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Tensegrity footbridge with prestressed deck

Stefano Punzo¹, Bruno Briseghella², Gian Felice Giaccu³, Alireza Hosseini¹, Wenjin Huang³ and Luigi Fenu¹.

¹ University of Cagliari, Cagliari 09124, Italy

² Fuzhou University, Fuzhou, China

³ University of Sassari, Sassari, Italy

lfenu@unica.it

Abstract. This paper presents an introduction to a tensegrity footbridge with a prestressed deck, focusing on its structural analysis and design. The inclusion of cable pretension leads to deck prestressing. Through finite element analysis, various aspects are investigated, including nonlinear geometric effects, key load cases, the influence of the main cable's rise, and modal analysis of this unique structure. The study reveals that the tensegrity bridge exhibits favourable structural behaviour, making it a suitable choice for footbridges, although it differs from traditional bridge designs. The paper concludes by summarizing essential recommendations concerning structural design, material properties, and the overall behaviour of the tensegrity bridge.

Keywords: tensegrity bridge, structural design, static behaviour, modal analysis.

1 Introduction

A tensegrity system is a spatial structure constituted by tensile components and compression members, the latter while only connected with tensile members [1,2].

Tensegrity is an elision of 'tension + integrity', a structural principle governing the structure of cells, radiolarian and other living beings. Some spiders' net work are tensegrity structures. It is not strange that, as the result of a Nature evolution, tensegrities are optimized structures that minimize struts, whose mass, to prevent buckling, is much less than that of ties [3,4].

Besides design objects, few tensegrity structures have been built since Buckminster Fuller [5,6] conceived the earliest tensegrity structures. Conversely, several studies have been done by Motro's [7] research group in Montpellier, Micheletti's in Rome, as well as by Kenneth Snelson and Buckminster Fuller

To build constructions with structure in tensegrity has some drawbacks as member congestion, high deflections, difficulties in building them for the number of pretensioned ties connected one another. Problems due to member congestion and great number of pretensioned ties connected one to the other can be less important if "not pure" tensegrities are also considered, that don't respect all the above assumptions for which a structure can be classified as a "pure tensegrity". For instance, a strut can be connected not only at its ends. In this way, some structures for architecture and civil engineering can be realized in tensegrity, and not only structures where high flexibility is allowed, as in antenna towers of which there are some examples in tensegrity.

The scientific literature reports some examples of “not pure” tensegrity bridges that can be considered as hybrid tensegrity structures. Arup designed and successively built a bridge only partly in hybrid tensegrity, the Kunipa Bridge in Brisbane.

This paper faces the design of a tensegrity footbridge with prestressed deck, that is compressed by the curved pretensioned cables acting as ties of the tensegrity structure and working in flexure induced by the deck permanent and applied loads. Moreover, the tensegrity ties acting as tension braces support the transverse beams stiffening the deck, while the curved shape of the top and bottom chord cables prestressing the deck is allowed by vertical struts of suitable length interposed between each other.

This tensegrity footbridge is of course a “not pure” tensegrity, that nonetheless has some advantageous characteristics as the very low self-weight, the absence of thrust at the abutments, as well as the aesthetic value of the typical relationship between form and structure of tensegrities.

2 Structure description

A tensegrity footbridge with span length of 30m has been studied. The footbridge layout without the composite corrugated sheet is shown in Fig.1.

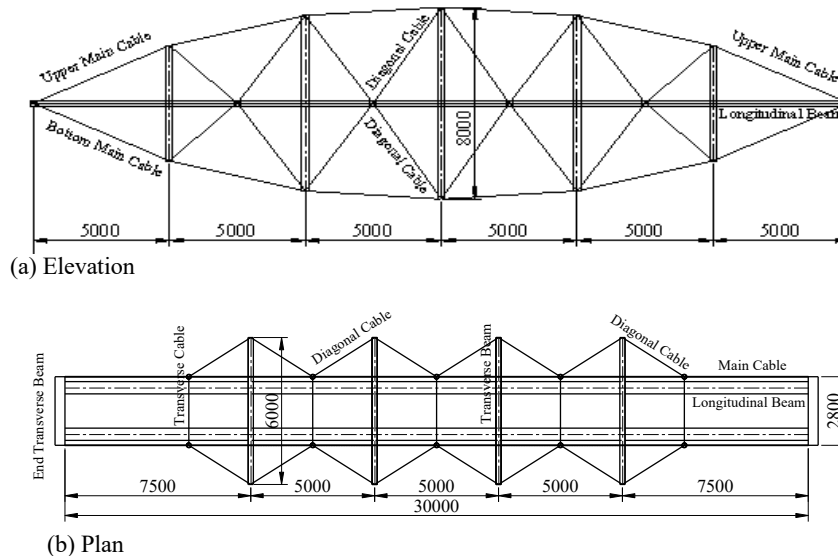


Fig.1 Dimension of the footbridge (unit: mm)

The tensegrity footbridge is made of a continuous cable-system connected to the ends of vertical columns and supporting the deck transverse beams, connecting two longitudinal box girders along the bridge span.

Both the rise of upper chord cables and the sag of the bottom chord cables are equal to 4m. The curved shape of the main cables is approximated by a polyline. The horizontal distance between the nodes of the polyline is 5 m. The mirrored nodes of the top and

bottom polyline are held in place by the vertical struts (columns), as well as by the cross-brace cables and by the transverse cables. Specifically, the cable system, consists of polyline longitudinal cables (main cables), cross-brace cables (diagonal cables) and transverse cables. The cross-brace cables prevent the longitudinal displacement of their end nodes, as it happens for vertical cross-braces in braced frame structures. Also, since they are inclined outward with respect to the plane passing on the plane curves of top and bottom main cables, they also prevent lateral displacements of the deck, that is hanged at the central node of the braces by means of its transverse beams. Considering the elongation during the prestressing procedure and fatigue life, a bundle of steel strands with a diameter of $10\phi 15.2$ was selected for the main cable section. They were designed to behave elastically at a stress level of 1000 MPa under service loads. Since their internal forces are not high, both transverse and diagonal ties are made of prestressed steel bars, thus giving a higher rigidity contribution. Their cross-sectional is 1100 mm^2 , with a pretensioning stress of only 300MPa. Vertical struts are made of steel tubes with 203mm in diameter and 16mm in thickness are chosen. They are subjected to axial compressive forces transmitted between upper and bottom main cables; due to this they could be also made by stone materials. About the deck, this is made by two longitudinal beams with sufficient flexural and axial stiffness to resist applied loads and normal stresses induced by the pretensioned cables, and by transverse beams through which the deck is hanged to the chord cables. I-shaped transverse beams with depth 294mm, flange width 302 mm, thickness of web and flange 12mm are used. Box sections are used in the longitudinal girders to resist torsion induced by transversally unbalanced loads, otherwise not absorbed by the cable structure. Concrete filled rectangular steel box sections with twin cells are adopted. Width and depth of the box section were 500mm and 200mm, respectively. Flange and web thickness is only 6mm, so that the deck is quite light before filling the concrete into the steel box, after the main cable has been prestressed for the first. After curing of the concrete poured in the box beams and cast on the corrugated sheet, the main cables are prestressed for the second time. Concrete class C 40/50 is used. Steel grade is S355. The corrugated sheet is placed over the flanges of the two longitudinal beams and connected to them through steel headed studs. No slip between sheet and flange are supposed. The two end transverse beams are designed to anchor the main cable and transfer the prestressing force to the longitudinal beams, so that the bridge is simply supported on the bank. The end transverse beams should be strong enough to prevent the longitudinal beam's out-of-plane bending due to huge compression forces. In addition, the core concrete would help to increase the damping ratio as well as provide enough self-weight to resist both displacement and vibration of the deck due to live loads such as crowd load and wind load. An axonometric view of the structural scheme of the bridge is shown in Fig.2.

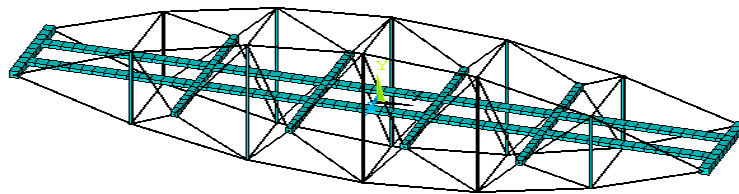


Fig.2. Structural scheme of the bridge

3 Static analysis

Due to the minimal dead load of a tensegrity bridge, the structural responses of the cable support system become significantly noticeable when subjected to crowd load. In the preceding analysis, various load cases were examined, revealing two unfavourable scenarios. Table 1 provides a comprehensive list of stress increments experienced by different members under crowd load. The "Full Deck" load case corresponds to the entire deck accommodating pedestrians, while the "Half Width Deck" load case pertains to only half of the deck accommodating pedestrians.

Table 1. Stress Increase of Members (MPa)

Load Cases	Variants		Cables				Strut		Beams			
			Main		Diagonal		Transverse		Longitudinal		Transverse	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
Full Deck	7%	-9%	18%	-48%	15%	-45%	-1%	0%	0%	0%	130%	103%
Half Width Deck	4%	-5%	14%	-19%	9%	-20%	0%	0%	30%	9%	72%	57%

Analysis of Table 1 reveals that the increase in minimum stress for cables surpasses that of the maximum stress. As decreased stress in cables leads to reduced structural stiffness, meticulous attention should be given to the minimum stress of cables during detailed structural design. Conversely, for longitudinal and transverse beams, the increase in maximum stress tends to exceed that of the minimum stress. However, since these beams undergo compression along their axes, the design is primarily governed by the minimum stress, which encompasses both axial direct stress and bending stress. Thus, the minimum stress assumes greater significance in the structural design of the beams. Furthermore, it is observed that the stress increase in the "Full Deck" load case is greater than that of the "Half Width Deck" load case. Consequently, in terms of stress levels, the design of the members would be primarily influenced by the "Full Deck" load case. Considering that the main cable is solely constrained by two end transverse beams and the diagonal cables connected to these beams, the displacement of cables, particularly in the transverse direction, is expected to be significantly greater than that of the beams. Table 2 provides the maximum displacements of cables under two critical load cases.

Table 2. Maximum Displacement of Cables

Load Cases	Variants		Absolute Value			Relative Value		
			Longitudinal	Vertical	Transverse	Longitudinal	Vertical	Transverse
Dead Load			16.8	22.1	6.9	100%	100%	100%
Full Deck			19.6	51.5	6.9	117%	233%	100%
Half Width Deck			18.3	48.0	71.1	109%	217%	1030%

Table 2 illustrates that both longitudinal and vertical displacements exhibit an increase in magnitude with the applied load. While crowd load slightly contributes to the longitudinal displacement of cables compared to the dead load, it significantly amplifies the vertical displacement. However, considering that both longitudinal and vertical displacements remain relatively low, it can be inferred that the tensegrity footbridge possesses adequate rigidity in these directions. Nevertheless, it is crucial to take note of the

transverse displacement under asymmetric loading conditions. The "Half Width Deck" load case induces an increase in transverse displacement, emphasizing the need to prioritize the transverse stiffness of the structure. Consequently, in future designs, the use of steel bars, instead of steel strands, to connect the main cables is recommended to enhance the transverse stiffness.

3.1 Nonlinear geometric effects

The behaviour of cables is directly linked to their applied force, and if the force falls below a critical value, the cable's rigidity significantly decreases, rendering it ineffective. Nonlinear geometric effects have a substantial impact on the structure, leading to changes in stress and displacement.

Table 3. Nonlinear Geometric Factor

Cables			Strut	Beams		Displacements		
Main	Diagonal	Transverse		longitudinal	Transverse	Longitudinal	Vertical	Transverse
-1%	-25%	-10%	0%	0%	30%	-32%	19%	-55%

Table 3 shows that while the stresses in the main cables remain relatively stable under large deformations, the diagonal and transverse cables experience significant reductions in stress. These cables play a crucial role in overall structural stiffness, and it's important to consider nonlinear geometric effects and design them accordingly. The vertical displacement increases due to large deformations, while the longitudinal and transverse displacements decrease due to the stiffness induced by the main cables' stress. Increasing the prestressing force in the cables can help limit transverse displacements, which is an important design objective. The transverse beam, supported by diagonal cables, behaves like a beam with four spring bearings. The bending moment in the transverse beam depends on the ratio between the diagonal cable's axial rigidity and the longitudinal beam's flexural rigidity. Nonlinear geometric effects increase the rigidity of the diagonal cables, resulting in stronger support at the ends of the transverse beams, followed by an increased axial force and a slightly increased bending moment in the beams. Large deformations have minimal impact on the stress levels of longitudinal beams, even if deflection of longitudinal beams varies by 19%. Struts, which brace the main cables, experience little change in stress under large deformations, despite slight variations in main cable stress, about 1%.

3.2 Influence of rise of main cable

The primary objective of applying prestressing force to the main cables is to achieve the necessary rigidity for the entire structure and ensure the desired geometry. However, there are practical limitations on the maximum prestressing force, taking into account joint configurations and the resistance of longitudinal beams. Theoretically, the prestressing force and the elevation of main cables are influenced to some extent by deck loads and prestressing stress losses. Increasing the rise of the main cables from 3m to 6m leads to corresponding variations in the maximum and minimum stresses of various structural members. Fig.3 illustrates the curves of minimum stress concerning the rise.

A larger rise in the main cable contributes to reducing the stress experienced by different members, except for the struts. The stresses in the transverse and diagonal cables notably decrease with an increase in the rise, while the force in the strut exhibits a linear increase with the rise. Conversely, the stresses in the main cables, transverse beams, and longitudinal beams undergo only minor changes with the rise.

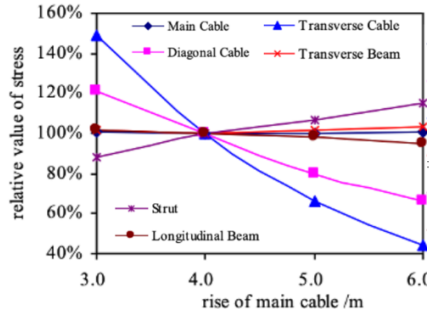


Fig. 3. Relationship of members' stresses with the rise

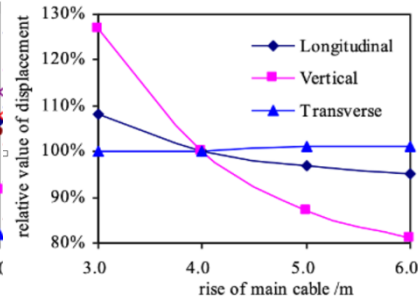


Fig. 4. Relationship of displacements with the rise

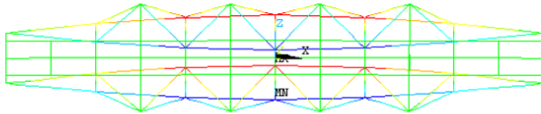
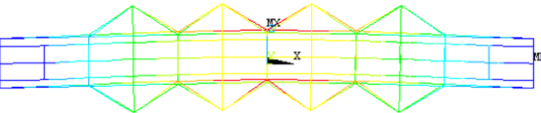


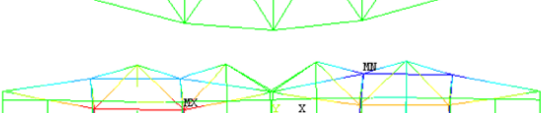
The correlation between the rise of the main cable and structural displacements exhibits a highly nonlinear behaviour, as depicted in Fig.4. As the rise increases, longitudinal and vertical displacements decrease, while the transverse displacement increases. However, beyond a rise value of approximately 4.5, the rate of diversification in displacements slows down. Since transverse displacement is a critical design consideration for a tensegrity bridge, it is important to be cautious with increasing the rise, as it can negatively impact the transverse stiffness of the entire structure. Moreover, excessively large rise values can lead to a loss of aesthetic appeal for the bridge and increase construction complexities. Consequently, the rise of the main cables should be limited to around 4.5 m to strike a balance between structural performance and practical constraints.

4 Modal analysis

Modal analysis was conducted to study the natural frequencies and mode shapes of the tensegrity bridge. The stress stiffening effect, which augments the nonlinear stiffness matrix caused by large strain or deflection, was taken into account by introducing an additional stiffness matrix referred to as the "stress stiffness matrix." This matrix was added to the regular stiffness matrix to obtain the total stiffness, considering the substantial static stress state identified in section 3.1, which required the inclusion of additional stiffness effects for accuracy. Furthermore, the prestressing effect was considered in the modal analysis. In the modal analysis, two options were evaluated for modelling the composite steel-concrete deck. The first option neglected the deck's rigidity, and the deck's weight was treated as added mass on the longitudinal beams. The second option involved a more detailed modelling approach using a beam grid system formed

by longitudinal and transverse beam elements, where the out-of-plane flexural rigidities were ignored. The longitudinal elements were modelled with a depth of 30mm, while the transverse elements had a depth of 80mm and a width of 100mm, with a spacing of 300mm. The modal analysis results, including the deck's rigidity modelling, were presented in Table 4.

Table 4. Partial natural frequencies and mode shapes

Order	Natural Frequency	Mode Shape	Description
1	1.54 Hz		Symmetric twist of the cable support system along the longitudinal direction with a half of full sine curve
2	2.14 Hz		Symmetric sway of the whole bridge along the lateral direction with a half of full sine curve
3	3.80 Hz		Symmetric in-plane bending of the deck and its support system with a half of full sine wave
4	4.71 Hz		Antisymmetric in-plane bending of the deck and its support system with a full sine curve
5	5.12 Hz		Antisymmetric twist of the cable support system along the longitudinal direction with a full sine curve

The analysis indicated that the first-order mode displayed a symmetric twist of the cable support system along the longitudinal direction, while the second-order mode showed vibration of the deck and cable support system in the transverse direction. Successive modes of vibration involved in-plane bending of the deck along the longitudinal direction, with out-of-plane bending or torsion of the deck occurring in higher-order modes. Consequently, the transverse stiffness played a crucial role in controlling the structure's deformation, as confirmed by the largest displacement observed under asymmetric deck load in a previous analysis (Table 2). The basic natural frequency of the system was approximately 1.5 Hz, corresponding to the twist vibration of the main cables. To enhance the dynamic behaviour, a damping system should be implemented to improve their response. Moreover, modifying the frequency of the second mode of vibration could be achieved by stiffening the deck.

5 Conclusions

The Finite Element Method (FEM) analysis of the tensegrity bridge has considered geometric nonlinearity, while assuming linear behaviour of materials. The results indicate a reasonably good structural behaviour of the bridge, with stress and deformation levels of various members well within acceptable limits. This suggests that building such a footbridge is a feasible option. Both static and dynamic analyses have emphasized the significance of accounting for nonlinear geometric effects in the structural design process. This highlights the importance of considering complex interactions between different elements and their responses to loading conditions. By incorporating these factors, the design can be optimized to ensure the bridge's stability, safety, and overall performance. Under deck loads, the stress increment of various members remains manageable, and it's worth noting the stress reduction in diagonal and transverse cables. To ensure overall structural stiffness, prestressing of these cable becomes essential. Asymmetrical loads significantly increase transverse displacements, underscoring the importance of transverse stiffening for the bridge using transverse and diagonal cables. Additionally, stiffening the deck against in-plane bending is particularly crucial. The rise of the main cable has a significant impact on structural responses. For a span length of 30m, the changes in member stress and structural displacement start to slow down when the rise exceeds 4.5m. This implies that excessively high-rise values are not advisable for this type of bridge, as they can lead to unfavourable structural behaviour.

6 References

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