

Incentive mechanisms for the secure integration of renewable energy in local communities: A game-theoretic approach

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ABSTRACT

In the context of local energy communities (LECs), prosumers are the main actors, as they can both produce and consume energy. Prosumers can interact with each other, and peer-to-peer (P2P) energy trading allows prosumers belonging to the same LEC to exchange energy with each other. This allows energy production to be consumed internally by the community, which has the benefits of reducing costs for energy consumption and reducing the amount of energy traveling from/to the external grid, which causes transmission losses and wears and tear to the grid itself. This paper proposes a design for the P2P market from a game-theoretical point of view, where prosumers are modeled as selfish agents whose goal is to maximize their own profits in energy trading. The purposes of this market design are to (i) discourage prosumers from curtailing their own energy production, (ii) avoid congestions as much as possible, (iii) encourage self-consumption from prosumers, and (iv) guarantee that the selfish behavior of prosumers allows for a common strategy. Furthermore, this work considers the possibility of prosumers making coalitions between themselves, and show how this still allows for the existence of a common strategy. Simulations of the proposed market design have been run on data from a grid in Cardiff, UK, and show how the proposed mechanism allows for cost reduction and encourages energy self-consumption. Experiments results show that the system discourages the formation of small coalitions, and encourages instead cooperation from all the prosumers in the community.

1. Introduction

In the last decades, more and more importance is being given to the issues regarding energy and climate; in particular, more attention is being given to renewable energies, as they can help from the perspective of preserving the environment, and are not exhaustible. For these reasons, more and more policies for the employment of renewable energy sources (RES) are being made. The principle of these policies is to reward RES producers for the energy they inject into the grid, either by paying them money for units of energy exported or by providing them energy at no cost later, in order to repay them for their RES investment. The effects of these policies in the increase of energy generation from RES can already be seen, as in 2016 it represented 30% of the total amount of energy generated in Europe.¹ However, there are some drawbacks to the utilization of this type of energy; in particular, the grid may present stability problems [1,2], and currently support policies cannot solve them, which limits their effectiveness. These problems are caused by the fact that renewable

energy production is difficult to control, so it is complicated for the grid to match consumption to production.

In the last years, there have been some new proposals for incentive mechanisms that take these problems into account [3–6]. These mechanisms are based on introducing payment support functions that would encourage grid users to change their daily energy consumption patterns so that energy consumption to the grid matches energy production.

Among these mechanisms, there are some that make use of market-based approaches [3,5]: they are based on local markets between prosumers who sell energy, and consumers who buy it. The approach in [4] is different: it proposes a mechanism called NRG-X-Change, who does not use an energy market, and lets prosumers directly inject their produced energy into the grid, while consumers may take it. This mechanism offers incentives to prosumers for encouraging them to produce energy, but also to consumers, in order to make their consumption follow the production pattern.

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¹ <http://www.eea.europa.eu/data-and-maps/indicators/overview-of-the-electricity-production-2/assessment>

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The *NRG-X-Change* mechanism is based around the utilization of a virtual currency called *NRGcoin*, which can give some important benefits to energy trading. However, the mechanism does not take into account some crucial aspects, like the management of congestion and encouraging energy self-consumption for prosumers. About this matter, it is important to make sure that prosumers will always consume all the energy they produce before either selling the excess produced energy or buying the extra energy they need.

For this reason, the aim of this work is to propose a new incentive mechanism that is based on *NRG-X-Change* but with new payment functions that overcome the critical points of the original mechanism and study its performance from a game-theory aspect. To do this, this paper builds on the previous works performed in [7,8], extending and consolidating the shown results. In particular, this paper analyzes the same problem taking into consideration both the case where the users operate selfishly, and the case where they form small coalitions.

More in detail, these are the contributions of this paper:

- Performing an analysis on the original *NRG-X-Change* mechanism and the mathematical aspects which determine its functioning (the functions which determine the price for buying and selling energy), and detecting the main issues about them.

- Proposing a solution to the mechanism's issues by designing new import and export price functions, which also take into account the issues related to non-cooperative game theory, by the exploitation of known results in this field.

- Making use of mathematical proof for demonstrating the effectiveness of the proposed approach, and performing simulations using real data from a grid in Cardiff in order to measure the efficacy of the proposal.

- Analyzing the behavior of the proposed mechanism when the users are allowed to form small coalitions and supporting the analysis with simulations.

This is how the rest of this work is organized: Section 2 contains an overview of the related work. Section 3 describes the problem in terms of game theory. Section 5 shows in detail the proposed solution of the problem, and describes the existing proposals and the newly proposed cost and reward functions so that a Nash Equilibrium is always guaranteed. Section 6 shows how to set the parameters of the proposed functions in order to be compliant with the self-consumption constraint.

Section 7 outlines the effectiveness of the proposal by showing the results of experiments made on real data from a grid in Cardiff (UK), and Section 8 sums up the conclusions and presents future work directions.

2. Related work

The *NRG-X-Change* mechanism has not been exploited in actual grids yet. However, some works in literature give a comprehensive description of it. The most extensive one is [4], which describes the advantages of exploiting a digital currency like *NRGcoin*, and how grid users would interact between themselves and with parties external to the local grid. The work in [9] describes how the mechanism works in a simulation of a local grid, in order to show how it would perform in a realistic scenario.

In [1] there is a survey on incentive mechanisms. It describes the mechanisms which are more used nowadays, such as net metering and feed-in tariffs, and their issues, such as overconsumption in certain periods of the year, overpayment, and underpayment to energy producers depending on the case. The paper provides then some new solutions and compares them. Among these, the auction-based mechanism Nobel [3] is based on energy trading at a local level and is based on the stock exchange model, while *PowerMatcher* [5] aims to balance demand and supply by the interaction of specific agents, whose name is *SD-matchers*.

Finally, the solution proposed in [6] is a negotiation-based mechanism in which prosumers and generating companies (*Gencos*) may sell energy and bargain the price with the buyers. It is displayed how the energy cost is affected by this mechanism, and also the effects of adopting learning strategies for buyers. It has to be said, however, that the previously cited papers are centered on market mechanisms based on bidding systems for prosumers.

Also worth mentioning is the work in [10] which used the same Cardiff dataset to evaluate new decentralized mechanisms designed to optimize energy exchanges in local energy communities. However, the focus of [10] is on coordinating prosumers' flexibility to maximize their benefits under a given payment scheme, while the goal of this paper is to propose such a payment scheme, making the two papers complementary.

Regarding P2P energy trading, there are some reviews that describe its state of the art. The one from [11] describes the contributions that can be given from this form of energy trading, and sorts the existing papers depending on the challenges they tackle. The review [12] describes the recent developments of P2P energy trading implementations and the advancements for it to establish as an energy management option, while the review [13] describes their characteristics, state of art, and challenges.

There are plenty of works in the smart grid domain that make use of game theory. The survey [14] describes in detail how game theory methods have been exploited in the smart grid context, and presents works in which this has been done, in particular for microgrids, for demand side management and for communication in smart grid systems. The review paper [15] describes several works for exploiting game theory in the context of P2P energy trading, as a mean for energy management. There is also another survey [16] about the usage of game theory for smart grid, which is mostly oriented in the description of cooperative games; the review [17] mostly refers to non-cooperative games instead, but in their introduction to smart grid architecture they explicitly take into account the possibility of energy generation. Conversely, the survey [18] generically describes the usage of game theory in smart grid problems, showing how the exploitation of Nash Equilibrium existence can be adopted for solving various energy grid issues.

The work [19] is very similar to the work proposed in this paper for what game theory exploitation is concerned, but its main purpose is to manage demand response, while this paper focuses more on the nature of the incentive mechanism. The model is very similar, but in this case, there is a much greater emphasis on cost/reward functions, and on the ability of the grid users to produce energy. Energy production is considered in [20], which uses a strategy conceptually very similar to the proposed paper in order to guarantee the existence of a Nash Equilibrium, even though its payoff functions have only the purpose of solving the game theory problem, and therefore has simpler models. Also, the works [21,22] have been of great relevance to the proposed work, as the game theory model presented in this paper has been significantly influenced by theirs. Both are focused on demand side management, although the first one shows how the decentralized design performs compared to the centralized one, while the second one proposes a novel cost function and two possible models for the scenario: one non-cooperative game, and one Stackelberg game. Another Stackelberg game, although more complex, has been modeled in [23], which is however referred to as the case of energy trading between different microgrids. The same type of game has been employed by [24], which however used it for modeling peer-to-peer energy trading inside a virtual microgrid: in this specific Stackelberg game, producers lead and consumers follow. On the topic of energy trading the work in [25] provides another example of game theory usage in a smart grid, whose purpose is regulating trading of energy between different storage units in a grid, with an approach based on double actions. Another work conceptually similar to ours is [26], which exploits game theory for managing user consumption at a grid level: the main difference is

that they use a Bayesian game model, and are based on a generic dynamic pricing system, while in this case the focus is on the pricing system itself. In [27] there is another game theoretic application in this domain, with the purpose of maximizing profits for utility companies in the smart grid environment, in which dynamic pricing mechanics are exploited.

As far as the cooperative games are concerned, the survey in [28] offers a comprehensive review of canonical coalition games, coalition formation games, and coalitional graph games, applied to the sector of communication. There is also another work [29] describing a coalition formation game in the specific environment of a microgrid, and studying the behavior of the grid with respect to environmental changes. Also, [30] proposes a cooperative game theory approach for actively encouraging smart grid users to participate the peer-to-peer energy trading, and found similar results to ours for the grand coalition in their model. Finally, although not strictly related to game theory, [31] describes a cooperative P2P energy trading technique for battery exploitation.

3. Modelization of the problem

This section describes the modelization of the problem. The idea is to perform an analysis of the effects of the incentive mechanism on the community while taking into account the behavior of the users: for this reason, a game theory model of the community has been created. The idea of this paper follows the models described in [21,22].

This paper does not describe in detail neither how the network is managed nor how the P2P trading actually happens. This work focuses on the game-theoretical aspects, and on interactions on the financial side. Regarding the management of network issues and constraints for the system proposed here, we refer the reader to [32].

3.1. Game theory approach to the smart grid problems

Notation for game theory will be as follows. A game $G = (U, S, Q)$ is defined, so that:

$U = \{U_1, \dots, U_N\}$ is the set of players.

$S = \{S_1, \dots, S_N\}$, where for any $i \in \{1, \dots, N\}$, S_i is the set of player U_i 's strategies.

$Q = \{q_1, \dots, q_N\}$, where for any $i \in \{1, \dots, N\}$, $q_i : \prod_{j=1}^N S_j$ is the payoff function for player U_i .

Given a grid with N users, a game is defined on it in the following way. First, the players will be the grid users, which will therefore be denoted by U_i , for $i \in \{1, \dots, N\}$. Then there will be the definitions of, in order, the payoff functions for the players and the set of strategies of each one of them.

For each of the users U_i , consider two vectors: \mathbf{c}_i , which corresponds to the energy consumption of U_i , and \mathbf{p}_i , which corresponds to the energy production of U_i . Each component of these two vectors is a non-negative real number. The length of these vectors is determined by how many time intervals the considered time horizon is made of: this number is denoted by T . For example, in the case that will be considered in the experiments, the time horizon is 24 h and the considered time intervals are 15 min long, therefore the total amount of time intervals is 96, and 96 will be the length of \mathbf{c}_i and \mathbf{p}_i .

From now on, each of the users will be assumed to consume all the energy he/she produces, before either taking the extra energy needed for consumption from the grid or exporting the excess produced energy to it. \mathbf{c}_i and \mathbf{p}_i are evaluated only after considering self-consumption: this means that, if $\mathbf{c}_i(t)$ is the t -th element of \mathbf{c}_i ,

$$\mathbf{c}_i(t) - \mathbf{p}_i(t) = 0 \quad \forall t \in \{1, \dots, T\} \quad (3.1)$$

for each $i \in \{1, \dots, N\}$. It stands to reason that if U_i is not an energy producer, \mathbf{p}_i is the zero vector. Later in this paper, there will be the

mathematical proof that every proposed payment function actively promotes self-consumption for energy producers.

The utility function q_i can now be defined for each user U_i as the combined utility over all the time units. More formally,

$$q_i = \sum_{t=1}^T q_i(t). \quad (3.2)$$

In this equation the term $q_i(t)$, i.e., the utility of a single user U_i at time $t \in T$, is

$$q_i(t) = g(\mathbf{p}_i(t), t_p(t)) - h(\mathbf{c}_i(t), t_c(t)) \quad (3.3)$$

where g and h are the two payment functions that characterize the incentive mechanism: g describes the reward for energy production, while h describes the cost for energy consumption. Also, for every $a, b \in \mathbb{R}$, $g(0, a) = h(0, a) = 0$. For every time t , t_p and t_c are defined in the following way:

$$\begin{aligned} t_p &= \sum_{i=1}^N \mathbf{p}_i(t) \\ t_c &= \sum_{i=1}^N \mathbf{c}_i(t). \end{aligned} \quad (3.4)$$

t_p and t_c describe respectively the total amount of energy produced and consumed at the considered time unit. It is important to notice that, for every time unit t , the value of t_p is dependent on each value $\mathbf{p}_i(t)$, and t_c is dependent on each value $\mathbf{c}_i(t)$. In particular, the utility q_i for each user U_i depends not only on production and consumption of U_i , but also on production and consumption of all the other users.

The idea of designing cost and reward functions depending on the total amount of produced and consumed energy comes from [4]: the approach used in that paper represents the baseline for this work.

The players of the game and the payoff functions have been defined: the only thing left to describe is the set of strategies. In order to do so, energy loads and the interaction with them must be defined. A similar approach has been taken in [22]. In this paper there are three main types of energy loads:

Production: It is denoted by the vector \mathbf{p}_i . It refers to the amount of energy produced by the user U_i . In some cases, energy production may have some degree of flexibility; however, in the work, this is a fixed vector.

Fixed consumption: It is denoted by the vector \mathbf{f}_i . It refers to the amount of energy consumed by the user U_i that cannot be shifted, and it is a fixed vector. In general, this quantity is not known in advance and has some degree of uncertainty over it: this is addressed by [33] by replacing the component of the consumption vector with probability distributions. However, for simplicity, in this paper, \mathbf{f}_i is assumed to be known in advance.

Shiftable load: It is denoted by \mathbf{h}_i^j the j -th shiftable load of the user U_i . A shiftable load is an energy load whose energy profile cannot in general be changed in amount but can be moved over time. If \mathbf{h}_i^j represents such a load, $r_k(\mathbf{h}_i^j)$ denotes the same load shifted k time units forward: in terms of operation on vectors, r_k is the rotation by k places. In particular, for $k = 0$, \mathbf{h}_i^j remains the same, and so $r_0(\mathbf{h}_i^j) = \mathbf{h}_i^j$. Unless there is continuity between days, the rotation cannot move a nonzero element from the last to the first place.

The vectors \mathbf{c}_i and \mathbf{p}_i will be calculated only after \mathbf{p}_i , \mathbf{f}_i and all the \mathbf{h}_i^j vectors have been defined, and the allocations for the flexible vectors have been chosen. Given a vector v , v^+ is defined as v whose negative elements have been replaced with 0, and v^- as v whose positive elements have been replaced with 0.

Consider the user U_i , and suppose it has n shiftable loads $\{\mathbf{h}_i^1, \dots, \mathbf{h}_i^n\}$. If U_i shifts each load \mathbf{h}_i^j by k_j time units respectively. With the notation

used, the vector \mathbf{c}_i will be

$$\mathbf{c}_i = \mathbf{f}_i + \sum_{j=1}^n r_{k_j} \mathbf{h}_i^{j^*} - \mathbf{p}\mathbf{f}_i)^+ \quad (3.5)$$

and the vector \mathbf{p}_i , likewise, is defined as

$$\mathbf{p}_i = -\mathbf{f}_i + \sum_{j=1}^n r_{k_j} \mathbf{h}_i^{j^*} - \mathbf{p}\mathbf{f}_i)^-. \quad (3.6)$$

The set of strategies for U_i is the set of all the possible vectors \mathbf{p}_i and \mathbf{c}_i that can be obtained by shifting $\{\mathbf{h}_1^1 \dots \mathbf{h}_1^n\}$ in every possible way.

3.2. Running example

In order to better explain the concepts that introduced in this section, this subsection proposes running example for a simple grid. It is defined as follows:

There are three grid users: U_1 , U_2 and U_3 . U_1 and U_2 are consumers, U_3 is a prosumer.

The time horizon is 8 time units. Each user U_i has a fixed consumption $\mathbf{f}_i = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$, and one shiftable load $\mathbf{h}_1^1 = [1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0]$. The vector $\mathbf{p}\mathbf{f}_i$ is $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$ for users 1 and 2, as they are consumers, while for user 3 it is $[2 \ 2 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0]$. For each vector, its t th element represents energy consumption or production at the t th time unit, and it is expressed in kWh.

Reward for energy production and cost for energy consumption are decided by hourly tariffs. For the sake of simplicity, this example defines an arbitrary, fixed tariff both for selling and buying energy: that is,

$$g(\mathbf{p}_i) = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot \mathbf{p}_i$$

and

$$h(\mathbf{c}_i) = [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot \mathbf{c}_i$$

where each element is expressed in EUR/kWh. So, in this simple case, both reward and cost depend linearly on the amount of consumed energy.

The vectors introduced in the previous chapter are now calculated. First, \mathbf{f}_i and $\mathbf{p}\mathbf{f}_i$ are written properly, as

$$\begin{aligned} \mathbf{f}_1 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \\ \mathbf{f}_2 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \\ \mathbf{f}_3 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \\ \mathbf{p}\mathbf{f}_1 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \mathbf{p}\mathbf{f}_2 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \mathbf{p}\mathbf{f}_3 &= [2 \ 2 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0]. \end{aligned} \quad (3.7)$$

Regarding \mathbf{h}_i^1 , for every i , it is a vector representing the shiftable load. The time horizon is eight time units, which means that there are eight possible rotations for the shiftable load. However, as indicated in the previous section for the shiftable load, the load cannot be interrupted until it finishes: this means that the time units where energy is consumed must be consecutive. Therefore, among the eight possible rotations, only the following four are valid.

$$\begin{aligned} r_k \mathbf{h}_1^1 &\in \{[1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0] \\ &\quad [0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0] \\ &\quad [0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0] \\ &\quad [0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1]\}. \end{aligned} \quad (3.8)$$

Now, \mathbf{c}_i and \mathbf{p}_i can be calculated as specified in Eqs. (3.5) and (3.6) respectively. It follows that

$$\mathbf{c}_i = \mathbf{f}_i + r_k \mathbf{h}_1^1 - \mathbf{p}\mathbf{f}_i)^+$$

So, replacing \mathbf{f}_i and $\mathbf{p}\mathbf{f}_i$ according to Eq. (3.7) and replacing $r_k \mathbf{h}_1^1$ with Eq. (3.8), the vector may assume one of those possible values after the calculation:

$$\begin{aligned} \mathbf{c}_1 &\in \{[2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1] \\ &\quad [1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] \\ &\quad [1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1] \\ &\quad [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2]\}. \end{aligned} \quad (3.9)$$

\mathbf{c}_2 and \mathbf{c}_3 are calculated in the same way. The values of \mathbf{c}_2 will be the exact same as for \mathbf{c}_1 , whereas for \mathbf{c}_3 there are the following possible values:

$$\begin{aligned} \mathbf{c}_3 &\in \{[0 \ 0 \ 0 \ 2 \ 2 \ 1 \ 1 \ 1] \\ &\quad [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 1 \ 1] \\ &\quad [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2 \ 1] \\ &\quad [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2 \ 2]\}. \end{aligned} \quad (3.10)$$

In the same way, \mathbf{p}_i is calculated. Since U_1 and U_2 have no production, it is already known that

$$\mathbf{p}_1 = \mathbf{p}_2 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

no matter how the flexible load is allocated. For \mathbf{p}_3 , calculating according to Eq. (3.6), it results that

$$\begin{aligned} \mathbf{p}_3 &\in \{[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad [1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]\}. \end{aligned} \quad (3.11)$$

Within this example, the game defined in Section 3 would be as follows. The set of players is $U = \{U_1 \ U_2 \ U_3\}$. The set of strategies is $S = \{S_1 \ S_2 \ S_3\}$, where each S_i contains every possible value for \mathbf{c}_i and, for each one of them, the corresponding value for \mathbf{p}_i . For example, this is S_1 .

$$\begin{aligned} S_1 &= \{ \mathbf{c}_1 = [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1] \ \mathbf{p}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_1 = [1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] \ \mathbf{p}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_1 = [1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1] \ \mathbf{p}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_1 = [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2] \ \mathbf{p}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \}. \end{aligned}$$

The set S_2 is defined by \mathbf{c}_2 and \mathbf{p}_2 , whose values are identical to the ones that \mathbf{c}_1 and \mathbf{p}_1 may assume. The set S_3 is defined by \mathbf{c}_3 and \mathbf{p}_3 , which however will have different values compared to the first two users due to the presence of energy production. More precisely:

$$\begin{aligned} S_3 &= \{ \mathbf{c}_3 = [0 \ 0 \ 0 \ 2 \ 2 \ 1 \ 1 \ 1] \ \mathbf{p}_3 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_3 = [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 1 \ 1] \ \mathbf{p}_3 = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_3 = [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2 \ 1] \ \mathbf{p}_3 = [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ &\quad \mathbf{c}_3 = [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2 \ 2] \ \mathbf{p}_3 = [1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0] \}. \end{aligned}$$

The payoff functions are defined as

$$q_i = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot \mathbf{p}_i - [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot \mathbf{c}_i.$$

So, for example, these are the possible values for each strategy of user U_1 , in the same order they have been presented earlier.

$$\begin{aligned} q_1 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1] = -32\text{EUR}. \\ q_1 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot [1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] = -31\text{EUR}. \\ q_1 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot [1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1] = -30\text{EUR}. \\ q_1 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \ 2] \cdot [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2] = -29\text{EUR}. \end{aligned}$$

Therefore, the best strategy for U_1 is the fourth listed, which allows the user to pay 29 EUR. The strategies for U_2 are the same as U_1 , so in this case, the minimum cost for energy is 29 EUR as well. Finally, the possible values for the payoff function for U_3 are as follows:

$$\begin{aligned} q_3 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \ [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2] \ [0 \ 0 \ 0 \ 2 \ 2 \ 1 \ 1] = -14\text{EUR}. \\ q_3 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \ [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2] \ [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 1] = -15\text{EUR}. \\ q_3 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \ [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2] \ [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2] = -16\text{EUR}. \\ q_3 &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \ [1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2] \ [0 \ 0 \ 0 \ 2 \ 2 \ 2 \ 2] = -17\text{EUR}. \end{aligned}$$

In this case, the best strategy is the first, which allows U_3 to pay 14 EUR. It is interesting to note that, for the producer, the best strategy is the one that maximizes self-consumption.

3.3. Cooperative game

The payment systems that will be described in the following section were born with the idea that each player behaves selfishly, with their actions depending on how other users would behave, but always with the purpose of maximizing their individual profits. Therefore, the idea is to study what would happen if users had the possibility to make coalitions. The term *coalition* denotes the agreement between members of the coalition to coordinate their energy profiles in such a way that the total economic profit between all of them is the highest possible. However, because of the way the game is structured, this may reduce other users' earnings, and may cause other unwanted effects such as a reduction in energy self-consumption through the grid. Another important aspect is to make sure that there is a fair distribution of profits between the users. Formation of a coalition may damage the profit of other users; also, users may threaten to join another coalition unless they receive a higher share of profit. For these reasons, it is important to make sure that, even if users are allowed to form coalitions, the proposed scheme does not actively encourage cooperation between them unless all the users cooperate together.

Consider a game $G = (U, S, Q)$ defined as in Section 3.1. The idea is to define a new game $G_{\overline{U}}$ where it is possible to make coalitions between users. Each coalition may be composed of one or more users, and a user is part of only one coalition: in more formal terms, coalitions define a partition on the set U . Once the coalitions are defined, let \overline{U} be the partition that represents them. The game $G_{\overline{U}} = (\overline{U}, \overline{S}, \overline{Q})$ is then defined in the following way:

$$\begin{aligned} \overline{U} &= \overline{U}_1 \dots \overline{U}_k \text{ is the set of players.} \\ \overline{S} &= \overline{S}_1 \dots \overline{S}_k \text{ is the set of strategies, where for every } i \in \{1 \dots k\}, \overline{S}_i = \bigtimes_{U_j \in \overline{U}_i} S_j \\ \overline{Q} &= \{\overline{q}_1 \dots \overline{q}_k\} \text{ is the set of utility functions, where for every } i \in \{1 \dots k\}, \overline{q}_i = \sum_{U_j \in \overline{U}_i} q_j. \end{aligned}$$

To be more precise, every element \overline{U}_i of the partition behaves like a single entity that can control all the loads of the users belonging to \overline{U}_i . For this reason, the set of the strategies of \overline{U}_i is the product of all the sets of strategies of those users, since the entity \overline{U}_i can adopt every possible combination of the strategies of the users belonging to it. The utility function of \overline{U}_i is the sum of all the utility functions of the users belonging to that partition; this is because \overline{U}_i is an entity representing the interests of all the users inside it, so its utility is the overall utility of these users.

In order to explain this better, consider the case of the running example from Section 3.2. It would be useful to show a case where there are at least two different coalitions, but at least one is non-trivial. For this reason, U_1 and U_2 are assumed to form a coalition, while U_3

operates on his/her own. Therefore, there are two players: the coalition $\overline{U}_{12} = \{U_1, U_2\}$, and the coalition $\{U_3\}$ (which, having only one user, will simply be denoted by U_3). Here, the set S_3 will be the same as in the previous example. The set \overline{S}_{12} is the cartesian product of S_1 and S_2 : more precisely, this means that every possible strategy for S_1 is coupled with every possible strategy for S_2 . As in the example from Section 3.2 there were four elements for both S_1 and S_2 , \overline{S}_{12} will have 16 elements, which are reported below (abbreviated).

$$\begin{aligned} \overline{S}_{12} &= \{ \mathbf{c}_{12} = [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] + [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] \\ &\quad \mathbf{p}_{12} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ &\quad \mathbf{c}_{12} = [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] + [1 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] \\ &\quad \mathbf{p}_{12} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \} \\ &\dots \\ &\quad \mathbf{c}_{12} = [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] + [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2] \\ &\quad \mathbf{p}_{12} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ &\quad \mathbf{c}_{12} = [1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 1] + [2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1] \\ &\quad \mathbf{p}_{12} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \} \\ &\dots \\ &\quad \mathbf{c}_{12} = [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2] + [1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2] \\ &\quad \mathbf{p}_{12} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \}. \end{aligned}$$

Regarding the utility function, q_3 will be the same as the previous example while calling q_1 and q_2 the utility functions of the previous example, $q_{12} = q_1 + q_2$. The value of q_{12} is calculated for each strategy, as in Section 3.2: after doing so, the strategy that allows for the lower cost will be the last one, for which

$$\begin{aligned} \mathbf{c}_{12} &= [2 \ 2 \ 2 \ 4 \ 4 \ 4 \ 4] \\ \mathbf{p}_{12} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \end{aligned}$$

and therefore,

$$\begin{aligned} q_{12} &= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \ [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] - \\ &\quad [3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2] \ [2 \ 2 \ 2 \ 4 \ 4 \ 4 \ 4] = -58\text{EUR}. \end{aligned}$$

If the value of the utility function for each strategy is computed, this is the best possible strategy for \overline{U}_{12} . The best strategy for U_3 is the same as Section 3.2, and it leads to a cost $q_3 = -14$ EUR. A strategy for dividing the combined profit/costs inside a coalition is outside the scope of this paper; however, the combined profit of U_1 and U_2 is the same as the non-cooperative case, as it is -58 EUR here, while in Section 3.2 the result was -29 EUR for each of them, for a total of -58 EUR. Also, the profit of U_3 does not change with respect to the non-cooperative case. For this reason, this coalition brings no advantage to the users.

4. Game events description

This section describes how the game described in Section 3 is actually run, and how the events in the game follow one another. Algorithm 1 describes the game flow when all the users are selfish.

Algorithm 1.

1. First, denote by N the number of players, and by N_p the number of prosumers among them: they are numbered as $U_1 \dots U_{N_p}$. For each user U_i , its predicted energy consumption and energy production is known in advance (and production will simply be zero if the user is not a prosumer). Each user is assumed to have one shiftable load, and the starting time for each load is randomly assigned.
2. The following will be performed by every user U_i one at a time, following their numeration: this means, starting from U_1 and going until U_N . First, the user U_i receives information on t_p and t_c depending on the choices made by the players operating

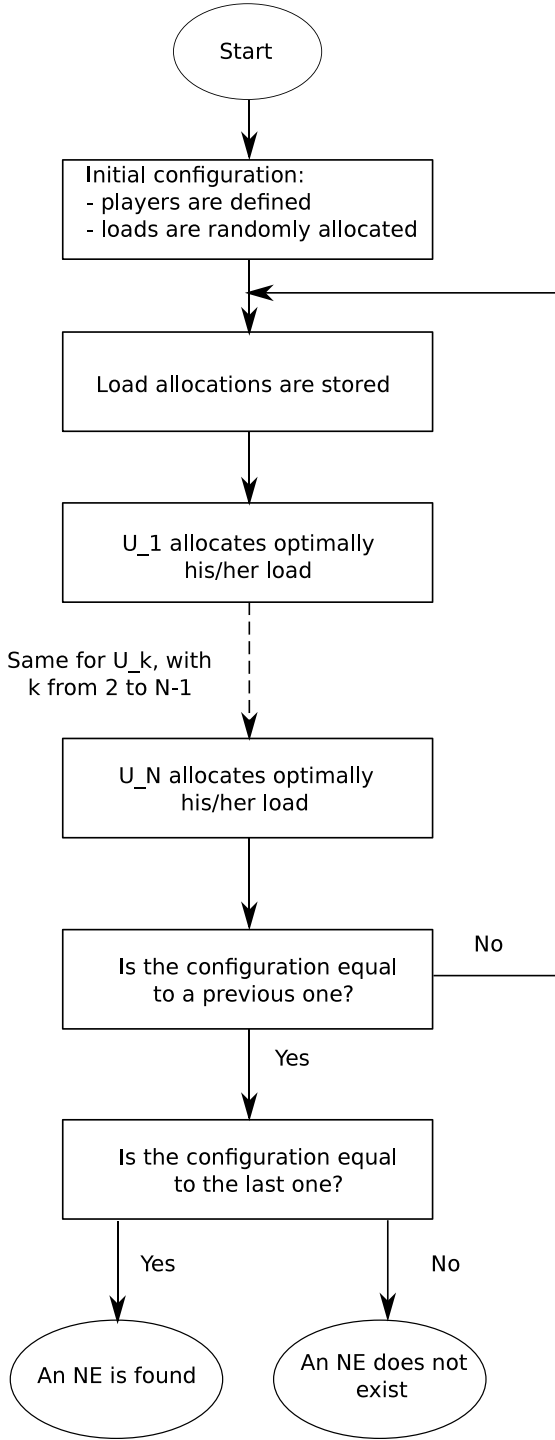


Fig. 1. Flowchart of the game.

- before him/her. Now, U_i can decide the starting time of his/her shiftable load. This choice is made by computing the payoff function q_i for every possible strategy (i.e., for every possible starting time of the flexible load), and then moving the load to the time which maximizes the value of q_i . Once all of the users $U_1 \dots U_N$ have chosen their strategy, an *iteration* has been performed.
3. The allocations of each shiftable load, before and after the current iteration, are compared. If no load has changed allocation,

the simulation ends. Otherwise, a new iteration is performed, going back to step 2 and through it all over again.

4. If, after the current iteration, the state of the game (i.e., the allocation for shiftable loads) is the same as one of the previous iterations, the game ends.

There are two possible outcomes of the game.

The first outcome happens if the allocations at the end of an iteration are the same as the previous iteration. In this case, the game has reached a NE: since no user has changed their load allocation, it means that the previous allocation was already the most profitable one for them, and repeating further interactions will lead to the same result, as the starting point would be the same. This corresponds to the simulation halting at step 3.

The second outcome happens if the allocations at the end of an iteration are the same as a previous iteration and different from the last one. In this case, the game cannot reach a NE. This is because the strategies that will be adopted during an iteration only depend on the state at the beginning of the iteration: therefore, the game will go on a loop and the same sequence of configuration changes will happen over and over again. This corresponds to the simulation halting at step 4.

It is important to say that the game will eventually reach one of those two outcomes since the number of possible configurations is finite.

The diagram in Fig. 1 describes how the game described in Algorithm 1 is played. The reader notices that Algorithm 1 can also be used, with some modifications, for describing the cooperative game. In the cooperative game case, at step 1, coalitions are considered, rather than single users. Also, as there is the assumption that each user has exactly one load, each coalition will have a number of loads equal to the number of users belonging to that coalition. For example, a coalition made by 3 users will be a player of the cooperative game with 3 loads.

5. Peer-to-peer market design

In this section, the proposed peer-to-peer market for energy trading is presented.

5.1. Purpose

The objective of this paper is to create a new model of an incentive mechanism for renewable energy usage in smart grids: the idea is to start from the concept of NRG-X-Change [4], and enhancing it in order to cope with its weaknesses. These are the weak points of the original mechanism that have been identified:

1. In some cases, the prosumers may be incentivized to curtail their own energy production. This should not happen, except when it is necessary for grid stability.
2. The original mechanism passively limited the possibility of congestion, but the functions did not have terms for dealing with this circumstance.
3. Prosumers may produce and consume energy at the same time. In this case, their ideal behavior would be to self-consume all the energy they produce, before either taking from the grid the extra energy they need or injecting into the grid the excess energy they produce. Unfortunately, this is not always encouraged.
4. If the behavior of the users is modeled as selfish agents, they may change their energy profile according to other users' choices. It is important that they reach an equilibrium.

The work done in [7] dealt with the first three points: in their approach, the authors created one new cost function for the consumers and one new reward function for the prosumers. The proposed idea is to create new functions which address all of the four weaknesses of the NRG-X-Change mechanism listed above.

5.2. The NRG-X-Change incentive mechanism

This subsection will describe the mechanism which this work aims to extend. The NRG-X-Change mechanism will be described: it is an incentive mechanism designed for the employment of NRGcoin, a digital currency [4].

This is how the mechanism works. Consider a specific time unit: at that time, some grid users may be producing energy, and others may be consuming energy. Consider a certain user U_i who produces energy at that time, and exports to the grid a certain quantity x of energy. In exchange for that, the user receives a certain amount of NRGcoins: this reward depends on a function named g , whose definition is as follows:

$$g(x, t_p, t_c) = x \cdot \frac{q}{e^{\frac{(t_p - t_c)^2}{a}}} \quad (5.1)$$

In this formula, q and a are two positive real numbers, with q determining the maximum value that the tariff can reach, and a regulating the shape of the function.

Now, suppose that U_i is a consumer and that at the considered time unit he/she takes an amount of energy y from the grid for consumption. In this case, he/she will have to pay a certain quantity of NRGcoins in exchange for that energy. This quantity is determined by a function named h , which is defined as

$$h(y, t_p, t_c) = y \cdot \frac{r \cdot t_c}{t_c + t_p} \quad (5.2)$$

In this case, r is a positive real number that corresponds to the maximum value that the tariff can reach.

The idea for the choice of this tariff is to encourage self-consumption at the community level: more specifically, prosumers are incentivized to increase their energy consumption when there is more local energy production than local consumption and to decrease their energy consumption when there is more local energy consumption than production. Other than this, the reward for energy production is higher when there is not much difference between the amounts of energy production and energy consumption at local level: this is an incentive to maintain those values close to each other.

This incentive mechanism, for how it is defined, presents several critical points. The work in [7] has identified some of these and proposed a solution for them by designing two new functions, one for defining costs for energy consumption and one for defining rewards for energy production. The functions have been designed as follows.

The notation t_p^{-i} and t_c^{-i} will denote the values of t_p and t_c when x and y are both zero. In more formal terms,

$$\begin{aligned} t_p^{-i} &= \sum_{j \neq i} p_j(t) \\ t_c^{-i} &= \sum_{j \neq i} c_j(t) \end{aligned} \quad (5.3)$$

From here, the new selling function g is

$$g(x, t_p, t_c) = P_{\max} \left(g_1(t, x, t_p^{-i}, t_c^{-i}) - g_1(t, 0, t_p^{-i}, t_c^{-i}) \right) - P \cdot x \cdot t_p^{-i} \cdot t_c^{-i} \quad (5.4)$$

in this case, $P \cdot x \cdot t_p^{-i} \cdot t_c^{-i}$ is a penalty term, which is greater than zero when the values of t_p^{-i} and t_c^{-i} correspond to the case of congestion (that is when $|t_p^{-i} + x - t_c^{-i}| > B$), and zero when they do not. P_{\max} is a positive real number that determines how high is the tariff. The function g_1 mentioned in the previous definition is

$$g_1(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ \frac{1}{1 + e^{\frac{2t-1}{2}}} & \text{if } t \in (0, 1) \\ 1 & \text{if } t \geq 1 \end{cases} \quad (5.5)$$

and has the purpose of defining the shape of g . Finally, t is a function such that

$$t = x \cdot t_p^{-i} \cdot t_c^{-i} = \frac{t_p^{-i} + x - t_c^{-i}}{2B} + \frac{1}{2} \quad (5.6)$$

In this definition B is a positive real number, corresponding to the maximum difference between the values of t_p^{-i} and t_c^{-i} for which congestion will not happen.

The newly proposed buying function h is

$$h(y, t_p^{-i}, t_c^{-i}) = Q_{\max} \cdot h_1 \left(\frac{t_c^{-i} + y - t_p^{-i}}{B} + 1 \right) \cdot y + P \cdot y \cdot t_p^{-i} \cdot t_c^{-i}. \quad (5.7)$$

Here, Q_{\max} and B are two positive real numbers, indicating respectively half of the maximum unitary cost for energy and the congestion threshold from Eq. (5.6); $P \cdot x \cdot t_p^{-i} \cdot t_c^{-i}$ is a non-negative penalty function whose value is greater than zero if and only if $t_c^{-i} + y - t_p^{-i} > B$.

In Eq. (5.7), the function h_1 is

$$h_1(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ \sqrt{t} & \text{if } t \in (0, 1) \\ 2 - \sqrt{2-t} & \text{if } t \in [1, 2) \\ 2 & \text{if } t \geq 2. \end{cases} \quad (5.8)$$

The selling and buying functions that have been described in this section deal efficiently with the problems described in Section 5.1. However, there is still the need to observe their behavior in terms of game theory; for this purpose, the problem will be described with the representation of Section 3, and the performances of both the original NRG-X-Change and the newly defined functions will be investigated in this context.

Regarding the functions described in the NRG-X-Change work, by exploiting Eq. (3.3), the payoff function $q_i(t)$ results to be

$$\begin{aligned} q_i(t) &= x \cdot \frac{q}{e^{\frac{(t_p - t_c)^2}{a}}} \quad \text{if } y = 0 \\ q_i(t) &= -y \cdot \frac{r \cdot t_c}{t_c + t_p} \quad \text{if } x = 0 \end{aligned} \quad (5.9)$$

Note that at least one between x and y is zero, according to Eq. (3.1).

Writing explicitly the formula for q_i is difficult [7]; however, either x or y must be equal to zero: this allows the formula to be written in an easier way. There are two cases.

$y = 0$. Here, the amount of energy produced by the user is higher than the amount of energy consumed by the user. Replacing Eq. (5.5) and (5.6) inside Eq. (5.4), the formula becomes

$$q_i(t) = \frac{P_{\max}}{1 + e^{\left(\frac{t_p^{-i} + x - t_c^{-i}}{B}\right)^2 - \frac{1}{4}}} - \frac{P_{\max}}{1 + e^{\left(\frac{t_p^{-i} - t_c^{-i}}{B}\right)^2 - \frac{1}{4}}}. \quad (5.10)$$

$x = 0$. Here, the amount of energy produced by the user is lower than the amount of energy consumed by the user. Replacing Eq. (5.8) inside Eq. (5.7), the formula becomes

$$\begin{aligned} & - \text{If } t_p^{-i} > t_c^{-i} \\ q_i(t) &= -y \cdot Q_{\max} \sqrt{\frac{t_c^{-i} + y - t_p^{-i}}{B} + 1}. \end{aligned} \quad (5.11)$$

$$\begin{aligned} & - \text{If } t_p^{-i} < t_c^{-i} + y, \\ q_i(t) &= -y \cdot Q_{\max} \left(2 - \sqrt{1 - \frac{t_c^{-i} + y - t_p^{-i}}{B}} \right). \end{aligned} \quad (5.12)$$

It is relevant to keep in mind that x is defined as the difference between production and consumption. Therefore, as the users cannot change their energy production value (as they cannot curtail their energy production), the only way they may change their value for x is by modifying their amount of consumed energy.

5.2.1. Numeric examples for the existing proposals

As a title of example, it is possible to see the values that the functions would assume in the case of the running example from Section 3.2. The values for f_i and pf_i can be seen in Eq. (3.7), while for the strategies, consider the shiftable load allocated in the second-last time unit, i.e.,

$$r_{k_1} h_1^1) = [0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0].$$

t_c and t_p are the total amount of, respectively, net consumption and net production in the grid: therefore, t_c is the sum of all the c_i , and t_p the sum of all p_i . This is the result of this calculation, taking the values from the third row of Eq. (3.9), Eq. (3.10) and (3.11) respectively:

$$\begin{aligned} t_c &= [2 \ 2 \ 4 \ 6 \ 6 \ 6 \ 6 \ 3] \\ t_p &= [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \end{aligned} \quad (5.13)$$

For example, in the first time unit, the user U_1 will consume 1 kWh. Let us see now the value of the cost function h with the two introduced mechanisms. Just to give an idea, the parameters will be fixed as follows. For the original NRG-X-Change functions, $q = 2.5$ $a = 1$ $r = 4.5$, each one expressed in its respective measure unit ($\frac{EUR}{kWh}$ for q and r , kWh^2 for a). For the improved functions from [7], $P_{max} = 6EUR$ $Q_{max} = 2.5EUR$ $B = 10$ kWh.

Now, for user U_1 at time 1, the cost value will be calculated by the original mechanism as:

$$h \ 1 \ 1 \ 2) = 1 \ \frac{2 \cdot 4.5}{3} = 3EUR$$

while, with the improved functions, it will be calculated as follows. First,

$$\frac{t_c^{-1} + y - t_p^{-1}}{B} + 1 = \frac{1 + 1 - 1}{10} + 1 = 1.1$$

then

$$h_1 \ 1.1) = 2 - \sqrt{0.9} = 1.0513$$

and finally

$$h \ 1 \ 1 \ 1) = 2.5 \cdot 1.0513 \cdot 1 = 2.63EUR.$$

Regarding the reward functions g , consider user U_3 at time 1. Here, the value for the original NRG-X-Change function would be:

$$g \ 1 \ 1 \ 2) = 1 \ \frac{2.5}{e^1} = 0.92EUR.$$

For the improved functions, first calculate

$$t \ 1 \ 0 \ 2) = \frac{0 + 1 - 2}{10} + \frac{1}{2} = 0.4$$

$$t \ 0 \ 0 \ 2) = \frac{-2}{10} + \frac{1}{2} = 0.3$$

then

$$g_1 \ 0.4) = \frac{1}{1 + e^{\frac{-0.2}{0.16-0.4}}} = \frac{1}{1 + e^{0.83}} = \frac{1}{3.29} = 0.30$$

$$g_1 \ 0.3) = \frac{1}{1 + e^{\frac{-0.4}{0.09-0.3}}} = \frac{1}{1 + e^{1.9}} = \frac{1}{7.69} = 0.13$$

and finally

$$g \ 1 \ 0 \ 2) = 6 \cdot 0.30 - 0.13) = 1.02EUR$$

5.3. New proposals

In Section 5.1 there is a list of four weak points of the original NRG-X-Change mechanism. Points 1 to 3 have been addressed up to now, and it is time to show how point 4 can be addressed. In other words, determining whether the existing mechanism always allows for the existence of a Nash Equilibrium (NE) and, if this is not the case, new cost and reward functions should be designed, in such a way that

the existence of a NE is always guaranteed, and points 1 to 3 from Section 5.1 are still addressed.

Regarding the existing systems, the game from Section 3.1 has been simulated with the previously existing cost and reward functions, in order to check whether a NE always exists or not. In case it did, the following step would prove mathematically that such a game always admits a NE. However, as shown in Section 7, this is not the case, and the previously existing functions cannot guarantee that there is always a NE.

Because of this, the objective is to design cost and reward functions for which it is possible to be sure that the game always reaches a NE. The idea was to start from a game theory result: if the payoff function of a game is concave, the existence of a pure NE is guaranteed [21]. Therefore, the objective is to design a concave reward function and a convex cost function. If this is done, the utility function (which has been defined in (3.3)) will be concave, and the result from [34] implies that a pure NE for the game always exists.

It is important that the new functions that will be created can also guarantee that points 1 to 3 from Section 5.1 are still addressed; in particular, the most critical ones are production curtailment prevention and self-consumption enforcement. Intentional energy production curtailment can be avoided by making sure that the reward function g is monotonic in x . Regarding self-consumption, it can be incentivized by making sure that the inequality

$$g \ x \ t_p \ t_c) < h \ x \ t_p \ t_c) \quad (5.14)$$

holds for every $x > 0$ and for every $t_p \ t_c \geq 0$.

Some functions that comply with the above requirements have been designed. In the remainder of the paper, Z will be defined as

$$Z = t_p^{-i} - t_c^{-i} + B. \quad (5.15)$$

The functions that have been designed are the following:

• For the reward function g :

[Logarithm] A well-known monotonous and concave function is the logarithm. A possible function based on this property is

$$g \ x \ t_p \ t_c) = k_1 \left(\ln \frac{x + Z + a_1}{Z + a_1} \right). \quad (5.16)$$

Here, a_1 and k_1 are real numbers, with $a_1 > 0$.

[n-th root] Another known monotonous and concave function is the n -th root function, where $n > 2$ is a real number. A possible buying function based on this could be

$$g \ x \ t_p \ t_c) = k_1 \ (x + Z + a_1)^{\frac{1}{n}} - (Z + a_1)^{\frac{1}{n}}) \quad (5.17)$$

Here, a_1 and k_1 are real numbers, with $a_1 > 0$.

For the cost function h :

[n-th power] A well-known monotonous and convex function is the n -th power function, where $n > 2$ is a real number. A possible function based on this property is

$$h \ y \ t_p \ t_c) = k_2 \ (y - Z + a_2)^n - (a_2 - Z)^n) \quad (5.18)$$

Here, a_2 and k_2 are real numbers, with $a_2 > 2B$.

[n-th root] Another known monotonous and convex function is the negative n th root function, where $n > 2$ is a real number. A possible selling function based on this could be

$$h \ y \ t_p \ t_c) = k_2 \ (Z + a_2)^{\frac{1}{n}} - (Z + a_2 - y)^{\frac{1}{n}}) \quad (5.19)$$

Here, a_2 and k_2 are real numbers, with $a_2 > 0$.

The actual functions are obtained by adding to the proposals listed above an additional term P , which represents the penalty for excess of energy production or consumption when this excess can cause congestion. For reward functions, the term P depends on x , t_p and t_c : the value of P is negative (i.e., reward is reduced) if $t_p - t_c > B$, zero otherwise.

Regarding cost functions, the term P depends on y , t_p and t_c : the value of P is positive (i.e., cost is increased) if $t_c - t_p > B$, zero otherwise.

The proposed functions will now be examined with respect to Z , which has been defined in Eq. (5.15) as the difference between total energy consumption and production, translated by the amount B : in other words, $Z = B$ corresponds to the case where total energy consumption and production are the same, $Z \leq 0$ to the case where congestion for overconsumption happens, and $Z \geq 2B$ to the case where congestion for overproduction happens. It can be seen that, for how the proposed functions have been made, when overall consumption is higher than production (i.e., $Z \in [0, B)$) energy generation has a higher value, as production does not completely match consumption and therefore it is desirable to increase it. Conversely, when the overall production is higher than consumption (i.e., $Z \in [B, 2B)$), there is already a surplus of produced energy and therefore the rewards for sellers become lower, as their energy is not needed as much as in the previous case. Regarding the prices for buying energy, they are higher in the case where overall energy consumption is higher than production: in this case, the energy produced locally is not enough to satisfy the needs of the consumers, and therefore more energy is introduced from outside, which causes an increase of prices for energy consumption.

Buying energy is cheaper when overall production is higher than consumption: the reason is that not all the energy produced locally is consumed, and therefore increased consumption is encouraged in order to match production, and the energy will be cheaper since it is produced from inside the community.

As will be seen now, the newly defined functions respect all the requirements listed in Section 5.1. Regarding point 1, for how the reward functions are defined, it is easy to notice that they are monotonic: that is, the reward for the prosumer increases as the amount of produced energy increases. This discourages the prosumer from intentionally reducing his/her energy production, as doing so would lower his/her revenues. However, if there is congestion for overproduction, the penalty term P causes the revenues for production to become lower, and encourages the prosumer to reduce his/her own energy production: this is the only case where curtailment should happen, and therefore point 1 is respected. Point 2 requires the functions to actively prevent congestion: this is achieved by adding the penalty terms P , which actively discourage the users from producing or consuming too much energy to the point of causing congestion. Point 3 requires prosumers to be actively encouraged to consume their own produced energy, before either selling their energy surplus or buying the remaining needed energy from outside: this point will be addressed in Section 6, where it will be shown that it is possible to choose the parameters for the proposed cost and reward functions so that this requirement is fulfilled. Finally, point 4 requires the game defined by those functions to always ensure the existence of a NE. This is a consequence of the theorem from [34], which ensures the existence of a NE provided that the payoff functions are concave: as the reward functions are convex and the cost functions are concave, and the payoff functions of the game are defined as in Eq. (3.3), it follows that the proposed payoff functions are concave, and the existence of a NE is guaranteed. At the end of Section 6, the values that those functions would take in the case of the running example from Section 3.2 will be shown.

5.4. Cooperative game with new proposals

It is important to ensure that the proposed solutions are still valid for the cooperative game. The NRG-X-Change issues described in [7] depended uniquely on how cost and reward functions were made, so this proposal solves them also in this case. However, in order to be able to ensure the existence of a pure Nash Equilibrium, the payoff functions of the cooperative game needs to be concave. For how the cooperative game is defined, its payoff functions are obtained by summing payoff functions of the selfish game: since the sum of concave functions is

still a concave function, the cooperative game will admit a pure Nash Equilibrium.

Denote by \bar{U}_{all} the partition whose only element is the set U itself: this partition corresponds to the scenario where there is only one coalition, formed by all the players. Denote by \bar{U}_{single} the partition with N elements, each of them being a single user: this corresponds to the situation where each user plays selfishly. The outcome of the games $G_{\bar{U}}$ changes depending on the choice of the partition \bar{U} , as in the following lemma:

Lemma 1. *For every possible partition \bar{U} of the set U the game $G_{\bar{U}}$ will reach a pure Nash Equilibrium: this Nash Equilibrium is an element of $\times_{i=1}^N S_i$ which is denoted by $s_{\bar{U}}$. Then for every possible partition \bar{U} of U*

$$\sum_{i=1}^N q_i(s_{\bar{U}}) \leq \sum_{i=1}^N q_i(s_{\bar{U}_{\text{all}}}) \quad (5.20)$$

Proof.

Chosen a partition $\bar{U} = \{\bar{U}_1 \dots \bar{U}_k\}$ of the set U , the game $G_{\bar{U}}$ admits a pure Nash Equilibrium, since its payoff functions are sums of concave functions, and therefore they are concave. The fact that $s_{\bar{U}}$ is a Nash Equilibrium means that, for every $i = 1 \dots k$, the inequality

$$\bar{q}_i(t) \leq \bar{q}_i(s_{\bar{U}}) \quad (5.21)$$

is true for every $t \in \times_{j=1}^N S_j$ such that, calling $S_{i-1} = \times_{j \notin \bar{U}_i} S_j$,

$$t|_{S_{i-1}} = s_{\bar{U}}|_{S_{i-1}}.$$

In the case of $G_{\bar{U}_{\text{all}}}$ there is only one coalition, containing all the users: Eq. (5.21) becomes then

$$\bar{q}_i(t) \leq \bar{q}_i(s_{\bar{U}_{\text{all}}}) \quad (5.22)$$

for every $t \in \times_{j=1}^N S_j$, since S_{i-1} is the empty set. Now, considering that by definition

$$\bar{q}_i = \sum_{U_j \in \bar{U}_i} q_j$$

and that every user belongs to the coalition, Eq. (5.20) follows. \square

This result, in economic terms, means the following. Given the possibility to create coalitions on a certain game $G = (U, S, Q)$, the partition generating the cooperative game with the overall best economic return for the whole community will always be \bar{U}_{all} .

Now, some considerations about general properties for cooperative games corresponding to other partitions. If $\bar{U} \neq \bar{U}_{\text{all}}$, no conclusion can be drawn *a priori* about $G_{\bar{U}}$. It may happen that $G_{\bar{U}}$ is more profitable for some alliances compared to $G_{\bar{U}_{\text{all}}}$, but the overall profit between all coalitions will always be lower or equal to the one obtained in $G_{\bar{U}_{\text{all}}}$. On the other hand, it may also happen that the overall profit between all coalitions in $G_{\bar{U}}$ is inferior to the one obtained in $G_{\bar{U}_{\text{single}}}$: in this case, the cooperative game is actually economically disadvantageous for the users compared to the selfish game.

Several simulations have been performed to further analyze how often is the formation of coalitions good for the users, both in terms of economic profit and energy self-consumption. Section 7.2 shows the used methods, and the results obtained.

6. Self-consumption constraint

This section shows that the newly proposed functions follow the self-consumption constraint, defined in Eq. (5.14), by proving that there is a choice for the parameters such that the inequality holds for all the values of the variable.

6.1. Candidate 1: logarithm selling n th power buying

Consider the following configuration: g , the selling function, is the one from Eq. (5.16) while h , the buying function, is the one from Eq. (5.18).

The objective is to verify that it is possible to choose the parameters of g and h in such a way that Eq. (5.14) holds for every x . In other words, given $n \geq 2$, the goal is to find values for k_1 , a_1 , k_2 and a_2 such that

$$k_1 \left(\ln \frac{x+Z+a_1}{Z+a_1} \right) < k_2 \left(x-Z+a_2 \right)^n - a_2 - Z^n$$

holds for every $x > 0$ and $Z \in [0, 2B]$.

Lemma 2. Let $n \geq 2$ be an integer number. For each $x > 0$, $Z > 0$, $a_1 \geq 1$ and $a_2 \geq 2B+1$ and choosing $k_1 = k_2 > 0$ the inequality

$$\ln \left(1 + \frac{x}{Z+a_1} \right)^{k_1} < k_2 x p_{a_2-Z}(x) \quad (6.1)$$

holds where $p_k(x)$ is the polynomial

$$p_k(x) = \sum_{m=0}^{n-1} \binom{n-1}{m} (x+k)^{n-1-m} k^m.$$

Proof. Take the exponential of both sides of Eq. (6.1): it then becomes

$$\left(1 + \frac{x}{Z+a_1} \right)^{k_1} < e^{k_2 x p_{a_2-Z}(x)}.$$

By hypothesis $k_1 = k_2$, so the exponents can be removed and

$$1 + \frac{x}{Z+a_1} < e^{x p_{a_2-Z}(x)}.$$

Finally, using the hypothesis $a_1 \geq 1$ and $a_2 \geq 2B+1$, the proof is completed by showing that

$$1 + \frac{x}{Z+a_1} \leq 1 + \frac{x}{Z+1} < 1+x < e^{x(x+1)} e^{x p_{a_2-Z}(x)}$$

which holds for every value of $x > 0$ and $Z \in [0, 2B]$. \square

Throughout this paper, it will be very important to remember the property

$$p_a(x) = \frac{x+a)^n - a^n}{x} \quad (6.2)$$

Proposition 1. Let $n \geq 2$ be a real number. The function from Eq. (5.16) with $a_1 = 1$ and the function from in Eq. (5.18) with $a_2 = 2B+1$ respect the self-consumption condition from Eq. (5.14) provided that $k_1 = k_2 > 0$.

Proof. First, suppose n is an integer number. Lemma 2 shows that, by choosing the parameters as earlier,

$$\begin{aligned} k_1 \ln \left(\frac{x+Z+a_1}{Z+a_1} \right) &= \ln \left(\frac{x+Z+a_1}{Z+a_1} \right)^{k_1} = \\ &= \ln \left(1 + \frac{x}{Z+a_1} \right)^{k_1} < k_2 x p_{a_2-Z}(x) = \\ &k_2 \left(x-Z+a_2 \right)^n - a_2 - Z^n \end{aligned}$$

for each $x > 0$, $Z \in [0, 2B]$.

Now, if n is a generic real number, it is easy to prove that $\left(x-Z+a_2 \right)^n - a_2 - Z^n > \left(x-Z+a_2 \right)^2 - a_2 - Z^2$. As the previous chain of inequalities holds for $n=2$, adding this inequality to that chain proves the proposition. \square

6.2. Candidate 2: n th root selling n th root buying

Here, the selling function g is as defined in Eq. (5.17), and buying function h is as defined in Eq. (5.19).

Proposition 2. Let $n \geq 2$ be a real number. It is possible to choose k_1 and a_1 in the selling function from Eq. (5.17) and for k_2 and a_2 in the buying function from Eq. (5.19) in such a way that Eq. (5.14) holds for those functions. In particular this happens by choosing $k_1 = k_2 > 0$ and $a_1 \geq a_2 + 2B$.

Proof. First, suppose that n is an integer. The self-consumption constraint is written as follows:

$$\begin{aligned} k_1 \left(x+Z+a_1 \right)^{\frac{1}{n}} - Z+a_1 &< \\ k_2 \left(Z+a_2 \right)^{\frac{1}{n}} - Z+a_2-x &^{\frac{1}{n}}. \end{aligned}$$

Now, replace $k_1 = k_2$. Dividing both sides by them, the inequality becomes

$$\left(x+Z+a_1 \right)^{\frac{1}{n}} - Z+a_1 < \left(Z+a_2 \right)^{\frac{1}{n}} - Z+a_2-x)^{\frac{1}{n}}.$$

By exploiting Eq. (6.2), this becomes equivalent to

$$\frac{x}{\sum_{m=0}^{n-1} \binom{n-1}{m} (x+Z+a_1)^{\frac{n-1-m}{n}} (Z+a_1)^{\frac{m}{n}}} < \frac{x}{\sum_{m=0}^{n-1} \binom{n-1}{m} (Z+a_2)^{\frac{n-1-m}{n}} (Z+a_2-x)^{\frac{m}{n}}}.$$

Since it needs to be proven for $x > 0$, denominators can be taken, and the above condition is equivalent to

$$\begin{aligned} \sum_{m=0}^{n-1} \binom{n-1}{m} (x+Z+a_1)^{\frac{n-1-m}{n}} (Z+a_1)^{\frac{m}{n}} &> \\ \sum_{m=0}^{n-1} \binom{n-1}{m} (Z+a_2)^{\frac{n-1-m}{n}} (Z+a_2-x)^{\frac{m}{n}}. \end{aligned}$$

Using the hypothesis $a_1 \geq a_2 + 2B$,

$$\begin{aligned} x+Z+a_1 &> Z+a_1 > a_1 > \\ 2B+a_2 &> Z+a_2 > Z+a_2-x. \end{aligned}$$

This is true for every $x > 0$, $Z \in [0, 2B]$, and, comparing the first two terms of the chain against the last two terms, the proof is complete.

Now, if $n = \frac{n_n}{n_d}$ for some integers n_n and n_d , the same argument could be replicated by multiplying the same exponents from above by n_n : this way, the inequality above can be proven for all rational values of $n \geq 2$ and, consequently, for all real values of $n \geq 2$, since the two compared functions are continuous in the considered interval. \square

6.3. Candidate 3: logarithm selling n th root buying

Consider now the selling function g as in Eq. (5.16), and the buying function h as in Eq. (5.19).

Proposition 3. Let $n \geq 2$ be a real number. It is possible to choose k_1 and a_1 for the selling function from Eq. (5.16) and k_2 and a_2 for the buying function from Eq. (5.19) in such a way that the self-consumption inequality defined in Eq. (5.14) holds. In particular this is true for the choice $a_1 \geq 1$, $a_2 = 1$ and $k_2 = k_1 n (2B+1)^{\frac{n-1}{n}}$.

Proof. First, assume that n is an integer. The self-consumption constraint can be written as

$$\begin{aligned} k_1 \left(\ln \frac{x+Z+a_1}{Z+a_1} \right) &< \\ k_2 \left(Z+a_2 \right)^{\frac{1}{n}} - Z+a_2-x &^{\frac{1}{n}}. \end{aligned}$$

For homogeneity, assume $k_1 = 1$ for now; k_2 is equal then to $n (2B+1)^{\frac{n-1}{n}}$, but it will be replaced later. Now,

$$\ln \left(1 + \frac{x}{Z+a_1} \right) < k_2 \left(Z+a_2 \right)^{\frac{1}{n}} - Z+a_2-x)^{\frac{1}{n}}.$$

Applying the exponential function to both sides, this becomes

$$1 + \frac{x}{Z + a_1} < e^{k_2 \frac{1}{n} (Z + a_2)^{\frac{1}{n}} - \frac{1}{n} (Z + a_2 - x)^{\frac{1}{n}}}.$$

Now, it is easy to apply the estimate

$$\frac{Z + a_2)^{\frac{1}{n}} - (Z + a_2 - x)^{\frac{1}{n}}}{\frac{x}{\sum_{m=0}^{n-1} \binom{n-1}{m} (Z + a_2)^{\frac{n-1-m}{n}} (Z + a_2 - x)^{\frac{m}{n}}}} > \frac{x}{n (Z + a_2)^{n-1}}$$

where the last inequality is true because the last term is obtained by assuming $x = 0$. So, the thesis can be proven by showing that

$$1 + \frac{x}{Z + a_1} < e^{k_2 \frac{x}{n (Z + a_2)^{n-1}}}.$$

Replacing now the value for k_2 and a_2 ,

$$k_2 \frac{x}{n (Z + a_2)^{n-1}} = x \frac{k_2}{n (Z + 1)^{n-1}} = x \frac{n (2B + 1)^{n-1}}{n (Z + 1)^{n-1}} > x$$

and so it becomes enough to prove

$$1 + \frac{x}{Z + a_1} < e^x.$$

If $a_1 \geq 1$,

$$1 + \frac{x}{Z + a_1} < 1 + \frac{x}{Z + 1} < 1 + x < e^x$$

which is true for every $x > 0$, $Z > 0$, and this proves the thesis.

Now, if n is a rational number, the same reasoning with the substitution presented in the proof of Candidate 2 can be followed, and if n is a real number, the thesis follows by an argument of continuity. \square

6.4. Candidate 4: n th root selling n th power buying

Consider now g to be the selling function defined by Eq. (5.17), while h will be the buying function defined in Eq. (5.18)

Proposition 4. Let $n \geq 2$ be a real number. It is possible to choose k_1 and a_1 for the function g described in Eq. (5.17) and k_2 and a_2 for the function h described in Eq. (5.18) in such a way that those functions respect the self-consumption constraint described in Eq. (5.14). More specifically this happens if $k_1 = k_2 > 0$, $a_1 = 1$ and $a_2 > 2B + 1$.

Proof. First, assume that n is an integer. The self-consumption constraint can be rewritten as

$$k_1 \left((x + Z + a_1)^{\frac{1}{n}} - (Z + a_1)^{\frac{1}{n}} \right) < k_2 \left((x - Z + a_2)^n - (a_2 - Z)^n \right).$$

Now, replace $k_1 = k_2$ and divide each side by them. This results in

$$(x + Z + a_1)^{\frac{1}{n}} - (Z + a_1)^{\frac{1}{n}} < x (a_2 - Z)^{n-1}. \quad (6.3)$$

Now, it is known that

$$\frac{(x + Z + a_1)^{\frac{1}{n}} - (Z + a_1)^{\frac{1}{n}}}{\frac{x}{\sum_{m=0}^{n-1} \binom{n-1}{m} (x + Z + a_1)^{\frac{n-1-m}{n}} (Z + a_1)^{\frac{m}{n}}}} < \frac{x}{n (Z + a_1)^{n-1}} \quad (6.4)$$

where the inequality is obtained by replacing $x = 0$. On the other hand, it is known that

$$x (a_2 - Z)^{n-1} > (a_2 - Z)x. \quad (6.5)$$

Replacing Eq. (6.3) with the estimates from Eq. (6.4) on the left side and Eq. (6.5) on the right side respectively, the thesis will be implied by proving that

$$\frac{x}{n (Z + a_1)^{n-1}} < (a_2 - Z)x.$$

Replacing the values for a_1 and a_2 , the following chain of inequalities holds:

$$\frac{x}{n (Z + a_1)^{n-1}} = \frac{x}{n (Z + 1)^{n-1}} < x < 2B + 1 - Z < (a_2 - Z)x.$$

This is true for every $x > 0$, $Z \in [0, 2B]$, and this completes the proof.

Now, if n is rational, the same proof as above with $n = \frac{n_n}{n_d}$ can be used. Because of the continuity of the two functions, the inequality holds for any real number $n \geq 2$ chosen a priori. \square

6.5. Numeric examples for the new proposals

This subsection will show an example of the values that the newly defined functions would have in the case of the running example described from Section 3.2. Like in the case of Section 5.2.1, the first time unit will be considered, and the cost for user U_1 and the profit for the user U_3 will be calculated. The values for the parameters will be arbitrary, and only have the purpose of showing how the calculation is made. Recall that the values for t_c and t_p have been calculated in Eq. (5.13). With the terms introduced now,

$$Z = B$$

for U_1 , and

$$Z = B - 2$$

for U_3 .

Candidate 1. The following parameters are used: $n = 2$, $B = 10$, $k_1 = 0.1$, $k_2 = 10$, $a_1 = 1$, $a_2 = 21$, in the respective measurement units (kWh for a_i and B , $\frac{EUR}{kWh}$ for k_i).

For the cost function for U_1 ,

$$h(1|1|2) = 0.1 (1 + 21 - 10)^2 - 21 - 10)^2 = -0.1 (12^2 - 11^2) = 2.3 EUR$$

For the reward function for U_3 ,

$$g(1|1|2) = 10 \ln \left(\frac{1 + 10 + 21}{10 + 21} \right) = 0.3 EUR$$

Candidate 2. The following parameters are used: $n = 2$, $B = 10$, $k_1 = 10$, $k_2 = 10$, $a_1 = 21$, $a_2 = 1$, in the respective measurement units (kWh for a_i and B , $\frac{EUR}{kWh}$ for k_i).

For the cost function for U_1 ,

$$h(1|1|2) = 10 (\sqrt{1 + 10} - \sqrt{1 + 9}) = 10 (\sqrt{11} - \sqrt{10}) = 1.54 EUR$$

For the reward function for U_3 ,

$$g(1|1|2) = 10 (\sqrt{1 + 21 + 8} - \sqrt{21 + 8}) = 10 (\sqrt{30} - \sqrt{29}) = 0.92 EUR$$

Candidate 3. The following parameters are used: $n = 2$, $B = 10$, $k_1 = 10$, $k_2 = 10$, $a_1 = 1$, $a_2 = 1$, in the respective measurement units (kWh for a_i and B , $\frac{EUR}{kWh}$ for k_i).

For the cost function for U_1 ,

$$h(1|1|2) = 10 (\sqrt{1 + 10} - \sqrt{1 + 9}) = 10 (\sqrt{11} - \sqrt{10}) = 1.54 EUR$$

For the reward function for U_3 ,

$$g(1|1|2) = 1 \ln \left(\frac{1 + 10 + 1}{10 + 1} \right) = 0.90 EUR$$

Candidate 4. The following parameters: $n = 2$, $B = 10$, $k_1 = 10$, $k_2 = 10$, $a_1 = 1$, $a_2 = 21$, in the respective measurement units (kWh for a_i and B , $\frac{EUR}{kWh}$ for k_i).

For the cost function for U_1 ,

$$h(1, 1, 2) = 0.1 \cdot (1 + 21 - 10)^2 - (21 - 10)^2 = 0.1 \cdot (12^2 - 11^2) = 2.3 EUR$$

For the reward function for U_3 ,

$$g(1, 1, 2) = 10 \cdot \sqrt{1 + 1 + 8} - \sqrt{1 + 8} = 10 \cdot \sqrt{10} - \sqrt{9} = 1.62 EUR$$

7. Simulations and results

The content of this section is as follows. Section 7.1 shows empirically that the previously existing (baseline) functions do not always allow the game to reach a NE, while the functions proposed here always allow it, as proven in the previous sections. Finally, Section 7.2 describes the results for the cooperative game, and shows that in general small coalitions are discouraged.

7.1. Results for non-cooperative game

This section has the purpose of evaluating the game defined in Section 3, showing empirically whether the existence of a NE always occurs or not. As it will be seen, this is not the case if the functions used are the original ones from the NRG-X-Change mechanism or the ones from [7]. Conversely, using the functions introduced in Section 5 always guarantees the existence of a NE, as proven in the previous sections.

The experimental evaluation has been made by simulating the game with a *Python* script. Real data from a grid from Cardiff, UK, have been used. The grid has 184 users, and 40 of them are capable of producing energy. Additional detail about the grid can be obtained from [35].²

The experiment has been run by simulating the game according to Algorithm 1. More specifically, at step 1 a random set of N users is chosen from the 184 users of the Cardiff grid: the only constraints are that N is a multiple of 10 up to 70, and N_p is either 10, 30 or 50 of N . Also, in case the game took too long to converge, an additional condition at step 4 has been added: the experiments end if a certain, pre-determined number of iterations has passed since the beginning of the game. This is because in some cases it may take many iterations before one of the two outcomes appears, and the computational time for the simulation may become excessive if this happens.

The simulation has been run for the following four use cases. As explained in Section 3, payoff functions are defined as the difference between reward and cost functions.

Cost and reward functions are the ones from the original NRG-X-Change mechanism. (**Original**)

Cost and reward functions are the ones described in [7]. (**Improved**)

Reward function is the logarithmic-based one defined in Eq. (5.16), cost function is the n th power-based one defined in Eq. (5.18). (**Log/Pw**)

Reward function is the n th root-based one defined in Eq. (5.17), cost function is the negative n th root-based one defined in Eq. (5.19). (**nRoot**)

For the n th power and n th root functions, $n = 2$ has been chosen for simplicity.

As shown in Section 6, there are many possible choices for the parameters of the functions. They are chosen so that, when $t_c = t_p$,

² To obtain the data please send an email to the MAS²TERING coordinator <https://www.mas2tering.eu/>.

Table 1

This table describes how many attempts for each approach have brought to a Nash equilibrium, out of a total of 245 for each case. The grid contains between 10 and 70 users. The column on the left describes the number of prosumers in percentage, while the upper row describes the approaches used.

	Original	Improved	Log/Pw	nRoot
10	20	203	245	245
30	114	232	245	245
50	242	245	245	245

Table 2

This table shows respectively: the approaches that have been tested (first column), the number of iterations needed on average for obtaining a NE (second column), and the average amount of self-consumption increase after the NE is reached (third column).

Functions	Iterations	SelfCons
Original	4.7255	15.7248
Improved	4.8554	20.2876
Log/Pw	4.0370	23.2854
nRoot	3.8872	23.3021

all the reward functions have the same value as the tariffs for selling energy in the Cardiff grid, and the cost functions have the same value as the tariffs for buying energy in the Cardiff grid.

Table 1 describes how often the game reaches a NE for each use case, depending on how many of the chosen grid users are producers (10, 30, or 50). Table 2 shows, for each use case, how many iterations are needed on average for the game to converge (**Iterations**), and by how many kWh the total amount of energy self-consumption in the grid increases after the loads are re-allocated (**SelfCons**).

The results lead to the following conclusions.

In the **Original** and **Improved** use cases, the game is not guaranteed to always reach a NE: it may happen, but its existence is not assured a priori. Conversely, it has been proven in the previous section that the game always reaches a NE for the newly proposed functions (i.e., the **Log/Pw** and **nRoot** use cases), and this is what happens in the simulations.

The functions from the **Improved** use case outperform the ones from the **Original** use case, both for convergence, as it happens more often for the **Improved** case, and for self-consumption, whose amount is higher in the **Improved** case. However, the **Original** case requires on average a slightly lower amount of interactions for convergence.

The functions defining the **Log/Pw** and **nRoot** use cases obtain better results both for convergence and self-consumption, compared to the ones for the **Original** and **Improved** cases. A NE is always obtained, convergence is obtained with a lower number of iterations, and total self-consumption is higher.

Results for the **nRoot** use case are on average slightly better compared to **Log/Pw**, both for the number of iterations needed for reaching convergence and for the amount of self-consumption.

As the functions have been designed to encourage self-consumption and avoid congestion, they are expected to affect the total grid consumption in terms of peak-shaving and valley-filling. Fig. 2 shows what happens regarding to this, in the specific case with $N = 40$ and $N_p = 4$. The two lines represent the total net consumption (i.e., the difference between energy consumption and energy production) at each time: the red line refers to the situation before the game, and the green line to the situation after the NE has been found.

The computational burden for this case is minimal: the time needed for completing an iteration scales linearly with the size of the considered grid. Anyway, even with a grid with 70 users, it takes less than 15 seconds to perform an iteration.

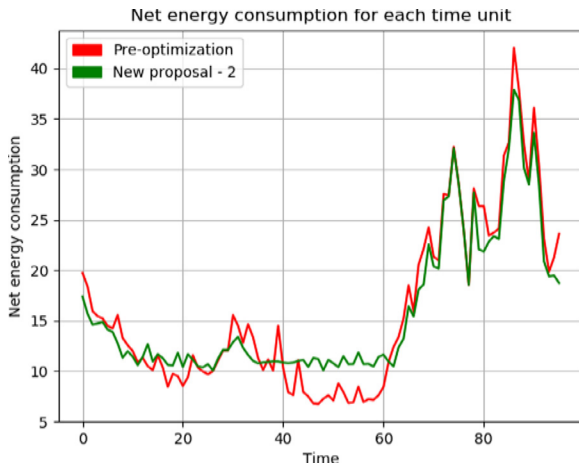


Fig. 2. This figure shows the time units within a day on the horizontal axis, and the net energy consumption through the grid on the vertical axis. The red line and the green line represent the consumption of a grid with 40 users, 4 of which are prosumers, respectively before and after the NE is reached.

7.2. Results for cooperative game

In this section, the impact of the formation of coalitions in a previously non-cooperative game will be analyzed, in terms of economic profits and energy self-consumption. The simulations focus on the case when, in a non-cooperative game, two users decide to form a coalition.

The simulations have been coded in a script in *Python* language, using the same data described in Section 7.1. The procedure has been based on Algorithm 1, by creating coalitions of two users inside the selfish game. This is how the simulations have been run.

1. A community $U = \{U_1 \dots U_N\}$ is generated, exactly in the same fashion as Step 1 in Algorithm 1; that is, U is a subset of the community with 184 users. In the cases considered for the experiments, N is either 10 or 20.
2. The procedure described in Algorithm 1 is then carried out and, for each user, their energy cost and their amount of energy self-consumption are noted.
3. For each couple of positive integers i, j such that $1 \leq i < j \leq N$, consider the game where U_i and U_j form a coalition, and every other user plays selfishly. Formally, the considered game is the game obtained by the partition where U_i and U_j belong to the same coalition, and every other coalition is formed by one single user.
4. Once the couple i, j has been chosen, the game for that partition is simulated like in Algorithm 1. For each user, their energy cost and their amount of self-consumption are noted.

It is checked how many times a user would have their economic profits increased or decreased after the formation of a coalition, and how many times would their self-consumption increase or decrease after the formation of a coalition. Also, the maximum increase in profits and of self-consumption that was possible to obtain with the formation of coalitions has been considered. The results are shown in Table 3 for the *Log/Pw* functions, and in Table 4 for the *nRoot* functions. In particular, the reported results refer to:

1. **Cases analyzed:** Total number of communities analyzed (the ones described in Step 1 of the procedure).
2. **Games run:** Total number of cooperative games run (the ones described in Step 3 of the procedure).
3. **MaxDecrease:** Given a community, when coalitions are formed, energy costs may increase or decrease for the players. This value indicates how much, on average, is the maximum possible

Table 3

Results for communities of respectively 10 and 20 users when coalitions of two users are formed. This table displays how often energy cost and self-consumption increase or decrease for the grid, and by which amounts. These results refer to the functions *Log/Pw*.

	Result	10 users	20 users
1	Cases analyzed	30	27
2	Games run	1350	5130
3	MaxDecrease	0.0015€	0.0056€
4	CasesIncrease	546	2948
5	CasesDecrease	799	2172
6	SelfCons(10)	0 kWh	5 10^{-4} kWh
7	SelfCons(30)	2.4 10^{-4} kWh	0.001 kWh
8	SelfCons(50)	0.02 kWh	0.002 kWh
9	SC increase	247	731
10	SC decrease	112	835

Table 4

Results for communities of respectively 10 and 20 users when coalitions of two users are formed. This table displays how often energy cost and self-consumption increase or decrease for the grid, and by which amounts. These results refer to the functions *nRoot*.

	Result	10 users	20 users
1	Cases analyzed	30	27
2	Games run	1350	5130
3	MaxDecrease	0.0013€	0.0033€
4	CasesIncrease	783	2749
5	CasesDecrease	558	2365
6	SelfCons(10)	0 kWh	0 kWh
7	SelfCons(30)	3.9 10^{-4} kWh	0.002 kWh
8	SelfCons(50)	0 kWh	0.004 kWh
9	SC increase	83	741
10	SC decrease	148	691

decrease of daily costs among all the coalitions of all the possible cooperative games that can be generated from one community.

4. **CasesIncrease:** Total number of players among all games for whom the energy cost after the cooperative game is higher than the energy cost in the non-cooperative game.
5. **CasesDecrease:** Total number of players among all games for whom the energy cost after the cooperative game is lower than the energy cost in the non-cooperative game.
6. **SelfCons(10):** Given a community, when coalitions are formed, the total amount of energy self-consumption through the grid can either increase or decrease. This value indicates how much, on average, is the maximum increase of self-consumption among all the cooperative games that can be generated from one community. This particular value refers to communities where the prosumers are the 10 of all the users.
7. **SelfCons(30):** Same as above, but it refers to communities where the prosumers are the 30 of all the users.
8. **SelfCons(50):** Same as above, but it refers to communities where the prosumers are the 50 of all the users.
9. **SC increase:** Total number of cooperative games in which the amount of self-consumption through all the community increases compared to the relative non-cooperative game.
10. **SC decrease:** Total number of cooperative games in which the amount of self-consumption through all the community decreases compared to the relative non-cooperative game.

The tables show several results. From what costs are concerned, it can be seen that the cooperative game does not always ensure a decrease in cost, as the cases where costs increase are more than the cases where costs decrease. This is true for both choices of functions (*Log/Pw* and *nRoot*), and the increase of costs happens more often if the *Log/Pw* functions are used. Furthermore, the maximum possible daily energy cost decrease for a coalition is on average 0.0056€(*Log/Pw*) or 0.0033€(*nRoot*), which has to be split between the members of the coalition itself, is marginal compared to the average energy cost for

a user and is very likely to decrease if new coalitions between the other users are formed. Self-consumption is not always encouraged: depending on the grid and on the functions used, a decrease in self-consumption can be more likely than an increase in self-consumption. In particular, if with the **nRoot** functions on a grid of 10 users, the self-consumption diminution is more likely than an increase, and the same is true if the **Log/Pw** functions are used for grids of 20 users. Also, when there is an increase in self-consumption, this is really low — at best, 0.0743 — compared to the increase that is obtained by the non-cooperative game from the initial situation.

Regarding the computational burden, the time needed for choosing a configuration for a coalition is less than 0.2 s in this case, and an iteration within a grid with 20 users takes less than 5 seconds overall.

The results that have been found suggest that the proposed system discourages the formation of coalitions inside the grid, as there is no guarantee that forming a coalition will be advantageous for the users, and even if it is, the advantages are minimum and can be negated by the eventual formation of opposite coalitions. The only case when a coalition is guaranteed to be advantageous is when all the users of the grid are in it, which is the objective that was been pursued.

8. Conclusions and future work

For the future of smart grids, local energy communities and peer-to-peer energy trading will be very important aspects. This work proposed new mechanisms for defining energy exchanges at the level of a local energy community. Two cases are investigated: the case where agents are considered to act on their own and the case where agents have the possibility to cooperate. The market is designed in order to encourage renewable energy production and self-consumption, and at the same time to match energy supply and demand at the community level.

The proposed mechanism has been designed with theoretical properties that are favorable compared to other existing market mechanisms. Simulations performed by using real-world data showed that the incentives granted by the proposed mechanism resulted in a change in the behavior of the local energy community for the better, in terms of economic costs and energy self-consumption. In particular, consumers are encouraged to schedule their loads in periods when prosumers inject a lot of energy into the grid, while prosumers are incentivized to not curtail their energy production by receiving higher rewards for it, except in critical cases like congestions.

This study on the cooperative game showed that forming small coalitions does not guarantee a decrease in costs or an increase in self-consumption, and therefore their formation is discouraged. The only coalition that ensures those benefits a priori is the global coalition.

Ideas for future work regard the investigation of this system when more types of flexible loads are available, for example, heating systems and storage units. Also, the impact of incentive mechanisms for bigger coalitions will be investigated.

CRedit authorship contribution statement

Fabio Lilliu: Conception and design of study, Acquisition of data, Analysis and/or interpretation of data, Writing – original draft. **Diego Reforgiato Recupero:** Conception and design of study, Writing – original draft, Writing – review & editing. **Meritzell Vinyals:** Conception and design of study, Writing – original draft, Writing – review & editing. **Roman Denysiuk:** Conception and design of study, Acquisition of data, Analysis and/or interpretation of data, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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