Large deformations induced in planar pantographic sheets by loads applied on fibers: experimental validation of a discrete Lagrangian model

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Abstract

In [1] a novel metamaterial has been designed and studied, whose performances include an enhanced toughness in extension: one of the problems to be solved in the further development of the concept involves the study of its deformation induced by loads concentrated on fibers (fiber push-out test). A continuum model seems particularly unfit for the description of this kind of phenomena. As a consequence, in order to design a campaign of experimental tests we resorted to numerical simulations based on a novel Lagrangian, finite dimensional model for pantographic sheets. The developed code has the robustness features necessary to supply reliable predictions also in presence of very large deformations and, indeed, produces very accurate predictions. The agreement between the presented three-elastic-parameters discrete Lagrangian model and the numerous experimental measurements performed is very accurate. Therefore we are confident that, once we will have improved the model to include damage onset, it will be possible to describe also the rupture and final failure of planar pantographic sheets.

Keywords: Pantographic structures, fiber push-out test, Lagrangian model, micro-mechanical model, second gradient continuum, nonlinear problems
2010 MSC: 74-XX, 70H03, 74-04, 74B20, 74S30, 74Q05

1. Introduction

To become mature for technological applications a new concept needs to be tested from a theoretical and experimental point of view. Crucial in this process, which leads from the initial scientific elaboration to the final technological application, is the systematic theoretical, numerical and experimental study of all properties of the novel artefact, which one wants to transform into an engineering solution. Recently, (see for instance [1, 2]) it has been proposed the design of a novel orthotropic metamaterial to which some exotic performances are requested: *i*) they must have a large compliance towards large deformations, still remaining in the largest possible elastic domain; *ii*) their ratio toughness over weight must as low as possible; *iii*) their ultimate toughness before failure in extension

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has to be extremely high.

Actually the class of proposed metamaterials, which have been named pantographic sheet or lattices, was conceived at the end of precisely directed mathematical investigations (which lasted several decades see *e.g.* [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]) whose aim was to prove the effective existence of materials which could be modeled as second gradient materials. Therefore the efforts were directed to prove the possibility to realize (or *synthesize*, as it is usual to say in some scientific milieux) a material whose continuum model, at a suitable large length scale, was outside the applicability scope of first gradient (so-called Cauchy) standard continuum mechanics, see [14]. Only recently these abstract investigations were proven to have a physical verification, as some results obtained using mathematical methods as for instance Gamma-convergence as in [15, 16, 17, 18] or more standard convergence methods as in [19] allowed for the design of real mechanical systems, which after some experimental investigations proved to have the expected properties, see [20, 21, 22, 23].

The construction of searched micro-structures were made possible by the advent of more advanced 3D printing technology. In particular, in [1] the actual feasibility and applicability of the concept leading to pantographic sheets seems to have been reasonably substantiated. One of the main problems to be solved next in the further development of considered concept involves the study of the deformation of pantographic sheets induced by loads concentrated on fibers: this is what is called in the theory of composite fiber reinforcements the *fiber push-out test*. The aware reader will immediately agree that a continuum model seems particularly unfit for the description of this kind of phenomena, although it is known (see [24, 25]) that second and higher gradient models are perfectly able to encompass boundary conditions including concentrated forces and imposed displacements of single points.

Therefore, in order to design a campaign of experimental tests, it has been considered more effective to resort to numerical simulations based on a novel Lagrangian, finite dimensional model for pantographic sheets [26]. In fact, the developed code has shown to have the robustness features which are necessary to supply, in a reasonable period of time, reliable predictions also in presence of very large deformations and, indeed, produces very accurate estimates of the reaction forces exerted by the applied hard devices. The agreement between the presented three-parameter discrete Lagrangian model (the reader will remark that we avoid the proliferation of material parameters) and the numerous experimental measurements performed in the reported campaign seems very promising and is really impressive.

2. Numerical model synoptic

The Hencky-type numerical model is completely defined by the strain energies for: *i*) axial springs, each one stores elastically an energy which depends, quadratically, on its length variation:

$$w_0 = \frac{1}{2} r_0 \left(\| p_j - p_i \| - \varepsilon \right)^2 \, , \tag{1}$$

where p_i and p_j are the actual position, see Fig. 1, of the particles distinguished by P_i and P_j in the reference configuration and connected by an extensional spring whose rigidity is denoted r_0; *ii*) bending springs whose interactions on three consecutive particles (along the arrays distinguished by the fiber directions α_1 and α_2) store energy which depends on the angle formed by the two consecutive segments connecting, in the actual configurations, the particles' positions:

$$w_{1,2} = r_{1,2} \left(\cos \gamma_{1,2} + 1 \right) \, , \tag{2}$$

where $\cos\gamma$, see Fig. 1, can easily be evaluated by using the Carnot's theorem from the actual position of the knots and $r_{1,2}$ denote the rigidities of the corresponding rotational springs; *iii*) shear springs which interact elastically *via* the interconnecting pivots between the two arrays of fibers: if the actual angle between the two arrays is γ_3 we assume that the stored energy is:

$$w_3 = \frac{1}{2} r_3 \left(\gamma_3 - \frac{\pi}{2} \right)^2 \, , \tag{3}$$

where, as shown in Fig. 1, the angles γ_3 is completely defined, *e.g.* by the Carnot's theorem, by the actual position of the knots.

The total potential energy is used to build in a straightforward manner its gradient, the so-called internal force vector, and the Hessian, the so-called stiffness tangent matrix, which are, when its stationarity condition is used, the the basic ingredient for a step-by-step procedure which reconstructs the complete nonlinear equilibrium path by using an incremental-iterative procedure, guided by the

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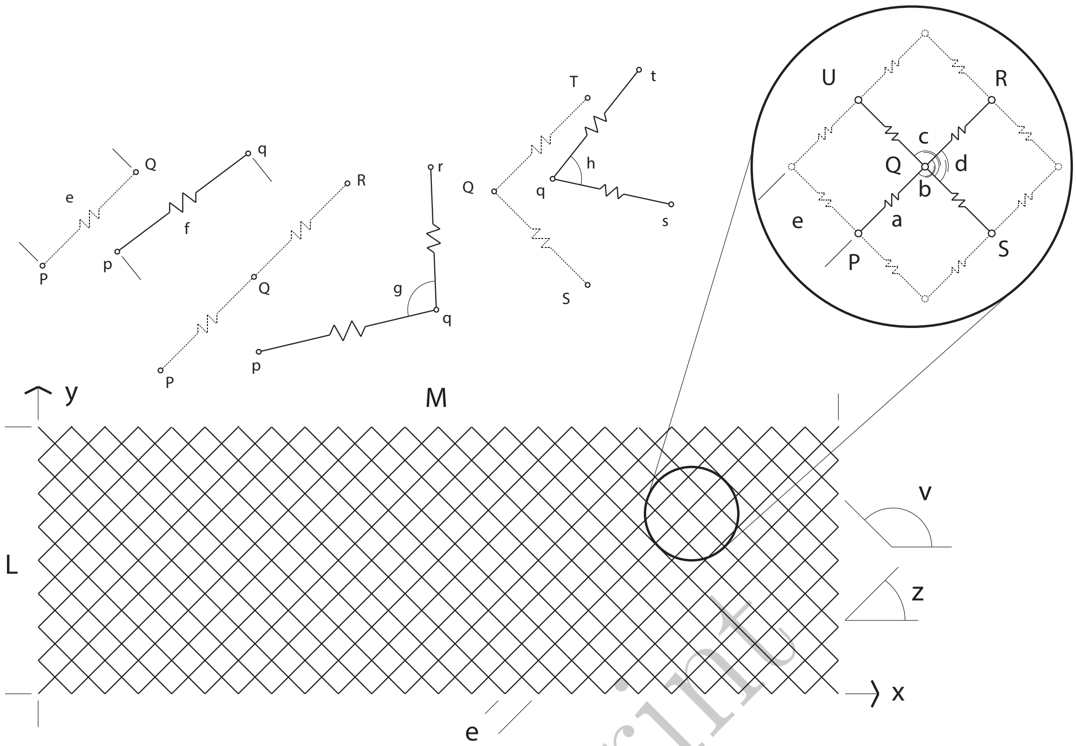

Figure 1: Hencky-type pantographic structure.

imposed displacements, based on the classic Newton's residual scheme. The interested reader will find a thoroughly description of the Hencky-model and some insights about the numerical code implementation in [26].

3. Experimental and numerical tests

By referring to Fig. 1, the underlying pantographic structure has $\ell = 69.3$ mm, $\varepsilon = 6.13$ mm, $\alpha_1 = \pi/4$ and $\alpha_2 = 3\pi/4$. As Young's modulus for the polyamide, we chose the value $E = 1600$ MPa. The cross-section of the fiber is 2.25 mm for the width and 1.6 mm for the height while the pivot are cylinder with diameter equal to 0.9 mm and height equal to 1 mm. From the geometrical and constitutive parameters described in the immediately above, the extensional and rotational stiffness parameters of each kind of springs were estimated by using the guidelines described in [27]. Consequently, for the numerical simulation reported below, we assume $k_0 = 265$ N/mm, $k_1 = k_2 = 238.2$ Nmm and $k_3 = 0.9739$ Nmm. The experimental tests were performed by using a MTS Bionix system strength machine able to impose the prescribed displacements at a controlled velocity of 5 mm/min.

The first test, identified in the following as T_1-test, is well-depicted in Fig. 2. Starting from the reference configuration (on the left), a displacement along a diagonal of the specimen is gradually imposed on its upper corner until the value $v_{\max} = 75.3$ mm (on the right) and taking a picture also at the intermediate point (in the middle), *i.e.* $v_i = 37.65$ mm. The results of the numerical simulation of this test are reported in Fig. 3 using the displacement parameter $\lambda = v/v_{\max}$ to individuate the intermediate, $\lambda = 0.5$, and the final configuration, $\lambda = 1$. The color bar gives an immediate representation of the level of the axial forces on the extensional springs. Fig. 4 reports the global reaction $R(\lambda)$ evaluated both by means of the experimental, in black, and numerical test, in red. We underline the good agreement between experimental and numerical values, in particular for $\lambda \leq 0.7$.

The second test, T_2-test, is reported in Fig. 5 (the reference configuration is reported on the left). A vertical displacement is gradually imposed along the larger side on a single fiber, the 6-th from the top, of the specimen until the value $v_{\max} = 40.7$ mm

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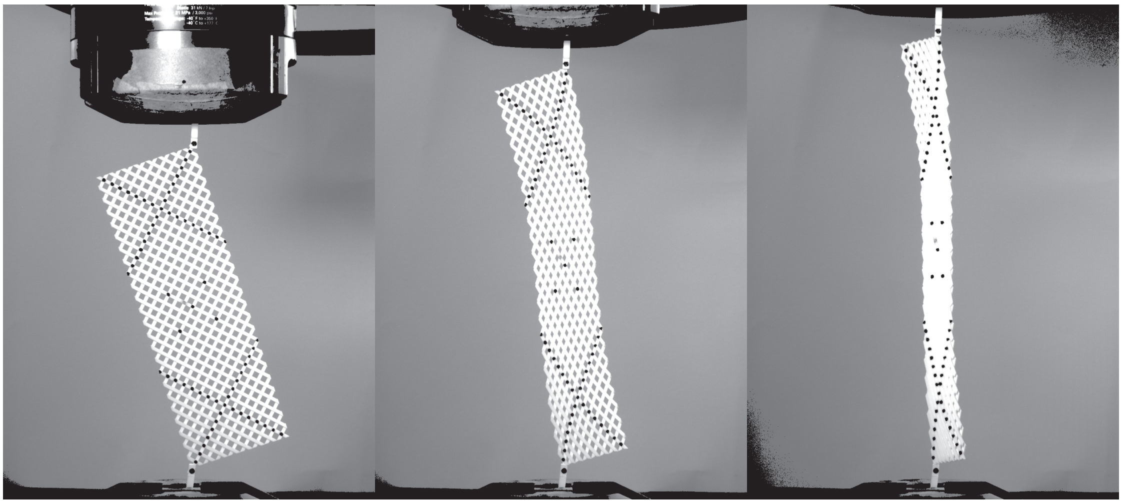

Figure 2: T_1 test: initial (left), intermediate (middle) and final (right) configuration.

(on the right) and taking a picture also at the intermediate point (in the middle), *i.e.* $v_i = 20.35$ mm. The numerical simulation of this test gives the plots reported in Fig. 6 using again the displacement parameter $\lambda = v/v_{\max}$ in order to distinguish the intermediate and the final configurations, and using the color bar to give evidence at a first glance to the axial forces on the extensional springs. In Fig. 7 the global reaction $R(\lambda)$ evaluated both by means of the experimental, in black, and numerical test, in red, is reported. Also in this case, there is a good agreement between experimental and numerical results for $\lambda \leq 0.6$.

Finally, we remarks that the numerical model could be produce results more close to the experimental one choosing strain energy laws, in particular that for the shear energy since it is the more relevant, different from that used, quadratic in angle variation for the shear strain energy.

4. Concluding remarks and perspectives

The used numerical code (for more details the reader is referred to [26]) has proven its predictive capability even in extreme cases where the modeling assumptions on which it is based may loose their applicability. The experimental campaign which has been performed has proven that the performances of pantographic sheets in the fiber pull-out test are really promising and that, therefore, this concept needs to be further investigated. There are several research direction where investigations are needed. The most important involves the modeling of damage onset and evolution. Also in this aspect the discrete model which we have introduced seems to be more efficient than continuum ones. We are confident that, once we will have suitably improved aforementioned discrete model to include damage onset, it will be possible to describe also the rupture progress and final failure of pantographic sheets.

Finally, we briefly list some research line which deserve to be further investigated: 1) lattice with curved elements using the tools deriving from the isogeometric approach, see [28, 29, 30, 31, 32, 33, 34, 35]; 2) global buckling modes could also be exploited by the proposed numerical model following the guidelines reported in [36, 37, 38]; 3) the Hencky-type model briefly sketched here could be interesting also to model the granular media interactions, see [39, 40, 41, 42], or generalized continua, see [43] for a general review and [44] for an interesting application for the masonries; 4) the identification of the spring rigidities for the pantographic structure could be fitted using the tools reported in [45, 46, 47, 48].

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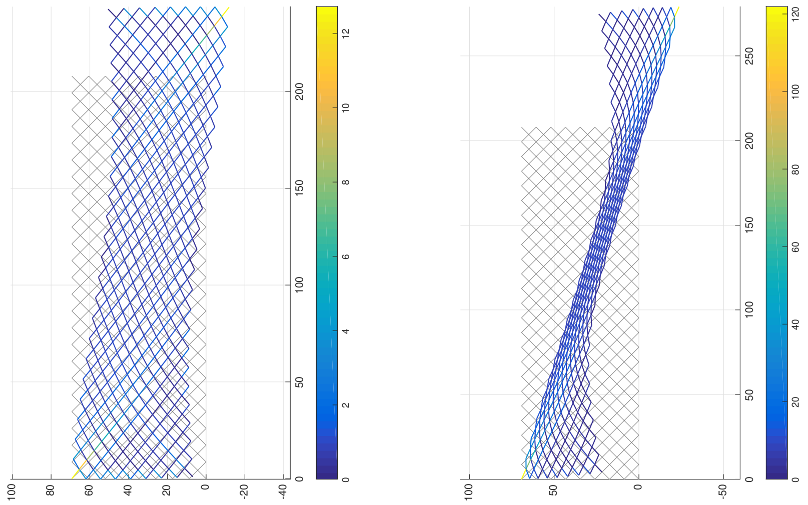

(a) $\lambda = 0.5$ (b) $\lambda = 1$

Figure 3: Numerical simulation of T_1 test: deformation and axial force level for the extensional springs at intermediate (on the left) and final (on the right) configuration (reference configuration is depicted in grey).

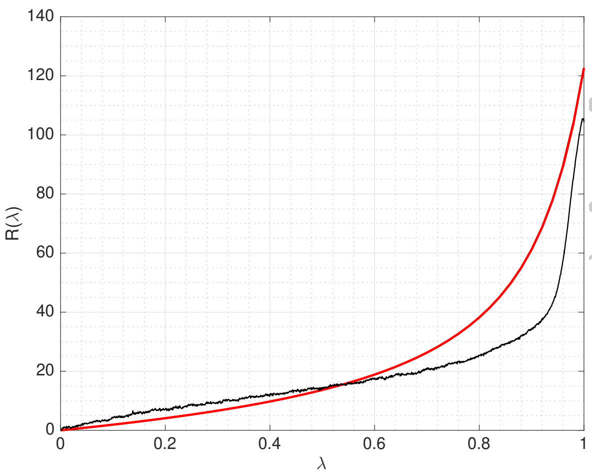

Figure 4: Comparison between experimental and numerical results for T_1-test: global structural reaction evolution $R(\lambda)$ *vs.* λ (in black for experimental results and in red for numerical results).

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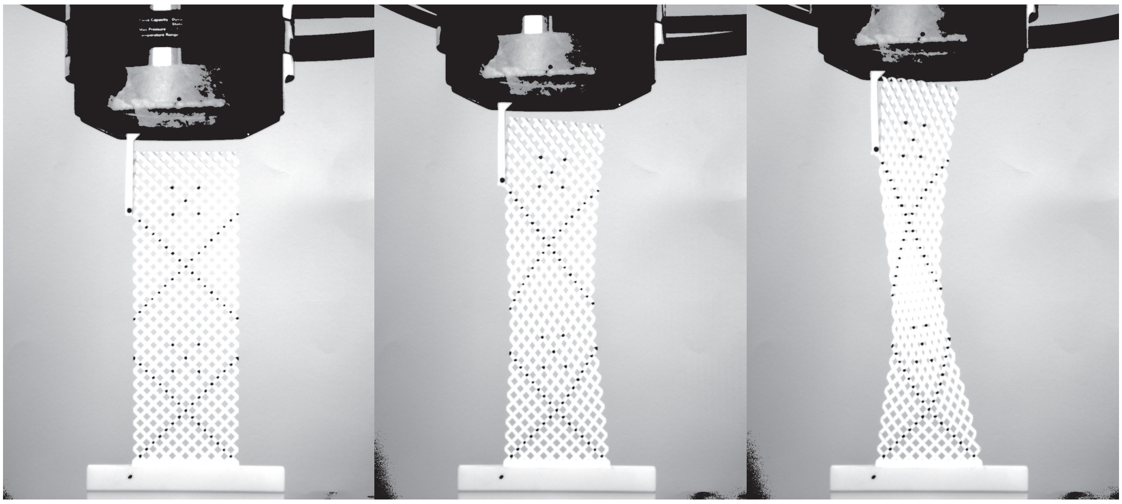

Figure 5: T_2 test: initial (left), intermediate (middle) and final (right) configuration.

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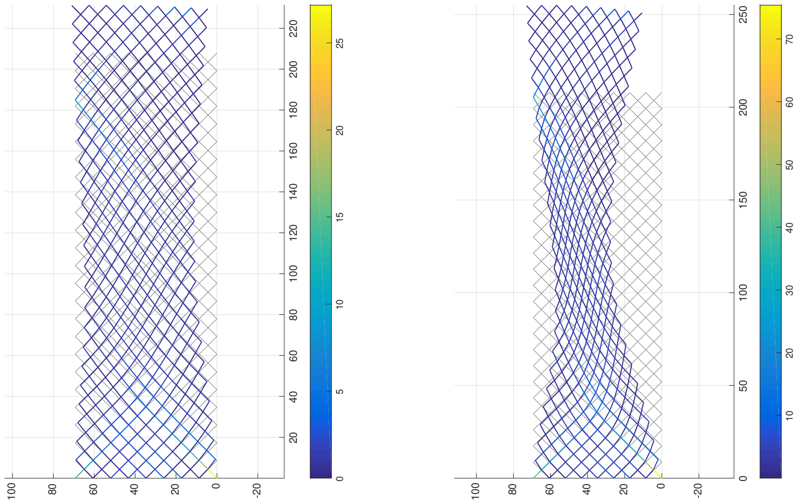
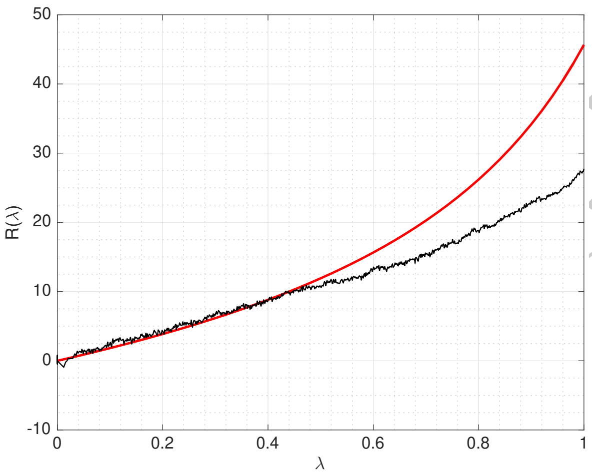


(a) $\lambda = 0.5$ (b) $\lambda = 1$

Figure 6: Numerical simulation of T_2 test: deformation and axial force level for the extensional springs at intermediate (on the left) and final (on the right) configuration.



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