Forward dynamics of the five-bar parallel mechanism in presence of singularities

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Abstract. Simulation of motion of dynamic systems is addressing the interest of the robotic and mechatronic communities with the goal to have a full digitalization of the physical system (digital twin). The final goal consists of exchanging data from/to the system and to use the model as predictor and even corrector of the system motion. The first step of the application described is to implement a dynamic model able to simulate the behaviour of the system under several conditions. This paper deals with the solution of the forward dynamics problem of a planar five-bar parallel mechanism in presence of singular configurations. In the paper the index-3 augmented Lagrangian formulation with velocity and acceleration projections is implemented to overcome the kinematic combined and the constraint singularities.

Keywords: multibody system · singularities · ODE.

1 Introduction

The two degrees of freedom five-bar planar parallel manipulator addressed in this paper, was extensively studied and finds applications for pick and place operations. A kinematic analysis was first developed by Liu et al. [1] who investigated the workspace, singularities of the mechanism. They solved the inverse and forward position problems determining the working and assembly modes of the mechanism. In the paper the maximal inscribed workspace (MIW) was detected as the maximum workspace with no singularities. Further, they conducted a deep investigation on the nature of the singularities of the mechanisms. They found the stationary (open chain) singularities, a.k.a. inverse, the uncertainty (closed) singularities, a.k.a. direct and the combined singularities. Campos et al. [2] built DexTAR, a double SCARA robot (a.k.a. five-bar parallel mechanism) with all four links with equal lengths. With this geometry the number of singularities increases such that the authors had to implement a strategy for avoiding them by switching working modes. DexTAR has a workspace at least one third larger than that of a similarly sized five-bar parallel robot with the ability of crossing the open chain singularities. Another mechatronic system dedicated to control a five-bar parallel mechanism was then proposed by Giberti et al. [3]. They built a laboratory prototype with coaxial motors, i.e., the ground link with zero length.

This geometry was selected as outcome of a kinematic synthesis based on the optimization of the condition number of the kinematic Jacobian.
Finally, it is worth mentioning the great attention addressed to this mechanism by the research groups in the field of the biomechanics. For example, recently, Yamine et al. [4] proposed the mechanism as an alternative design to the planar rehabilitative robots as the MIT-Manus. The mechanism seems to be an acceptable compromise in terms of workspace symmetry with respect to the sagittal plane, relatively large workspace, portability and affordability. Although the five-bar planar parallel manipulator has been extensively studied and has numerous applications, there is a lack of work that addresses the solution of the forward dynamics problem, that can be useful as simulation tool, but also as a step of a design process, not based only on kinematic parameters, and it can even become mandatory for applications where the mechanism is driven by the operator as reported in [5] for the rehabilitation mirror therapy, i.e., rehabilitative machines, haptic devices costituited by spatial mechanisms as reported in [6]. Further it may represent a first step needed to build a digital twin of the system to finally predict the behaviour and eventually the faults. In other words, solution of the forward dynamic problem may allow building a cyber-physical sytem.
However, simulation of the motion of a mechanism may become challenging for the presence of singular configurations [7]. Indeed, the goal of this paper is to solve the forward dynamics problem for the five-bar mechanism by implementing a numerical algorithm able to cross the singular configurations. Further, in the paper, the singular configurations of the mechanism are clearly classified distinguishing between kinematic and constraint singularities.
The paper is organized as follows. First a review of the numerical algorithm implemented is presented, then a decription of the mechanism with its geometry and with some details on the nature of the singularities is given. Finally, the results from the simulations are shown and the conclusions are drawn.

2 Index-3 Augmented Lagrangian Formulation

In this section a brief description of the method used to model the dynamics of the mechanism is presented. The method was developed by Bayo [8–10] and extensively applied in [11–13].
Equation of motion of a mechanical system described by a set of n Cartesian coordinates $\mathbf{q}(t)$ subjected to m holonomic kinematic constraints $\mathbf{\Phi}(\mathbf{q}(t))$ can be expressed as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T \boldsymbol{\lambda} = \mathbf{f}. \tag{1}$$

In Eq. (1), $-\mathbf{\Phi}_q^T \boldsymbol{\lambda}$ represents the constraints forces with $\mathbf{\Phi}_q = \partial \mathbf{\Phi} / \partial \mathbf{q}$ the Jacobian of the constraints and $\boldsymbol{\lambda}$ vector of the unknown Lagrangian multipliers. $\mathbf{M}$ is the mass matrix that becomes constant according to the coordinates used, $\mathbf{f}$ is the vector of forces applied to the system.
Eq. (1) can be treated according to numerous formulations in order to convert the differential-algeabric into the standard ordinary-differential equations (ODE). In

this paper we follow the penalty formulation combined with the Newmark integration scheme to obtain the index-3 Lagrangian augmented formulation. In the penalty formulation the constraint is modeled as a spring-damper unit such that

$$\boldsymbol{\lambda} = \alpha(\ddot{\boldsymbol{\Phi}} + 2\mu\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi}), \tag{2}$$

where α is the penalty factor, μ and ω are the Baumgarte stabilization parameters. By substituting $\ddot{\boldsymbol{\Phi}} = \dot{\boldsymbol{\Phi}}_q\dot{\mathbf{q}} + \boldsymbol{\Phi}_q\ddot{\mathbf{q}}$, $\dot{\boldsymbol{\Phi}} = \boldsymbol{\Phi}_q\dot{\mathbf{q}}$ into Eq. (2), we obtain the ODE:

$$(\mathbf{M} + \boldsymbol{\Phi}_q^T\alpha\boldsymbol{\Phi}_q)\ddot{\mathbf{q}} = \mathbf{f} - \boldsymbol{\Phi}_q^T\alpha(\dot{\boldsymbol{\Phi}}_q\dot{\mathbf{q}} + 2\mu\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi}) \tag{3}$$

Eq. (3) can be written by following the Lagrangian multiplier method leading to the index-1 iterative algorithm:

$$\begin{aligned}(\mathbf{M} + \boldsymbol{\Phi}_q^T\alpha\boldsymbol{\Phi}_q)\ddot{\mathbf{q}} + \boldsymbol{\Phi}_q^T\boldsymbol{\lambda}^* &= \mathbf{f} - \boldsymbol{\Phi}_q^T\alpha(\dot{\boldsymbol{\Phi}}_q\dot{\mathbf{q}} + 2\mu\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi}) \\ \boldsymbol{\lambda}_{i+1}^* &= \boldsymbol{\lambda}_i^* + \alpha(\ddot{\boldsymbol{\Phi}} + 2\mu\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi})\end{aligned} \tag{4}$$

The algorithm in Eq. (4) is then combined with the implicit single-step trapezoidal rule:

$$\dot{\mathbf{q}}_{k+1} = \frac{2}{h}\mathbf{q}_{k+1} + \hat{\dot{\mathbf{q}}}_k \quad \text{with} \quad \hat{\dot{\mathbf{q}}}_k = -(\frac{2}{h}\mathbf{q}_k + \dot{\mathbf{q}}_k),$$

$$\ddot{\mathbf{q}}_{k+1} = \frac{2}{h^2}\mathbf{q}_{k+1} + \hat{\ddot{\mathbf{q}}}_k \quad \text{with} \quad \hat{\ddot{\mathbf{q}}}_k = -(\frac{2}{h^2}\mathbf{q}_k + \frac{4}{h}\dot{\mathbf{q}}_k + \ddot{\mathbf{q}}_k).$$

and with the velocity and acceleration projections such that dynamic equilibrium can be established at time step $k+1$ as:

$$\mathbf{M}\mathbf{q}_{k+1} + \frac{h^2}{4}\boldsymbol{\Phi}_{q_{k+1}}^T(\alpha\boldsymbol{\Phi}_{k+1} + \boldsymbol{\lambda}_{k+1}^*) - \frac{h^2}{4}\mathbf{f}_{k+1} + \frac{h^2}{4}\mathbf{M}\hat{\ddot{\mathbf{q}}}_k = \mathbf{0} \tag{5}$$

The system of nonlinear equations in Eq. (5) is then solved via Newton-Raphson iterative approach (i is the iteration counter)

$$\left[\frac{\partial\mathbf{f}(\mathbf{q})}{\partial\mathbf{q}}\right]_i \Delta\mathbf{q}_{i+1} = [\mathbf{f}(\mathbf{q})]_i\,,$$

with

$$[\mathbf{f}(\mathbf{q})] = \frac{h^2}{4}(\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_q^T\alpha\boldsymbol{\Phi}_q + \boldsymbol{\Phi}_q^T\boldsymbol{\lambda}^* - \mathbf{f})$$

and the approximated tangent matrix

$$\left[\frac{\partial\mathbf{f}(\mathbf{q})}{\partial\mathbf{q}}\right] = \mathbf{M} + \frac{h^2}{4}(\boldsymbol{\Phi}_q^T\alpha\boldsymbol{\Phi}_q).$$

What is relevant in the formulation presented is that the leading matrix in Eq. (3) is always invertible although the constraint Jacobian $\boldsymbol{\Phi}_q$ becomes rank deficient whenever the mechanism crosses a constraint singular configuration.

3 Description of the mechanism

The planar five-bar parallel mechanism is shown in Figure 1. The mechanism is commonly used for positioning point Q on the plane. The model considered has the moving links with $L = 210\ mm$ with uniform mass $m = 0.017\ Kg$ and moment of inertia of $I = 6.23 \cdot 10^{-5}\ Kgm^2$. In order to implement the algorithm

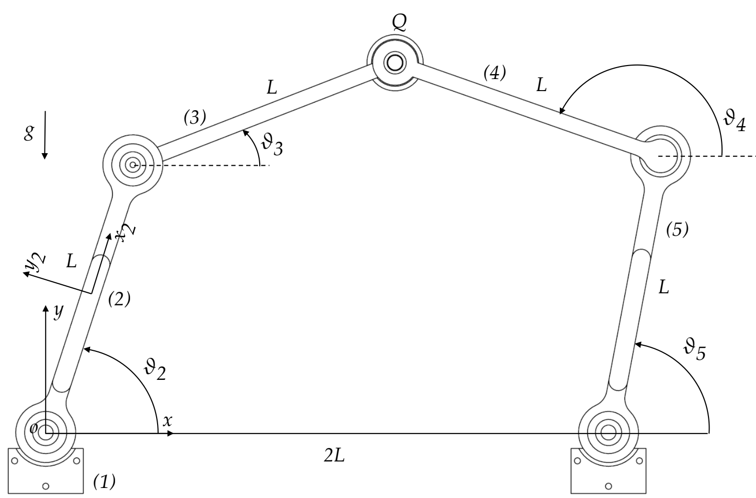

Fig. 1. Planar five-bar parallel mechanism.

presented, a reference system was attached to each link, thus the mechanism motion was parametrized by a vector

$$\mathbf{q} = \left(r_{x_1}, r_{y_1}, \theta_1, r_{x_2}, r_{y_2}, \theta_2, \ldots, r_{x_{n_b}}, r_{y_{n_b}}, \theta_{n_b}\right)^{\mathrm{T}},$$

where $(r_{x_i}, r_{y_i}, \theta_i)$, $i = 1, \ldots, n_b$, identifies the position of the reference system with origin in the center of mass of the ith body and $n_b = 5$.

3.1 Constraint and Kinematic Singular Configurations

The ability of the the algorithm used to simulate the mechanism motion is to overcome the constraint singularities. Constraint singularities occur whenever $\mathbf{\Phi}_q$ loses full rank. That means that the constraint equations are no longer linear independent, one or more constraints are needless. This fact has impact on the mobility of the mechanism that, in a singular configuration, may change its dofs and may switch the working mode. On the other hand, the mechanism motion

presents kinematic singularities, as well. Usually, kinematic and constraint singular configurations are addressed by the kinematic and dynamic communities independently. In this case we treat them all together to try to clarify their connection in the case of the five-bar mechanism.
It is well known that for a parallel mechanism the velocity mapping is expressed by

$$\mathbf{J}_x \dot{\mathbf{x}} = \mathbf{J}_q \dot{\mathbf{q}}.$$

$\mathbf{x} = (x, y)^{\mathrm{T}}$ is the vector of the point Q coordinates, $\mathbf{J}_x$, $\mathbf{J}_q$ are the Jacobian matrices given as:

$$\mathbf{J}_x = \begin{bmatrix} L(yc_{\theta_2} - xs_{\theta_2}) & 0 \\ 0 & L(yc_{\theta_4} - xs_{\theta_4} + 2Ls_{\theta_4}) \end{bmatrix},$$
$$\mathbf{J}_q = \begin{bmatrix} x - Lc_{\theta_2} & y - Ls_{\theta_2} \\ x - Lc_{\theta_4} - 2L & y - Ls_{\theta_4} \end{bmatrix} \quad (6)$$

and

$$x = r_{x_3} + \frac{L}{2} c_{\theta_3}, \quad y = r_{y_3} + \frac{L}{2} s_{\theta_3}.$$

The mechanism can attain three kinematic singularities during its motion:

- Direct singularity: the null space of $\mathbf{J}_x$ is not empty, that is $det(\mathbf{J}_x) = 0$;
- Inverse singularity: the null space of $\mathbf{J}_q$ is not empty, that is $det(\mathbf{J}_q) = 0$;
- Combined singularity: both the null space of $\mathbf{J}_x$ and of $\mathbf{J}_q$ are not emplty, that is $det(\mathbf{J}_x) = det(\mathbf{J}_q) = 0$. In the combined singular configurations the number of solutions either of the forward or inverse kinematics change.

For the geometry under study, combined singular configurations take on importance as it was found from the simulations that they coincide with the constraint singularities, hereafter they will be called combined-constraint singularities.

4 Numerical Simulations

Two simulation tests were performed. The former was run to validate the implementation of the algorithm i3-ALP in our own code, the latter to stress our code when the mechanism passes through combined-constraint singularities. In the i3-ALP algorithm we chose $\alpha = 10^7$ and a fixed time integration step of $h = 0.001\ s$.

4.1 Code validation

In this case an external torque of 0.25 Nm was applied to link (2) with initial configuration given by $\theta_2 = \theta_5 = \pi/2$ and $\dot{\mathbf{q}} = \mathbf{0}$. The simulation time was set to $t_{span} = 2.5\ s$. The results from the i3-ALP algorithm were compared with those obtained by a code written via a commercial software, namely Simscape, with Runge-Kutta order 4 as integration method and same fixed time integration

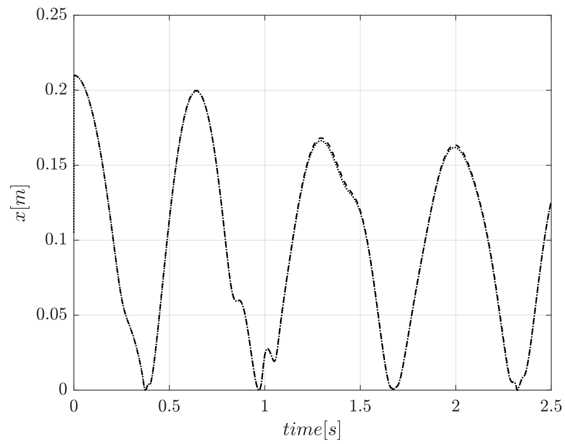

Fig. 2. $x(t)$: $--$ RK4 commercial, $\cdots$ i3-ALP.

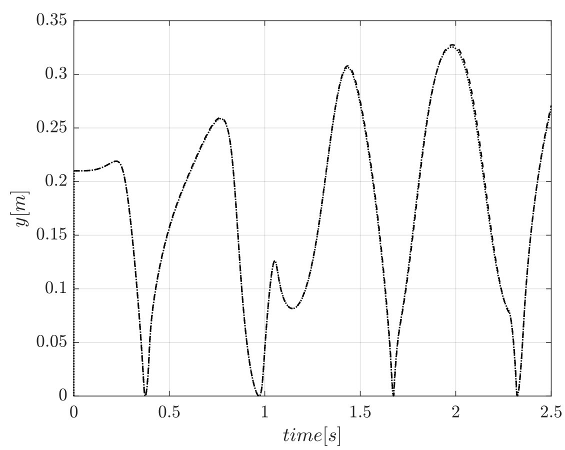

Fig. 3. $y(t)$: $--$ RK4 commercial, $\cdots$ i3-ALP.

step.
Figures 2, 3 show $x(t)$, $y(t)$ obtained from the simulations. It is evident from the plots that our code produces results almost coincident with those from the commercial software.

4.2 Tests on Singularities

In this case the mechanism was subjected only to gravity leaving it to fall freely. This motion can be considered very challenging from a numerical point of view as the mechanism may attain any kind of singular configurations. The types of singularities met was checked calculating the singular values of the Jacobians, $\mathbf{J}_x$,

$\mathbf{J}_q$ and $\boldsymbol{\Phi}_q$. The simulation parameters were held identical to those in the validation test except for the simulation time that was extended to $t_{span} = 50\ s$. The combined-constraint singularities occured, for example, at $t = 41.84\ s$ without stopping abrutely the computation. Figures 4, 5 show $x(t)$ and $y(t)$, respectively. The plots show that the trajectories have no discontinuities at the singular configuration. Figure 6 shows the mechanism at the combined-constraint singular

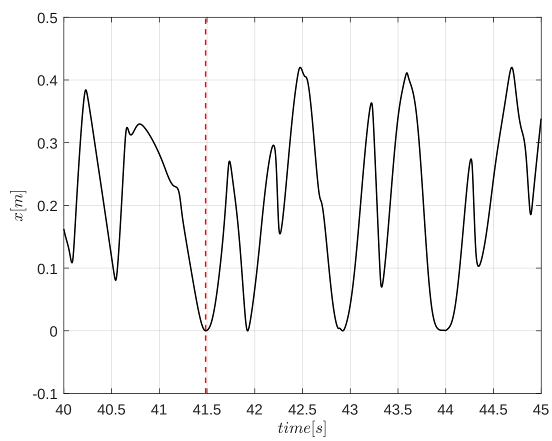

Fig. 4. x(t) crossing the combined-constraint singularity.

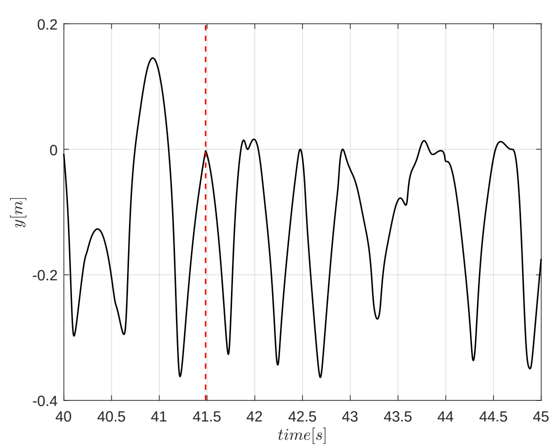

Fig. 5. x(t) crossing the combined-constraint singularity.

configuration. As it was said, this is a bifuraction point as the mechanism can

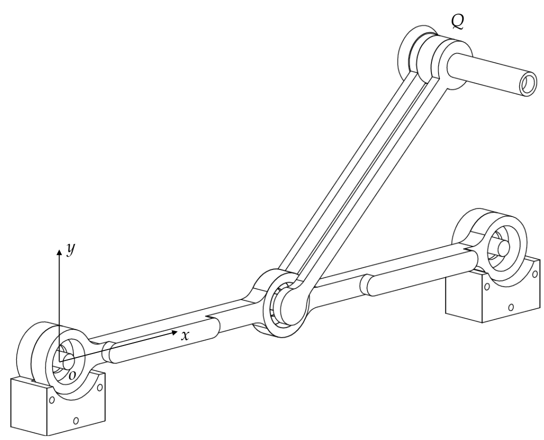

Fig. 6. Costrained-Combined singularity configuration.

either continue to move with 2 dof or to switch to 1 dof motion.

5 Conclusions

The paper presents the solution of the forward dynamics problem for a planar five-bar mechanism in the presence of singular configuations. The algorithm i3-ALP was used successfully allowing us to simulate the mechanism motion when crossing the constraint singularities. For the geometry studied, it was found that the constraint singular points coincide with the kinematic combined singularities. Future work will be dedicate to arrange a mechatronic system with a 3D printed prototype. The final goal will be dedicated to develop a digital twin of the printed mechanism to have a cyber sytem working as predictor/corrector of the prototype motion.

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