Physics-based Demand Model and Fragility Functions of Industrial Tanks under Blast Loading

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Abstract

Blast hazards represent a serious threat to industrial facilities. Past explosion incidents highlight the severe consequences of such events. A probabilistic approach can help industries and designers mitigate the consequences of blast loading by better organizing industrial plants. In this paper, we propose a physics-based probabilistic demand model and formulate the reliability problem for industrial steel tanks under blast loading. Starting from a deterministic Single-Degree-of-Freedom (SDOF) model based on Donnell shallow-shell theory, we develop a correction term that improves the model accuracy due to the simplified representation of the SDOF model. We use Bayesian inference to estimate the unknown model parameters in the correction term and model error, combining predictions from the SDOF model with experimental data and any prior information. To illustrate, we estimate the reliability of an example cylindrical steel tanks subject to blast loading considering three damage levels. The reliability analysis yields a set of fragility curves that represent the conditional probability of the bending failure of the tank given a scaled distance, as the load intensity measure. Then, as an example, we use the developed fragility functions to estimate the reliability of a chemical industrial facility considering different explosion scenarios.

Keywords

Steel tank; blast load; fragility; demand model; SDOF

1 Introduction

Blast loading represents a severe threat to industrial facilities. During their service lives, the component of industrial facilities such as tanks might be subject to blast loading from malicious or accidental explosions (Antonioni et al. 2009, Misuri et al. 2020, Ding et al. 2022, Ovidi et al. 2022, Tugnoli et al. 2022). Past events have shown the vulnerability of industrial tanks to blast loading (e.g., Ibitayo et al. 2004, Zio et al. 2013.) Similarly, past events have highlighted the severe consequences of blast loading in industrial facilities, which can include fatalities, economic losses, and environmental impacts due to the release of hazardous materials (e.g., Marsh et al. 2016, Damle et al. 2021, Araki et al. 2021, Suarez-Paba et al. 2022). The organization of industrial facilities requires optimizing the production characteristics considering possible natural or man-made threats to key structural elements, columns (Singh et al. 2020) and beams (Stochino et al. 2021), or to the vulnerable components, steel tanks, and their consequences (e.g., Salvado et al. 2017, Mayorga et al. 2019, Zhou et al. 2020, Lan et al. 2022). In this context, a probabilistic approach is necessary to account for the relevant uncertainties like aleatory uncertainties in the characteristics of the threats and epistemic uncertainties due to scarcity of data typical of low probability, high consequence events (Gardoni et al. 2002, Murphy et al. 2011). Cylindrical steel tanks represent a typical storage system characterized by a slender structure prone to buckling failure under blast loading (Hansen et al. 2010, Dadashzadeh et al. 2013, Sochet et al. 2014). Cylindrical steel tanks are typically characterized by a large ratio between thickness and height, which may yield flexural buckling in case of a horizontal load orthogonal to the tank. The performance of a cylindrical steel tank subject to blast loading represents a complex nonlinear

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behavior due to the characteristics of the blast loading, i.e., a high amount of energy over a short time.

Past research focused on experimental tests on cylindrical steel tanks under blast loading. For example, Duong et al. (2012a,b) presented an experimental and analytical analysis of the dynamic response of a cylindrical shell under blast load while studying both the blast pressure and the structural behavior. Jiang et al. (2020) studied the structural behavior of monolithic steel tanks and polyurea-coated tanks under real field blast loading. Chen et al. (2016) analyzed full-scale validation tests of far-field air blast loading acting on deformable steel cylinders.

Similarly, modeling the response of cylindrical steel tanks subject to blast loading has also received much attention. Mathematical representations like differential equations can rigorously represent cylindrical steel tanks under blast loading (Donnell 1934). Similarly, the increasing computational capabilities and advances in numerical techniques have also enabled the modeling of the response of cylindrical steel tanks subject to blast loading using high-fidelity computational models of structural dynamics (e.g., Ameijeiras et al. 2016, Chen et al. 2016).

However, such approaches have several limitations in their use in the reliability analysis of cylindrical steel tanks subject to blast loading. For example, mathematical representations like differential equations pose a significant computational challenge when incorporating the uncertainties in the response of cylindrical steel tanks under blast loading. Similarly, high-fidelity models present a high computational cost, limiting their use in the reliability analysis with small failure probabilities.

Alternatively, available approaches (e.g., Duong et al. 2021a,b), including current blast design guidelines (e.g., PDC-TR 06-08), model the response of cylindrical steel tanks under blast loading

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using a generalized single-degree-of-freedom (SDOF). The use of a generalized SDOF in reliability analysis based on sampling methods such as the Monte Carlo (MC) simulation to estimate the failure probabilities (Ditlevsen and Madsen 1996, Gardoni 2017). However, sampling methods are generally computationally expensive. Lastly, the developed models are structure-specific; thus, the developed models for a specific cylindrical steel tank cannot generally be used for other tanks.

This paper proposes a physics-based probabilistic demand model and formulates the reliability problem for cylindrical steel tanks under blast loading. In this paper, we refer to *risk* as a metric that captures the probability of occurrence and the associated consequences of a set of hazardous scenarios (Bedford and Cooke, 2001), as well as the source or cause of the hazardous scenarios (Gardoni and Murphy 2014). Similarly, *vulnerability* is defined as the susceptibility to adverse impacts and propensity to limited or delayed recovery within a hazardous scenario (Wang et al. 2021). Lastly, *reliability* is defined as the probability that a system remains available or functional (Gardoni 2017). The proposed physics-based probabilistic models estimate the quantities of interest as a function of state variables that define the structural system (e.g., geometric quantities and material properties.) The proposed model combines a computationally convenient representation of the governing physical laws with an analytical correction term. In this paper, we first derive a generalized SDOF model based on Donnell (1934) shallow-shell theory. Then, we develop a correction term that improves the model accuracy due to the simplified representation of the SDOF model. The proposed physics-based probabilistic model also includes a model error term that captures the remaining model's uncertainty (Gardoni et al. 2002). We use Bayesian inference to estimate the unknown model parameters in the correction term and model error,

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combining predictions from the generalized SDOF system with experimental data and any prior information. Using the proposed physics-based probabilistic model, we then formulate the reliability problem of cylindrical steel tanks under blast loading. To illustrate, we conduct a reliability analysis of an example cylindrical steel tank subject to blast loading considering three possible damage states. The reliability analysis yields a set of fragility curves conditioned on the scaled distance, which defines a load intensity measure. Then, as an example, we use the developed fragility functions to estimate the reliability of an industrial facility considering different explosion scenarios.

Following this Introduction, the rest of the paper is organized into six sections. Section 2 presents the experimental database used in the model calibration. Section 3 briefly reviews the general mathematical formulation to develop probabilistic physics-based models. Section 4 presents the proposed physics-based probabilistic demand model for the cylindrical steel tanks under blast loading. Section 5 develops the physics-based fragility functions, while Section 6 presents the example.

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2 Experimental database

The experimental data used for calibrating the proposed probabilistic model are from Ameijeiras et al. (2016), Chen et al. (2016), Jiang et al. (2020), Duong et al. (2021a,b).

Table 1: Range of the values of the blast load and structural variables in the database.

Variable	Symbol	Range
Scaled distance [m/kg$^{1/3}$]	z	2.24-18.19
Reflected peak pressure [kPa]	P_{ref}	12.1-594.8
Positive phase duration [msec]	t_d	0.32-43.41
Steel yielding strength [MPa]	f_{stat}	235-305
Steel elastic modulus [MPa]	E_s	170-225
Tank diameter [m]	d	0.6-70.0
Tank height [m]	h	0.9-35.0
Tank thickness [mm]	e	1.0-35.5

The collected data include the dynamic response of 27 cylindrical steel tanks subject to blast loading. Table 1 summarizes the variables of interest and their range of values.

The number of training data required to estimate the unknown model parameters mainly depends on the number of the unknown model parameters (Stone, 1996) . Developing physics-based probability models, in general, alleviates the need for (experimental) data by incorporating knowledge in other forms like existing (deterministic) models, and rules of physics and mechanics. In addition, as more data become available, the model can also be updated using Bayesian inference (more details in Section 4).

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3 Review of probabilistic physics-based models

Gardoni et al. (2002, 2003) proposed a general mathematical formulation for developing probabilistic physics-based models while accounting for the various sources of information (i.e., from the governing rules of physics and mechanics to the data from laboratory experiments and field measurements), as well as the different sources of uncertainty. In this section, we illustrate the general mathematical formulation for developing probabilistic physics-based models, the Bayesian model calibration, and the model selection procedure.

3.1 General formulation of the probabilistic model

Following Gardoni et al. (2002, 2003), we can write the generic q-dimensional multivariate probabilistic physics-based model as

$$T_k[Y_k(\mathbf{x}, \boldsymbol{\Theta}_k)] = T_k[\hat{y}_k(\mathbf{x})] + \gamma_k(\mathbf{x}, \boldsymbol{\theta}_k) + \sigma_k \varepsilon_k, \qquad k = 1, \dots, l \tag{1}$$

where $T_k(\cdot)$ is a transformation function; $Y_k(\mathbf{x}, \boldsymbol{\Theta}_k)$ is the k^{th} quantity of interest (e.g., capacity or demand); $\hat{y}_k(\mathbf{x})$ is an existing deterministic model to predict Y_k that is typically based on rules of physics and mechanics; $\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$ is a correction term constructed to improve the prediction of $\hat{y}_k(\mathbf{x})$ and capture any possible bias in $\hat{y}_k(\mathbf{x})$; $\mathbf{x}$ is a vector of basic variables that characterize Y_k; $\boldsymbol{\Theta}_k = (\boldsymbol{\theta}_k, \sigma_k)$ is a vector of unknown model parameters that need to be estimated. Also, if we let $\boldsymbol{\Sigma}$ denote the covariance matrix of $(\sigma_1\varepsilon_1, \dots, \sigma_l\varepsilon_l)$, the collection of all unknown model parameters can then be written as $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \boldsymbol{\Sigma})$, where $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_l)$.

There are three main assumptions in the generic multivariate probabilistic physics-based model as written in Equation (1). The $\sigma_k\varepsilon_k$ is assumed an (additive) model error (additivity assumption), in which σ_k is assumed not to depend on $\mathbf{x}$ (homoskedasticity assumption), and ε_k is a standard normal random variable (normality assumption). The transformation $T_k(\cdot)$ is used to

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(approximately) satisfy these three assumptions within the range of the data used to calibrate the model. For example, Box and Cox (1964) proposed a parameterized family of suitable transformations that can guide the selection of $T_k(\cdot)$.

Following Gardoni et al. (2002, 2003), we can write the correction term as

$$\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k) = \sum_{j=1}^{N_{h,k}} \theta_{k,j} h_{k,j}(\mathbf{x}), \qquad k = 1, \dots, l \tag{2}$$

where $h_{k,j}(\mathbf{x}), j = 1, \dots, N_{h,k}$, are explanatory functions that are (also) partially constructed based on the rules of physics and mechanics. A distinctive feature of the physics-based probabilistic model in Equation (1) is that it takes advantage of both first principles (i.e., the governing rules of physics and mechanics) as well as empirical information. The use of statistical tools guides in tailoring the empirical portion of the model to provide the most accurate predictions. For instance, Gardoni et al. (2002) proposed a stepwise deletion process (discuss next) to construct a parsimonious, yet accurate predictive model in which $\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$ includes only the terms that significantly contribute to the prediction of Y_k.

3.2 Bayesian model calibration and model selection

After formulating the probabilistic physics-based model in Equation (1), one can use a Bayesian approach to estimate $\boldsymbol{\Theta}$. The Bayesian approach combines the prior information about $\boldsymbol{\Theta}$ with the objective information from the observed data. Also, the Bayesian approach can be used to update the physics-based model as new data become available (e.g., Choe et al. 2007; Gardoni et al. 2007). To calibrate the probabilistic physics-based model, one can use both real data like data from laboratory experiments (Gardoni et al. 2002, 2003; Zhong et al. 2009), and virtual data like computer simulations (Gardoni et al. 2003; Huang et al. 2010). Also, one can use an

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experimental design to identify the most informative cases to test or run and reduce the statistical uncertainty (Huang et al. 2010; Tabandeh and Gardoni 2015, Tabandeh et al. 2020).

Using the Bayesian approach, the statistical uncertainty in the estimates of $\boldsymbol{\Theta}$ is captured through its posterior probability density function (PDF), $f(\boldsymbol{\Theta})$. Specifically, to estimate $\boldsymbol{\Theta}$, we use the Bayes theorem

$$f(\boldsymbol{\Theta}) = \kappa L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) \quad (3)$$

where $f(\boldsymbol{\Theta})$ is the posterior PDF of $\boldsymbol{\Theta}$ containing the updated state of knowledge about $\boldsymbol{\Theta}$; $L(\boldsymbol{\Theta})$ is the likelihood function that captures the objective information on $\boldsymbol{\Theta}$ contained in the data; $p(\boldsymbol{\Theta})$ is the prior distribution reflecting our state of knowledge about $\boldsymbol{\Theta}$ before collecting the data; and $\kappa = \left[\int L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) d\boldsymbol{\Theta}\right]^{-1}$ is a normalizing constant.

In the most general case, the likelihood function allows including lower-bound, upper-bound, and equality data (Gardoni et al. 2002). Data can be defined as lower-bound data if the measured values are less than the actual values of the quantity of interest. Similarly, data can be defined as upper-bound data if the measured values are greater than the actual values of the quantity of interest. Finally, data are defined as equality data if the measured values are the actual values of the quantity of interest. Following Gardoni et al. (2002), we can write the general expression of $L(\boldsymbol{\Theta})$ as

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$$L(\boldsymbol{\Theta}) \propto \prod_{\substack{\text{observation}\\ i}} P\left\{ \bigcap_{\substack{\text{equality}\\ \text{data } k}} [\sigma_k \varepsilon_{ki} = r_{ki}(\boldsymbol{\theta}_k)] \bigcap_{\substack{\text{lower-bound}\\ \text{data } k}} [\sigma_k \varepsilon_{ki} > r_{ki}(\boldsymbol{\theta}_k)] \bigcap_{\substack{\text{upper-bound}\\ \text{data } k}} [\sigma_k \varepsilon_{ki} < r_{ki}(\boldsymbol{\theta}_k)] \right\} \quad (4)$$

where $r_{ki}(\boldsymbol{\theta}_k) = T_k[Y_k] - T_k[\hat{y}_k(\mathbf{x}_i)] - \gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$ is the residual of the prediction for the i^{th} datum. Under the assumptions of exact measurements and statistically independent observations, we can write the likelihood function as

$$L(\boldsymbol{\Theta}) \propto \prod_{\substack{\text{equality}\\ \text{data}}} \left\{ \frac{1}{\sigma_k} \varphi\left[\frac{r_{ki}(\boldsymbol{\theta}_k)}{\sigma_k} \right] \right\} \times \prod_{\substack{\text{lower-bound}\\ \text{data}}} \left\{ \Phi\left[-\frac{r_{ki}(\boldsymbol{\theta}_k)}{\sigma_k} \right] \right\} \times \prod_{\substack{\text{upper-bound}\\ \text{data}}} \left\{ \Phi\left[\frac{r_{ki}(\boldsymbol{\theta}_k)}{\sigma_k} \right] \right\} \quad (5)$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the standard Normal probability density function and the standard Normal cumulative density function. A discussion on the use of data when measurement errors are present can be found in Gardoni et al. (2002).

The prior distribution $p(\boldsymbol{\Theta})$ can incorporate any additional information to reflect the state of knowledge about $\boldsymbol{\Theta}$ before collecting the data. When no such information is available, one should use a prior distribution that has minimal influence on $f(\boldsymbol{\Theta})$. Therefore, in the absence of additional information, we can use a noninformative prior distribution $p(\boldsymbol{\Theta}) \cong p(\boldsymbol{\Sigma})$, where $p(\boldsymbol{\Sigma})$ can be written as (Gardoni et al. 2002)

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$$p(\boldsymbol{\Sigma}) \propto |\mathbf{R}|^{-(l+1)/2} \prod_{k=1}^{l} \frac{1}{\sigma_k} \tag{6}$$

where $\mathbf{R}$ is the correlation matrix. In the case $l = 1$, i.e., for a univariate physics-based probabilistic model, Equation (6) can be simplified in $p(\boldsymbol{\Theta}) \propto 1/\sigma$.

For practical use of the developed physics-based probabilistic model, the model selection should aim for an accurate and unbiased model that can be easily adopted in practice. Also, from a statistical standpoint, the inclusion of insignificant terms in $\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$ would lead to a loss of precision of the estimates of $\boldsymbol{\Theta}$ due to data over-fitting. To develop a parsimonious form of $\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$, Gardoni et al. (2002) proposed a stepwise deletion process that can be summarized as follows. We start with the complete set of explanatory functions that we believe would enhance the predictive ability of $\hat{y}_k(\mathbf{x})$ based on rules of physics and mechanics. After calibrating the physics-based probabilistic model, we eliminate the $h_{k,j}(\mathbf{x})$ whose coefficient $\theta_{k,j}$ has the largest posterior Coefficients Of Variation (COV). At each iteration, i.e., after each elimination, we re-calibrate the remaining $\boldsymbol{\Theta}$. We then repeat this process until the posterior mean of σ_k has not increased by an unacceptable amount. If the posterior mean of σ_k increases by an unacceptable amount, the reduction is not desirable and the model form at one iteration before is as parsimonious as possible. After defining the final form of the physics-based probabilistic model, we can further combine highly correlated model parameters (e.g., if $|\rho_{\theta_{k,i}\theta_{k,j}}| \geq 0.7$) as

$$\hat{\theta}_{k,i} = \mu_{\theta_{k,i}} + \rho_{\theta_{k,i}\theta_{k,j}} \frac{\sigma_{\theta_{k,i}}}{\sigma_{\theta_{k,j}}} \left(\theta_{k,j} - \mu_{\theta_{k,j}}\right) \tag{7}$$

where $\mu_{\theta_{k,i}}$ and $\sigma_{\theta_{k,i}}$ are the posterior mean and standard deviation of $\theta_{k,i}$. As discussed in Gardoni et al. (2002), Equation (7) provides the best linear prediction of $\theta_{k,i}$ as a function of $\theta_{k,j}$.

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4 Proposed probabilistic demand model for industrial tanks under blast loads

In this section, we use the general formulation in Equation (1) to develop the proposed probabilistic demand model. In the proposed probabilistic demand model, the predicted quantity is the maximum displacement due to the blast load, i.e., $Y_k(\mathbf{x}, \boldsymbol{\Theta}_k) = \delta w_{MAX}(\mathbf{x}, \boldsymbol{\Theta})$, with $k = 1$. In the following, we first formulate $\hat{\delta} w_{MAX}(\mathbf{x})$ using a semi-analytical model for the dynamic elastic buckling of cylindrical tanks. Then, we design the correction term $\gamma(\mathbf{x}, \boldsymbol{\theta})$ and calibrate the probabilistic model with the experimental data introduced in Section 2.

4.1 Deterministic model

In this section, we discuss the blast loading model and the structural semi-analytical deterministic model for the dynamic elastic buckling of cylindrical tanks based on Duong et al. (2012a,b).

4.1.1 Blast Loading Model

In this paper, we consider the general case of a hemispherical explosion where the effects of ground reflection and the formation of the so-called Mach stem are considered (UFC 3-340-02). Following Friedlander (1946), we can write the blast loading time history, $q(t)$ as

$$q(t) = P_{ref}\left(1 - \frac{t}{t_d}\right)\exp\left(\frac{-2t}{t_d}\right) \tag{8}$$

where P_{ref} is the reflected peak pressure, and t_d is the duration of the positive phase, i.e., $q(t) \geq 0$, $\forall t \in [0, t_d]$. Figure 1 shows the blast loading time history in Equation (8), highlighting the values of P_{ref} and t_d.

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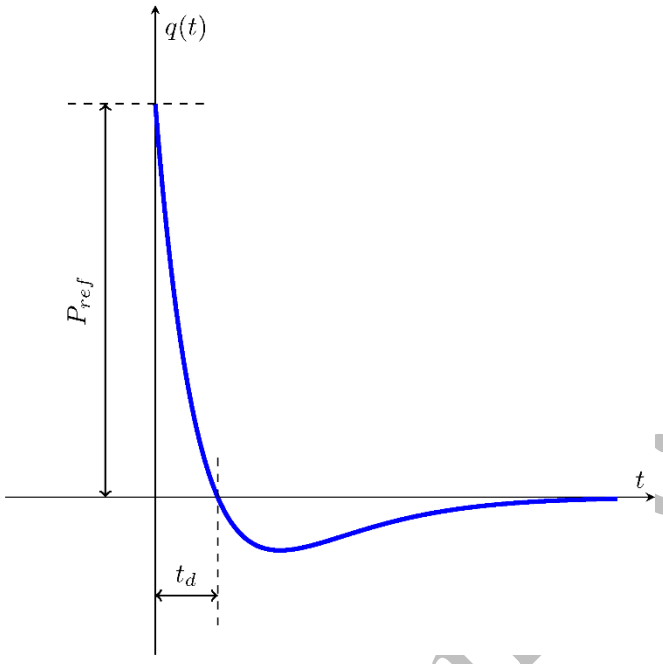

Figure 1: Blast loading time history.

Following Olmati et al. (2014), we can then write P_{ref} as

$$P_{ref} = 2 \cdot P_{so} \left(\frac{7P_{atm} + 4P_{so}}{7P_{atm} + P_{so}} \right) \tag{9}$$

whereP_{so} is the stand-off pressure (expressed in MPa), and P_{atm} is the standard atmosphere pressure, i.e., $P_{atm} = 0.1\ MPa$. The stand-off pressure P_{so} can be calculated as (Mills 1987)

$$P_{so} = 1.772 \left(\frac{1}{z^3} \right) - 0.114 \left(\frac{1}{z^2} \right) + 0.108 \left(\frac{1}{z} \right) \tag{10}$$

where z is the scaled distance, i.e., the ratio between the distance from the explosive charge to the cylindrical steel tank and the cubic root of the explosive charge. We can write the scaled distance as

$$z = \frac{R}{W^{\frac{1}{3}}} \tag{11}$$

where R is the stand-off distance and W is the mass of explosives in kg of equivalent TNT (Held 1983; UFC 3-340-02.) We assume a triangular impulse, and we obtain t_d from

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$$t_d = \frac{2\, I_{sO}}{P_{sO}} \tag{12}$$

where I_{sO} is the incident impulse, which can be written according to Held (1983) as

$$I_{sO} = 4.8 \cdot 10^5 \cdot \frac{W^{2/3}}{R} \tag{13}$$

In the discussed blast loading model, we assume that the blast wave is planar and corresponds to a uniformly distributed load on the cylindrical steel tank if the distance between the charge and the building is sufficiently large (e.g., $R \geq h$).

4.1.2 Structural model

Let us consider a cylindrical tank of height h, diameter d, and thickness e, in the cylindrical coordinate system (s, ω, a) as shown in Figure 2. We can express the radial displacement as $w(s, \omega, a)$, the axial displacement as $u(s, \omega, a)$, and the circumferential displacement as $v(s, \omega, a)$.

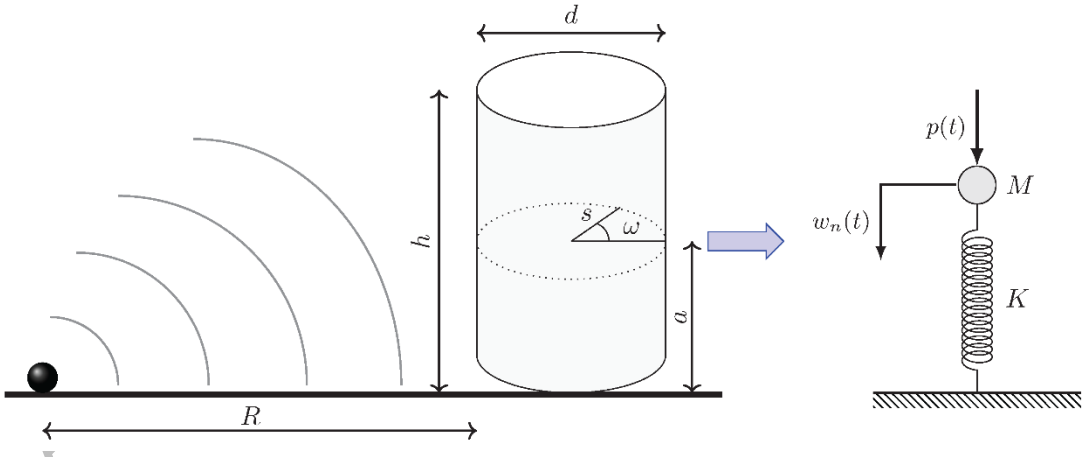

Figure 2: Geometrical characteristics of the problem (left) and structural model (right): M, K, q respectively denote the equivalent mass, stiffness, and load.

Furthermore, based on Donnell (1934) shallow-shell theory, we enforce the Kirchhoff-Love hypotheses with further simplifications justified by the slenderness of the cylindrical steel tanks. It follows that u and v are negligible compared to w, the curvature variations and the partial derivatives of w are small, and the product of the partial derivatives of w have the same order of magnitude of strains.

If the shell is initially in a membrane state under a lateral pressure q so that there is a distributed circumferential stress N_ω (while the axial and shear stress are negligible $N_s = N_{s\omega} = 0$), we can write the dynamic equilibrium equation as follows, see Duong (2012b):

$$D\nabla^4 \delta w = -\frac{2q}{d}\frac{\partial^2 \delta w}{\partial \omega^2} + \frac{2}{d}\frac{\partial^2 \delta F}{\partial s^2} - \rho e \frac{\partial^2 \delta w}{\partial t^2} \tag{14}$$

where ρ is the specific mass, $F_A(s, \omega, a)$ is the Airy stress function defined in the next Equation (15):

$$N_s = \frac{4}{d^2}\frac{\partial^2 F_A}{\partial \omega^2}, \qquad N_\omega = \frac{\partial^2 F_A}{\partial s^2}, \qquad N_{s\omega} = -\frac{2}{d}\frac{\partial^2 F_A}{\partial \omega \partial s} \tag{15}$$

where N_ω is the circumferential stress, N_s is the axial stress and $N_{s\omega}$ denote the shear stress, B is a stiffness parameter equal to $E_s e^3/(12(1-\nu^2))$, E_s is the steel elastic modulus, ν is the Poisson coefficient, e is the tank thickness.

Also, to account for the strain rate effects due to the high loading rate of blast loading, we use a Dynamic Increase Factor $DIF = \frac{f_{dyn}}{f_{stat}}$ =1.17, where f_{dyn} is the dynamic yielding strength and f_{stat} is the static one (UFC 3-340-02).

We assume that, during buckling, the cylindrical tank will present a doubly periodic surface expressed by the sum of m half-waves in the axial direction and n full waves in the circumferential

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one. Thus, we write the out-of-plane displacement field δw and the Airy function δF by using the Fourier expansion representation as

$$\delta w(s,\omega,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \delta w_{mn}(t) \sin\left(\frac{m\pi s}{h}\right) \sin(n\omega) \tag{16}$$

$$\delta F_A(s,\omega,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \delta F_{A_{mn}}(t) \sin\left(\frac{m\pi s}{h}\right) \sin(n\omega) \tag{17}$$

We then define $\lambda = 2nh/\pi d$, and using the Fourier expansion representation in Equation (14), we can write

$$\left(\frac{B\pi^4}{h^4}(m^2+\lambda^2)^2 + \frac{4m^4 Ee}{d^2}(m^2+\lambda^2)^{-2} - \frac{q(t)d\pi^2}{2h^2}\lambda^2\right)\delta w_{mn}(t) = \rho e \delta \ddot{w}_{mn}(t) \tag{18}$$

Following Duong et al. (2012b), we assign an initial radial shape imperfection δw_n^i to each mode n. Then, we study the elastic dynamic buckling based on the amplification of such imperfections. In this paper, we use $\delta w_n^i = 8.33 \cdot 10^{-7} \cdot h/e$ as suggested in Duong (2012b).

Experimental results show that $m = 1$ and $n \gg 1$ so that $1 + \lambda^2 \approx \lambda^2$ and Equation (18) becomes:

$$\left(\frac{16B\pi^4}{d^4} + \frac{Ee\pi^4 d^2}{4n^4h^4}\right)\delta w_n(t) - \frac{2q(t)n^2}{d}\left(\delta w_n(t) + \delta w_n^i\right) = -\rho e \delta \ddot{w}_{mn}(t) \tag{19}$$

Equation (19) is the Equation of motion of a single degree of freedom mass-spring system (Ruiz et al. 1989; Duong et al. 2012b), which can be written as

$$\delta w_n(t) - \frac{q(t)}{Q_n}\left(\delta w_n(t) + \delta w_n^i\right) = -\frac{\ddot{w}_{mn}(t)}{\Omega_n^2} \tag{20}$$

where $Q_n = (16Bn^4/d^4 + Ee\pi^4 d^2/(4n^4h^4))/(\rho e)$, and $\Omega_n = (16Bn^4/(d^4) + Ee\pi^4 d^2/(4n^4h^4))/(2n^2/d)$.

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We solve Equation (20) using an explicit finite difference scheme, obtaining the coefficients of the Fourier expansion defining $\delta w(s, \omega, a)$ in Equation (16). Using this solution, we then obtain the time history of the displacements, as well as the location and the estimates of the maximum displacement $\hat{\delta} w_{MAX}(\mathbf{x})$.

4.2 Correction terms

In this section, we design $\gamma(\mathbf{x}, \boldsymbol{\theta})$ to capture the physical characteristics not fully represented by $\hat{\delta} w_{MAX}(\mathbf{x})$. As the candidate explanatory functions, we define $h_1^{'}(\mathbf{x}) = 1$ that captures the potential bias in $\hat{\delta} w_{MAX}(\mathbf{x})$; $h_2(\mathbf{x}) = z/d$; $h_3(\mathbf{x}) = f_{stat}/E_s$; $h_4(\mathbf{x}) = h/e$; $h_5(\mathbf{x}) = h/d$; $h_6(\mathbf{x}) = (t_d \cdot P_{ref})/I_{sO}$; $h_7(\mathbf{x}) = (d \cdot I_{sO})/(t_d \cdot e \cdot f_{stat})$. In the proposed model, the explanatory functions are then standardized as follows

$$h_j(\mathbf{x}) = \frac{h_j^{'}(\mathbf{x}) - \mathsf{E}\left[h_j^{'}(\mathbf{x})\right]}{\mathsf{Var}\left[h_j^{'}(\mathbf{x})\right]}, j > 1 \tag{21}$$

Table 2 lists the minimum and maximum values, the mean, and the standard deviation of the designed explanatory functions. The proposed probabilistic model can be used for cylindrical steel tanks whose explanatory functions fall within the ranges in Table 2. However, if additional data become available, the model parameters can be updated to expand the range of applicability of the model.

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Table 2: Statistics of the designed explanatory functions.

Explanatory function	Min	Max	Mean	Standard deviation
$h_2^{'}(\mathbf{x})$	0.04	35.95	5.83	9.16
$h_3^{'}(\mathbf{x})$	0.00104	0.00145	0.00128	0.00016
$h_4^{'}(\mathbf{x})$	905	2400	1121.35	463.95
$h_5^{'}(\mathbf{x})$	0.51	4	2.10	1.32
$h_6^{'}(\mathbf{x})$	3.52	11.44	5.49	1.80
$h_7^{'}(\mathbf{x})$	0.005	0.94	0.18	0.25

4.3 Bayesian model calibration and model selection

In this section, we reduce the number of $h_j(\mathbf{x})$' s in $\gamma(\mathbf{x}, \boldsymbol{\theta})$ using the stepwise deletion process described in Section 3.2 to obtain a parsimonious model. Figure 4 shows the posterior COV of the model parameters $\boldsymbol{\theta}$ and the posterior mean of σ at each iteration in the stepwise deletion process.

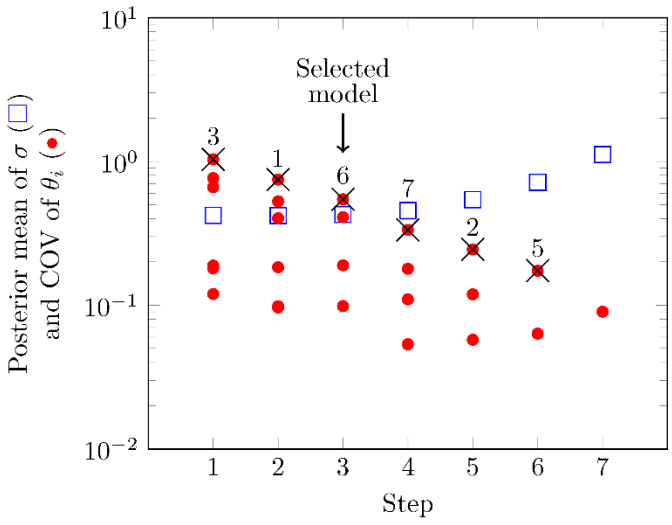

Figure 3: Stepwise deletion process of the proposed probabilistic model.

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We stop the stepwise deletion process at Step 3, resulting in the final form $\gamma(\mathbf{x}, \boldsymbol{\theta})$ as

$$\gamma(\mathbf{x}, \boldsymbol{\theta}) = \theta_2 h_2(\mathbf{x}) + \theta_4 h_4(\mathbf{x}) + \theta_5 h_5(\mathbf{x}) + \theta_7 h_7(\mathbf{x}) \quad (22)$$

Table 3 lists the posterior statistics of $\boldsymbol{\Theta}$.

Table 3: Posterior statistics of the proposed probabilistic model.

Parameter	Mean	Standard deviation	Correlation coefficient				
			θ_2	θ_4	θ_5	θ_7	σ
θ_2	0.73	0.13	1				
θ_4	–1.99	0.11	0.46	1			
θ_5	1.29	0.14	0.59	0.45	1		
θ_7	0.43	0.14	0.63	0.35	0.71	1	
σ	0.45	0.07	0.03	0.02	0.05	0.04	1

Figure 5 shows a comparison between the measured and predicted values of the maximum displacements for all the cylindrical tanks in the database. In Figure 5, the left plot shows the predictions based on the deterministic model, whereas the right plot shows the predictions based on the probabilistic model. For the probabilistic model, median predictions ($\varepsilon = 0$) are shown, and the dotted lines delimit the region within 1 standard deviation of the median.

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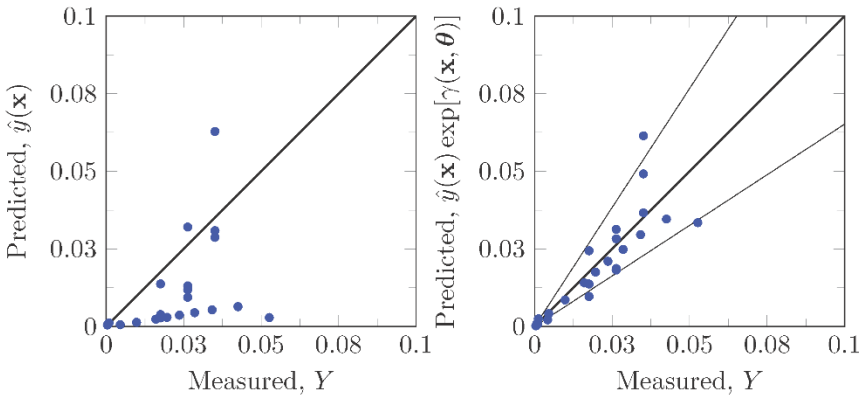

316

317 *Figure 4: Comparison between measured and predicted values based on deterministic (left) and probabilistic (right)*
318 *models.*

319

320 The deterministic model on the left is strongly biased since it underestimates the maximum
321 displacement due to the blast load, i.e., most of the data lie below the 1:1 line. Instead, the
322 probabilistic model on the right corrects this bias and properly accounts for all the prevailing
323 uncertainties.

324 **5 Fragility functions using the proposed probabilistic demand model**

325 Fragility functions are defined as the conditional probability of attaining or exceeding a specified
326 performance level given a (set of) intensity measure(s) (Gardoni et al. 2002, 2003, Nocera et al.
327 2019). Gardoni et al. (2002, 2003) proposed to develop fragility functions by conducting a
328 rigorous reliability analysis considering limit-state function(s) expressed in terms of the structural
329 capacity and the corresponding imposed demands. After developing the physics-based
330 probabilistic demand model according to Equation (1), we can write the limit-state function as
331 (Gardoni 2017)

$$g(\mathbf{x},\boldsymbol{\Theta}) = C(\mathbf{x}) - D(\mathbf{x},\boldsymbol{\Theta}) \tag{23}$$

332 where C and D are the capacity and demand models associated with the considered failure mode.

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For the capacity, performance thresholds (e.g., PDC-TR 06-08) can be defined considering the value of bending rotation α, see Table 4. Alternatively, one could develop a probabilistic capacity model following the general form in Equation (1).

Table 4: Bending rotation performance thresholds.

Component Damage Level	α [°]
Blowout	12
Hazardous Failure	6
Heavy Damage	2

Similarly, we translate the estimates of the maximum displacement into the corresponding bending rotation. We linearize the curvature distribution around the maximum displacement, and we estimate the bending rotation α via geometrical considerations. Figure 3 shows the bending rotation scheme along the cylindrical tank perimeter and the geometrical characteristics used to estimate α. In Figure 3, δw_{MAX} is the tank maximum radial displacement pointed by ω_c in the cylindrical coordinate system and δw_{min} represents the closest minimum distance between the deformed tank and the tank center. Both δw_{MAX} and δw_{min} are obtained with the model presented in the previous sections.

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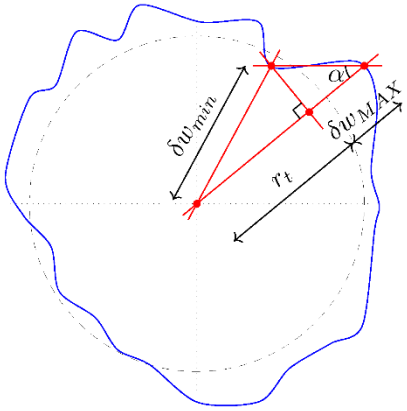

Figure 5: Bending rotation scheme along the cylindrical tank perimeter.

We can then write α as

$$\alpha = \tan^{-1} \frac{(r_t - \delta w_{min}) \sin(\omega_c)}{r_t + \delta w_{MAX} - (r_t - \delta w_{min}) \cos(\omega_c)} \tag{24}$$

where r_t is the tank radius.

The proposed performance thresholds can be linked to currently applied approaches for defining release scenarios in the case of quantitative risk analysis. As a first approximation, we can refer to the Loss Of Containments events LOCs for stationary single-containment atmospheric tanks (see Uijt et al. 2005, and Landucci et al. 2020). For example, the blowout threshold can be equal to the instantaneous release of the complete inventory to the atmosphere, the Hazardous Failure threshold can be equal to the continuous release of the complete inventory to the atmosphere at a constant rate of release in 10 min, while the Heavy Damage threshold is equal to the continuous release from an equivalent release diameter of 10-50 mm.

Next, let $\mathbf{x} = (\mathbf{r}, z)$ where $\mathbf{r}$ is the set of state variables that characterizes the system like material properties, and z is the scaled distance that represent chosen intensity measure, we can then write the fragility function as

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$$F(z, \boldsymbol{\Theta}) = P[g(\mathbf{x}, z, \boldsymbol{\Theta}) \leq 0 | z, \boldsymbol{\Theta}] \quad (25)$$

362 Depending on the treatment of uncertainty in $\boldsymbol{\Theta}$, we can obtain two estimates of fragility
363 functions (Gardoni et al. 2002). The first option is to neglect the uncertainty in $\boldsymbol{\Theta}$ and use a fixed
364 value $\widehat{\boldsymbol{\Theta}}$ (e.g., the posterior mean or mode of $\boldsymbol{\Theta}$) and obtain a point-estimate of the fragility $\hat{F}(z) =$
365 $F\left(z, \widehat{\boldsymbol{\Theta}}\right)$. The second option is to account for the uncertainty in $\boldsymbol{\Theta}$ and estimate a predictive estimate
366 of the fragility $\tilde{F}(z)$ as

$$\tilde{F}(z) = \int F(z, \boldsymbol{\Theta}) f(\boldsymbol{\Theta}) d\boldsymbol{\Theta} \quad (26)$$

367 Furthermore, following Gardoni et al. (2002), we obtain confidence bounds on the estimate in
368 Equation (10). We can define the reliability index as

$$\beta(z, \boldsymbol{\Theta}) = \Phi^{-1}[1 - F(z, \boldsymbol{\Theta})] \quad (27)$$

369 where $\Phi^{-1}[\cdot]$ is the inverse of the standard Normal cumulative density function. The variance of
370 $\beta(z, \boldsymbol{\Theta})$ can then be estimated as (Gardoni et al. 2002)

$$\sigma_{\beta}^2(z) \cong \nabla_{\boldsymbol{\Theta}} \beta(z) \boldsymbol{\Sigma}_{\boldsymbol{\Theta}\boldsymbol{\Theta}} \nabla_{\boldsymbol{\Theta}} \beta(z)^{\mathsf{T}} \quad (28)$$

371 where $\nabla_{\boldsymbol{\Theta}} \beta(z)$ is the gradient of $\beta(z, \boldsymbol{\Theta})$ evaluated at the mean value; and $\boldsymbol{\Sigma}_{\boldsymbol{\Theta}\boldsymbol{\Theta}}$ is the estimated
372 covariance matrix. The gradient vector $\nabla_{\boldsymbol{\Theta}} \beta(z)$ is obtained by performing a first-order reliability
373 method (FORM) analysis (Ditlevsen and Madsen 1996). Therefore, we can obtain bounds on $\tilde{F}(z)$
374 in terms of the desired number of standard deviations away from the mean. For example, bounds
375 representing approximately 15% and 85% probability levels can be expressed as

$$\left\{\Phi\left[-\tilde{\beta}(z) - \sigma_{\beta}(z)\right], \Phi\left[-\tilde{\beta}(z) + \sigma_{\beta}(z)\right]\right\} \quad (29)$$

376 where $\tilde{\beta}(z) = \Phi^{-1}[\tilde{F}(z)]$.

377

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6 Fragility functions of an example tank and reliability of an industrial plant

In this section, we develop the fragility functions of an example cylindrical steel tank subject to blast loading. A significative case study is represented by the water tanks for cooling tower systems in chemical industries. Then, we use the developed fragility functions to estimate the reliability of a typical industrial plant. Table 5 summarizes the geometry of the considered cylindrical steel tank and material properties along with their statistical information. The distance between the tank and the TNT explosive charge is 20 m to satisfy the planar blast wave hypothesis. The scaled distance z varies from 3.5 to 8 $\mathrm{m/kg}^{1/3}$ by changing the mass of the TNT explosive charge.

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Table 5: Geometry and material properties of the example tank.

Variable	Value	COV	Distribution
$d\ [m]$	18	-	-
$h\ [m]$	12	-	-
$e\ [mm]$	5	-	-
$E_s\ [GPa]$	210	-	-
$f_{yd}\ [MPa]$	235	0.05	Lognormal
ν	0.3	-	-
$\rho\ [kg/m^3]$	7850	-	-

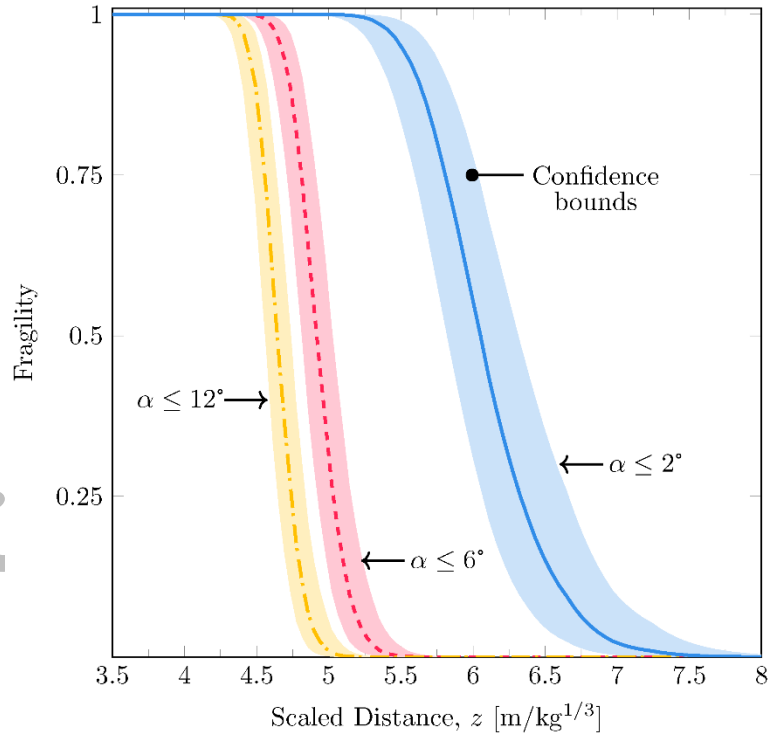

Figure 6: Fragility functions for the three capacity performance thresholds.

Figure 6 shows the computed fragility functions for the three capacity performance thresholds listed in Table 4. The solid curves in the figure represent the predictive estimate of the fragility

function $\tilde{F}(z)$. The shaded area around the predictive estimate represents the 15 and 85% confidence bounds due to the uncertainty in the estimate of the model parameters $\boldsymbol{\Theta}$.

The computed fragility functions can be useful in the early design of a tank, in the risk analysis of an industrial plant containing the tank, or, in a more general setting, in a supply chain reliability analysis where the tank could represent one of the elements in the supply chain. To illustrate, we use the developed fragility functions to estimate the reliability of a typical industrial plant for three different threat scenarios. Figure 7 shows the layout of the example industrial plant, including the location of the tanks and their relative distances and the location of the TNT explosive charge for the three considered threat scenarios. Tables A1-A3 (in the Appendix) show the values of z for each tank. In this example, we assume the tanks contain non-inflammable material (i.e., water) so that cascading explosive charges do not occur. However, the formulation can be expanded to account for domino effects studies because (i) the formulation is developed by bringing in the physics of the problem, and (ii) the modeling is done considering state variables, which describe the dynamic state of tanks. For instance, the initial explosion of a tank may affect the state variables in the developed demand model. Specifically, shock deterioration models (e.g., Iannacone and Gardoni 2019) can be tailored to the case of industrial steel tanks under blast loading to implement the proposed approach in a more comprehensive quantitative risk analysis framework.

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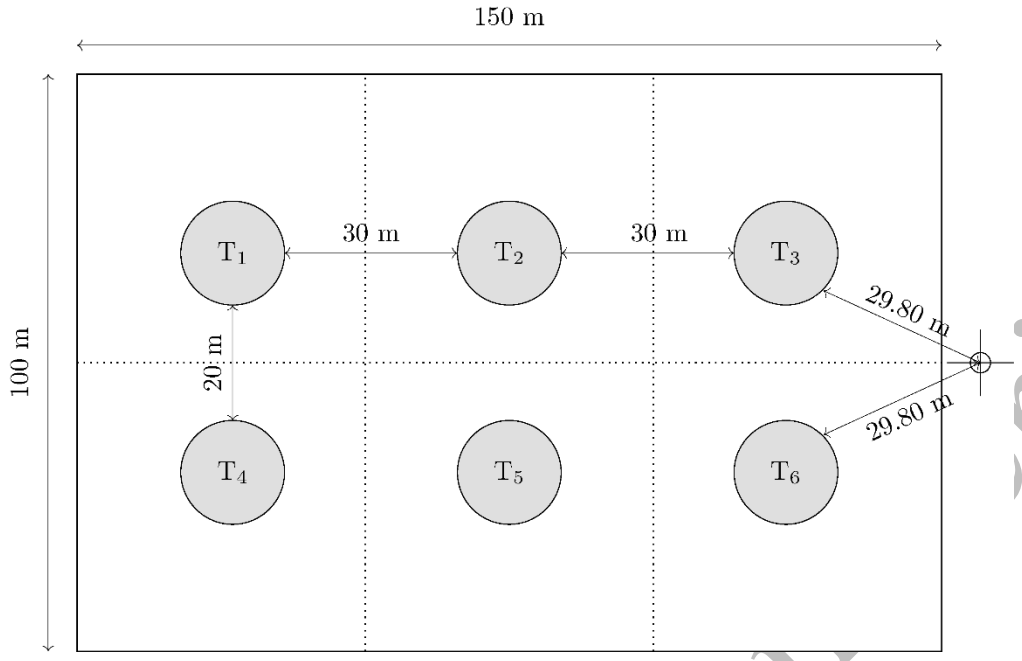
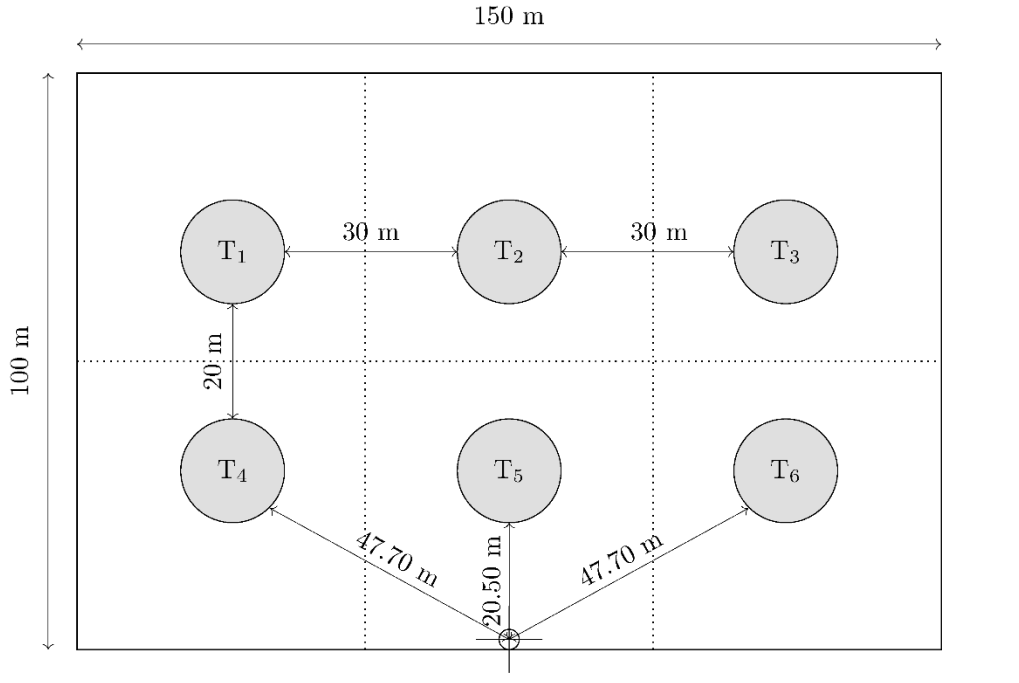


(a) First scenario



(b) Second scenario

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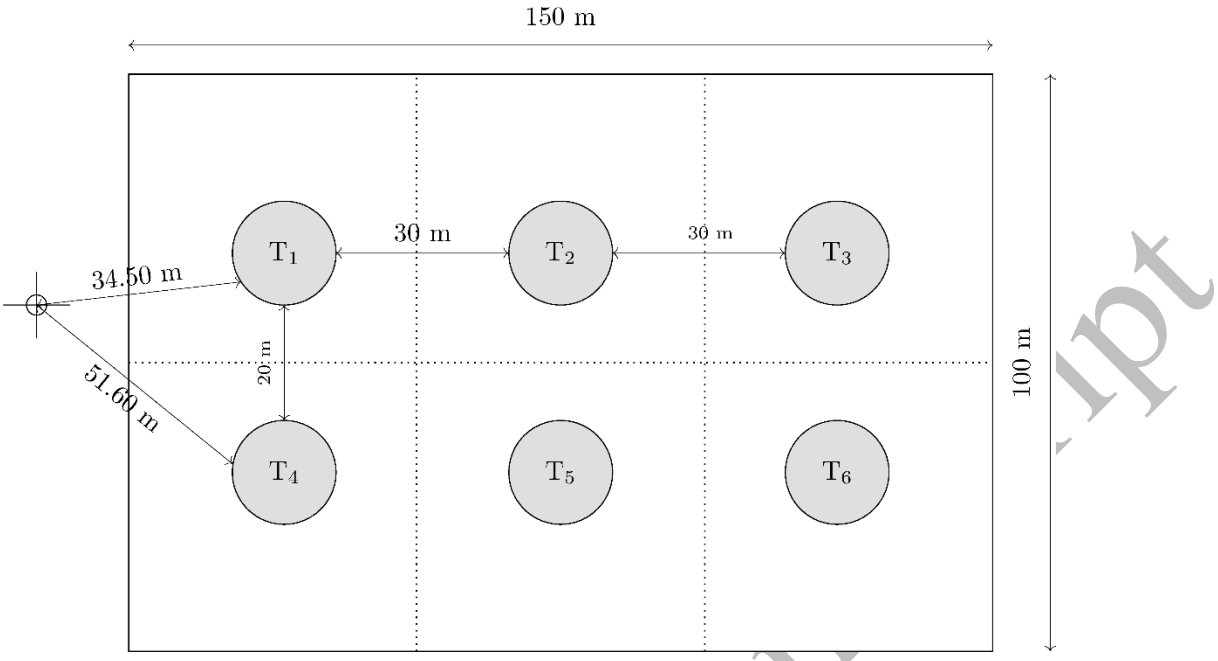

(c) Third Scenario

Figure 7: Layout of the example industrial plant and location of the TNT explosive charge for the three considered scenarios.

Using the fragility functions in Figure 6, we then estimate the probabilities of each tank being in each damage state based on the definition in Table 4, i.e., heavy damage, hazardous failure, and blowout. Figure 8 shows the considered tanks in the industrial plant labeled according to the most likely damage state due to the blast loading.

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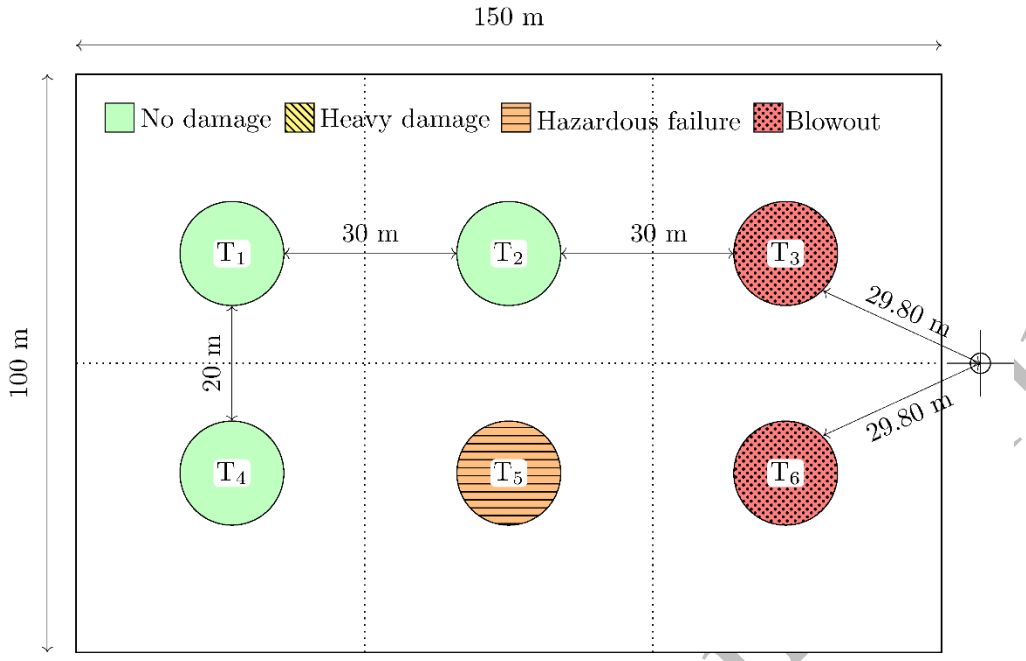
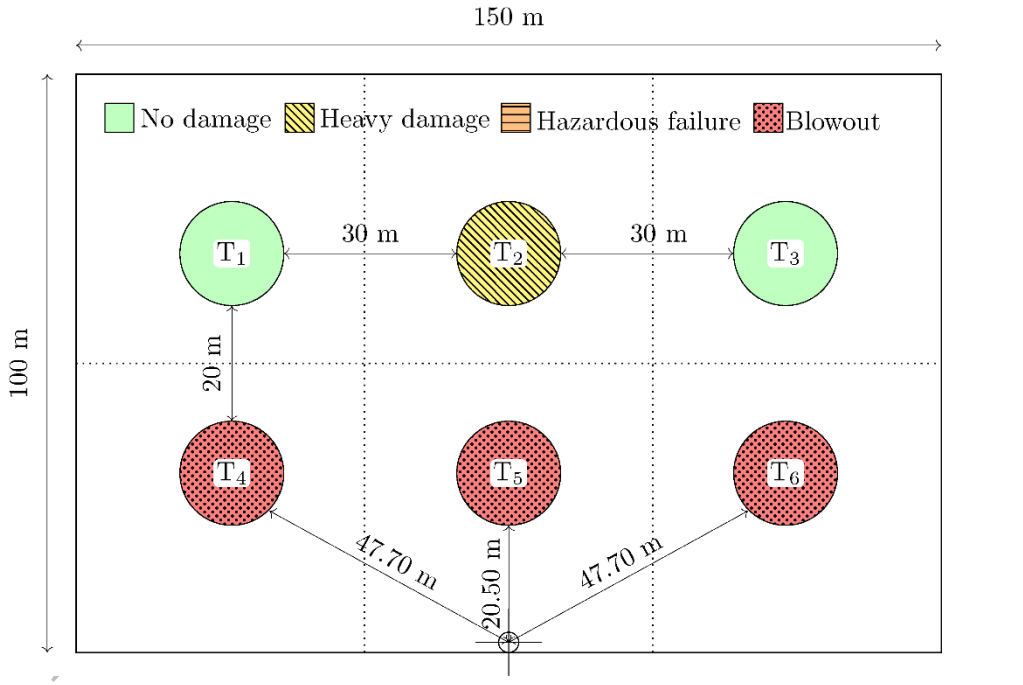


(a) First scenario



(b) Second scenario

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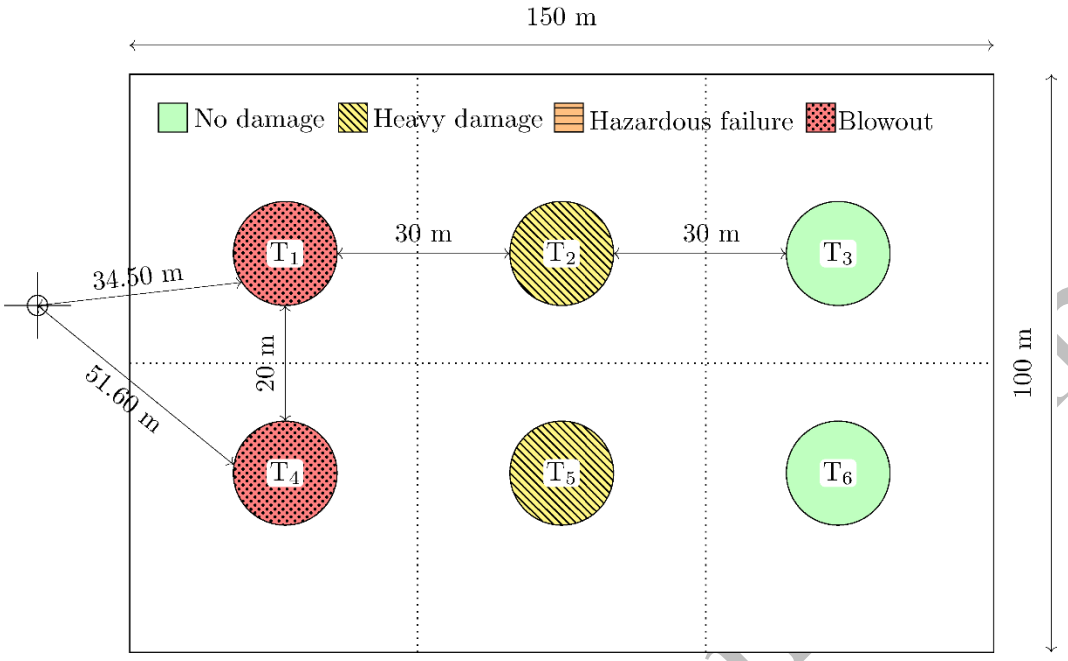

(c) Third scenario

Figure 8: Most likely damage state of each tank in the example industrial plant.

In the first scenario, Tanks 1, 2 and 4 do not suffer any damage; Tanks 3 and 6 are most likely to be in the blowout damage state, whereas Tank 5 is most likely to be in the hazardous failure domain. Similarly, in the second scenario considered, Tanks 1 and 3 do not suffer any damage, whereas Tank 2 is most likely heavily damaged. All the remaining tanks are expected to be in the blowout damage state. Lastly, in the third scenario, Tanks 3 and 6 do not suffer any damage; Tanks 2 and 5 are most likely to be heavily damaged, and Tanks 1 and 4 are expected to be in the blowout damage state. In all the considered scenarios, if the scaled distance z is at least equal to 6.20 the tanks are expected not to experience any damage. Similarly, for values of z less or equal to 4.60 the tanks are expected to be in the most severe damage state.

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7 Conclusions

Blast loading represents a severe threat to industrial facilities. During their service lives, the component of industrial facilities such as tanks might be subject to blast loading from malicious or accidental explosions. The paper proposed a novel probabilistic demand model for industrial steel tanks under blast loading. The proposed probabilistic demand model combines the predictions of a simplified deterministic model with an analytical correction term. We used a generalized single-degree-of-freedom (SDOF) system as the simplified deterministic model based on shallow-shell theory. Then, we developed an analytical correction term that captures the physical characteristics not fully represented by the deterministic model. The model also included an error term that captures unexplained uncertainty. We used Bayesian inference to estimate the unknown model parameters in the correction and the error terms from experimental data found in literature. Then, we used the proposed demand model to estimate the fragility functions of a typical cylindrical steel tank. The computed fragility functions can be useful in the early design of a tank, in the risk analysis of an industrial plant containing the tank, or, in a more general setting, in a supply chain reliability analysis where the tank could represent one of the components in the supply chain. To illustrate, we used the derived fragility functions to estimate the reliability of a typical industrial plant under three different malicious explosion scenarios.

Further developments of this research are expected considering the cascading effects of the damage to the tanks on the performance of industrial facilities at the plant level and the supply chain.

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612 **Nomenclature**

a	Vertical abscissa in the cylindrical coordinate system
B	Numerical parameter equal to $E_s e^3/(12(1-\nu^2))$,
$C(\mathbf{x})$	Capacity model associated the performance thresholds
$D(\mathbf{x},\boldsymbol{\Theta})$	Demand model
d	Tank diameter
E_s	Steel elastic modulus
e	Tank thickness
$F_A(s,\omega,a)$	Airy stress function
$F(\mathbf{s},\boldsymbol{\Theta})$	Fragility function
$\hat{F}(\mathbf{s})$	Point-estimate of the fragility
$\tilde{F}(\mathbf{s})$	Predictive estimate of the fragility
f_{dyn}	Steel dynamic yielding strength
f_{stat}	Steel static yielding strength
$f(\boldsymbol{\Theta})$ i	posterior PDF of $\boldsymbol{\Theta}$
h	Tank height [m]
$h_{k,j}(\mathbf{x})$	Explanatory functions
$g(\mathbf{x},\boldsymbol{\Theta})$	Limit-state function
I_{sO}	Incident impulse
K	Equivalent stiffness
$L(\boldsymbol{\Theta})$	Likelihood function that captures the information on $\boldsymbol{\Theta}$

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M	Equivalent load
N_s	Distributed axial stress
$N_{s\omega}$	Distributed shear stress
N_ω	Distributed circumferential stress
P_{ref}	Reflected peak pressure [kPa]
P_{atm}	Atmospheric pressure
P_{sO}	Stand-off pressure
$p(\boldsymbol{\Theta})$	Prior distribution reflecting our state of knowledge about $\boldsymbol{\Theta}$
Q_n	Numerical parameter equal to $(16Dn^4/d^4 + Ee\pi^4d^2/(4n^4H^4))/(\rho e)$
q	Blast load pressure
$\mathbf{R}$	Correlation matrix
R	Stand-off distance
$r_{ki}(\boldsymbol{\theta}_k)$	Residual of the prediction for the i^{th} datum
r_t	Tank radius
s	Radial abscissa in the cylindrical coordinate system
t	Time
t_d	Positive phase duration
$T_k(\cdot)$	Transformation function
$Y_k(\mathbf{x}, \boldsymbol{\Theta}_k)$	k^{th} quantity of interest
$\hat{y}_k(\mathbf{x})$	Existing deterministic model to predict Y_k
u	Axial displacement

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v	Circumferential displacement
W	Mass of explosive in kg of equivalent TNT
w	Radial displacement
$\mathbf{x}$	Vector of basic variables that characterize Y_k
z	Scaled distance
α	Bending rotation
$\beta(\mathbf{s}, \boldsymbol{\Theta})$	Reliability index
$\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k)$	Correction term constructed to improve the prediction of $\hat{y}_k(\mathbf{x})$
ε_k	Standard normal random variable
$\boldsymbol{\theta}$	Vector of model parameters
$\boldsymbol{\Theta}_k = (\boldsymbol{\theta}_k, \sigma_k)$	Vector of unknown model parameters
κ	Normalizing constant.
λ	Parameter equal to $2nh/\pi d$
$\mu_{\theta_{k,i}}$	Posterior mean of $\theta_{k,i}$
ν	Poisson coefficient
ρ	Specific mass
$\boldsymbol{\Sigma}$	Covariance matrix
σ_k	Standard deviation of the model error
$\sigma_{\theta_{k,i}}$	Standard deviation of $\theta_{k,i}$.
$\Phi(\cdot)$	Standard Normal cumulative density function
$\varphi(\cdot)$	Standard Normal probability density function
Ω_n	Numerical parameter equal to $(16Dn^4/(d^4) + Ee\pi^4d^2/(4n^4H^4))/(\frac{2n^2}{d})$

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ω	Angular coordinate in the cylindrical coordinate system
ω_c	Angular distance between maximum and minimum deflection

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Appendix

Table A1. Values of z for each tank in the first scenario

Tank	z
1	8.20
2	4.20
3	4.61
4	7.23
5	4.89
6	2.69

Table A2. Values of z for each tank in the second scenario

Tank	z
1	6.66
2	5.57
3	6.61
4	3.78
5	1.61
6	3.71

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619

620 Table A3. Values of z for each tank in the third scenario

Tank	z
1	2.26
2	5.40
3	8.56
4	3.40
5	6.02
6	8.97

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