Use of the cantilever beam vibration method for determining the elastic properties of maritime pine cross-laminated panels

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Abstract

This paper presents a study aimed to assess the modulus of elasticity $E_0$ and the rolling shear modulus $G_{\theta 0}$ of three-layer Maritime Pine Cross Laminated Timber (CLT) panels. The proposed methodology is based on a vibration test carried out on panels in cantilever configuration and on the results of a sensitivity study conducted via FE model analyses, which highlights a straightforward relationship between the first two vertical natural frequencies and the aforementioned elastic properties of the panels. The procedure has been developed and applied to a specific configuration of CLT panels made of Sardinian Maritime Pine (Pinus Pinaster). Nevertheless, the same approach could be easily extended to any cantilever three layers CLT panel with different dimensions, providing a new sensitivity study is carried out. The results suggest that the proposed methodology can be effectively used as a dynamic identification process for quality control in industrial production chains, where the role of non-destructive controls is becoming increasingly important.

Keywords: Laminated Timber panels, Free vibration methods, Modulus of elasticity, Rolling shear stiffness.
1. Introduction

Cross Laminated Timber (CLT) is a promising and relatively new wood-based structural product which is increasingly used on the building market for the several advantages it can offers [1]. CLT panels are wooden structural products consisting of laminations of finger-jointed boards, arranged crosswise and glued together according to different lay-ups. This cross-lamination process brings several structural advantages including: stability of long panels with respect to swelling and shrinkage, reduced scatter of original lumber properties, reduced influence of anatomic defects, two-way behavior in slabs, and relatively high transverse and in-plane stiffness [2].

CLT technology is particularly suitable for modular buildings, since it can be employed in horizontal elements (floors and roofs) and vertical load-bearing systems (wall panels), according to different structural and finishing quality design [3]. CLT panels allow for large spans and good behavior with respect to fire [4] and earthquake resistance [5,6]; moreover, the ease of installation of modular systems ensures the effectiveness of the construction process [7].

On the other hand, the use of CLT technology requires careful consideration of serviceability design, including deflection control [8] due to low bending stiffness, high creep and shrinkage deformations [9-11]. Furthermore, vibration control may be an issue, as well as possible low acoustic insulation properties [12] due to low mass of wood. Therefore, additional timber layers have sometimes to be used to increase stiffness and acoustic separation [13-15]. The assessment of the elastic mechanical properties of CLT panels thus represents a crucial issue for a proper design of CLT buildings.

The linear model [16] is generally used in CLT design as the plasticization of timber in compression is usually negligible. However, in the case of reinforced elements more complex non-
linear three-dimensional models can be used [17]. Furthermore, new studies have focused on modeling the flexural behavior of hybrid timber-steel and timber-concrete beams through sophisticated non-linear models [18,19]. Since the goal of this paper is the evaluation of global elastic properties of CLT panels, a linear constitutive model has been considered throughout the numerical investigation.

The mechanical behavior of CLT is rather complex due to the intrinsic anisotropy of timber, its anatomic defects and the cross-wise arrangement of panel layers. Significant variations in mechanical properties can be noticed in CLT depending on the layers orientation [20,21]. This fact can be ascribed to the dramatic difference between the Young’s modulus of the boards parallel ($E_0$) and perpendicular ($E_{90}$) to the grain direction, and the shear moduli of the boards in the parallel (longitudinal shear modulus $G_0$) and orthogonal planes (rolling shear modulus $G_{90}$), the latter being usually very small. This issue can cause a mechanical complexity in CLT panels which may be not effectively tackled by using the standard beam theory [16,22].

In this respect, several analytical approaches have been presented in literature for the evaluation of an effective or equivalent bending stiffness depending on timber layers elastic properties, thickness and grain orientation [16,23,24]. The most referenced ones are: the “Mechanically jointed Beams Theory”, or $\gamma$-method (GM), proposed in the Annex B of Eurocode 5 [24], the “Composite Theory”, or k-method (KM) [25] and the “Shear Analogy Method”(SAM) [26]. Recently, Thiel et al. proposed a Timoshenko beam approaches a simplified alternative [23].

Several authors proposed innovative approaches in order to take the influence of shear into account for composite elements. As an example, Kaci et al. [27] developed an exact analytical solution for the analysis of the post-buckling non-linear response of simply supported deformable
symmetric composite beams. Kada et al. presented in [28] the results of a static flexure analysis of laminated composite plates by utilizing a higher order shear deformation theory in which the stretching effect is incorporated. Moreover, the vibrations and bending responses of carbon nanotube-reinforced composite plates resting on the Pasternak elastic foundation are analyzed in [29]. Belabed et al. [30] and Zidi et al. [31] discussed new hyperbolic plate theory for the free vibration analysis of functionally graded material (FGM) sandwich plates. Zine et al. [32] and Chikh [33] performed the bending and free vibration analysis of multilayered plates and shells, and a thermal buckling analysis of cross-ply laminated composite plates, respectively, by utilizing a new higher order shear deformation theory (HSDT). They also compared the results of their approach against existing theories for investigating the static and dynamic response of isotropic and multilayered composite shell and plate structures and the thermal buckling behavior of laminated composite plates. Moreover advanced theories which take into account of the shear deformation effects on beams and plates are discussed in the recent works [34-36].

The experimental determination of wood products elastic properties can be performed using different methodologies. Either destructive or non-destructive techniques can be used for the determination of elastic properties of CLT panels [37]. However, some of these properties, like the aforementioned rolling shear modulus $G_{90}$, are currently determined via empirical correlations with other properties such as the longitudinal elastic modulus $E_0$, since their direct evaluation requires special tests and measurement equipment [37]. The indirect evaluation of elastic properties of CLT elements is therefore important and represents one of the goals of this paper.

In recent years researchers have focused the attention on non-destructive techniques [38-40] and several dynamic testing methods have been proposed for the determination of both elastic and shear moduli of wood composite panels. Dynamic testing has been proved to be an effective
procedure for performing elastic identification of CLT panels [41-43] and cantilever beam vibration methods have been listed as reliable approaches in [42,43]. Nevertheless, these approaches are usually based on a Euler-Bernoulli beam analysis [41], which can lead, as illustrated in [41,44], to systematic errors in evaluating the CLT panel elastic modulus \( E_0 \), and to disregard the role of shear deformations. In this respect, the determination of the rolling shear modulus is commonly based on static tests, in accordance with EN 408 [45]. However, as previously mentioned, a rather complex experimental set-up is required to carry out the tests. Furthermore, when applied to CLT panels, these approaches may lead to localized measurements usually not representative of the global elastic behavior of the specimens due to the intrinsic heterogeneity of the CLT panels [20].

This paper illustrates an innovative dynamic identification procedure, developed for three-layer CLT panels, and based on the results of sensitivity analyses performed by means of a 3D solid Finite Element (FE) model. The proposed procedure enables the determination of the main elastic properties of laminations forming the CLT specimen, including the dynamic modulus of elasticity \( E_0 \) and the dynamic rolling shear modulus \( G_{90} \).

The paper is organized as follows: Section 2 illustrates the theoretical background regarding the beam theory and the FE model approach; Section 3 describes the tested specimens, the related sensitivity analyses and the dynamic identification procedure; numerical results are discussed in Section 4; conclusions are summarized in Section 5.

2. Theoretical background

The proposed methodology is based on a set of sensitivity analyses, performed using the FE model illustrated in Section 2.2, aimed at the indirect determination of the elastic properties. Nevertheless, as a comparison tool, a free vibration approach based on Euler-Bernoulli beam
theory is discussed in Section 2.1. The comparison between classic beam theories and FE models can be a useful mean for understanding the limits of the Euler-Bernoulli beam approach [16] often employed in dynamic identification techniques [41,43] and in structural design of timber construction.

2.1 Free vibration Euler Bernoulli beam theory

The theoretical relationship between the dynamic modulus of elasticity (MOE) parallel to the grain orientation of face boards and the first natural vibration frequency $f$ of the panel can be assessed using the cantilever beam free vibration theory, provided a proper transformed moment of inertia is employed; for this configuration the dynamic MOE can be calculated through the following formula [46]:

$$E_0 = \left(2\pi f\right)^2 \frac{M}{L l^4}$$  \hspace{1cm} (1)

where $E_0$ (Pa) is the dynamic MOE of the panel parallel to the grain orientation of face boards, $f$ (Hz) is the panel first natural vibration frequency, $M$ (kgf) is the weight of the specimen, $L$ (m) is the length of the panel, $I$ (m$^4$) is the cross-section effective moment of inertia with respect to the centroid, while $l$ (m) denotes the effective span of the cantilever [16,43]. The panel MOE can therefore be easily evaluated through an experimental determination of the first natural frequency. Regarding the evaluation of the effective bending stiffness, since the CLT panel cross-section consists of differently (usually 0°/90°) oriented and mutually glued layers, the following issues need to be considered. Firstly, the 90° oriented layers offer a minimum contribution to the panel longitudinal bending stiffness due to the low value of $E_{90}$ compared to $E_0$ (usually a ratio $E_{90}/E_0=1/30$ [24] is assumed for softwood). Secondly, for low slenderness elements, bending deformations are considerably affected by the shear stiffness and in particular, by the rolling shear modulus of
inner 90° oriented layers, which is conventionally assumed as 1/10 of the longitudinal shear modulus $G_0$ [24]. Analytical models for the evaluation of an actual or equivalent bending stiffness depending on layers elastic properties, thickness and grain orientation have already been mentioned in Section 1. Among them, only GM and SAM can handle shear deformations, while KM only accounts for bending stiffness and thus, it should be used only for high span-to-depth ($L/H$) panel ratios. In the following, Eq. (1) is employed with an effective moment of inertia evaluated using the transformed section method, assuming $E_{90}/E_0=1/30$, and neglecting the influence of the shear deformation.

2.2 Finite Element model

This section describes the adopted FE model, which represents an improved tool for an accurate dynamic modeling of the CLT panels, directly depending on the elastic properties of the boards forming the panel layers. For this purpose, a 3D model has been implemented in Abaqus 2017 software package and a numerical investigation has been carried out. The model is characterized by a structured mesh of 80640 isoparametric brick elements (7.5x5x5 mm), selected after a convergence test with subsequent mesh refining, until a variation threshold of the higher frequency smaller than 1% was achieved. Despite the small grooves in each lamination boards and the vertical gaps in the inner layer introduced during production to prevent excessive cupping and cracking, full continuity between the layers has been assumed in the FE mesh. As shown in a previous work [2], such grooves and gaps do not considerably affect the overall panel behavior. Similarly, the presence of the glue film among timber layers has been neglected, since it does not affect the panel deformations.

As a result of its anatomy, timber shows an anisotropic constitutive behavior, usually regarded as orthotropic, which is based on a grain-oriented orthogonal system for each timber layer, as if
growth rings were flattened. This assumption introduces a certain conventionality in the modeling.

Therefore, only the following five elastic parameters have been accounted for in the constitutive model:

\[ E_l = E_0, \quad E_t = E_{90}, \quad G_{lt} = G_{0}, \quad G_{lr} = G_{90}, \quad v_l = v_t = v_r = v \]  

where the subscripts \( l \), \( t \) and \( r \) signify the longitudinal, the tangential and the radial directions respectively. Given the above assumptions, the layer orientations in the panel layup have been modeled by rotating the lamination local material orientations according to the 0°/90° panel directions as shown in Fig. 1.

![Fig. 1. Schematic of the three layers CLT panel.](image)

The cantilever configuration has then been modeled by imposing zero displacement boundary conditions at the fixed edge of the specimen, as shown in Fig. 2b, in accordance with the experimental set-up.

![image](image)
Fig. 2. (a) Cross section of 60PF specimens (60×240×1280 mm), (b) schematic of the experimental set-up, (c) FEM model of three layers cantilever CLT specimen.

3. Material and methods

3.1 Specimens and testing program

The investigated CLT panels are composed by three layers of solid timber finger-jointed boards, previously graded, crosswise packed (0°-90°-0°) and glued together. For this study, a total number of 14 panels 240 mm wide, 60 mm thick and 1280 mm long, made of Sardinian Maritime Pine (Pinus Pinaster) has been used. The panels (60-PF series) are made of 20 mm thick boards, with a global panel thickness of 60 mm. The cross-section geometry of the 60PF series panels is shown in Fig.2a. Maritime Pine boards have been previous classified, according to EN 338 [47] strength classes, as C16 and C14 for the outer and the inner layers respectively. Classification is based on the visual grading rule recently developed by the Department of Civil, Environmental Engineering and Architecture (DICAAR) of the University of Cagliari [48,49], in cooperation with
the CNR-IVALSA of Florence. Table 1 lists the main properties of the tested panels. The specimen has been rigidly clamped for a length of 230 mm in cantilever position with a free span of 1050 mm as illustrated in Fig. 2.

Table 1

Specimen dimensions, weight and density.

<table>
<thead>
<tr>
<th>Specimen code</th>
<th>B (Breadth) [mm]</th>
<th>H (Depth) [mm]</th>
<th>L (Length) [mm]</th>
<th>Weight [g]</th>
<th>Mean Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60PF 8</td>
<td>2460.50</td>
<td>62.09</td>
<td>1280.00</td>
<td>8759.50</td>
<td>4480.12</td>
</tr>
<tr>
<td>60PF 9</td>
<td>2427.00</td>
<td>61.87</td>
<td>1279.00</td>
<td>9048.00</td>
<td>4711.85</td>
</tr>
<tr>
<td>60PF 10</td>
<td>2430.00</td>
<td>61.80</td>
<td>1280.00</td>
<td>9131.50</td>
<td>4750.48</td>
</tr>
<tr>
<td>60PF 11</td>
<td>2437.00</td>
<td>62.44</td>
<td>1280.00</td>
<td>9503.50</td>
<td>4880.20</td>
</tr>
<tr>
<td>60PF 12</td>
<td>2435.50</td>
<td>61.88</td>
<td>1279.00</td>
<td>9634.00</td>
<td>4999.31</td>
</tr>
<tr>
<td>60PF 13</td>
<td>2450.00</td>
<td>61.76</td>
<td>1280.00</td>
<td>9018.00</td>
<td>4656.15</td>
</tr>
<tr>
<td>60PF 14</td>
<td>2467.00</td>
<td>61.83</td>
<td>1280.00</td>
<td>8997.00</td>
<td>4608.94</td>
</tr>
<tr>
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<td>61.85</td>
<td>1279.00</td>
<td>9502.50</td>
<td>5010.79</td>
</tr>
<tr>
<td>60PF 16</td>
<td>2457.50</td>
<td>61.54</td>
<td>1280.00</td>
<td>8812.00</td>
<td>4553.91</td>
</tr>
<tr>
<td>60PF 17</td>
<td>2460.00</td>
<td>61.59</td>
<td>1280.00</td>
<td>9736.50</td>
<td>5020.24</td>
</tr>
<tr>
<td>60PF 18</td>
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<td>61.77</td>
<td>1280.00</td>
<td>8645.00</td>
<td>4499.33</td>
</tr>
<tr>
<td>60PF 19</td>
<td>2430.00</td>
<td>61.96</td>
<td>1280.00</td>
<td>9323.00</td>
<td>4837.58</td>
</tr>
<tr>
<td>60PF 20</td>
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<td>9504.50</td>
<td>4927.78</td>
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<tr>
<td>60PF 21</td>
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<td>1280.00</td>
<td>9188.00</td>
<td>4768.18</td>
</tr>
</tbody>
</table>

3.2 Sensitivity analysis (numerical modeling)

A comprehensive set of sensitivity analyses aimed to assess the global dynamic behavior of the panels depending upon the material elastic properties has been carried out. The dimensions of the 60-PF specimens have been chosen according to the commercial thickness of the boards forming the panel (20mm), with a Length-to-Depth ratio ($L/H$) high enough to ensure that the shear deformation can be regarded as negligible in cantilever configuration.

The analyses have been specifically conducted for the 60-PF specimen type illustrated in Fig. 2 and are therefore fully reliable only for that configuration; nevertheless the same approach could be easily extended to any cantilever three-layer CLT specimen with different dimensions, by means new sensitivity studies. Similar trends of sensitivity analyses are expected for this case.
The proposed methodology has been applied to a benchmark model having the initial elastic properties shown in Table 2 and evaluated as function of $E_0$, according to EN384 [50] ($E_{90} = E_0/30$, $G_{0} = E_0/16$, $G_{90} = G_0/10$ and $\nu = 0.40$).

Table 2

<table>
<thead>
<tr>
<th>$E_0$ [MPa]</th>
<th>$E_{90}$ [MPa]</th>
<th>$G_0$ [MPa]</th>
<th>$G_{90}$ [MPa]</th>
<th>$\nu$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7500.00</td>
<td>250.00</td>
<td>468.75</td>
<td>46.88</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The analyses have been conducted for the first four natural frequencies by varying the elastic properties in a suitable range, displayed in Table 3; the natural frequencies have been assessed as function of each considered property, by varying the elastic properties once at a time while keeping the other elastic properties constant to the initial values displayed in Table 2.

Table 3

<table>
<thead>
<tr>
<th>$E_0$ [MPa]</th>
<th>$E_{90}$ [MPa]</th>
<th>$G_0$ [MPa]</th>
<th>$G_{90}$ [MPa]</th>
<th>$\nu$ [-]</th>
</tr>
</thead>
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<tr>
<td>2000-12000</td>
<td>100-1000</td>
<td>200-1000</td>
<td>20-150</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Figure 3 shows the first four mode shapes obtained by the FE model of the specimens in cantilever configuration.
Fig. 3. Mode shapes of the specimens in cantilever configuration (FE): (a) first vertical mode, (b) first horizontal mode, (c) first torsional mode, (d) second vertical mode.

The results of the sensitivity analyses are displayed in Figs. 4-5. Figure 4 shows the natural frequencies of the specimens as a function of $E_0$ (Fig. 4a) and $E_{90}$ (Fig. 4b). The elastic properties affect the global dynamic behavior differently with respect to the different vibration modes. Fig. 4a reveals that $E_0$ affects the vertical and horizontal modes but not the torsional one; on the other hand, it is noteworthy that $E_{90}$ does not have any influence in the four examined modes (Fig. 4b), by varying the parameters in the whole examined range. This result can be justified by observing that $E_{90}$ is not exploited in any of the considered modes, with exception of the central layer which, on the other hand, provides a minor contribution to the overall panel stiffness due to its proximity to the neutral axis. Moreover, $E_{90}$ values are much lower with respect to $E_0$. This result confirms the possibility to simply neglect the central layer in the evaluation of the effective bending stiffness as commonly done in current design procedures.
Fig. 4. Natural frequencies of the specimens as function of elastic properties (FEM) of the benchmark model: (a) $E_0$ and (b) $E_{90}$.

Figure 5 displays the natural frequencies of the specimens depending upon $G_0$ (Fig. 5a) and $G_{90}$ (Fig. 5b) variations; $G_0$ directly affects horizontal and torsional modes, hardly influences the second vertical mode and does not affect the first vertical mode at all. Conversely, the second vertical mode, which is almost independent of $G_0$, is markedly influenced by $G_{90}$ (Fig. 5b). This occurrence can be explained by the high slenderness of the panels (Length-to-Depth ratio $L/H = 17.5$), which makes shear deformations negligible for the first vertical mode; on the other hand, for a lower Length-to-Breadth ratio ($L/B = 4.4$), the first horizontal mode shows a clear dependency by on $G_0$. Conversely, shear clearly affects the second vertical mode (bending mode), but due to the low $G_{90}/G_0$ ratio, in this case deformations are mostly governed by rolling shear deformations in the central layer rather than longitudinal shear deformations in the outer layers.

![Graph showing natural frequencies](image)

Fig. 5. Natural frequencies of the specimens as function of elastic properties (FEM) of the benchmark model: (a) $G_0$ and (b) $G_{90}$.

From a global examination of the results shown in Fig. 4 and Fig. 5, it can be concluded that first mode is influenced only by the modulus of elasticity parallel to the grain $E_0$, while the second vertical mode is affected by both $E_0$ and $G_{90}$. The torsional mode does not show any dependence.
upon $E_0$, but is affected by the shear modulus, especially $G_0$. These conclusions can be effectively used for the dynamic identification of the specimens according to the procedure described in the next section.

3.3 *Comparison between the elementary beam theory and the sensitivity results*

As a first step, a comparison between Finite Element and Euler-Beam models has been carried out as a way for understanding the limits of the beam theory applied in Eq.(1). Fig. 6 shows a direct comparison between theoretical (Euler-Beam) and computational (FE) frequencies of the first vertical and horizontal modes of the panel as a function of the modulus of elasticity $E_0$.

Figure 6 compares the theoretical and the computational frequency obtained respectively by Eq.(1) and the FE model, showing the first vertical and horizontal natural frequencies of the panel as function of the modulus of elasticity $E_0$.

![Vertical vs Horizontal Modes](image)

Fig. 6. Comparison between theoretical and computed (FEM) natural frequencies of the first vertical and the first horizontal bending mode of the panel, as function of the modulus of elasticity $E_0$: (a) first vertical mode and (b) first horizontal mode.

It appears that Eq. (1) theoretical approach agrees well with the FE model for the first vertical mode; on the other hand, a larger error affects the horizontal mode, leading to the conclusion that the error provided by the use of Eq. (1) is not acceptable in the last case. This result can be easily
explained by the limitations of the Eq. (1), which is reliable when Euler-Bernoulli beam hypothesis is fulfilled [16,43], namely in the case of high slenderness, whereby shear deformations are negligible. As anticipated, for the considered horizontal mode, (low Length-to-Breadth ratio $L/B=4.4$), shear contribution to deformations cannot be disregarded, when the rolling shear modulus of the orthotropic material model is markedly low. This behavior is confirmed by Fig. 5a which illustrates that the natural frequency of the horizontal mode is clearly influenced by the shear modulus $G_0$. [44]. Nevertheless, the elementary beam theory can be effectively employed when determining $E_0$ from the first vertical mode frequency in slender specimens [44].

3.4 Dynamic identification procedure (determination of $E_0$ and $G_{90}$)

A dynamic identification procedure for the determination of the longitudinal MOE and rolling shear modulus $G_{90}$ has been derived from the results of the sensitivity analyses conducted via the FE model described in the previous Section. Various dynamic identification procedures have been recently proposed by several authors [22,41,43]; these methodologies are based on Euler-Bernoulli beam free vibration theory and are generally restricted to the evaluation of $E_0$ only.

The proposed approach mainly aims to extend the aforementioned procedures to the determination of the rolling shear modulus $G_{90}$ of the specimen timber layers. As discussed in Section 3.2, the three-layer panel dynamic behavior, in particular the vertical modes, is mainly influenced by the elastic properties $E_0$ and $G_{90}$. A direct inspection of Fig. 5 (b) shows that, for a predefined value of $E_0$ ($= 7500$ MPa), the second vertical mode frequency is directly related to the rolling shear modulus $G_{90}$. A similar analysis has been carried out for discrete values of $E_0$, this time restricted to a suitable range of 3500 to 7500 MPa, leading to the results shown in Fig.7, where a second vertical mode sensitivity analysis is shown in terms of frequencies with respect to
$G_{90}$. For the sake of clearness, the axes of Fig. 7 have been inverted with respect to the previous Figs. 5-6 and a larger range of $G_{90}$ has been considered.

Fig. 7. Rolling shear $G_{90}$ as function of the frequency of the second vertical mode, for different values of $E_0$.

The first step of the proposed procedure is therefore based on the determination of $E_0$ by means of the sensitivity analysis approach displayed in Fig. 4a. As it can be deduced by Figs. 5-6, $E_0$ is sufficiently decoupled from the other elastic properties $E_{90}$, $G_0$ and $G_{90}$ for the given panel configuration. It can therefore be independently calculated through the experimental evaluation of the panel first mode frequency. The results are shown in Fig. 4a. The second step requires the experimental determination of the second vertical mode frequency of the specimen. Such trend, evaluated by the FE modeling, has been plotted in Fig. 7 as a function of $G_{90}$ for different values of $E_0$. Finally, the rolling shear modulus $G_{90}$ is assessed, (see also section 3.6), after a proper interpolation for the actual value of $E_0$. It is noteworthy that the above second step relies on the hypothesis, discussed in section 3.2, that the second vertical mode frequency is hardly affected by
Due to the high ratio $G_0/G_{90}$, which causes the shear deformations to mostly depend upon the latter rolling shear modulus. An example of identification procedure applied to the 60PF-16 specimen is illustrated in section 3.6.

3.5 Testing apparatus

An experimental setup for the dynamic identification of the CLT panels has been prepared at the laboratory of DICAAR, where a vibration test equipment has been assembled for the non-destructive evaluation of the mechanical properties of the specimens (Fig. 8). Specimens have been arranged in a cantilever configuration by fastening the fixed end to a rigid support through appropriate clamps. In order to assess the effectiveness of the restraint, accelerations of the clamped support have been carefully monitored in preliminary studies performed on the testing apparatus, proving the reliability of the restraint assumptions. A proper impulsive mechanical load has been subsequently applied to the specimen in order to selectively excite the different panel vibration modes (Figs 8(a) and 8(b)).
The vertical and horizontal accelerations have been detected by a set of #4 PCB 333B40 accelerometers whose properties are summarized in Table 4.

**Table 4**

<table>
<thead>
<tr>
<th>Weight [gm]</th>
<th>Sensitivity [mV/g]</th>
<th>Frequency range [Hz]</th>
<th>Measurement Range [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>500</td>
<td>0.5-3000</td>
<td>±98</td>
</tr>
</tbody>
</table>

Accelerometers were located in proper positions on the specimen in the experimental setup shown in Fig. 8. The distribution has been chosen in order to properly identify the modes of interest. For example, accelerometer #4 has been placed in the node of the second vertical mode (refer to Fig. 3(d) and Fig. 8), to identify the selected mode when almost zero amplitude is detected in the normalized Power Spectral Density PSD.
The signals have been acquired by the accelerometers through a data acquisition card and processed by a LabVIEW-based software to directly elaborate the desired natural frequencies of the panels. The software has been properly implemented to process vibrations signals, allowing the determination of $E_0$ and $G_{90}$ according to the proposed procedure.

3.6 Application of the identification procedure to a case study

In the present section, the proposed dynamic identification procedure is illustrated for the 60-PF16 specimen, however the same procedure has been repeated for all the remaining specimens. As already mentioned, the procedure is based on the results of the sensitivity analyses conducted by the FE model shown in Figs. 4 and 5. The identification approach requires, as first step, the determination of $E_0$ through the cantilever vibration method [43].

![Fig. 9. Power Spectral Density response of the 60-PF16 panel.](image)

Fig. 9 shows the normalized PSD function of the 60-PF16 panel specimen corresponding to a mechanical impulse applied as illustrated in Fig. 8 and aimed at exciting vertical modes. The figure highlights the first two vertical natural frequencies of the panel $\alpha_I$ and $\alpha_{II}$ needed for the identification procedure.
It can be noted that the horizontal accelerometer #2 (see Fig. 9) does not detect any evident peak. Conversely similar trends of PSD are shown by accelerometers #1 and #3 (see Figs. 8a and 8c), since the vertical mechanical impulse has been applied on the longitudinal symmetry axis of the panel and no torsional mode has been excited.

Accelerometer #4 has been placed in proximity of the expected node of the second vertical mode. Thus, as can be seen in Fig. 9, the corresponding PSD does not highlight any evident peak in correspondence of the second vertical natural frequency. Some “spurious” peaks are present as well, displayed by accelerometer #3 in the interval between 100 and 150 Hz. These peaks have been attributed to the torsional mode, slightly excited by small eccentricities of the mechanical impulse and/or by anatomic and geometric imperfections of the specimens.

The first frequency \( \alpha_1 \) is then processed for the determination of \( E_0 \) (Fig. 10), which in the present case results in a value of 4318 MPa.

![Illustrative example of the first step of the proposed dynamic identification procedure for the determination of the modulus of elasticity \( E_0 \) of the 60-PF16 specimen.](image)

Once \( E_0 \) has been evaluated, the second step requires a linear horizontal interpolation procedure (Fig. 11) leading to the dark dashed curve corresponding to the above measured \( E_0 \) of the panel.
(60-PF16). A subsequent step allows the determination of $G_{90}$ for the considered specimen as a function of the measured second vertical mode frequency $\alpha_{II}$ (Fig.11).

![Illustrative example of the second step of proposed dynamic identification procedure for the determination of $G_{90}$ of the 60-PF16 panel with $E_0=4318$ MPa](image)

It should be noted (Fig. 11) that the proposed approach requires, especially for low values of $E_0$, a high accuracy on $E_0$ [41,43,44], due to the high precision required by the horizontal interpolation (dark dashed curve) and to the increasing slope of the curves for lower values of $E_0$.

4. Results and discussion

The above illustrated procedure has been used for a total of 14 3-layered CLT specimens made of Sardinian Maritime Pine. The results obtained by means of the proposed identification technique are discussed in the following. As mentioned in the previous section, the proposed technique relies on the accuracy of the calculated $E_0$ which represents the starting point of the elaboration. Therefore, for the purpose of evaluating the rolling shear modulus $G_{90}$, the use of FE model results is strongly recommended for the first step of the proposed procedure. Moreover it has to be pointed out that despite classic beam theory is commonly used to perform this task [41], the theoretical
approach tends to underestimate the elastic modulus $E_0$ by 4% for the considered 60-PF typology (refer to Fig.12 and Table 5).

![Graph](image)

**Fig. 12.** Distribution of mechanical properties of the considered specimens, (a) Modulus of elasticity $E_0$, (b) Rolling shear modulus $G_{90}$.

Results show a rather low Young’s modulus $E_0$ than other softwood species [43,51,52], which is not surprising given the low mechanical quality of the examined wood species. As previously mentioned, a classification and the evaluation of the elastic properties of the boards forming the panel has been already performed by the authors, e.g. [43,48,49] where a mean value of modulus of elasticity of the panel boards forming the 60-PF specimens has been estimated as $E_0=5190$MPa [43]; must be noted that such value matches very well the elastic modulus shown in Table 5 calculated by FE approach ($E_0=5125.80$ MPa). As previously anticipated (refer also to section 3.3), such value differs by about 4% [41,44] from the elastic modulus calculated by classic beam theory ($E_0=4898.06$MPa).

**Table 5**

Properties of the specimens: statistics (refer to Fig.12).

<table>
<thead>
<tr>
<th>Property</th>
<th>$\rho_0$ [kg/m$^3$]</th>
<th>$E_0$ (beam theory) [MPa]</th>
<th>$E_0$ (FE) [MPa]</th>
<th>Error $E_0$ [%]</th>
<th>$G_{90}$ (FE) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>476.46</td>
<td>4898.06</td>
<td>5125.80</td>
<td>-4.12%</td>
<td>208.79</td>
</tr>
</tbody>
</table>
As displayed in Fig. 12 and Table 5, the rolling shear modulus shows the highest scatter, due to the influence of several parameters related to the intrinsic anisotropy of timber; e.g. Fellmoser and Blass [53] discussed the relationship between rolling shear modulus and strength and stiffness of timber elements; Görlacher [54] highlighted the influence of annual growth rings orientation on the value of $G_{90}$; Ehrhart et al. [55] identified in the sawing pattern and the boards geometry the main features affecting the rolling shear properties. These authors presented a comprehensive study summarizing the values of $G_{90}$ reported in literature for different European timber species including pine, confirming the high level of scattering of this parameter. Moreover, the study of Ehrhart et al. pointed out that European pine $G_{90}$ ranges between 90 MPa and 210MPa; it should be noted that, the values of rolling shear modulus reported in Fig.12b fall within this range, placing themselves towards its highest end. Thus, Sardinian maritime pine confirms a relatively high rolling shear modulus, if compared with other European species of pine.

It should be noted that the elastic properties evaluated using the proposed approach have to be considered as mean global properties of the panel. The proposed technique is therefore suitable for assessing the global behavior of the panel and may be conveniently exploited for CLT panel design and production control.

Figure 13 shows the correlation between the mean values of elastic properties $E_0$, $G_{90}$ and density $\rho_0$ for the tested specimens. It can be noted that both the correlations $\rho_0-E_0$ and $\rho_0-G_{90}$ are insignificant; this is consistent with similar results found by other authors ($R^2=0.13$) [55] for softwood species, especially for conifers, confirming the independence of elastic properties of Sardinian Maritime Pine of density. Better correlations were found in the case of hardwood
species; for example, in the cases of beech and ash, the correlation coefficient raise to $R^2=0.34$ and $R^2=0.50$ respectively [55].

![Graphs showing correlation between density and elastic properties of the considered specimens.](image)

**Fig. 13.** Correlation between density and elastic properties of the considered specimens, (a) density vs Modulus of elasticity $E_0$, (b) density vs rolling shear Modulus $G_{90}$.

### 5. Summary and conclusions

In the present study, a new cantilever free vibration-based procedure has been introduced to determine the elastic properties of a three-layer CLT panel made of Sardinian Maritime Pine.

The study has been focused on the determination of the Modulus of Elasticity $E_0$ and the Rolling Shear Modulus $G_{90}$ of the boards composing the specimens, identified as the most significant elastic properties for the examined panel layup.

The proposed procedure is based on the results of sensitivity studies conducted via FE model analysis on a specific panel configuration. However the same approach could be easily extended to any cantilever three-layer CLT specimen with different dimensions, provided that a new sensitivity study is carried out for the new panel dimensions.

A straightforward relationship between the first two vertical mode frequencies of the CLT panels and the elastic properties of their boards has been highlighted. The process enables a direct
comparison between the elastic properties and the corresponding free vibration frequencies of the panels. The elastic moduli $E_0$ and $G_{90}$ have been thus assessed for 14CLT specimens made of Maritime Pine, showing the possibility of replacing destructive static tests with non-destructive and quicker dynamic tests. In this respect, the proposed methodology has been proved to be an effective approach to assess the elastic properties of CLT panels in a straightforward way, which makes it a powerful tool to be used during quality control procedures in CLT manufacturing. More specifically, the proposed method focuses on the rolling shear modulus of the central layer of the specimen, whose direct experimental determination is more demanding.

The dynamic response of CLT specimens has been evaluated through a set of accelerometers suitably placed on the specimen surfaces. Alternatively, a faster and more effective data acquisition system based on laser sensor devices could be employed for a quality control procedure.

The discussed procedure assesses the average global elastic properties of the panels ($E_0$ and/or $G_{90}$) rather than local values, as addressed by standard procedures defined in European regulations. This approach somehow overcomes the scatter of elastic parameters at the local level, even within the same element, and the influence of defects, and thus represents a more useful evaluation.

Future studies will aim to possibly extend the proposed approach to the other elastic properties of CLT panels, e.g. $G_0$ or $E_{90}$, or to different panel layups (such as 5-layers CLT panels).

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