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A MULTISCALE/MULTIDOMAIN MODEL FOR THE FAILURE ANALYSIS OF MASONRY WALLS: A VALIDATION WITH A COMBINED FEM/DEM APPROACH

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An accurate and fast failure simulation for masonry walls is still an active field of research, due to its fundamental role in predicting the overall response of masonry structures under seismic and other extreme natural and man-originated events. Multiscale models have been successfully exploited for achieving this task, being characterized by high computational efficiency, especially in the presence of strong nonlinearities due to multiple microcrack initiation and propagation. In this paper, a novel multiscale/multidomain approach for nonlinear analysis of masonries is presented, based on a couple-stress homogenization for undamaged regions and an adaptive strategy for triggering the macro-to-micro switching operations. An extended validation of the proposed approach is presented, via suitable comparisons with a micromechanical model, here regarded as a benchmark model, that finely describes the microstructure, based on the combined finite/discrete element method (FEM/DEM). A critical discussion of the obtained numerical results has shown the efficacy of the proposed models as well as their limits of application.

KEY WORDS: *masonry modeling, multiscale methods, multidomain approach, couple-stress continua, FEM/DEM, nonlinear analyses, in-plane failure*

1. INTRODUCTION

As is well known, there are several difficulties in masonry modeling, related to the peculiar nature of this composite material and to the specific characteristics of masonry structures. Even if in the last decades the research community paid great attention to this topic (Adam and Lourenço, 2011; Calladine, 1992; Milani, 2012; Sacco et al., 2018) there are still some open questions: Which model may be more appropriate for studying the behavior of masonry material and masonry structures? Is it better to adopt a discrete or a continuous model? How can a discrete system like masonry material can be modeled as a continuum? How can the kinematic and static descriptors and the constitutive prescriptions be transferred to the continuum model? Moreover, in the framework of continuous models, which continuum is more suitable: a standard or a generalized continuum (Trovalusci and Pau, 2014)? The choice of the most appropriate model for masonry is still an unresolved issue.

Masonry is a composite material characterized by heterogeneity, nonlinear behavior, and different responses in compression and in tension. To a usually high strength in compression is coupled a very low or uncertain tensile strength, due to the poor resistance of mortar joints to tension stresses. This aspect plays an important role in the case of historical masonry, where the joints could be considered completely cracked: this led in the past to the formulation of specific models for the so-called no-tension material that has been adopted for the modeling of masonry structure representing, in particular, a wide part of the architectural heritage (Como, 1992; Del Piero, 1989, 1998; Di Pasquale, 1992; Heyman, 1966).

Nevertheless, the mechanical behavior of real masonry is more complex and it is strongly influenced by its constructive features: quality of constituent materials, size of blocks, thickness of joints, arrangement of blocks, etc. All these characteristics play a relevant role in the mechanical behavior of masonry, and furthermore they vary considerably from one case to another, even inside the same masonry structure. This calls for the need of specific mechanical descriptions to be properly adopted case-by-case.

Despite the mentioned complexities in masonry modeling, in the common practice simplified phenomenological models are usually adopted, in which masonry structures are modeled as simplified structural systems, such as a system of nonlinear beams (Bucchi et al., 2013; Lagomarsino et al., 2013; Magenes and Calvi, 1997; Roca et al., 2005; Turnsek and Tomazevic, 1980).

On the other hand, the scientific literature is wide ranging and several specific models have been proposed. With a particular focus on brick/block masonry, it is possible to point out two different main approaches, developed over the years but still extremely current:

- the discrete approach — in which masonry is modeled as a discrete system of elements (blocks, joints, and/or interfaces) exhibiting different constitutive behaviors (Baggio and Trovalusci, 1993, 1998, 2000; Casapulla et al., 2018; Cecchi and Sab, 2004; Lemos, 2007; Portioli et al., 2013, 2014);

- the continuum approach — in which masonry is modeled as an equivalent continuum, whose constitutive model may be defined either through a simplified phenomenological approach, i.e., smeared cracking models (De Carvalho Bello et al., 2017; Ghiassi et al., 2013; Lourenço and Rots, 1997) — or through homogenization techniques for the derivation of both classical (Addessi et al., 2014; Anthoine, 1995; Caporale et al., 2015; Casalegno et al., 2013; Cecchi and Sab, 2002; De Buhan and De Felice, 1997; Lourenço et al., 2007; Luciano and Sacco, 1997; Milani, 2011) and generalized continua (Bacigalupo and Gambarotta, 2011; Baraldi et al., 2015a; De Bellis and Addessi, 2011; Forest and Sab, 1998; Leonetti et al., 2018; Masiani and Trovalusci, 1996; Pau and Trovalusci, 2012; Salerno and de Felice, 2009; Stefanou et al., 2008; Sulem and Muhlhaus, 1997; Trovalusci and Masiani, 2003). In particular, among various generalized continua formulations, the micropolar model has been widely proven to be able to properly account for the size and the orientation of the elements (Masiani et al., 1995; Trovalusci and Pau, 2014).

Overall, on one hand, micromechanical models are able to completely reproduce the masonry microstructure in order to provide reliable results, but, due to high computational costs, their applicability should be limited to small portions of building. On the other hand, the adoption of macromechanical models obtained through multiscale homogenization procedures may provide high computational efficiency keeping an appropriate numerical accuracy.

We consider masonry as a composite material, obtained by the union of at least two different components—units and joints—with different constitutive behaviors and arranged in several possible ways. In order to take into account this internal microstructure we conside[illegible] different mechanical models:

- a micromechanical model [“fine” model, marked (a) in the following), based on the discrete element method (DEM) (Cundall, 1988), that accurately takes into account the effective micro-structure of masonry material (Baraldi et al., 2013, 2015b, 2018b; Reccia et al., [illegible]) ;

- a multiscale/multidomain continuous model [“fine/coarse” model, marked (b) in the following), in which the the characteristics of masonry emerging at microscale are accounted for at the macroscale by means of homogenization procedures (Greco et al., 2016, 2017), here applied to a couple-stress continuum description (Leonetti et al., 2018).

The multiscale/multidomain model provides a computational strategy, whose aim is to reduce the typically high computational cost exhibited by fully microscopic numerical analyses. The model is based on the combination of two approaches: (i) a couple-stress (Mindlin and Tiersten, 1962; Toupin, 1964) homogenization technique to derive the equivalent continuum to be used as a coarse model for undamaged masonry; (ii) an adaptive multidomain decomposition technique, which allows us to automatically zoom-in the zones incipiently affected by damage onset. The associated model refinement criterion requires the determination of microscopically informed first failure surfaces, which take into account both classical and bending deformation effects, by taking advantage of the above-mentioned couple-stress based homogenization technique.

The aim of this work is to propose a comparison between these two different modeling approaches in the failure analysis of masonry panels. In particular, we provide a systematic validation of the results of the model (b) using the comparison with the results obtained via a micromechanical model (a), that finely describes the microstructure. The related accuracy and computational performances of the latter methodology are investigated via suitable numerical experiments on different masonry panels with various sizes and aspect ratios, subjected to both vertical and lateral actions.

The outlines of the paper are as follows. Section 2 provides the theoretical background of the proposed models: the basic assumptions of the two models are reported, attention is paid to the opportunity, as well as the computational convenience, of validating a nonclassical continuum multiscale/multidomain model, specifically conceived for the gross modeling of masonry nonlinear behavior, with a more refined discrete model that has been proven to well represent the failure behavior. In Section 3 the results of the numerical comparative analyses are reported. A first analysis has been performed on a masonry wall panel subjected to in-plane horizontal displacements applied on the top, with the purpose of calibrating the mechanical parameters to be adopted in the two models. Afterwards, attention has been focused on the shape/scale effect, by changing both dimensions and shape of the panel but keeping fixed dimensions and arrangement of bricks. Finally, in Section 4 some concluding remarks are presented.

2. THE FINE AND COARSE DESCRIPTIONS

In this section attention is focused on the basic theoretical assumptions of the two proposed descriptions for brick/block masonry. The ‘fine’ nonlinear discrete model (a) (Section 2.1), that well represents the microstructure in detail, is assumed as a benchmark model for validating the ‘fine/coarse’ couple-stress continuum model (b), enriched by the heterogeneous description in the nonlinear regions of damage/failure via a multiscale/multidomain strategy (Section 2.2). The former model has been widely proved to well represent the masonry nonlinear behavior, but with high computational cost. The latter exploits the advantages of nonclassical continuum modeling, that, for instance, relies on the possibility to take into account scale effects (Masiani et al., 1995; Trovalusci and Masiani, 2003; Trovalusci and Pau, 2014), enriched by the multidomain decomposition strategy, that allows a detailed description of masonry behavior with reduced computational costs.

2.1 A Combined Finite/Discrete Element Model (FEM/DEM) (a)

A micromechanical model, based on the use of combined finite and discrete element methods (FEM/DEM) is here adopted as a benchmark model for validating the multiscale/multidomain model described in Section 2.2.

The discrete element method (DEM) (Cundall, 1988) belongs to a specific class of discrete models which have to satisfy the following criteria: (Cundall and Hart, 1992): (i) discrete elements that can move independently; (ii) contacts between the elements that may vary during the analysis, so that different elements can come in or lose contact; (iii) contact detection governed by a molecular dynamics algorithm; (iv) analysis are performed under the hypothesis of large displacements.

DEM was originally developed for the study of particulates, jointed rock, granular assemblies (Cundall and Strack, 1979) and, in general, of nonlinear problems characterized by the mutual movement of rigid bodies, eventually interacting through contact. For these reasons, it has been successfully adopted also for masonry modeling (Baraldi et al., 2015a; Cecchi and Sab, 2004, 2009; Lemos, 2007).

The combination of the finite element method (FEM) with the discrete element method (DEM) had been already introduced in the initial improvements of the original discrete model, in order to describe the deformability of the elements by means of FE discretization (Cundall and Hart, 1985, 1992). The combined FEM/DEM approach here adopted is based on the original method developed by Munjiza (2004) and improved by the Toronto Geo Group

(Mahabadi et al., 2010). In FEM/DEM, which is based on the discrete element method, the DEs are meshed into FEs with embedded crack elements that activate whenever the peak strength is reached.

The governing equations in a FEM/DEM system are

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} = \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{M}$ is the lumped mass m[illegible] $\boldsymbol{C}$ is the damping diagonal matrix, $\boldsymbol{u}$ is the vector of the nodal displacements, and $\boldsymbol{f} = \{\boldsymbol{f}_{ext}, \boldsymbol{f}_{int}, \boldsymbol{f}_{def}, \boldsymbol{f}_{joint}\}^T$ is the vector of the nodal forces, which includes several contributions: external forces $\boldsymbol{f}_{ext}$; internal forces $\boldsymbol{f}_{int}$ related to the interaction between the elements; deformation forces $\boldsymbol{f}_{def}$ [illegible]puted under [illegible]ssumption of isot[illegible] linear elasticity; and crack forces $\boldsymbol{f}_{joint}$ [illegible]rces transmitted throug[illegible] elements ar[illegible]d to simulate m[illegible]l failure. An explicit second-order finite-[illegible]ence time integration scheme is applied to solve the equations of motion for the discretized system and to update the nodal coordinates at each simulation time step. FEs allow modeling elastic deformation, while DEs are able for modeling interaction, fracture, and fragmentation processes. In the normal direction, body impenetrability is enforced using a penalty method (Munjiza and Andrews, 2000), while in the tangential direction, discontinuity frictional behavior is simulated by a Coulomb-type friction law (Mahabadi et al., 2012).

The FEM/DEM approach here adopted was originally introduced in soil mechanics problems (Munjiza et al., 1995); however, it has been successfully extended to the field of masonry structures by some of the authors (Baraldi et al., 2013, 2015b, 2018b; Reccia et al., 2012, 2016) and by other research groups (Miglietta et al., 2016, 2017; Smoljanović et al., 2013, 2015, 2017).

2.1.1 FEM/DEM Approach for Masonry Panels

The approach here adopted follows the lines developed in previous works of some of the authors, and in particular in (Baraldi et al., 2018a). Masonry panels are modeled as an assemblage of deformable bricks connected by zero-thickness Mohr-Coulomb interfaces. A mesh of triangular constant strain triangle (CST) FEs is adopted to model the whole masonry panel, under the hypothesis [illegible]e stress. Each brick is modeled by 16 FEs; deformability is governed by elastic modulus and Poisson's ratio. [illegible]onlinear behavior is modeled by crack elements, [illegible]dded cohesive elements embedded among all the triangular FEs. The crack elements allow us to model the mortar [illegible], but they can also represent inner brick subdivisions, so that cracks may occur both in the bricks and in the joints between adjacent bricks. However, in order to compare the FEM/DEM approach to the multiscale/multidomain model (b) described in Section 2.2, in this work the bricks are assumed to be infinitely resistant to tensile and shear forces; therefore damage is limited to the interfaces (mortar joints) between adjacent bricks. The mesh adopted and the two different crack elements inside and between bricks are shown in Fig. 1(a). It is possible to notice that, the mortar joints being modeled as zero-thickness interfaces, the dimensions of the bricks are increased by half of the joints' thickness (extended bricks), with reference to the running bond arrangement that will be considered in the numerical simulations (Section 3).

Interfaces are governed by a tensile strength criterion, $(\sigma_\perp \le f_t)$, and by a Mohr-Coulomb yield criterion, $(|\ \sigma_\parallel\ | \le f_s)$, with $f_s = c - \sigma_\perp tan\phi$, where $(\boldsymbol{\sigma}) = \{\sigma_\perp\ \ \sigma_\parallel\}^T$ is the stress vector, with the normal and tangential

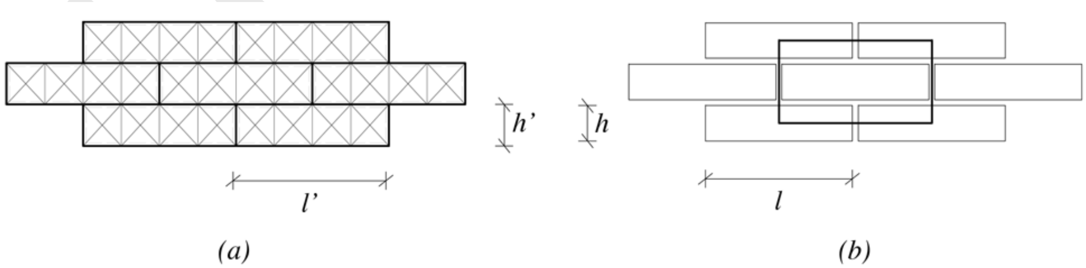

FIG. 1: Mesh adopted for FEM/DEM model (a) RVE defined for the continuous multiscale model (b)

components, respectively, over a generic interface depending on the relative displacements between two adjacent blocks. The mechanical parameters involved in the joints are cohesion $c > 0$, friction angle ϕ $(0 < \phi < \pi \setminus 2)$, and the tensile f_t and shear strength f_s.

The cohesive law adopted in the FEM/DEM model is a modified version proposed by Lisjak et al. (2013). When the value of cohesion c or tensile strength f_t is reached, the crack elements are activated. Depending on the local stress and deformation field, fractures can develop and grow in mode I (decohesion mode), mode II (slippage mode), or in mixed mode I-II. The nonlinear behavior of crack elements is governed by two fracture energies: mode I, G_{Ic}, related to the decohesion mechanism, and mode II, G_{IIc}, related to the slippage mechanism, corresponding to the areas under the bonding stress-softening curves reported in Fig. 2. This cohesive law has been also implemented in the multiscale/multidomain model.

Numerical simulations are performed utilizing the open source computer code Y2D/Y-GUI (Mahabadi et al., 2010) for generating input files and Y-Geo (Mahabadi et al., 2012) for running numerical analyses. Results have been processed by means of ad hoc MATLAB scripts and by means of spreadsheets.

2.2 Multiscale/Multidomain Model for Masonry Structures (b)

The continuum model is a linearly elastic couple stress model, obtained using a homogenization procedure, enriched by the discrete description in the regions in which failure mechanisms occur. The macroscopic couple stress description is the key ingredient of the adopted multiscale/multidomain framework, allowing for the injection of the characteristic length of masonry in its overall mechanical response, both in the linear and nonlinear ranges.

The numerical results are obtained using an original finite element (FE) discretization, formulated for couple-stress media and implemented within the commercial code COMSOL Multiphysics®, which exploits an adaptive multilevel domain decomposition strategy, initially proposed in Greco et al. (2015, 2016, 2017) for classical continua and then extended to couple-stress continua (Leonetti et al., 2018). The proposed multiscale/multidomain model allows the nonlinear analysis of masonry structures. The fracture simulation is performed under the assumptions of quasistatic loading conditions, isothermal evolution over time, and small displacements. The macroscopic problem is discretized by a coarse mesh of finite elements which cannot capture the nonlinear microstructural behavior. Indeed, initiation and propagation of cracks (mainly occurring at the mortar joints) are only represented at the fine scale. It follows that linear homogenization is sufficient for deriving the macroscopic constitutive model of undamaged masonry.

In the critical regions where the scale separation assumption is not satisfied, the homogenization step is by passed and a concurrent multiscale method based on domain decomposition is adopted. In these regions, the microscopic model is directly solved in a strongly coupled manner with the macroscopic model, as discussed in Leonetti et al. (2018). The proposed concurrent multiscale model for masonry structures is equipped with adaptive capabilities,

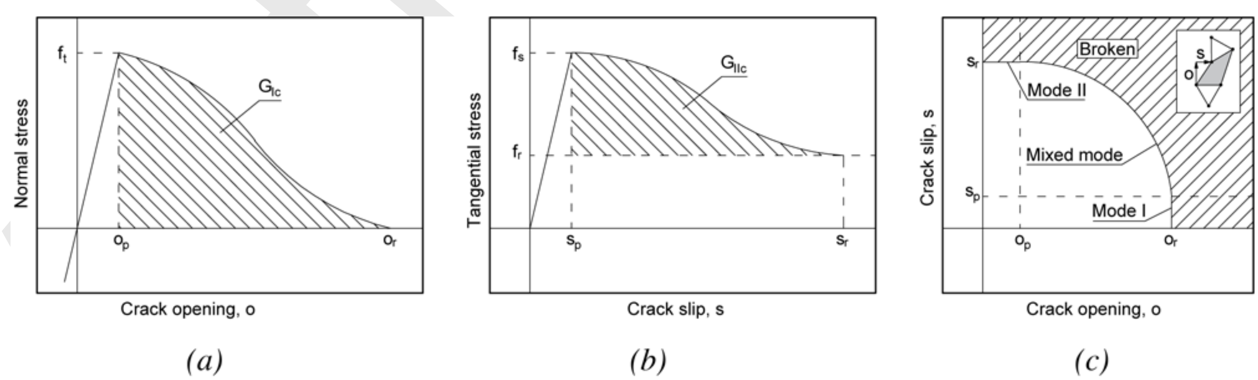

FIG. 2: Constitutive behavior of the crack elements (Lisjak et al., 2013): (a) mode I, decohesion; (b) mode II, slippage; (c) mixed mode I-II. Subscripts p and r refer to onset and complete separation, respectively.

with the aim of reducing the overall computational cost of the associated numerical method for the solution of the underlying nonlinear problem.

2.2.1 Couple-Stress Homogenization

As a homogenized model for undamaged masonry, a couple-stress model is adopted here, regarded as a modified Cosserat model obtained by introducing an additional internal constraint (constrained Cosserat model).

In the linearized kinematics setting, the consistent strain measures of a couple stress continuum are the (symmetric) strain tensor and the curvature tensor:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}}\right], \qquad \boldsymbol{\chi} = \nabla \boldsymbol{\omega}, \tag{2}$$

where the rotation vector $\boldsymbol{\omega}$ is not independent as in the Cosserat model, but is related to the displacement vector $\boldsymbol{u}$ as follows:

$$\boldsymbol{\omega} = \frac{1}{2}\mathrm{curl}\,(\boldsymbol{u}) = \frac{1}{2}\left[\nabla \boldsymbol{u} - (\nabla \boldsymbol{u})^{\mathrm{T}}\right]. \tag{3}$$

It is worth noting that Eq. (2) represents the equality between the Cosserat microrotation and the local rigid rotation (macrorotation) as defined for classical media.

By neglecting the body couple per unit volume of the continuum, the differential form of the equilibrium equations for the couple-stress theory can be written as

$$\mathrm{div}\,(\boldsymbol{\sigma}) + \boldsymbol{b} = 0, \quad \mathrm{div}\,(\boldsymbol{\mu}) = 0, \tag{4}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ are the stress and couple-stress tensors, div$(\cdot)$ denotes the divergence operator, and $\boldsymbol{b}$ is the body force. It is worth noting that the stress tensor $\boldsymbol{\sigma}$, the dynamical work-conjugated counterpart of the strain measure $\boldsymbol{\varepsilon}$, is symmetric.

By standard variational arguments, it follows that the virtual work principle for a couple-stress continuum occupying a Euclidean region $\mathcal{P}$ bounded by a surface $\partial\mathcal{P}$ can be written as

$$\int_{\mathcal{P}} (\boldsymbol{\sigma}\cdot\delta\boldsymbol{\varepsilon} + \boldsymbol{\mu}\cdot\delta\boldsymbol{\chi})\,\mathrm{dV} = \int_{\partial\mathcal{P}_{\mathrm{t}}} \mathbf{t}\cdot\delta\mathbf{u}\,\mathrm{dS} + \int_{\partial\mathcal{P}_{\mathrm{m}}} \mathbf{m}\cdot\delta\boldsymbol{\omega}\,\mathrm{dS} + \int_{\mathcal{P}} \mathbf{b}\cdot\delta\mathbf{u}\,\mathrm{dV}, \tag{5}$$

where $\boldsymbol{t}$ and $\boldsymbol{m}$ are the traction and surface couple imposed on the portions $\partial\mathcal{P}_t$ and $\partial\mathcal{P}_m$, respectively, and δ denotes the usual variational operator.

The elastic energy density function $W = W(\boldsymbol{\varepsilon}, \boldsymbol{\chi})$ takes the general form

$$W = \left(\frac{1}{2}\boldsymbol{A}\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon} + \boldsymbol{B}\boldsymbol{\varepsilon}\cdot\boldsymbol{\chi} + \frac{1}{2}\boldsymbol{D}\boldsymbol{\chi}\cdot\boldsymbol{\chi}\right). \tag{6}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ are constitutive tensors. Under the hypothesis of hyperelasticity, it follows that the constitutive equations for the couple-stress model are

$$\boldsymbol{\sigma} = \frac{\partial W}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{A}\boldsymbol{\varepsilon} + \boldsymbol{B}\boldsymbol{\chi}, \quad \boldsymbol{\mu} = \frac{\partial W}{\partial \boldsymbol{\chi}} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{\varepsilon} + \boldsymbol{D}\boldsymbol{\chi}. \tag{7}$$

In order to derive the macroscopic elastic moduli tensor for the undamaged masonry, a repeating cell is considered [Fig. 1(b)], subjected to five (three Cauchy and two bending) independent pure macrodeformation modes. Periodic boundary conditions are imposed for the Cauchy modes, whereas the special mixed boundary conditions shown in Fig. 3 are prescribed for the bending modes, involving a combination of periodic, antiperiodic, and zero fluctuation on different portions of the repeating cell (RC) boundary.

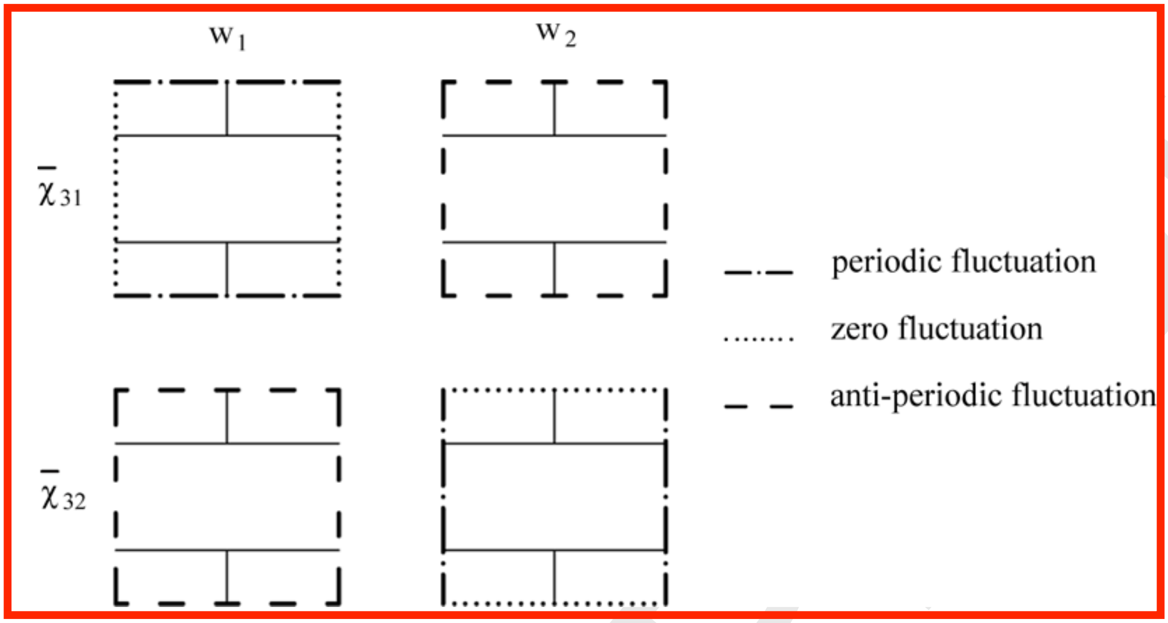

FIG. 3: Boundary conditions required for bending macrodeformation modes ($\overline{\chi}_{31}$, $\overline{\chi}_{32}$ macroscopic curvatures; w_1, w_2 horizontal and vertical components of the microscopic fluctuation field over the RC boundary)

Let us consider a two-dimensional framework. The homogenized response of undamaged masonry is assumed to be orthotropic; thus only six independent moduli must be identified. The homogenized constitutive law can be written in the following matrix form (overlined symbols indicate the homogenized quantities):

$$\begin{bmatrix} \overline{\sigma}_{11} \\ \overline{\sigma}_{22} \\ \overline{\sigma}_{12} \\ \overline{\mu}_{31} \\ \overline{\mu}_{32} \end{bmatrix} = \begin{bmatrix} \overline{A}_{1111} & \overline{A}_{1122} & 0 & 0 & 0 \\ \overline{A}_{1122} & \overline{A}_{2222} & 0 & 0 & 0 \\ 0 & 0 & \overline{A}_{1212} & 0 & 0 \\ 0 & 0 & 0 & \overline{D}_{3131} & 0 \\ 0 & 0 & 0 & 0 & \overline{D}_{3232} \end{bmatrix} \begin{bmatrix} \overline{\varepsilon}_{11} \\ \overline{\varepsilon}_{22} \\ \overline{\varepsilon}_{12} \\ \overline{\chi}_{31} \\ \overline{\chi}_{32} \end{bmatrix}, \tag{8}$$

where $\overline{\sigma}_{ij}$ and $\overline{\varepsilon}_{ij}$ (with $i, j = 1, 2$) represent the macroscopic (homogenized) Cauchy stress and strain components, respectively, whereas $\overline{\mu}_{3i}$ and $\overline{\chi}_{3i}$ (with $i = 1, 2$) are the macroscopic couple-stress and curvature components, respectively. Moreover, $\overline{A}_{ijhk}$ (with $i, j, h, k = 1, 2$) denote the homogenized Cauchy moduli, whereas $\overline{D}_{3i3i}$ (with $i = 1, 2$) are the homogenized bending moduli, related to the unit dimensions, which are responsible for the size dependence of the overall mechanical response of the medium.

It is worth noting that the couple-stress model usually requires a C^1 continuity for the discretized displacement field, when employed in a standard finite element setting. To avoid this strong requirement, the couple-stress model is directly derived from an unconstrained Cosserat model, in which displacements and rotations are taken as independent nodal variables. The additional internal constraint enforcing the equality between macrorotations (i.e., computed from displacement gradients) and microrotations (i.e., local rotations) is introduced in the variational formulation by means of a penalty approach, as proposed in Garg and Han (2013).

2.2.2 Multiscale/Multidomain Formulation

At the microscopic level, regular masonry may be regarded as a two-phase composite material, made of units (e.g., bricks or blocks) and mortar joints periodically arranged. We analyze the in-plane behavior of two-dimensional (2D) assemblies, due to the assumed small thickness. It follows that the given structure is described by a periodic heterogeneous continuum, occupying a bounded open set $\Omega \subset \mathbb{R}^2$. Its external boundary $\partial\Omega$ is made of two disjoint parts, denoted as $\partial_D\Omega$ and $\partial_N\Omega$, where Dirichlet and Neumann boundary conditions are prescribed, respectively. The units behave as linearly elastic, whereas the mortar joints are modeled as cohesive interfaces, collectively denoted as Γ_c, equipped with a mixed-mode softening constitutive law. According to this approach, referred to as simplified micro-modeling, artificially expanded units are modeled by bulk elements, whereas the joints are lumped into zero-thickness elements placed in between the units.

Even under the hypotheses of zero body forces, small deformations, and quasistatic loading conditions, the mechanical response of the considered masonry structure subjected to in-plane external actions can be obtained by solving a highly nonlinear boundary value problem (BVP), due to the presence of damageable interfaces and unilateral contact conditions along the crack faces. This nonlinear problem is cumbersome to solve at the fine scale in the standard finite element setting, due to the excessively high number of associated degrees of freedom (DOFs).

In order to reduce the complexity of such a purely microscopic problem, the multiscale model proposed in Leonetti et al. (2018) is here adopted, based on a two-level domain decomposition approach used in combination with a couple-stress homogenization technique. According to this model, a homogenized couple-stress model for masonry is adopted everywhere, except for critical regions, where nonlinear damage phenomena occur, thus requiring a detailed microstructural modeling.

As a consequence, the original problem is replaced by the equivalent multiscale/multidomain problem shown in Fig. 4, leading to the partition of Ω in two nonoverlapping subdomains, i.e., the macroscopic and the microscopic domains denoted as Ω_M and Ω_m, respectively. Because of this partition, additional internal boundaries are introduced, collectively referred to as the micro/macro interface and indicated by Γ_{int}.

Let $\boldsymbol{u}_m$ and $\boldsymbol{u}_M$ represent the micro- and macroscale displacement fields, belonging to the spaces of admissible solutions, U_m and U_M, respectively. The resulting boundary value problem (BVP) turns out to be an interface problem, represented by the following variational formulation: Find $(\boldsymbol{u}_m, \boldsymbol{u}_M, \boldsymbol{\lambda}) \in U_m \times U_M \times \Lambda$ such that

$$\int_{\Omega_m\setminus\Gamma_c} \boldsymbol{\sigma}_m \cdot \boldsymbol{\varepsilon}(\delta\boldsymbol{u}_m)\mathrm{d}\Omega + \int_{\Gamma_c} \mathbf{t_c}([\![\mathbf{u_m}]\!], \mathbf{d}) \cdot [\![\delta\mathbf{u_m}]\!]\mathrm{d}\Gamma + \int_{\Gamma_{\mathrm{int}}} \boldsymbol{\lambda} \cdot \delta\boldsymbol{u}_m \mathrm{d}\Gamma$$

$$= \int_{\partial_N\Omega_m} \boldsymbol{t}^\star_m \cdot \delta\boldsymbol{u}_m \mathrm{d}\Gamma, \quad \forall\delta\mathbf{u}_m \in$$

$$\int_{\Omega_M} (\boldsymbol{\sigma}_M \cdot \boldsymbol{\varepsilon}(\delta\boldsymbol{u}_M) + \boldsymbol{\mu}_M \cdot \boldsymbol{\chi}(\delta\boldsymbol{u}_M))\mathrm{d}\Omega - \int_{\Gamma_{\mathrm{int}}} \boldsymbol{\lambda} \; \delta\mathbf{u}_M \mathrm{d}\Gamma$$

$$= \int_{\partial_N\Omega_M} \boldsymbol{t}^\star_M \cdot \delta\boldsymbol{u}_M \mathrm{d}\Gamma, \quad \forall\delta\mathbf{u}_M \in V_M$$

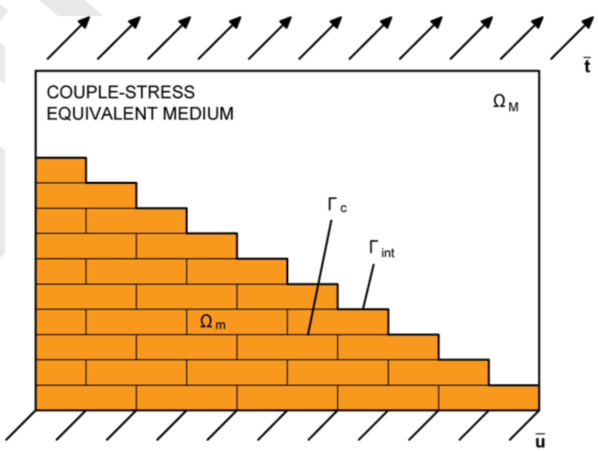

FIG. 4: Multiscale/multidomain scheme for a masonry structure

$$\int_{\Gamma_{\text{int}}} \delta\boldsymbol{\lambda} \cdot (\boldsymbol{u}_m - \boldsymbol{u}_M)\, \mathrm{d}\Gamma = 0, \qquad \forall \delta\boldsymbol{\lambda} \in \Lambda. \tag{9}$$

The first two equations represent the equilibrium conditions of the portions Ω_m and Ω_M, respectively, while the third equation is the kinematic compatibility condition at the micro/macro interface. In Eq. (9), $\boldsymbol{\lambda}$ is the Lagrange multiplier field, belonging to a properly defined space Λ, whereas δ denotes the usual variational operator. Moreover, V_m and V_M are the spaces of test functions for $\boldsymbol{u}_m$ and $\boldsymbol{u}_M$, respectively. Furthermore, $\boldsymbol{t}_c$ are the cohesive tractions; $\boldsymbol{t}_m^\star$ and $\boldsymbol{t}_M^\star$ are the boundary tractions imposed on the microscopic and macroscopic portions of $\partial_N\Omega$, respectively. Finally, the double square brackets denote the jump of the enclosed quantity across the cohesive boundary Γ_c. It is worth noting that the curvature $\boldsymbol{\chi}$ and the couple-stress $\boldsymbol{\mu}_M$ tensors are defined only over Ω_M.

The required constitutive law must then be computed from the couple-stress based homogenization explained in Section 2.2.1. Furthermore, as shape functions for the Lagrange multiplier field $\boldsymbol{\lambda}$, Dirac functions have been chosen, so that the displacement continuity along Γ_{int} is enforced pointwise, resulting in a strong coupling between micro- and macrodisplacements [see Lloberas-Valls et al. (2012) for additional details].

The above formulation takes into account a mixed-mode cohesive law, $\boldsymbol{t}_c\left([\![\boldsymbol{u}_m]\!], d\right)$, of the intrinsic type, which involves a scalar state variable d indicating the current damage level and ranging from 0 (for perfect interfaces) to 1 (for completely failed interfaces). Such an irreversible cohesive law has been properly calibrated to be perfectly equivalent to that used within the FEM/DEM approach. In the elastic range, the interface constitutive law can be written as $\boldsymbol{t}_c = \boldsymbol{K}[[\boldsymbol{u}_m]] = diag\{K_n, K_s\}$, K_n and K_s being the normal and tangential stiffness components, respectively. Moreover, the inelastic parameters of the interface elements are the tensile, f_t, and shear, c, cohesive strengths; the friction angle, φ; and both the mode I, G_{Ic}, and mode II, G_{IIc}, fracture energies. The elastic stiffness components are not regarded here as penalty values, but possess a precise physical meaning, being determined from the elastic properties of masonry constituents and the joint thickness, according to Lourenço and Rots (1997).

2.2.3 Adaptive Model Refinement Technique

The previously described multiscale/multidomain formulation is used in combination with an adaptive framework, which is mainly responsible for the overall efficiency of the proposed multiscale method.

The model refinement strategy adopted here is based on the fulfillment of a properly defined zooming-in criterion. According to this criterion, the zone of interest, directly influenced by fracture phenomena, is continuously updated to push the micro/macro interface away from the already damaged zones.

Similarly to the well-known adaptive mesh refinement, the model refinement strategy consists in replacing the homogenized coarse-scale model by the heterogeneous fine-scale model in the spatial regions where homogenization ceases to be valid because of the occurrence of strain localization phenomena. Indeed, in this situation both macrohomogeneity and perfect periodicity conditions are no longer valid. The macro-to-micro switching operations are based on the detection of subdomains where the first nonlinearity is predicted. To this end, a suitable first failure surface defined on the macroscopic strain space is derived for the given repeating cell. Operationally, this limit locus is constructed by points after solving a number of (linear) microscopic problems by varying the prescribed macrostrain path direction, under plane-stress conditions. All the related numerical details are explained in Leonetti et al. (2018).

At the beginning of the multiscale analysis, the undamaged masonry is described as a homogenized material and the computational domain is decomposed into a finite number of coarse-level elements (here referred to as macro-elements) arranged in a rectangular grid. At a given load level, a global nonlinear boundary value problem is solved, and all the macro-elements for which the above-mentioned switching criterion is fulfilled are flagged as critical and replaced by finely meshed subdomains representing the underlying masonry microstructure with damageable mortar joints. The latter step is repeated at fixed external load according to the predictor-corrector algorithm shown in Fig. 5, until the macro-strain level in all the remaining macro-elements is kept below its critical value. After that, the external load is further incremented and the overall loop is repeated within the adopted (displacement-based) continuation strategy until the final collapse of the structure.

It is worth noting that the extension of coarse- and fine-level meshed subdomains is expected to vary during the multiscale simulation, and therefore the assumed scalar damage variable must be projected from the previous to the current mesh.

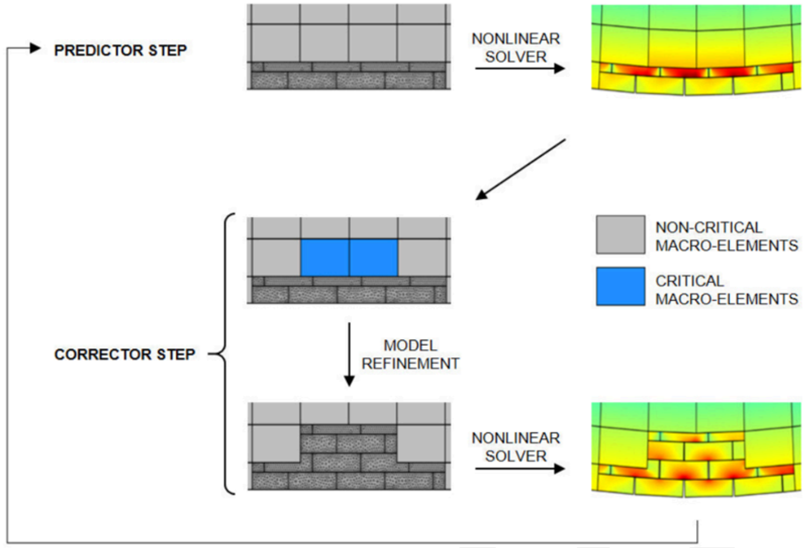

FIG. 5: Predictor-corrector algorithm for the adaptive multiscale/multidomain model refinement

The multiscale/multidomain model presented here has been implemented within the commercial finite element software COMSOL Multiphysics®, used in combination with related product LiveLink® for MATLAB. In fact, an ad hoc developed MATLAB code has been linked to the main computational environment to implement the previously described predictor-corrector algorithm for model refinement within the built-in continuation solver.

3. NUMERICAL SIMULATIONS

A series of numerical analyses on masonry walls has been performed with the purpose of comparing the two approaches, evaluating their advantages and limits of applicability. The main differences between the two models are synthesized in Table 1.

The two models adopt the same mechanical parameters at the microscale, in particular, as stated in Section 2, the same cohesive law adopted in FEM/DEM, developed by Lisjak et al. (2013) and described in Section 2.1, has been introduced in the multiscale/multidomain model. At the macroscale, the multiscale model requires the macroscopic elastic moduli, which characteriz[illegible] the constitutive ~~to the constitutive~~ l[illegible]ing from homogenization.

A first analysis has been carried out with the purpose of calibrati[illegible]e parameters to be adopted in the two models. With this aim, a benchmark example (Raijmakers and Vermeltfoort, 1992), widely studied by several authors, has been considered. It consists of a square masonry panel with a running bond arrangement of bricks, tested in the laboratory by applying increasing horizontal displacements on the top of the wall until the collapse. Then, panels of different dimensions and shape, made of bricks of constant size, shape, and arrangement, have been considered for the comparison between the two models. The panels analyzed are reported in Fig. 6.

TABLE 1: Comparison between the main features of the two models

FEM/DEM (a)	Multiscale/Multidomain (b)
Discrete Model	Continuum Model
Molecular Dynamic	Quasi-static FEM Analysis
Finite-Displacement Contact	Cohesive Contact
Large Displacements	Small Displacements

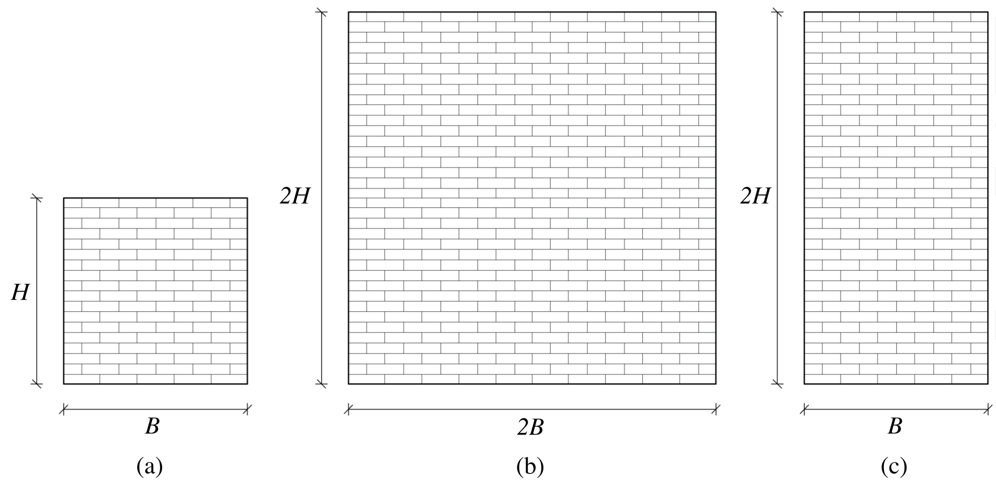

FIG. 6: Panels: benchmark panel (a); square panel double size (b); rectangular panel (c)

3.1 Calibration of the Parameters: Benchmark Panel

A first analysis is performed on a square panel with a side of $H = B =$ 1.10 m, made of 18 layers consisting of five bricks, each one arranged in a running bond pattern, as shown in Fig. 6(a). The dimensions of the bricks are (210 $\times$ 52) mm^2 with a width of 100 mm. Both vertical and horizontal mortar joints have a thickness of 10 mm. The analysis reproduces the tests made by Raijmakers and Vermeltfoort (1992) and afterwards studied by several authors. In particular, the works of Lourenço (1996) and Lourenço and Rots (1997) have been considered as a reference for the mechanical parameter adopted for constituents and joints, summarized in Tables 2 and 3.

The panel is fixed at the base and subjected to an increasing horizontal displacement on the top, as shown in Fig. 7. Vertical displacements are fixed on the top in order to reproduce the test setup, in which horizontal displacements were applied through a loaded steel beam along their upper edge. Self-weight is taken into account.

The results of the analysis are reported in Fig. 8, where the reaction at the base is plotted versus the displacement applied on the top. The dashed line is referred to the solution provided by FEM/DEM (a) while the solid line is referred to the solution of the multiscale/multidomain model (b). The kinematic mechanisms of collapse are shown for different values of displacements on top, identified on the curves by red circles. The two mechanisms marked with (a) are referred to the FEM/DEM model: figures on the left provide the FE mesh, in blue; the crack elements activated, in black; and the cracked joints, in red; the figures on the right, for the sake of clearness, show only the cracked joints. The two mechanisms marked with (b) are referred to the multiscale/multidomain model: in gray are the areas in which the macro-elements are replaced by finely meshed subdomains, representing the underlying masonry microstructure with damageable mortar joints; in red are the cracked joints. In Fig. 8 the mechanisms in the nonlinear phase (at a value of 0.001 m of the horizontal displacement at the top) and at the collapse (at 0.002 m) are reported.

TABLE 2: Elastic properties of masonry components

Component	**E [MPa]**	**ν**
Bricks	16700	0.15
Mortar	782	0.14

TABLE 3: Inelastic parameters of mortar joints

f_t **[MPa]**	c **[MPa]**	f_c **[MPa]**	ϕ **[deg]**	GI_c **[N/mm]**	GII_c **[N/mm]**
0.25	0.35	10.5	36.9	0.018	0.125

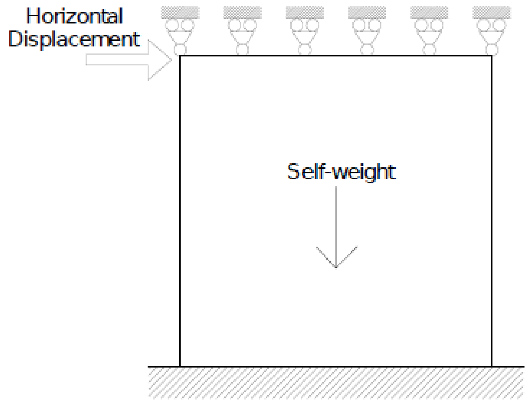

FIG. 7: Boundary conditions and load applied on the square panel

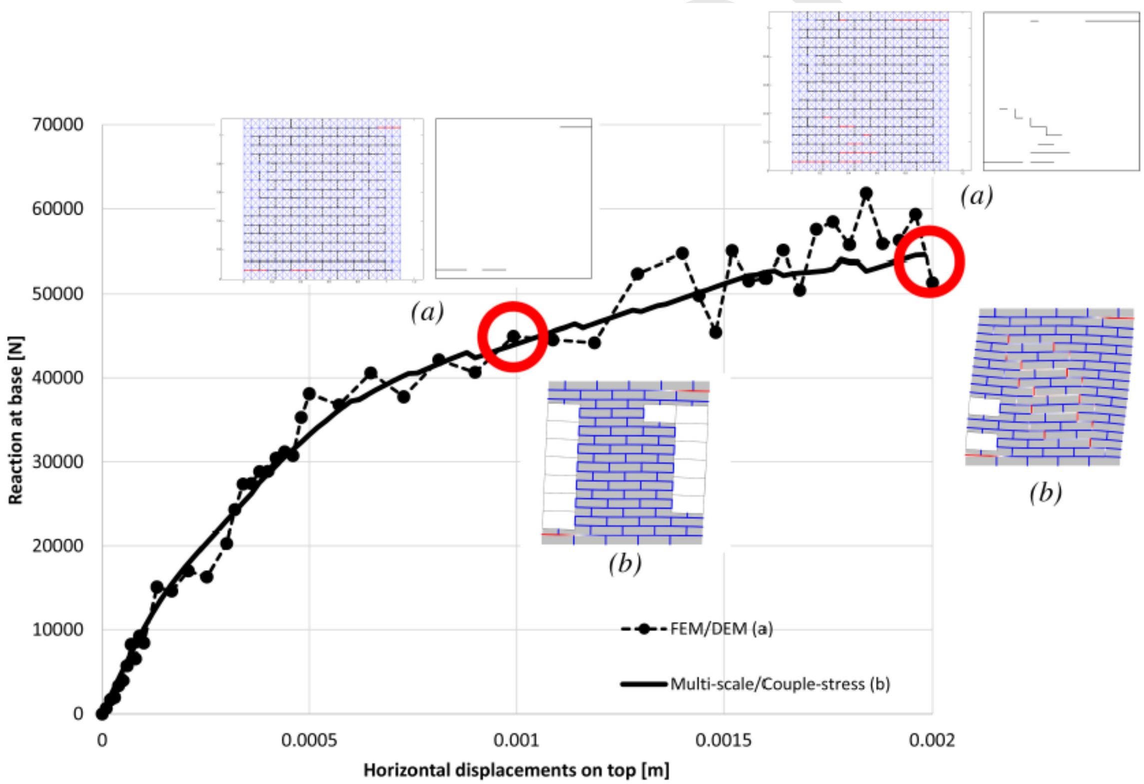

FIG. 8: Square panel $H = B = 1.10$ m: Horizontal displacements on top vs reaction at base. Collapse mechanisms at 1 mm and at 2 mm obtained by FEM/DEM model (a) and multiscale/multidomain model (b).

The results of the two approaches are in good agreement; however, some differences between them may be pointed out. The FEM/DEM model is a discrete model, in which the equation of motion is solved by an explicit second-order finite-difference time integration scheme, leading to a discontinuous nonsmooth curve. The multi-scale/multidomain model, instead, is a continuous model adopted to perform a quasistatic FEM analysis, resulting in a continuous smooth curve. However, the two curves exhibit a very similar trend and in both cases the value of collapse load, measured as reaction at the base, is around 50 kN, also comparable is the collapse displacement (0.002 m).

The collapse mechanisms provided by the two models are similar and in good agreement with the ones observed in the tests (Raijmakers and Vermeltfoort, 1992) and in the literature (Lourenço and Rots, 1997). At the beginning of the nonlinear phase, 0.001 m of displacement, failure occurs in the upper and lower horizontal joints, close to the corners. Subsequently, at the collapse, 0.002 m of displacement, cracking spreads out in the panel along the main diagonal, which is a typical shear damage pattern. However, some slight differences may be noticed: while in FEM/DEM the damage affects both vertical and horizontal joints, leading to a mechanism in which sliding occurs, in the multiscale/multidomain model cracking along the diagonal appears only in the vertical joints, with the predominant rotation mechanism. The comparison between the results allows us to consider reliable the adopted parameters for the two models, and in particular the cohesive law implemented in both models (Lisjak et al., 2013).

3.2 Sensitivity to Shape/Scale Effects

Two cases of study have been analyzed for evaluating the sensitivity of the two models with respect to the shape and scale effects, by varying dimensions of the panel and keeping constant size and arrangement of bricks and boundary conditions (Fig. 6). The first case study is a square panel with side $H = B = 2.20$ m (double size with respect to the benchmark panel), made of 36 layers consisting of 10 bricks, each one arranged in a running bond pattern [Fig. 6(b)]. The second case study is a rectangular panel with the same base of the benchmark panel, $B = 1.10$ m, but double height, $H = 2B = 2.20$ m, made of 36 layers consisting of five bricks, each one arranged in a running bond pattern [Fig. 6(c)]. The results of all the analyses performed are reproduced according to Fig. 8 with the same information provided and the same symbols adopted.

3.2.1 Case 1—Double Size Square Panel (H = B = 2.20 m)

The results of the analysis are reported in Fig. 9. The results of the two approaches are in good agreement, the value of reaction at the base of the collapse is around 90 kN, corresponding to 0.0026 m of displacements on the top; however, some differences between them may be highlighted. The differences between the two curves are greater than in the benchmark case, in particular close to the collapse. The multiscale/multidomain model shows an increasing trend of the curve also after 0.0015 m of displacements, where the behavior becomes completely nonlinear. The reaction at the base increases until the convergence cannot be reached at the collapse of the panel. Instead, in the same steps, FEM/DEM provides more oscillations in the results, due to the activation of the crack elements, to the breaking of joints, to the changes in the contacts between the elements and to the dynamic field of analysis. This is typical in the case of an explicit solution of the equation of motion. Moreover, it may be observed that the discontinuous curve of FEM/DEM shows the beginning of softening.

The mechanisms of collapse are similar to the benchmark case. In FEM/DEM model (a), a mixed shear-sliding mechanism develops, with two diagonal cracks occurring in the areas above and below the cracks in the horizontal joints close to the corners. In the multiscale/multidomain model (b) the nonlinear behavior affects the diagonal area of the panel—in which the microstructure is activated—however, cracking occurs only in the horizontal joints close to the corners.

3.2.2 Case 2—Rectangular Panel (H = 2B = 2.20 m)

The differences between the two models are related to their specific characteristics and are emphasized by the results of the analysis performed on a rectangular panel. The results of the analysis are reported in Figs. 10 and 11, and provide the same information as the previous figures. In Fig. 10 attention is focused on the linear phase, while in Fig. 11 the results of analysis up to the collapse are provided.

In Fig. 10 the results in terms of the reaction at the base versus displacements on the top provided by the two models are very similar. At a displacement of 0.0036 m the panel is still in the linear phase; the reaction is around 50 kN. The reaction increases almost linearly, except for a slight change of slope due to the opening of the cracks close to the corners. However, looking at the mechanisms of collapse, it is possible to notice that in the multiscale/multidomain only the corners are affected by nonlinearity, while in FEM/DEM crack elements activate in the whole diagonal.

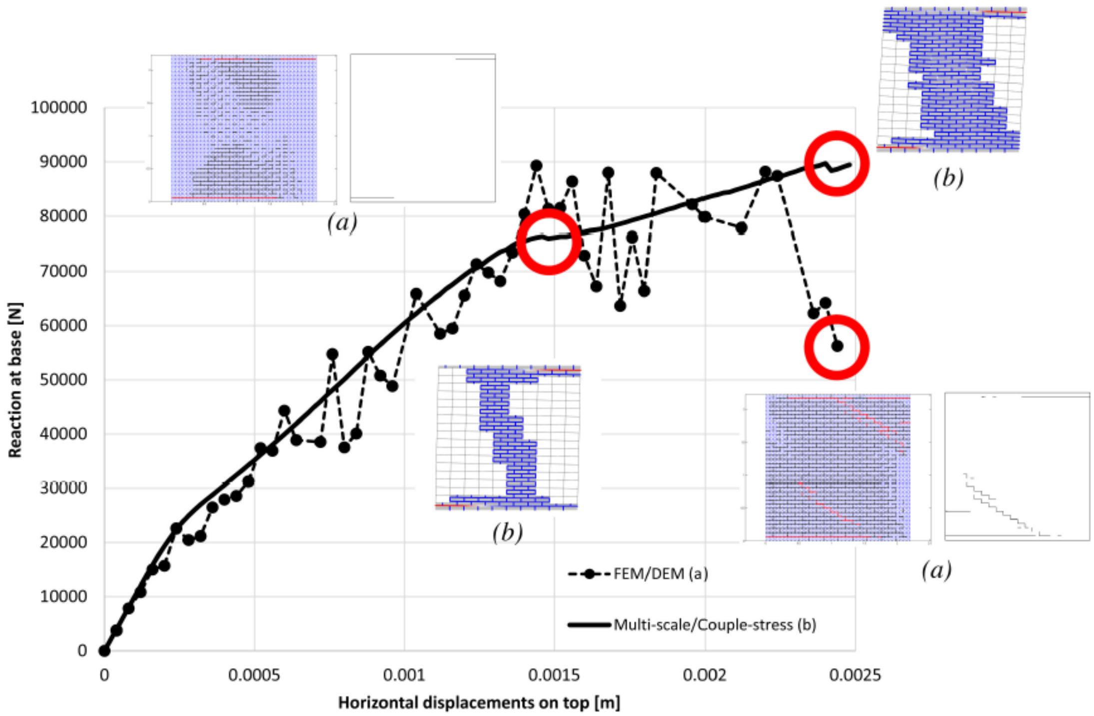

FIG. 9: Double size square panel ($H = B = 2.20$ m): horizontal displacements on top vs reaction at base. Collapse mechanisms at 1 and 2.6 mm obtained by FEM/DEM model (a) and multiscale/multidomain model (b).

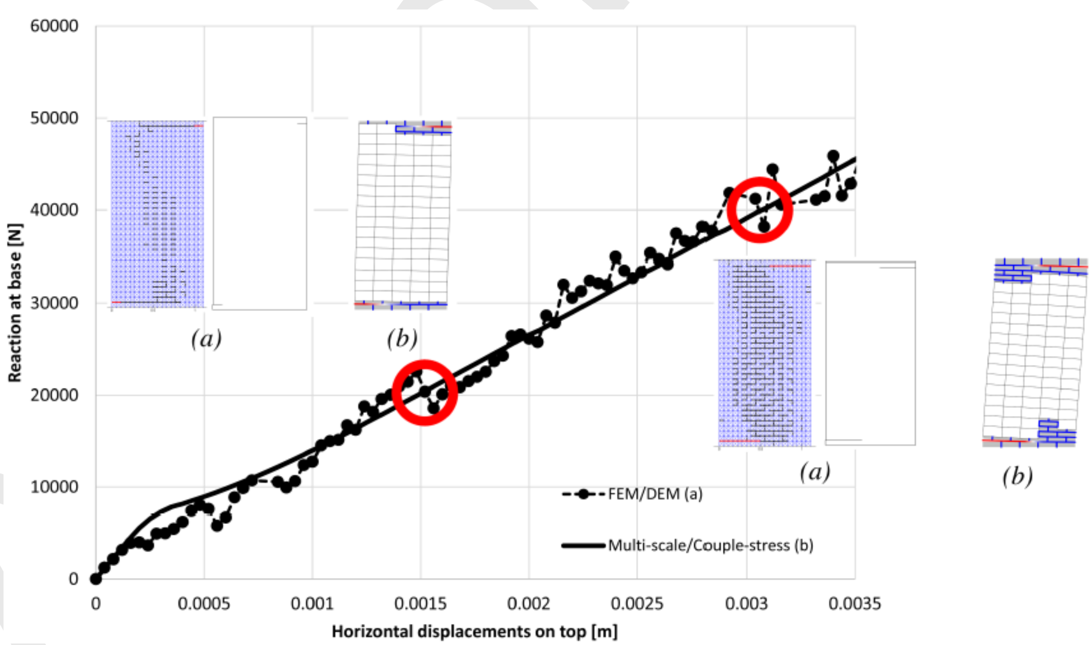

FIG. 10: Rectangular panel $H = 2B = 2.20$ m: Horizontal displacements on top vs reaction at base. Collapse mechanisms in the elastic phase obtained by FEM/DEM model (a) and multiscale/multidomain model (b).

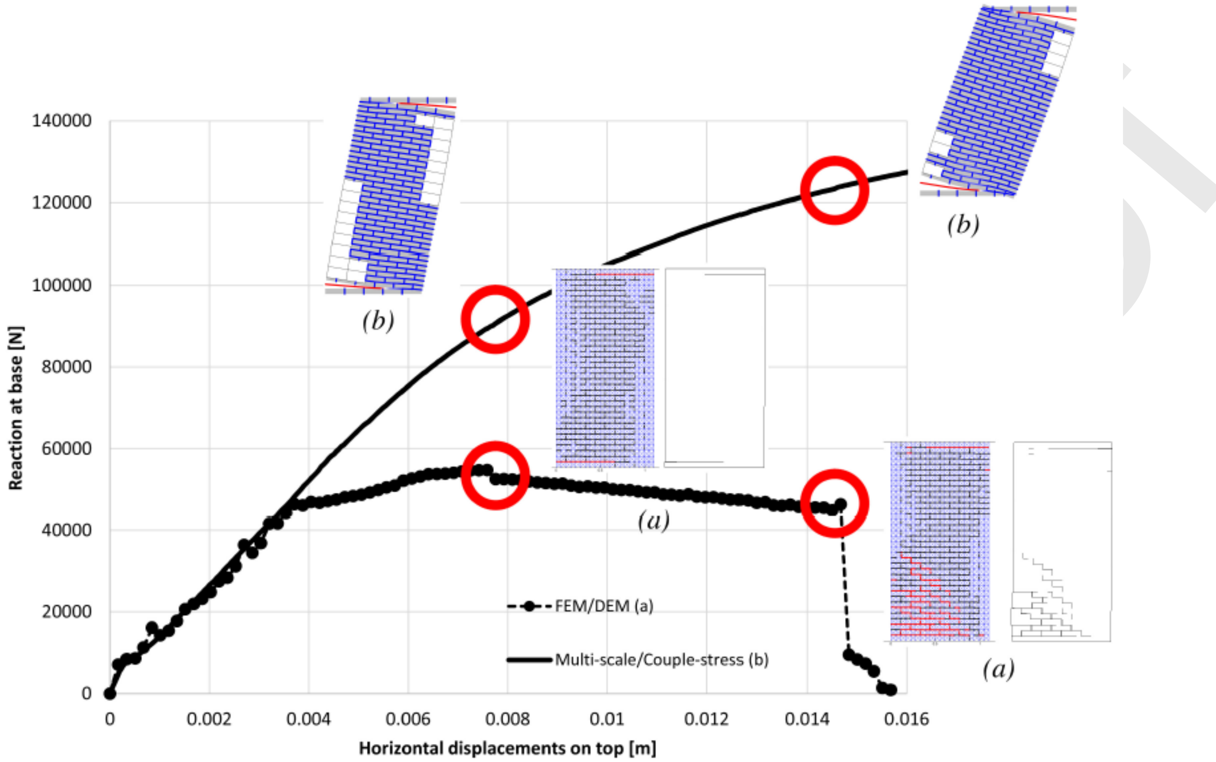

FIG. 11: Rectangular panel $H = 2B = 2.20$ m: Horizontal displacements on top vs reaction at base. Collapse mechanisms in the nonlinear phase obtained by FEM/DEM model (a) and multiscale/multidomain model (b).

As the analysis goes on, until 0.015 m of displacement on top, the two models provide completely different results as shown in Fig. 11. The curve of multiscale/multidomain model constantly increases, while in FEM/DEM the activation of the mixed shear-sliding mechanisms lead to softening. The distance between the two curves grows as the displacements on top increase, and it is more evident when large displacements are reached. In the multiscale/multidomain model, the field of analysis is in fact limited to the small displacements hypothesis. Even if the microstructure is activated where joints break and the panel behaves nonlinearly, the wall remains a deformable continuous heterogeneous panel. Instead, as FEM/DEM is a fully discrete model, once the cracks determine the separation of portions of the panel, these portions move separately from each other.

This aspect is more clear looking at the mechanisms of collapse: even if the nonlinearity spreads out along the panel, in the multiscale/multidomain model (b) cracking still affects [illegible] two corners, while in FEM/DEM (a), cracking affects the whole lower area of the panel. This difference be[illegible] the two models is made more evident by the prevented rotation of the upper side of the panel. As is already known, in the case of slender masonry panels with bricks of this aspect ratio the global mechanism of collapse is overturning (Baggio and Trovalusci, 1993; Baraldi et al., 2018b; Masiani et al., 1995), but in this case this mechanism is not allowed.

4. CONCLUSIONS

In this work, two different approaches to the modeling of masonry materials have been compared: (a) a micromechanic discrete model, based on a mixed finite/discrete element method, namely, FEM/DEM, and (b) a multiscale continuous model, based on a couple-stress homogenization and on a multidomain decomposition. The aim of this work is the validation of the latter model by the comparison with the former one, here regarded as a benchmark. The main features of the two models in comparison may be synthesized: (i) discrete versus continuum model; (ii) molecular dynamics versus quasistatic FEM analysis; (iii) finite displacements contact versus cohesive contact; (iv) low versus high computational efficiency.

The two models have been compared by analyzing the behavior of masonry panels subjected to incremental horizontal displacements applied on top. With this aim, a square panel has been modeled with reference to the test carried out by Raijmakers and Vermeltfoort (1992), that is a benchmark example widely studied by several authors. This comparison allowed us to calibrate the mechanical parameters adopted in the two models. Subsequently, two different cases of study have been considered: a square panel with double size with respect to the benchmark panel and a rectangular panel, keeping fixed in both cases size and arrangement of bricks. The purpose was the evaluation of the sensitivity of the two models to shape and scale effects.

Some final remarks may be pointed out:

- both the FEM/DEM model and the multiscale/multidomain model model are able to provide reliable results for the in-plane failure analysis of masonry panels;
- mechanical properties at macroscale obtained by couple-stress homogenization allow taking into account the characteristics of masonry material, in particular the size of heterogeneity;
- the FEM/DEM model allows performing analysis under the large displacements hypothesis, but with high computational costs that limit its application to small structures;
- the application of the present multiscale/multidomain model is limited to the small displacements hypothesis;
- the multiscale model conjugates accuracy in results with high computational efficiency, leading to the possibility of analyzing large size real buildings;
- the adopted multiscale/multidomain model generally leads to an overestimation of both strength and stiffness of masonry with respect to the FEM/DEM approach. In particular, the appearance of the softening branch is delayed, due to the small displacement assumption, being too restrictive in real-life situations, where large-scale contact plays a fundamental role in stress redistribution.

ACKNOWLEDGMENTS

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