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Experimental Comparison of Models of the Drying-Cooling Process of Flatbreads for Optimized Automated Production: the Case Study of *Carasau* Bread

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***Abstract*— This paper presents a data-driven experimental approach to identify models and design efficient automation strategies that optimize the drying-cooling process of 2D-shaped bread during the redesign of low-automated production systems. Thin-layer drying-cooling equations are shown to be suitable for describing the water and heat transfer dynamics in flatbreads. These equations can be used to predict the time required to achieve desired levels of moisture and temperature as a function of external temperature. This knowledge is then used to derive a discrete-time model of the drying-cooling process, which helps identify the most suitable re-engineering actions to improve production and quality while reducing waste.**

The case study involves a bakery that produces *Carasau* bread, which is a flat and dry bread typical of the Sardinian tradition with a long history and renewed interest in recent years. In this scenario, the drying-cooling process occurs during transportation via conveyor belts. We discuss how the process can be optimized using real-time measurements an adaptative control of the speed given the length of the conveyor belt. This represents a significant step forward in automating the manufacturing process of Carasau bread.

***Index Terms*— Industry 4.0, Data-driven identification, Thin-layer bread, Carasau, Cooling and drying processes, Agrifood.**

I. Introduction

Flatbreads are a staple food consumed by approximately one-third of the world's population, with a global market valued at $ 38.8 billion in 2018 and projected to reach $ 62.8 billion by 2026 [1], [2]. Made from a simple mixture of flour, water, and salt, flatbreads vary in shape, size, and preparation method across different cultures [3]. Examples include *Lavash* in the middle-east [4], *Chapati* in India [5], and *Tortillas* in Central Americas [6], to name a few.

Most research has focused on traditional 3D-shaped bread [7]–[10], leading to a lack of theoretical knowledge about the baking and drying/cooling processes of flatbreads. Up to our knowledge, one of the few studies focused on flatbreads is that of Salari et al. in [4], which is mainly interested in the baking-drying kinetics. This knowledge gap is compounded by the fact that many types of flat bread are typical of small geographic areas where traditional production plants are not fully automated, creating difficulties in meeting the increasing market demand. In many small businesses, although some automation has been implemented, a significant portion of production still involves non-ad-hoc machinery or manual labor, affecting production rates and labor time [11]. To address these challenges, we propose data-driven monitoring and control methods to increase production, automation, and reliability of low-automated flat bread bakeries, with a focus on the *Carasau* bread [12]–[17]. Carasau bread is a round, flat bread of ancient origin from Sardinia, Italy, known as "carta musica" (music sheets) due to its thin, translucent appearance [18].

Problem of interest. The production process of Carasau bread involves several stages. First, the dough is prepared by mixing flour, water, salt, and yeast. The dough is then shaped into thin, round sheets, which are baked in an oven. During this phase, the Carasau bread inflates due to the sudden change in temperature of its air and water content. After the baking process, the sheets are moved onto a conveyor belt for the drying-cooling process. This is a critical step that requires precise levels of temperature and moisture to ensure that the sheets are ready for the next step, which is the separation process. In the production plant of our case study, after the drying-cooling process, the sheets are moved directly into an automated separation machine that split each sheet into two separate sheets. The correct functioning of the separation machine needs specific levels of temperature and moisture in the sheets to minimize the amount of product wasted due to incorrect separation and successive separation of the flatbread sheets. Understanding the moisture and heat transfer dynamics is essential for the system's automation efficiency and for the final quality of the bread [19]–[21].

The **main contribution** of this paper is the identification of the most appropriate models for the water/heat transfer dynamics during the drying-cooling process of Carasau bread, potentially applicable to other flatbreads as well. We will discuss how such models can be exploited in the online design of the transportation system to achieve the desired moisture and temperature levels at the end of the drying-cooling process, using real-time data collected from the plant, including the current state of the bread and environmental conditions. The proposed strategy is instrumental to improve the efficiency and reliability of the production process, reducing the need for human supervision, and improving the quality of the finished product.

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Fig. 1. Pictures of Carasau production process showing the steps of baking in the oven, cooling/drying over the conveyor belt, and automated separation.

Outline of the paper. In Section II we discuss how thin-layer equations, which are usually employed in the study of other kinds of food [22]–[24], well approximate the drying-cooling dynamics of flatbreads. Moreover, we identify the most suitable approximate model for the Carasau Bread by experimental results. Section III is instead devoted to the design of the conveyor, i.e., determining its length and how to tune its speed based on the initial temperature and moisture of the incoming sheets.

II. DRYING-COOLING DYNAMICS OF CARASAU BREAD

The production process of Carasau bread can be divided into several steps, which are pictured in Fig. 1 and schematized in Fig. 2: mixing the ingredients to obtain a dough; rolling out the dough into round sheets of about 1 mm thickness; baking the sheets in an oven and then leaving them to cool down and dry; separating the cooled and dried sheets into two separate sheets; re-baking the separated sheets.

This work focuses on the automatization of the dying-cooling process and the subsequent separation process. In our plant, the sheets are left to cool down and dry over a conveyor belt that brings the sheets directly into the separation machine that cut and split them apart. Note that, while traditionally the separation process is done manually, in our plant it has been designed a custom separation machine that automates this process. To ensure it functions correctly, the separation machine relies on precise levels of temperature and humidity of the sheets.

To develop a model that can capture the dynamics of these factors, we start by describing the geometric properties of the Carasau bread and by stating our assumptions during the drying-cooling process:

- (a) Each dough has a cylindrical shape, with radius r much larger than height h, i.e., $r \gg h$.
- (b) Each dough is homogeneous and isotropic;
- (c) The material characteristics are constant and the shrinkage/expansion effect is neglected;
- (d) The heat transfer is much faster than the moisture transfer, thus we consider the effect of time alone on the dependent variables of moisture M and temperature T [22], [25];
- (e) The heat and moisture transfers occur by convection between the product and the surrounding air [26], [27], thus neglecting the transfer due to the conduction with the bearing surface, which is usually much slower;

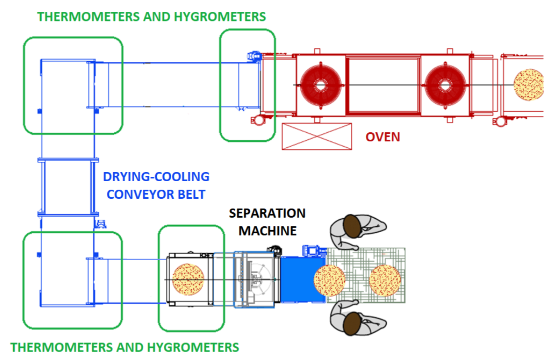

Fig. 2. A schematic representation of the cooling/drying process of Carasau bread, showing the steps of baking in the oven,over a conveyor belt, and automated separation.

- (f) The heat and moisture transfers occur equally at both surfaces (e.g., the dough is flipped periodically);
- (g) Effective moisture and thermal diffusivity are constant versus moisture content and temperature.
- (h) The pressure variations are neglected during the drying process.

A. Analytical model

Let x, t be the variables modeling the space and time, respectively. Under assumptions (a)-(h), the Luikov equations for planar geometries become [28]

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial c(x,t)}{\partial x^2} \tag{1}$$

where $c(x,t)$ denotes the generic local concentration of the *agent* (either moisture or heat) at space x and time t, and D denotes its diffusivity, assumed constant both in space and time. The Eq. (1) is known as the Fick's second law for the mass transfer (in our case moisture) and the Fourier's second law for heat transfer, respectively.

The transient-state solutions should be otained by solving Eq. (1) with the help of the appropriate initial condition and boundary conditions. According to several authors [23], [28], [29], since by assumption $r \gg h$, we consider an infinite slab of finite thickness h and the following boundary conditions:

- (j) Initial concentration is uniform within the product,

$$c(x,0) = c_0, \quad x \in (0,h).$$

(k) Surface diffusion only happens at one surface and not on the other one, which may then be assumed to be at equilibrium,

$$c(0,t) = c_0, \quad t \geq 0,$$
$$c(h,t) = c_e, \quad t \geq 0.$$

Let us define the dimensionless concentration $c_r(t)$ as

$$c_r(t) = \frac{\bar{c}(t) - c_e}{c_0 - c_e}, \tag{2}$$

where $\bar{c}(t) = h^{-1} \int_0^h c(x,t)dx$ denotes the average concentration within the product at time t. Then analytical solutions of Eq. (1) under assumptions (a)-(k) are given by [28]

$$c_r(t) = \sum_{i=1}^{\infty} \frac{8}{\pi^2} \frac{1}{(2i-1)^2} \exp\left[-\frac{(2i-1)^2 \pi^2 D}{4h^2} t\right]. \tag{3}$$

B. Approximate models

The theoretical model in Eq. (1) clearly explains the water/heat transfer outside the product and can be used for all process conditions, and the solution accuracy in Eq. (3) is very high. However, errors in estimating the diffusion coefficient D, the equilibrium agent content c_e, and possible errors in the measurement of the dough thickness h can affect the accuracy of the moisture/temperature ratio calculation, and the combined error will depend on the number of terms considered in the series.

For this reason, the most widely used thin layer equations (which are given in Table I) provide an approximation of the solution in Eq. (3), considering only a finite number n of terms and neglecting higher terms, $i > n$, leading to the following approximate solution

$$c_r(t) \approx \sum_{i=1}^{n} A_i \exp\left[-k_i t\right],$$

where constants A_i and k_i depend on the specific flatbread product. Moreover, it can be noticed that this model assumes a correct physical meaning if the quantity $c_r(t)$ is monotonically decreasing and satisfies

$$c_r(0) = 1, \qquad \lim_{t\to\infty} c_r(t) = 0. \tag{4}$$

Table I presents several simplified models that can accommodate these boundary conditions through a suitable choice of parameters. These models are mainly based on the next three models:

1) Newton Model $c_r(t) = \exp\left[-k_0 t\right]$;
2) Page Model $c_r(t) = \exp\left[-k_0 t^n\right]$;
3) Overhults Model $c_r(t) = \exp\left[-(k_0 t)^n\right]$.

Some variations of these models are also referred to as *empirical* because their derivation was influenced by experimental data. We consider three empirical models:

1) Wang and Sing Model $c_r(t) = 1 + at + bt^2$;
2) Weibull Model $c_r(t) = a - b\exp\left[-k_0 t^n\right]$;
3) Thompson Model $t = a\ln(c_r(t)) + b\ln^2(c_r(t))$.

C. Experimental results on the Carasau bread

To determine which of the models listed in Table I best describe the drying and cooling behavior of Carasau bread, we conducted experiments to measure the temperature/weight decay of seven sheets under different conditions, as specified in Table II. Note that moisture content can be calculated by subtracting the weight of all ingredients used for each sheet from the total weight of the sheet.

Fig. 3 shows the normalized temperature $T_r(t)$ and moisture $M_r(t)$ of the sheets. The average temperature $\bar{T}(t)$ and moisture $\bar{M}(t)$ have been normalized as follows, according to Eq. (2):

$$T_r(t) = \frac{\bar{T}(t) - T_e}{T_0 - T_e}, \quad M_r(t) = \frac{\bar{M}(t)}{M_0},$$

where T_0 and M_0 are the initial average temperature and moisture, and T_e denotes the environmental temperature, which is the equilibrium temperature achieved by the sheets. Note that the moisture content at equilibrium is zero.

We determined the optimal parameters of each model in Table I using the function `fit` in Matlab that implements the Nonlinear Least Squares method. Table III shows the minimum and average root-mean-square errors and the standard deviation for all cases, indicating that the best models are Noomhorm & Verma, Diffusion Approach, Verma, and Three Term Exponential.

Model Name	Formula	Num. Parameters
Newton (Lewis)	$c_r(t) = \exp\left[-k_0 t\right]$	1
Logarithmic*	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)$	2
Two Term exponential	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-ak_0 t\right]$	2
Diffusion Approach	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-bk_0 t\right]$	3
Verma	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-k_1 t\right]$	3
Noomhorm & Verma*	$c_r(t) = a\exp\left[-k_0 t\right] + b\exp\left[-k_1 t\right] + (1-a-b)$	4
Three Term exponential	$c_r(t) = a\exp\left[-k_0 t\right] + b\exp\left[-k_1 t\right] + (1-a-b)\exp\left[-k_2 t\right]$	4
Page	$c_r(t) = \exp\left[-k_0 t^n\right]$	2
Midilli and Kucuk	$c_r(t) = \exp\left[-k_0 t^n\right] + bt$	3
Modified Kaleta	$c_r(t) = a\exp\left[-k_0 t^n\right] + (1-a)\exp\left[-k_1 t^n\right]$	4
Overhults	$c_r(t) = \exp\left[-(k_0 t)^n\right]$	2
Demir*	$c_r(t) = a\exp\left[-(k_0 t)^n\right] + (1-a)$	3
Wang and Sing	$c_r(t) = 1 + at + bt^2$	2
Weibull*	$c_r(t) = (1-a) - a\exp\left[-k_0 t^n\right]$	3
Thompson	$t = a\ln(c_r(t)) + b\ln^2(c_r(t))$	2

TABLE I

APPROXIMATE THIN-LAYER MODELS; THOSE MARKED WITH * HAVE BEEN REFORMULATED TO ACCOMPLISH THE BOUNDARY CONDITIONS IN EQ. (4).

The simplest of these models is the Verma model [30], [31], which is a model of order 2 with 3 parameters: $k_0, k_1 \geq 0$ are the modes of the system and the coefficient $a \in (0,1)$ quantifies which mode is prevalent. The same model is suitable for both temperature and moisture decay:

$$\begin{aligned} T_r(t) &= a_T \exp\left[-k_{0,T}t\right] + (1-a_T)\exp\left[-k_{1,T}t\right], \\ M_r(t) &= a_M \exp\left[-k_{0,M}t\right] + (1-a_M)\exp\left[-k_{1,M}t\right]. \end{aligned} \tag{5}$$

In Table IV we summarize the model's coefficients that best fit all the experiments, both for the temperature $c_r(t) = T_r(t)$ and the moisture $c_r(t) = M_r(t)$. It can be noticed that there are two main modes:

- Temperature $T_r(t)$:
  - The slowest mode $k_{0,T} \approx 2 \div 5 \cdot 10^{-3}$;
  - The fastest mode $k_{1,T} \approx 2 \div 4 \cdot 10^{-2}$.
- Moisture $M_r(t)$:
  - The slowest mode $k_{0,M} \approx 1 \div 4 \cdot 10^{-4}$;
  - The fastest mode $k_{0,M} \approx 5 \div 9 \cdot 10^{-3}$.

According to what can be noticed by directly looking at the temperature and moisture decay curve in Fig. 3, the first three experiments reveal only the slowest mode, while the last 4 experiments reveal both modes.

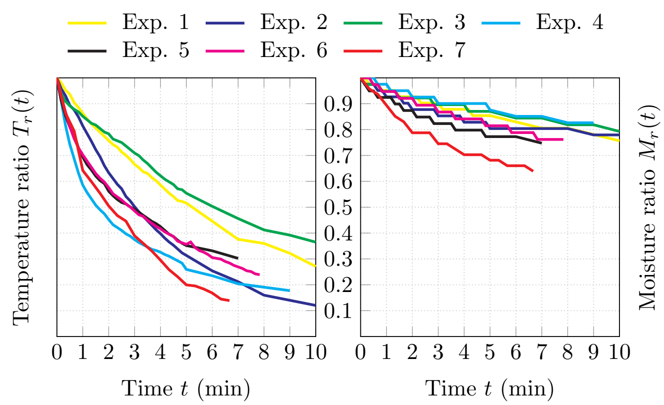

Fig. 3. Temperature and moisture decaying of bread sheet for the experiments detailed in Table II.

	Exp. 1 (yellow)	Exp. 2 (blue)	Exp. 3 (cyan)	Exp. 4 (green)	Exp. 5 (black)	Exp. 6 (magenta)	Exp. 7 (red)
Month	Feb	Feb	Feb	Feb	Jul	Jul	Jul
Ext. temperature	22°C	22°C	22°C	22°C	32°C	32°C	32°C
Init. temperature	55°C	63°C	50°C	67°C	64°C	63°C	80°C
Revolution	NO	NO	NO	NO	NO	NO	YES
Tray holes	NO	NO	YES	YES	YES	YES	YES
Tray material	Carton	Carton	Carton	Carton	Carton	Plastic	Plastic

TABLE II

Cooling conditions of the experiments depicted in Fig. 3. The colors denote the color of the curves in this figure.

	Temperature			Moisture		
Model name	**Min Error**	**Mean Error**	**Std. Dev.**	**Min Error**	**Mean Error**	**Std. Dev.**
Newton (Lewis)	0.0084	0.0375	0.0364	0.0056	0.0110	0.0051
Logarithmic	0.0068	0.0192	0.0176	0.0030	0.0034	0.0003
Two Term exponential	0.0069	0.0227	0.0218	0.0033	0.0068	0.0036
Diffusion Approach	**0.0038**	**0.0085**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Verma	**0.0038**	**0.0085**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Noomhorm & Verma	**0.0036**	**0.0083**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Three Term exponential	**0.0036**	**0.0080**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Page	0.0064	0.0127	0.0106	0.0028	0.0040	0.0013
Midilli and Kucuk	0.0064	0.0124	0.0102	0.0028	0.0040	0.0013
Modified Kaleta	0.0064	0.0090	0.0065	0.0027	0.0035	0.0006
Overhults	0.0064	0.0127	0.0106	0.0028	0.0040	0.0013
Demir	0.0064	0.0124	0.0111	0.0027	0.0032	0.0004
Wang and Sing	0.0110	0.0429	0.0451	0.0032	0.0047	0.0012
Weibul	0.0064	0.0124	0.0111	0.0027	0.0032	0.0004
Thompson	0.2643	0.3297	0.0542	0.2618	0.3230	0.0415

TABLE III

Fitting results for the experiments.

	Param.	Exp. 1 (yellow)	Exp. 2 (blue)	Exp. 3 (green)	Exp. 4 (cyan)	Exp. 5 (black)	Exp. 6 (magenta)	Exp. 7 (red)
$\boldsymbol{T_r(t)}$	a_T	≈ 1	≈ 1	≈ 1	0.563	0.738	0.754	0.888
	$k_{0,T}$	$2.1 \cdot 10^{-3}$	$3.8 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$	$2.3 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
	$k_{1,T}$	useless	useless	useless	$2.4 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	$4.1 \cdot 10^{-2}$
$\boldsymbol{M_r(t)}$	a_M	≈ 1	0.807	≈ 1	≈ 1	0.841	0.892	0.700
	$k_{0,M}$	$3.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$3.0 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$3.8 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
	$k_{1,M}$	useless	$6.7 \cdot 10^{-3}$	useless	useless	$9.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	$6.9 \cdot 10^{-3}$

TABLE IV

Optimal coefficients of the Verma models in Eq. (5) describing the temperature and moisture decay.

III. CONVEYOR BELT DESIGN FOR TEMPERATURE AND MOISTURE CONTROL OF THE BREAD

In the previous section, we derived that the Verma model in Eq. (5) is the most appropriate model to describe the decay of temperature and moisture in sheets of Carasau bread. Moreover, we have discussed how to identify the parameters of the model using curve-fitting techniques with real measurements. Using this information, we can now determine the amount of wait time t^* the bread sheet must remain over the drying/cooling conveyor belt to reach the desired temperature T^* and moisture M^*. To do so, one needs to solve the following system of inequalities:

$$\begin{cases} a_T \exp[-k_{0,T}t^*]+(1-a_T)\exp[-k_{1,T}t^*] \leq \frac{T^*-T_e}{T_0-T_e}, \\ a_M \exp[-k_{0,M}t^*]+(1-a_M)\exp[-k_{1,M}t^*] \leq \frac{M^*}{M_0}, \end{cases} \tag{6}$$

where T_e is the temperature of the surrounding air, T_0, M_0 are the initial temperature and moisture levels, and a_T, a_M, $k_{0,T}$, $k_{1,T}$, $k_{0,M}$, $k_{1,M}$ are the parameters of the Verma models in Eq. (5) obtained offline through curve-fitting techniques.

We consider the length of the conveyor belt to be fixed, while its speed can vary within a certain range to be determined, which is the quantity of interest to be controlled. In the remainder of this section, we outline the procedure to design the length of the conveyor belt and show how to control its speed online to meet the desired levels of temperature and moisture at the end of the drying-cooling process and right before the separation process.

A. Speed constraints

The throughput of the cooking and cooling process must be equal, and we can obtain the speed v_{conv} of the conveyor belt using the following equation $v_{conv} = \frac{\delta_{conv}}{\delta_{oven}} v_{oven}$, where δ_{conv} and δ_{oven} are the distances between two consecutive sheets on the conveyor belt and in the oven, respectively, and v_{oven} is the speed of the oven conveyor belt. In order to guarantee that two consecutive sheets do not overlap, such distances must be greater than the diameter of the sheets, i.e., $2r$. Let then $\delta_{oven} > 2r$ be fixed, then the speed of the conveyor belt must be designed such that also $\delta_{conv} > 2r$, yielding to the following lower bound $v_{conv} \geq \frac{2r}{\delta_{oven}} v_{oven}$. On the other hand, the conveyor belt cannot be too fast for two different reasons: first, the sheets may be deformed and lose their original shape; secondly, the separation machine requires some time to perform the cutting and splitting of the sheets. Thus, letting v_{max} a suitable upper bound to the speed of the conveyor belt leads to the following design constraints

$$v_{conv} \in \left[\frac{2r}{\delta_{oven}} v_{oven},\, v_{max} \right], \quad \delta_{oven} > 2r. \tag{7}$$

B. Length design of the conveyor belt

Let us now move forward with determining the suitable length of the conveyor belt, which is considered to be fixed, as is typically the case in industrial processes. This requires taking into account both the constraintsimposed on the conveyor belt speed v_{conv}, given in Eq. (7), and the desired time t^* necessary to achieve optimal drying and cooling, which is the maximum time satisfying the system of inequalities in Eq. (6). With these parameters, the length ℓ^* of the conveyor belt can be designed by

$$\ell^* = v_{conv} t^*, \tag{8}$$

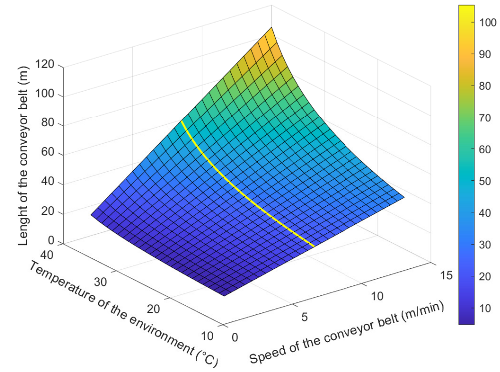

Fig. 4. Length of the conveyor belt depending on air temperature and speed. The yellow curve is the working speed of $8.5\ m/min$.

Fig. 4 displays the computed length of the conveyor belt obtained from Eq. (8) under the following choices:

- The desired temperature and moisture of the bread sheet at the end of the cooling process are $T^* = 45°$ C and $M^* = 20$ g. These values have been empirically determined from real data measurements during the normal execution of the process.
- Since in our plant the speed of the conveyor belt is usually about 8.5 meters per minute, it has been chosen to range within $v_{conv} \in [2, 15]$.
- The air temperature has been chosen to range between 15 and 40 degrees, according to the environmental temperatures reached in our plant during winter and summer times.
- The parameters of the models in Eq. (6) for the temperature and the moisture have been considered according to the results of Exp. 7 in Table IV, recalled next

$$\begin{array}{lll} a_T = 0.888, & k_{0,T} = 4.7 \cdot 10^{-3}, & k_{1,T} = 4.1 \cdot 10^{-2}, \\ a_M = 0.700, & k_{0,M} = 2.7 \cdot 10^{-4}, & k_{1,M} = 6.9 \cdot 10^{-3}. \end{array} \tag{9}$$

C. Temperature and moisture control

The minimum length of the conveyor belt required to tune the speed within the desired range in winter and summer can be inferred by the data in Fig. 4. Once the length of the conveyor belt is designed, the corresponding speed needs to be set at given time intervals within the design bounds given in Eq. (7). Real-time control of the conveyor belt speed can be used to fine-tune the temperature and moisture of the bread before entering the automated separation machine when the bread temperature and humidity at both the exit of the oven and in the environment surrounding the conveyor belt are available, e.g., through thermometers and hygrometers.

The control law to update the conveyor belt speed every given sampling interval can be chosen as follows

$$v_{conv}(t) = \begin{cases} \frac{2r}{\delta_{oven}} v_{oven} & \text{if } \frac{t^*(t)}{\ell^*} < \frac{2r}{\delta_{oven}} v_{oven} \\ \frac{t^*(t)}{\ell^*} & \text{if } \frac{t^*(t)}{\ell^*} \in \left[\frac{2r}{\delta_{oven}} v_{oven}, v_{max}\right] \\ v_{max} & \text{if } \frac{t^*(t)}{\ell^*} \geq v_{max} \end{cases},$$

where $t^*(t)$ is the waiting time needed by the bread sheet arrived at time t to achieve the desired temperature T^* and moisture M^* levels, computed by numerically solving Eq. (6) with parameters as in Eq. (9), which depends on its initial temperature T_0 and moisture M_0 measured at the exit of the oven and the ambient temperature T_e measured in the close proximity of the conveyor belt. The speed can be updated online to account for changes in the ambient temperature during the day, providing closed-loop control of the process.

IV. Conclusions

We compared different models for the water/heat transfer dynamics during the drying-cooling process of flatbreads and tested them experimentally to choose the best applicable to Carasau. The "Verma" model yields the best fitting of its parameters with the available real data and has the best accuracy when used to predict the future temperature of the bread (up to 10 minutes after the first baking of the bread), given ambient temperature and temperature at the instant of exit from the oven. The proposed strategy improves the efficiency and reliability of the production process because it allows the correct design of the length and speed of the conveyor belt which transports the flatbread from the oven to the separation machine. Correct temperature and humidity of the flatbread at the instant of separation allow to vastly reduce product wasted because of the incorrect separation of the bread by the human operator due to excessive stickiness which leads to rupture and tears in the flat bread.

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