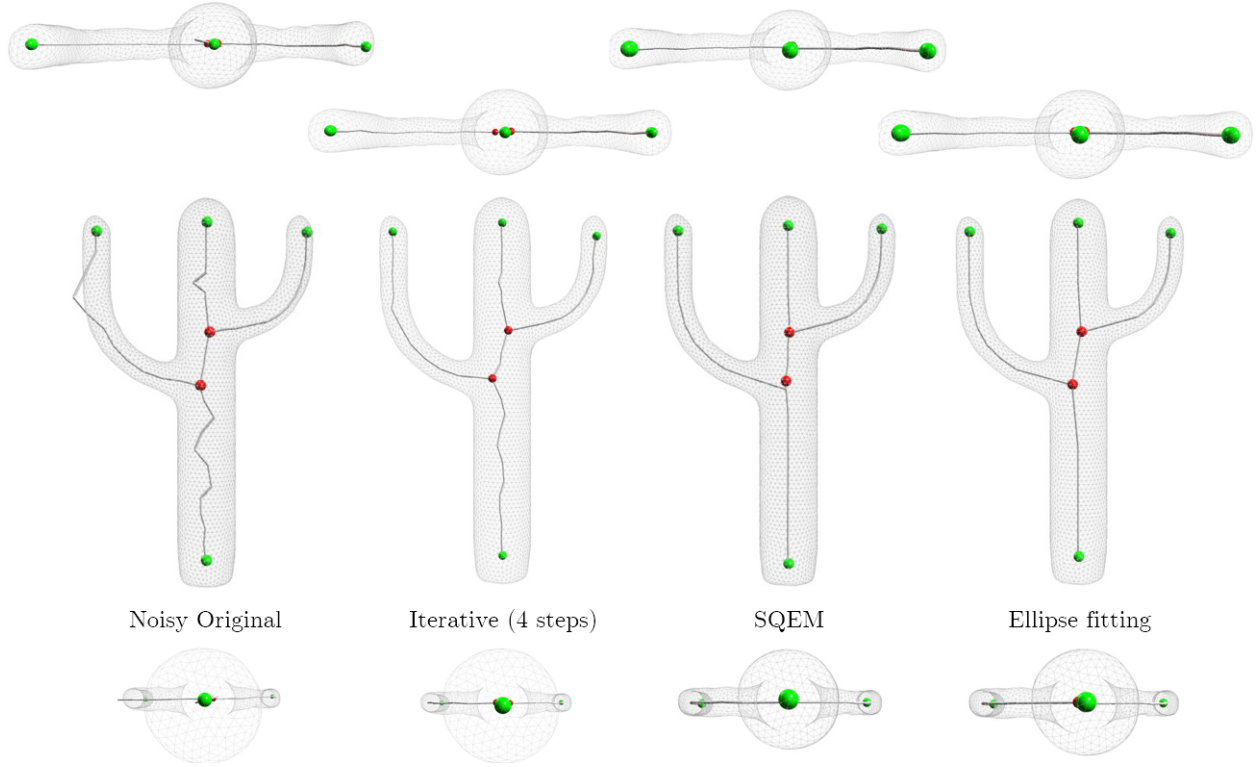


Figure 6: A comparison of the three recentering methods available in our tool. In the leftmost image we show a skeleton artificially deformed adding noise.

509 Figure 6. This problem arises due to the fact that the branching
 510 parts of a shape should be entirely represented by a single **Bn**,
 511 but skeletonization algorithms are usually not able to differenti-
 512 ate these parts. One possible strategy to alleviate this issue is
 513 to not perform the recentering procedure to all nodes at once,
 514 but use a two step approach. In the first step the positions of all
 515 the nodes that fall inside each **Bn**'s Zone of Influence can be
 516 kept unchanged. At the second step their new position can be
 517 computed using linear interpolation .

518 4.4.4. Posing the skeleton

519 While editing a skeleton, the user might need to move a set
 520 of nodes and at the same time preserving the proportion and the
 521 relative distance of the branch. This allows a user to manually
 522 create a curve-skeleton in a canonical pose that can be modified
 523 once the work is finished.

524 For this reason, the users are allowed to move a section of
 525 the branch from an **An** (or a **Bn**) to a **Ln**, while keeping the
 526 hierarchy intact, namely the distance and the relative position
 527 with the nodes in between.

528 To achieve this, the user selects an **An** or a **Ln**. Let us identify
 529 this node with N_i^b , where b identifies the branch and i is the
 530 position of the node in the branch (with N_0^b as the **Bn** and N_n^b
 531 as the **Ln**).

Then, the following set of nodes will be marked:

$$\text{marked} = \{N_j^b \mid 0 \leq j < l\}$$

532 where N_l^b is the next **An** from N_i^b toward the **Bn**. If there are no
 533 other **An**'s, then N_l^b will be N_0^b , namely the **Bn**. These marked

534 nodes are then involved in a constrained rotation centered on
 535 N_l^b .

536 This method uses the **An**'s to pose just the desired portion of
 537 the branch (see an example in Figure 7). The **An**'s however, are
 538 not required in case of leaf-ending nodes: if there are no **An**'s,
 539 the entire branch will be posed.

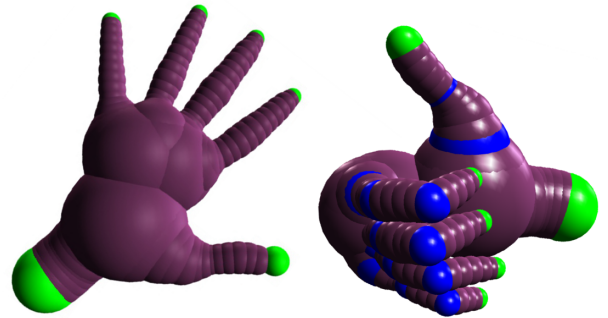


Figure 7: An example of how the user can pose a hand rotating the mesh around the **An**'s (in blue in the "thumb up" model).

540 5. User Evaluation

541 A user test has been carried out in order to evaluate the skele-
 542 ton editing tool with regard to usability, performance, cognitive
 543 load and the overall experience. The aim was to get feedback
 544 on the current status of the tool and to collect suggestions for
 545 further development of its interface.

5.1. Test design

The test was organized in order to provide both quantitative and qualitative data on different aspects of the tool. Before starting the trial, the users were asked to read a short document describing SkeletonLab, together with a short help on how to activate the different tool functionalities. After reading the document, which can be found in the additional material, and providing some demographic information, the users were asked to execute different tasks, which we divided in two categories, namely A and B.

In the A category we included a set of eight introductory tasks. The tasks A1 to A7 required the users to test a single function of the tool, while task A8 is a warming-up activity before starting the evaluation on complex tasks. The aim of this group was to help the users to understand the capabilities of SkeletonLab and to prepare them for next phase. The tasks in the A category are the following:

- A1** Creating a new node and changing its position.
- A2** Changing the radius of the maximal ball associated to a node.
- A3** Loading a skeleton from the disk and saving it into another file.
- A4** Activating the branch mode, deleting a terminal node and the entire branch.
- A5** Resampling a branch.
- A6** Loading the *Cactus* mesh, its skeleton and aligning them ¹
- A7** Deleting a branching node from a skeleton and transfer its links to another one.
- A8** Modifying an existing skeleton to obtain a specific shape (the user was provided with the image of the desired configuration).

After each task, the user was asked to complete the Subjective Mental Effort Question (SMEQ)[35] for evaluating the task load. The evaluator registered the completion time.

After finishing the first group, the user was asked to complete a set of three additional tasks, which required the composition of different functionalities for reaching the desired configuration. Each task represent a typical use case for SkeletonLab. The users were provided with an image showing the desired output, to be considered as success criterion. The tasks in the B category are the following:

- B1** Loading the mesh *Guy* and creating its skeleton.
- B2** Cleaning the skeleton of the octopus as showed in figure 11 (reducing the number of nodes, deleting spurious branches etc.)

¹Please note that task A6 is different from the recentering operator. Task A6 required the users to rigidly move the skeleton in order to align it with the mesh, while recentering independently moves each node of the skeleton in order to improve their centeredness. An example of task A6 can be seen in the accompanying video and at https://youtu.be/iGhfPvzk_uc

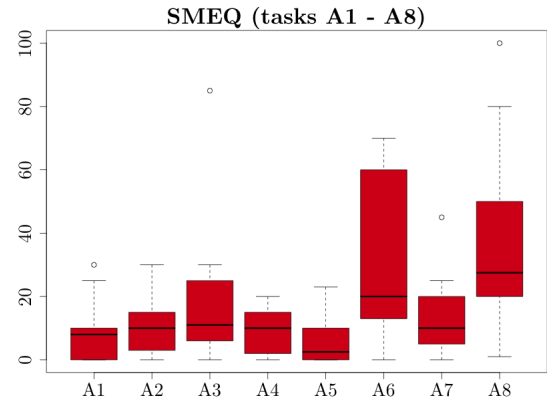


Figure 8: SMEQ ratings for group A tasks. The minimum value is 1, while the maximum is 150.

- B3** Loading the skeleton of a hand and modifying it in order to get the thumb up pose (figure 7).

After finishing each task, the users were asked to fill the NASA-TLX questionnaire for assessing the task load [36] and the evaluator tracked the completion time. At the end of the test, the users were requested to fill the SUS [37] questionnaire in order to evaluate the overall usability. The test is composed of three open-ended questions, regarding which aspects they liked and disliked in the tool and improvement suggestions for the interface.

5.2. Test results

Fourteen users participated to the test, 11 males and 3 females, aged between 30 and 21 ($\bar{x} = 25.07$, $s = 2.8$). Two of them have a high school degree, 3 a bachelor, 8 a master degree and one a PhD. They have an average experience in editing 3D meshes ($\bar{x} = 3.14$, $s = 1.61$ in a 1-5 Likert scale), while they had little knowledge on creating and manipulating curve skeletons ($\bar{x} = 1.57$, $s = 0.85$).

5.2.1. Introductory tasks

All users were able to complete the introductory tasks (type A). The box plot in figure 8 shows the SMEQ ratings. All the single function tasks required little effort for the users, who rated the tasks from A1 to A5 and A7 less than 13 on average. According to [35], all these tasks are *not very hard to do*. The difficulty of the other ones (A6 and A8) is between a *bit hard to do* and *fairly hard to do*.

We expected that modifying an existing skeleton to obtain a specific shape (A8) was the most difficult task. Indeed, it requires several operations for obtaining the desired result. We registered a high variability also for the recentering operation (A6), caused by the changes of the scene point of view needed for checking and modifying the alignment.

The analysis of the completion time (figure 9, red boxes) confirms the conclusions on SMEQ scores: completing task A6 and A8 takes longer than the other ones. We found a significant difference between the completion time of A8 and all the other tasks but A6. The difference was significant also between A6

Task A on Completion Time:

$$F(3.96, 47.48) = 12.78, p = 10^{-11}, \eta_{partial}^2 = 0.51, \epsilon = 0.56$$

Pair	95% c.i.	p	Pair	95% c.i.	p
A6-A1	[77s; 300s]	.001	A8-A1	[126s; 512s]	.007
A6-A2	[55s; 255s]	.01	A8-A2	[96s; 475s]	.02
A6-A4	[31s; 243s]	.04	A8-A4	[76s; 459s]	.03
A6-A5	[100s; 289s]	.001	A8-A5	[138s; 513s]	.003
A6-A7	[21s; 264s]	.001	A8-A7	[76s; 470s]	.02

Table 1: Summary of the one-way ANOVA for repeated measures on the task completion time (type A). Since the repeated measures violated the sphericity assumption, we applied the Greenhouse-Geisser procedure [38] for correcting the degrees of freedom of the F-distribution (ϵ). We applied the Bonferroni correction for counteracting the multiple (pairwise) comparison problem [39]. We report only the significant differences ($p < .05$).

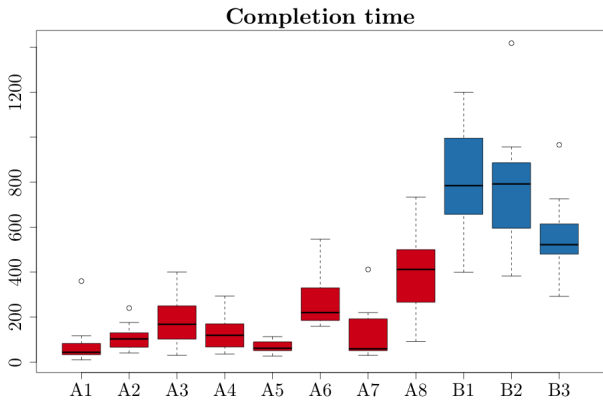


Figure 9: Completion time for task groups A (red) and B (blue) in seconds.

and the other tasks except A8 and A3. We report the details on the one-way ANOVA for repeated measures in table 1.

5.2.2. Use cases

The second part of the test included a set of more complex tasks (B1 to B3). The first task (creating a new skeleton) was completed by all users, two users abandoned B2 (cleaning a skeleton) and one B3 (modifying a skeleton pose). Users that abandoned the tasks shared the opinion that it was caused by the inability to figure out the sequence of actions for reaching the desired configuration. Considering that the experience level with the editing of curve skeletons was low for all users, this may be motivated with the need of a longer learning phase for solving complex tasks.

Figure 10 shows the results of NASA-TLX ratings for B1, B2 and B3. We report the overall difficulty evaluation (Tot) and the rating for each of the six factors considered in the questionnaire: the mental demand (MD), the physical demand (PD), the temporal demand (TD), the effort (Eff), the overall performance (Pr) and the frustration level (Fr). The task load is acceptable for all tasks, considering a scale from 1 to 100, the average values are between 33 and 43.

The analysis of each factor highlighted a significant effect of the considered task in the overall performance ratings. However, the pairwise comparison resulted only in a small difference between B1 and B2. The task has a significant impact on the

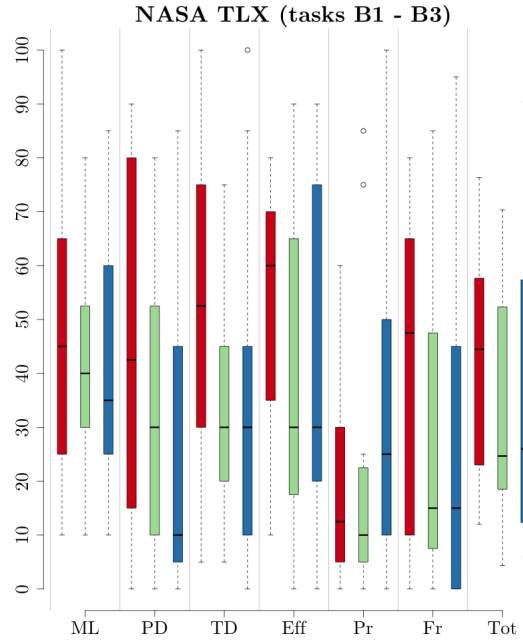


Figure 10: NASA-TLX ratings for group B1 (red), B2 (green) and B3 (blue). The results for the overall performance (Pr) have been reversed for consistency with the other dimensions (the lower, the better).

completion time, and in particular the difference in time between B1 and B3 is around 4 minutes. Additional details on the analysis are available in Table 2.

We conclude that users require a similar effort in all the envisioned use cases. They seem to be more efficient in changing the configuration of a skeleton rather than performing editing operations.

5.2.3. Post test

The SUS post test questionnaire shows that SkeletonLab has an average overall usability ($\bar{x} = 63.04, s = 16.35$). In particular, the users gave low ratings to question 3 (*I thought the system was easy to use*, $\bar{x} = 2.0, s = 0.92$) and question 7 (*I would*

Task B on Performance (Pr) ratings:

$$F(1.32, 13.2) = 11.45, p = .003, \eta_{partial}^2 = 0.055, \epsilon = 0.66$$

Pair	95% c.i.	p
B1-B2	[-28.8; 25.9]	.007

Task B on Completion Time:

$$F(1.32, 13.2) = 11.45, p = .003, \eta_{partial}^2 = 0.055$$

Pair	95% c.i.	p
B1-B3	[15s; 480s]	.08

Table 2: Summary of the one-way ANOVA for repeated measures for evaluating the effect of the task (type B) on the overall performance ratings (Pr in the NASA TLX questionnaire) and completion time. We applied the Greenhouse-Geisser correction [38] for sphericity on performance ratings and the Bonferroni correction [39] on the pairwise comparisons. We report the ANOVA results and we list the task pairs having a practically significant difference ($p < .1$) between them.

665 *imagine that most people would learn to use this system very*
 666 *quickly*, $\bar{x} = 2.0$, $s = 1.03$). The latter result may be explained
 667 again considering our users' low knowledge on curve skeletons.

668 We can explain the low ratings to question 3 through the an-
 669 swers to the open-ended questions: almost all users suggested
 670 to modify the keyboard shortcuts that are difficult to remember
 671 and they found some combinations difficult to be performed
 672 (especially the cmd+alt+shift ones). However, two users ac-
 673 knowledged that, once learned, the shortcuts allowed them to
 674 perform the operations quickly and they felt confident in using
 675 them. Therefore, we will consider to change some key combina-
 676 tions and to include buttons and an explicit graphical guidance
 677 for performing the basic operations on skeletons. We expect
 678 that, after the learning phase, most of the users will perform the
 679 manipulations through keyboard shortcuts.

680 The users found additional difficulties in changing the scene
 681 point of view while manipulating the skeleton, especially during
 682 the alignment operation. In this case we should consider to
 683 include predefined view camera positions (e.g. front, side and
 684 top) in order to help the users while aligning the skeleton to an
 685 existing mesh. Finally, the procedure for transferring the links
 686 to another branching node was not intuitive for them.

687 The users appreciated the node color feedback and the overall
 688 manipulation of the skeleton parts.

689 6. Results and Discussion

690 In this section we show how some automatically extracted
 691 curve-skeletons have been processed with our tool in order to be
 692 used as input in the pipeline presented in [6].

693 Figure 12 shows how the skeleton of the *Stanford Dragon*
 694 model has been edited in order to compute its quad layout. It has
 695 been one of the most challenging skeletons we have faced. The
 696 skeleton originally had 1050 nodes and, as can be seen in the
 697 close-up, some of the branches of the horns were disconnected.
 698 The skeleton has been simplified resampling most of its branches
 699 and removing 6 spurious ones, and it is now composed of 193
 700 nodes. The complete editing process, from a) to c), took about
 701 15 minutes.

702 Another example showing how some state-of-the-art methods,
 703 while successfully completing the skeletonization, may fail at
 704 conveying the correct semantic information can be seen in Figure
 705 11. Since a human would usually describe an octopus as a living
 706 being composed of a big head and 8 tentacles starting from the
 707 bottom side of the head, we edited the skeleton removing all the
 708 spurious branches that was present inside the head and one of the
 709 tentacles. The original skeleton contained 23 branches while the
 710 result shown on the right of Figure 11 contained the 9 branches
 711 one would expect. The complete editing took less than 4 minutes
 712 to a trained user. It is also important to note that in this specific
 713 case, using the original skeleton the resulting quad-layout would
 714 have been completely wrong, while the edited version brought
 715 to an optimum. In our experiments the tool shown to be versatile
 716 enough to be used for both quick-fixes and complex editing, and
 717 for repairing tasks.

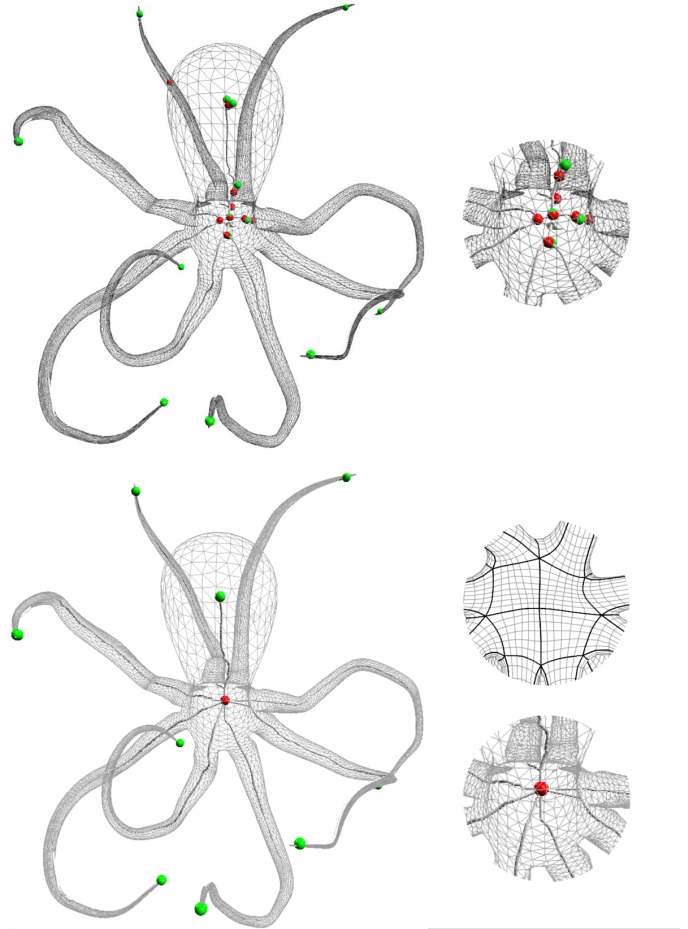


Figure 11: Skeleton of an octopus model extracted using [11] (top) and its edited version (down) with all the spurious branches collapsed. Close-ups on right side of the image show how the unnecessary **Bn**'s were removed in order to obtain the quad layout depicted on the lower right.

718 Our tool relies on the Qt Framework, libQGLviewer [40] and
 719 CGAL [32], and it is able to load curve-skeletons produced by
 720 the available implementations of [11, 17, 21].

721 7. Concluding remarks

722 In this paper we presented a novel tool for the interactive
 723 processing of curve-skeletons in order to make them fit the ap-
 724 plication in which they will be used. We observed that the
 725 evaluation of what is a good curve-skeleton is strictly depen-
 726 dent to the application in which it will be used and none of the
 727 existing skeletonization methods provide curve-skeletons that
 728 can be directly used in every application bringing to optimal
 729 results. This is not a defect of the proposed approaches. The
 730 topological and semantic information that a skeleton encodes are
 731 not easy to capture algorithmically, and fine-tuning the param-
 732 eters required by the skeletonization methods is often necessary
 733 but not resolute. While developing the method presented in
 734 [6] we have observed that an effective and practical approach
 735 to obtain optimal results in our pipeline, was to automatically
 736 extract the curve-skeleton (choosing the method we experienced

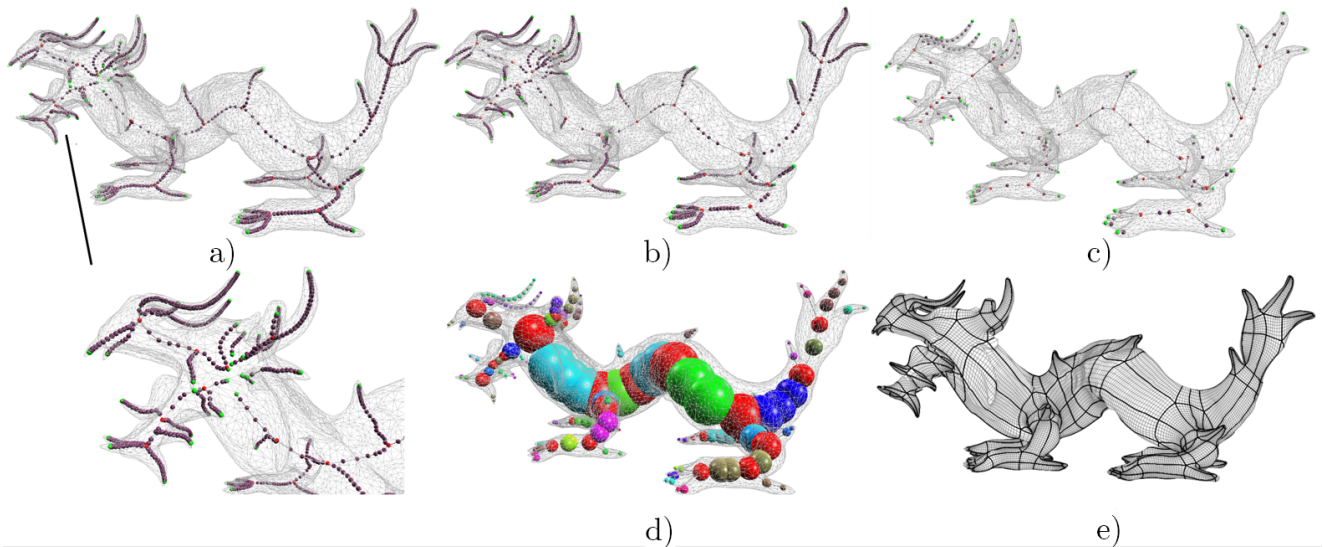


Figure 12: Editing process of a complex skeleton extracted with [17], a) has been computed with a number of disconnected branches (close-up) due to a buggy implementation or erroneous parameter setting. The skeleton was first reconnected b), and simplified c). In the second row we show the skeleton with its associated maximal balls d) and the quad-layout obtained with it e).

to be the best suited for the considered shape using standard parameter values) and process it with the presented tool in order to optimally fit our pipeline requirements. The tool has also been made publicly available both as source code and binaries in order to allow other researchers and practitioners to use it in their applicative scenarios.

Future work

Interesting directions for future work include the possibility to extend Skeleton Lab implementing a modified version of the method presented in [6] in order to be used as an inverse skeletonization tool, providing high quality base quad meshes with a low and controllable number of irregular vertices. Moreover it can be extended in order to produce rigged meshes. Loading a mesh and a skeleton (either automatically extracted or hand-crafted), the latter can be edited according to the user's needs, decimated retaining only **Bn**'s, **Ln**'s and **An**'s. The rigging weights can then be computed in order to obtain a rigged mesh ready to be used for animation purposes.

[1] Blum H. A Transformation for Extracting New Descriptors of Shape. In: Models for the Perception of Speech and Visual Form. 1967, p. 362–380.
 [2] Tagliasacchi A. Skeletal Representations and Applications. arXiv preprint arXiv:13016809 2013;abs/1301.6809.
 [3] Baran I, Popović J. Automatic Rigging and Animation of 3D Characters. ACM Trans Graph 2007;26(3).
 [4] Hilaga M, Shinagawa Y, Kohmura T, Kunii TL. Topology Matching for Fully Automatic Similarity Estimation of 3D Shapes. In: Proceedings of SIGGRAPH '01. 2001, p. 203–212.
 [5] Bærentzen JA, Abdrashitov R, Singh K. Interactive Shape Modeling Using a Skeleton-mesh Co-representation. ACM Trans Graph 2014;33(4):132:1–132:10.
 [6] Usai F, Livesu M, Puppo E, Tarini M, Scateni R. Extraction of the Quad Layout of a Triangle Mesh Guided by Its Curve Skeleton. ACM Trans Graph 2015;35(1):6:1–6:13.
 [7] Liu L, Zhang Y, Liu Y, Wang W. Feature-preserving T-mesh construction using skeleton-based polycubes. Computer-Aided Design 2015;58:162–172.

[8] Zhang Y, Bazilevs Y, Goswami S, Bajaj CL, Hughes TJ. Patient-specific vascular {NURBS} modeling for isogeometric analysis of blood flow. Computer Methods in Applied Mechanics and Engineering 2007;196(29–30):2943–2959.
 [9] Abeyinghe SS, Ju T. Interactive skeletonization of intensity volumes. The Visual Computer 2009;25(5-7):627–635.
 [10] Barbieri S, Meloni P, Usai F, Scateni R. Skeleton Lab: an Interactive Tool to Create, Edit, and Repair Curve-Skeletons. In: Smart Tools and Apps for Graphics - Eurographics Italian Chapter Conference. 2015, p. 121–128.
 [11] Tagliasacchi A, Alhashim I, Olson M, Zhang H. Mean Curvature Skeletons. Comp Graph Forum 2012;31(5):1735–1744.
 [12] Jalba A, Sobiecki A, Telea AC. An Unified Multiscale Framework for Planar, Surface, and Curve Skeletonization. Pattern Analysis and Machine Intelligence, IEEE Transactions on 2016;38(1):30–45.
 [13] Tagliasacchi A, Zhang H, Cohen-Or D. Curve Skeleton Extraction from Incomplete Point Cloud. ACM Trans Graph 2009;28(3):71:1–71:9.
 [14] Huang H, Wu S, Cohen-Or D, Gong M, Zhang H, Li G, et al. L1-medial Skeleton of Point Cloud. ACM Trans Graph 2013;32(4):65:1–65:8.
 [15] Sobiecki A, Jalba A, Telea A. Comparison of curve and surface skeletonization methods for voxel shapes. Pattern Recognition Letters 2014;47:147–156.
 [16] Serino L, Sanniti di Baja G, Arcelli C. Distance-Driven Skeletonization in Voxel Images. IEEE Transactions on Pattern Analysis and Machine Intelligence 2011;33(4):709–20.
 [17] Dey TK, Sun J. Defining and Computing Curve-skeletons with Medial Geodesic Function. In: Proceedings of SGP '06. 2006, p. 143–152.
 [18] Cornea ND, Silver D, Min P. Curve-Skeleton Properties, Applications, and Algorithms. IEEE Transactions on Visualization and Computer Graphics 2007;13(3):530–548.
 [19] Sobiecki A, Yasan HC, Jalba AC, Telea AC. Qualitative Comparison of Contraction-Based Curve Skeletonization Methods. In: Mathematical Morphology and Its Applications to Signal and Image Processing; vol. 7883 of Lecture Notes in Computer Science. 2013, p. 425–439.
 [20] Livesu M, Guggeri F, Scateni R. Reconstructing the Curve-Skeletons of 3D Shapes Using the Visual Hull. Visualization and Computer Graphics, IEEE Transactions on 2012;18(11):1891–1901.
 [21] Livesu M, Scateni R. Extracting curve-skeletons from digital shapes using occluding contours. The Visual Computer 2013;29(9):907–916.
 [22] Kustra J, Jalba A, Telea A. Probabilistic View-based 3D Curve Skeleton Computation on the GPU. In: Proceedings of the VISAPP '13. 2013, p. 237–246.
 [23] Au OKC, Tai CL, Chu HK, Cohen-Or D, Lee TY. Skeleton Extraction by Mesh Contraction. ACM Trans Graph 2008;27(3):44:1–44:10.

- 816 [24] Bærentzen JA, Misztal MK, Welnicka K. Converting skeletal structures to
817 quad dominant meshes. *Computers & Graphics* 2012;36(5):555 – 561.
- 818 [25] Ji Z, Liu L, Wang Y. B-Mesh: A Modeling System for Base Meshes of
819 3D Articulated Shapes. *Comp Graph Forum* 2010;29(7):2169–2178.
- 820 [26] Hijazi Y, Bechmann D, Cazier D, Kern C, They S. Fully-automatic
821 Branching Reconstruction Algorithm: Application to Vascular Trees. In:
822 *Proceedings of SMI '10*. 2010, p. 221–225.
- 823 [27] Autodesk. 123D. <http://www.123dapp.com/>; 2016. [Online; accessed
824 29 January 2016].
- 825 [28] ZBrush. ZSpheres. [http://pixologic.com/zbrush/features/
826 zspheres/](http://pixologic.com/zbrush/features/zspheres/); 2016. [Online; accessed 29 January 2016].
- 827 [29] Serino L, Sanniti di Baja G, Arcelli C. Object Decomposition Via Curvi-
828 linear Skeleton Partition. In: *Pattern Recognition (ICPR)*, 2010 20th
829 International Conference on. 2010, p. 4081–4.
- 830 [30] Heckbert PS. Bilinear Coons Patch Image Warping. In: *Graphics gems*
831 *IV*. 1994, p. 438–446.
- 832 [31] Douglas DH, Peucker TK. Algorithms for the reduction of the number of
833 points required to represent a digitized line or its caricature. *International*
834 *Journal for Geographic Information and Geovisualization* 1973;10(2):112–
835 122.
- 836 [32] Cgal. computational geometry algorithms library. [http://www.cgal.
837 org](http://www.cgal.org); 2016. [Online; accessed 29 January 2016].
- 838 [33] Fitzgibbon AW, Pilu M, Fisher RB. Direct least-squares fitting of el-
839 lipses. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*
840 1999;21(5):476–480.
- 841 [34] Thierry JM, Guy E, Boubekeur T. Sphere-Meshes: Shape Approx-
842 imation using Spherical Quadric Error Metrics. *ACM Trans Graph*
843 2013;32(6):178:1–178:12.
- 844 [35] Zijlstra FRH, van Doorn L. The Construction of a Scale to Measure
845 Subjective Effort. Tech. Rep.; Delft University of Technology, Department
846 of Philosophy and Social Sciences; Delft, Netherlands; 1985.
- 847 [36] Hart SG, Staveland LE. Development of NASA-TLX (Task Load In-
848 dex): Results of Empirical and Theoretical Research. In: *Human Mental*
849 *Workload*; vol. 52 of *Advances in Psychology*. 1988, p. 139 – 183.
- 850 [37] Brooke J. SUS: A “quick and dirty” usability scale. In: *Usability Evalua-*
851 *tion in Industry*. 1996, p. 189–194.
- 852 [38] Geisser S, Greenhouse SW. An Extension of Box’s Results on the Use of
853 the *F* Distribution in Multivariate Analysis. *The Annals of Mathematical*
854 *Statistics* 1958;29(3):885–91.
- 855 [39] Dunn OJ. Multiple Comparisons among Means. *Journal of the American*
856 *Statistical Association* 1961;56(293):52–64.
- 857 [40] libGLViewer. <http://libqglviewer.com/>; 2015. [Online; accessed
858 29 January 2016].