UNICA UNIVERSITÀ DEGLI STUDI DI CAGLIARI

UNICA IRIS Institutional Research Information System

This is the Author's *accepted* manuscript version of the following contribution:

Y. Li, Y. Tong and A. Giua, "Safe Optimal Train Formation Control in Virtual Coupling Using Control Barrier Functions," *2024 IEEE 20th International Conference on Automation Science and Engineering (CASE)*, Bari, Italy, 2024, pp. 647-652.

© 2024 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

The publisher's version is available at:

http://dx.doi.org/10.1109/CASE59546.2024.10711420

When citing, please refer to the published version.

This full text was downloaded from UNICA IRIS https://iris.unica.it/

Safe Optimal Train Formation Control in Virtual Coupling Using Control Barrier Functions

Yike Li, Yin Tong, Alessandro Giua

Abstract— **This paper investigates the optimal train motion control during the Virtual Coupling (VC) convoy formation where the following train (simply called the follower) needs to timely catch up with the velocity of the leading train (simply called the leader) while strictly adhering to the spacing principle that is undergoing change. We employ the Control Barrier Functions (CBFs) and Control Lyapunov Functions (CLFs) to demonstrate the control objectives within the formation progress, construct a safety control scheme composed of two controllers for the follower to achieve the coupling goals safely and apply the Quadratic Programming (QP) to efficiently compute the optimal constrained control input for the follower's motion control. The effectiveness of the proposed control method is demonstrated through numerical simulations of different types of convoy formation cases.**

I. INTRODUCTION

Virtual Coupling (VC) is an advanced signaling concept developed to substantially increase the line capacity and reduce the train headway, leveraging the advancements in Train-to-Train (T2T) communication technology. Rather than using the traditional physical coupler made of steering axles, trains operated in VC are connected with each other by means of a Wi-Fi connection or 5G network, and run together at similar speed and shorter distance to form a convoy [1].

The concept of VC was first introduced in [2] but was hindered by the techniques at the time for further development. In recent years, motivated by the growing capacity demands and emergent communication technologies, the industry and research institutions worldwide have devoted to demonstrate its market potentials and application benefits. However, technical challenges of using VC in practice still exist, and one of them is the research on longitudinal motion control in convoys [3]. The convoy formation control incorporates a desired intertrain spacing principle and the maintenance of a consistent velocity to prevent any risk of collision. This principle establishes a specific distance between trains, guaranteeing that any train operating within the speed limits will have sufficient time to stop. Work [4] presented a distributed model predictive control (MPC) method for maintaining the stable and safety spacing in the VC convoy. Considering the measurement errors in the velocity and position, Park *et al.* [5] applied sliding mode control, and proposed a robust controller that computes a spacing distance reference for a VC convoy of two trains. Based on the train-following model of k trains operating in the multiple VC scenarios, Quaglietta *et al.* [6] proposed a dynamic safety spacing model that contains various risk factors.

Among the recent studies on VC convoy safety control, MPC is widely used as the optimal control method in VC studies [7]–[9]. Su *et al.* [10] focused the centralized control for a convoy, using MPC to implement the safety and stable spacing policy. Also known as receding horizon control, MPC performs the optimization in an iterative way, which brings a high computational cost on both time and hardware resource. However for the real-world VC operation, real-time controllers are essential [9]. Particularly, in the VC, fast computing speed is necessary for the follower to calculate a safety distance that prevents collisions and regulate the train's speed to maintain that distance.

To meet the real-time requirements of control, Control Barrier Functions (CBF) is used in this paper, to deal with the VC train formation control problem. Control Lyapunov Function (CLF) is used to enforce exponential stability control objectives, turning constrained optimal control problems into quadratic programming (QP) problems. CBF inspired by and as a direct extension of the CLF [11], has shown great promise in providing a computationally efficient method for safe control of nonlinear systems in complex scenarios. Similar to MPC, CBF is a model-based control design method that can be formulated as a sequence of QP problems and solved online using real-time capable solvers. Numerous recent studies apply CBFs and CLFs to constrained optimal control problems, such as the control of Connected and Automated Vehicles (CAVs) in road traffic, as well as the control of the bipedal robots, which have shown favorable experimental results [12]. To the best of our knowledge, so far no studies have applied CBF or CLF in the control for VC. In this paper, we propose a spacing principle for the VC convoy formation, and adopt CBF and CLF to formulate QP problems. These problems are solved to compute the optimal control outputs for the train's motion, offering faster computation compared to existing studies that use MPC.

This work has been supported by the project co-funded by the European Union – Next Generation Eu - under the National Recovery and Resilience Plan (NRRP), Mission 4 Component 1 Investment 4.1 - Decree No.351 (9th April 2022) of Italian Ministry of University and Research - Concession Decree No. 2152 (28th December 2022) of the Italian Ministry of University and Research, Project code D93C22000860005, within the Italian National Program PhD Programme in Autonomous Systems (DAuSy), by the Project SERICS (PE00000014), and by the National Natural Science Foundation of China under Grant No. 62373313.

Y. Li is with the Department of Electrical and Electronic Engineering, University of Cagliari, Cagliari 09123, Italy; and the Department of Electrical and Information Engineering, Polytechnic of Bari, Bari 70125, Italy yike.li@unica.it; y.li@phd.poliba.it

Y. Tong (*Corresponding Author) is with the School of Information Science and Technology, Southwest Jiaotong University, Chengdu 611756, China yintong@swjtu.edu.cn

A. Giua is with the Department of Electrical and Electronic Engineering, University of Cagliari, Cagliari 09123, Italy giua@unica.it

This paper is organized as below. In the second section, we recall the notions of the CLF and CBF. In Section III,, we introduce the background of VC and then design the spacing control principle for the VC convoy formation. Section IV will presents the proposed approach that formulates the spacing control principle to the CBF and CLF as the constraints on the control input. Finally, effectiveness of the proposed approach is demonstrated through simulations in the cases with different initial conditions.

II. PRELIMINARIES

In this section, basic notions on CBF and CLF are introduced. For more details, we refer the reader [13].

In this paper, we focus on control affine systems, defined as

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t)) + g(\boldsymbol{x}(t))\boldsymbol{u}(t) \tag{1}$$

with $t \in T$, $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ locally Lypschitz, $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector and $\boldsymbol{u} \in U \subset \mathbb{R}^m$ is the control input vector. In the following, unless ambiguity arises, we omit explicitly stating the time dependence of signals $\boldsymbol{x}$ and $\boldsymbol{u}$.

This paper addresses control problems for such a class of systems, focusing in particular on stability and safeness.

A. Control Lyapunov Function

Generally, Control Lyapunov Functions (CLFs) are designed for reaching a target state (or set), whereas Control Barrier Functions (CBFs) are designed for avoiding an unsafe set. CBFs are a direct extension of CLFs and are used to enforce stability or state convergence requirements.

Definition 1: (Lie derivative) Consider a continuous function $y : \mathbb{R}^n \to \mathbb{R}$, the derivative $\dot{y}$ along the dynamic of (1) is given by

$$\dot{y} = \frac{\partial y}{\partial \boldsymbol{x}}[f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}] \overset{\text{def}}{=} L_f y(\boldsymbol{x}) + L_g y(\boldsymbol{x})\boldsymbol{u}, \tag{2}$$

where

$$L_f y(\boldsymbol{x}) = \frac{\partial y}{\partial \boldsymbol{x}} \cdot f(\boldsymbol{x}) \tag{3}$$

is called the Lie derivative of y with along to f.

Definition 2: (Control Lyapunov Function) Let $V : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable, positive definite, and radially unbounded function. If there exists a positive constant λ such that

$$\inf_{\boldsymbol{u}\in\mathcal{U}} [L_f V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u} + \lambda V(\boldsymbol{x}))] \leq 0, \tag{4}$$

then $V(\boldsymbol{x})$ is an exponentially stabilizing Control Lyapunov Function (ES-CLF) for system (1).

And then, any Lipschitz continuous control law $\boldsymbol{u}$ that satisfies (4) exponentially stabilizes system (1). Parameter λ serves as a decay rate of an upper bound of $V(\boldsymbol{x})$.

B. Control Barrier Function

Definition 3: (Class $\mathcal{K}$ Function) A continuous function $\alpha : [0, a) \to [0, \infty), a > 0$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Definition 4: (Forward Invariant Set) A set $\mathcal{C}$ is forward invariant for system (1) if its solutions starting at any $\boldsymbol{x}(0) \in \mathcal{C}$ satisfy $\boldsymbol{x}(t) \in \mathcal{C}$ for $\forall t \geq 0$.

Consider a set $\mathcal{C}$ (the safe set) defining by a continuously differentiable function $b : \mathbb{R}^n \to \mathbb{R}$ as follows:

$$\mathcal{C} := \{\boldsymbol{x} \in \mathbb{R}^n : b(\boldsymbol{x}) \geq 0\}. \tag{5}$$

Definition 5: (Control Barrier Function) Given the set $\mathcal{C}$ defined by (5), $b(\boldsymbol{x})$ is a Control Barrier Function (CBF) for system (1) if there exists a class $\mathcal{K}$ function α such that

$$\sup_{\boldsymbol{u}\in\mathcal{U}} [L_f b(\boldsymbol{x}) + L_g b(\boldsymbol{x})\boldsymbol{u} + \alpha(b(\boldsymbol{x}))] \geq 0, \forall \boldsymbol{x} \in \mathcal{C}. \tag{6}$$

It is assumed that $L_g b(\boldsymbol{x}) \neq 0$ when $b(\boldsymbol{x}) = 0$.

Given a CBF b associated with the set $\mathcal{C}$ from (5), any Lipschitz continuous controller $\boldsymbol{u}(t)$ with $t \geq t_0$ satisfying (6) makes set $\mathcal{C}$ forward invariant for (1), which is also called control invariant or safe. In practice, $\mathcal{K}$ function α usually takes the form of a positive coefficient γ. In this case, γ serves as a maximal decay rate of $b(\boldsymbol{x}(t))$, i.e. $b(\boldsymbol{x}(t))$ is not allowed to decay faster than the exponential decay curve $\dot{b}(\boldsymbol{x}(t)) = -\gamma b(\boldsymbol{x}(t))$ so that potentially unsafe behaviour smoothes out as it approaches the boundary of set $\mathcal{C}$.

From the above definitions, it can be seen that both the CBF constraints and the CLF constraints are affine in the control input $\boldsymbol{u}$, and thus the CBF and CLF in the optimal control problem (OCP) can be used as constraints in the minimum specification controller to form the QP problems, which are defined as CBF-CLF-QP. The formulation of CBF-CLF-QP is given as

$$\arg\min_{\boldsymbol{u}(t)} \|\boldsymbol{u}(t)\|^2 + p\delta^2, \tag{7}$$

subject to:

$$\begin{aligned} L_f b(\boldsymbol{x}) + L_g b(\boldsymbol{x})\boldsymbol{u}(t) + \gamma(b(\boldsymbol{x})) \geq 0, \quad & \text{(CBF constraint)} \\ L_f V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u}(t) + \lambda V(\boldsymbol{x}) \leq \delta, \quad & \text{(CLF constraint)} \\ \text{and } \boldsymbol{u}_{min} \leq \boldsymbol{u}(t) \leq \boldsymbol{u}_{max}, \quad & \text{(control constraint)} \end{aligned}$$

where $\boldsymbol{u}_{min}$ and $\boldsymbol{u}_{max}$ respectively represent the lower and upper bounds of $\boldsymbol{u}$, δ is the slack variable that serves as the relaxation on the CLF constraint to prevent it from conflicting with the CBF constraint, and p is the penalty.

III. VIRTUAL COUPLING SPACING CONTROL IN FORMATION PROCEDURE

In this section, we first briefly introduce the background on VC, including its main idea, and the general scenario of VC. Next, we analyse the spacing control principle during the convoy formation procedure and establish the spacing principle for convoy formal control.

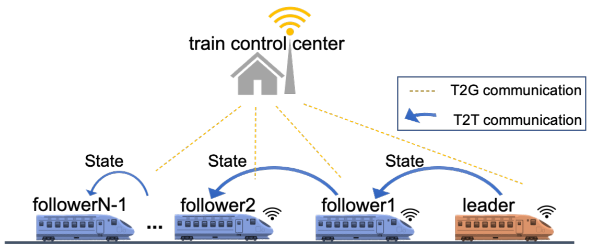


Fig. 1. A VC convoy of N trains.

A. Introduction on Virtual Coupling

1) The VC Convoy Topology: A VC convoy has a typical topology shown in Fig. 1. The convoy is formed by N trains, one leader, and $N-1$ followers. Each train computes its driving strategy according to the instant information sent from the train ahead via the T2T communication layer, which includes speeds, location, acceleration, etc. Meanwhile, the trains constantly communicate with the train control center to receive operational task orders and realize centralized dispatching management. Trains in a VC convoy move synchronously at a short distance from each other and work together as one in a cooperative manner.

For the sake of simplicity but without loss of generality, in the rest of this paper, we consider a VC convoy with two trains, namely one leader and one follower. Their velocities are denoted as v_l and v_f, respectively. Our study focuses on the spatial headway D, which is the distance between the leader's rear and the follower's front. In the next section, the spacing principle imposed on D will be discussed.

2) Virtual Coupling Scenario Analysis: Table I presents a general scenario in VC, which consists of four phases in terms of convoy status. In the beginning, the two adjacent trains operate at the moving block (i.e. trains are running in separated blocks that are defined by the latter's braking distance), communicating over the T2T channel. The train control center, responsible for dispatching, confirms the coupling need and sends the coupling order to the trains. Both trains must confirm their compliance with the conditions necessary for VC. These conditions cover various operational aspects, including but not limited to the sufficient overlap in their originally planned paths, no ongoing tasks or speed limitations that conflict with the coupling order, and sound communication with the neighboring train and the train control center. If the required conditions are all met, the operation mode of both trains transitions from the moving block to "Coupling".

Once the follower attains a speed close to that of the leader within a predefined threshold and remains within a distance range from the desired separation, VC is established between them. Afterward, the operation mode of the follower and leader will transition from "Coupling" to "Coupled". Suppose either the follower or the leader receives a separation order from the train control center, or any violations against conditions for VC such as communication loss or location failures occur. In that case, trains in convoy become separated, known as "Decoupling". When Decoupling occurs, the control mode of both trains is downgraded to the moving block.

TABLE I
VIRTUAL COUPLING TRAIN CONVOY FORMATION, RUNNING AND SEPARATION PROCESSES

PHASE	ACTION	MODE
Preparation	The follower and leader receive the coupling task, and check the conditions for coupling.	Moving block
Convoy formation	The follower accelerates, then decelerates, while the leader maintains its speed or remains stationary, until the follower matches both the desired distance and the leader's velocity.	VC: Coupling
Convoy running	The follower and the leader run synchronously.	VC: Coupled
Convoy separation	Any VC condition is violated.	VC: Decoupling

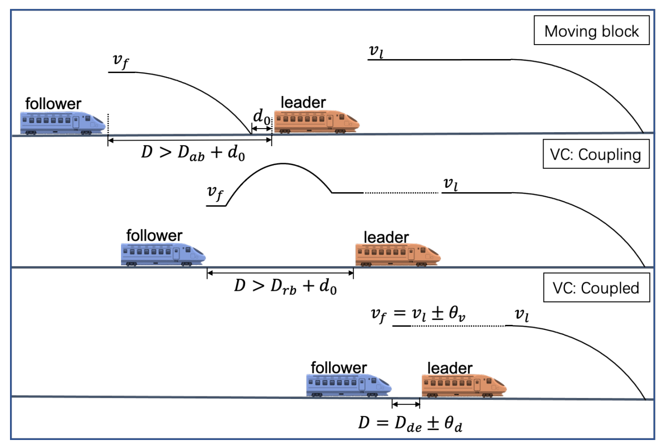


Fig. 2. Convoy Formation in Virtual Coupling

The above description provides an overview of the general scenario in which VC is established based on the existing studies' common sense. Next, we analyze and define ambiguous or undefined key factors in the process of the convoy formation and how to determine its completion.

B. Spacing Control Principle Analysis for Convoy Formation

As mentioned in Section III.A, the spacing principle determines the minimum safe distance between the trains. This paper focuses on the follower's motion control, addressing the problem of VC convoy formation under the assumption that no delay occurs in T2T communication. It is assumed that the follower's velocity $v_f(t)$ is time varying while the velocity v_l of the leader is constant. To form the VC convoy, the constraints on the spacing distance are defined as follows.

First, even if two adjacent trains are both in a standstill, they should be separated by a default margin d_0 [14]. To prioritize safety, there should be sufficient distance for collision avoidance. In the moving block, apart from d_0, the distance between two successive trains should be longer than the braking distance of the behind train, i.e. the distance it travels from the moment when the brake starts until the train completely stops, which is termed as the absolute braking distance D_{ab}. When calculating the braking distance, we assume the final velocity is zero. Based on this, the absolute braking distance can be calculated using the following equation

$$D_{ab}(t) = \frac{v_f^2(t)}{2a_f}, \tag{8}$$

where a_f is the deceleration rate of the following train, which is positive. Therefore, the spatial headway $D(t)$ should be greater than the total of d_0 and $D_{ab}(t)$ as shown in Fig. 2.

During the convoy formation phase, the safety distance in the spacing principle for collision avoidance turns to the relative braking distance D_{rb}, which equals to the difference between the braking distances of the two trains. Specifically, we consider the worst case where the leading train uses the emergency braking rate a_{eb}, while the following train uses the service braking rate a_{sb} and $a_{eb} > a_{sb}$. Therefore, we define the safety spacing principle for the formation phase as:

$$D(t) > D_{rb}(t) + d_0 = \frac{v_f^2(t)}{2a_{sb}} - \frac{v_l^2}{2a_{eb}} + d_0. \tag{9}$$

Through enforcing (9) as a hard constraint, even if the follower's velocity is much larger than that of the leader aiming to catch up with the leader quickly, it is still guaranteed that the follower can avoid rear-end collision in the case of emergency braking by the leader, even when the follower decelerates in a lower service braking rate.

The ultimate purpose of VC is to increase line capacity by minimizing train spacing distance. If (9) is the only spacing principle in the VC convoy formation, this may fail for the follower to close up the distance to the leader. Therefore, we introduce a *desired distance* as the target distance that guides the follower to quickly shorten the distance with the leader and form the convoy. This distance must adhere to the safety principle and meets our expectations on the convoy running in the ideal status: the trains run synchronously at a consistent distance. Consequently, the relative braking distance $D_{rb}(v_l = v_f)$ between trains after they are successfully coupled and the convoy has an overall consistent velocity, , is chosen as the desired distance, denoted as $D_{de}(t)$, and equals

$$\begin{aligned} D_{de}(t) &= d_0 + \frac{v_f^2(t)}{2a_{sb}} - \frac{v_f^2(t)}{2a_{eb}} \\ &= d_0 + \frac{(a_{eb} - a_{sb})v_f^2(t)}{2a_{sb}a_{eb}}. \end{aligned} \tag{10}$$

Let

$$k = \frac{a_{eb} - a_{sb}}{a_{eb}a_{sb}},$$

consequently (10) can be written as

$$D_{de}(t) = d_0 + \frac{kv_f^2(t)}{2}. \tag{11}$$

To determine if the formation of a VC convoy is complete, the following two conditions must be met:

1) The following train attains a desired distance from the leading train, i.e. $|D - D_{de}(t)| \leq \theta_d$.
2) The following train matches the speed of the leading train, i.e. $|v_f(t) - v_l| \leq \theta_v$.

Error thresholds θ_d and θ_v are set to accommodate for interference terms such as location errors, communication delays, sampling noise, etc. Additionally, we define the time of the convoy formation as t_C, i.e., the time from the start of the formation phase to its completion (when the two conditions above are simultaneous fulfilled).

The spacing principle described by (9) and (10) represents the safe constraint and the control objectives of the follower. In this section, we combine it with the train dynamics to establish the optimal and safe control problem of the follower during the convoy formation by applying the CBF-CLF-QP.

IV. CBF-CLF Forced Optimal Controllers Design for VC Convoy Formation

Before applying the CBF-CLF-QP technique to solve the optimal control of the follower for the formation scenario in VC, the control-affine nonlinear model of the following train's dynamic is first formalized.

A. Train Following Model

The moving train on the track is viewed as a mass point, and its longitudinal movement can be described by the dynamic model below [15].

$$m\frac{dv}{dt} = F_{tb} - F_r, \tag{12}$$

where m $[kg]$ is the mass, and v $[m/s]$ denotes the velocity. The involved external forces are traction and braking force (T/B force) F_{tb} $[N]$ and running resistance force F_r $[N]$.

The running resistance consists of the two components: fundamental resistance and additional resistance. The latter refers to the resistance generated by unfavorable track conditions including slopes, curves and tunnels. This paper concentrates on the general train control method in VC, hence we neglect the additional resistance, and assume that the trains run on the mainline with relatively desirable conditions. The fundamental resistance F_r can be calculated by the 2-order empirical equation (13) which is derived from experimental data [16].

$$F_r(v) = A \cdot m + B \cdot mv + C \cdot mv^2, \tag{13}$$

where B $[Ns/m \cdot kg]$ and C $[Ns^2/m^2 \cdot kg]$ are the coefficients related to mechanical and aerodynamic characteristics, respectively. The parameter A $[N/kg]$ refers to the starting resistance, which can be neglected when the speed is greater than $3m/s$ in our case.

Let $\boldsymbol{x} = (x_1, x_2)$ where $x_1(t) = D(t)$ is the distance between the leader and the follower, and $x_2(t) = v_f(t)$ is the follower's velocity. It holds that $\dot{x}_1 = v_l - x_2$. Taking the T/B force F_{tb} in (12) as the control input u, based on the prototype of the control affine system (1) and the longitudinal model (12), we model the VC train following model as follows:

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{\dot{\boldsymbol{x}}}=\underbrace{\begin{bmatrix}-x_2+v_l\\ -\frac{1}{m}F_r\end{bmatrix}}_{f(\boldsymbol{x})}+\underbrace{\begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}}_{g(\boldsymbol{x})}\underbrace{F_{tb}}_{u}. \tag{14}$$

Note that the leader's velocity v_l is considered a known constant and thus can be regarded as a parameter rather than an input in the model. Besides, although (14) is written in a linear form, the model is nonlinear because F_r depends on the quadratic form of x_2 in (13).

B. Control Objectives

As discussed in the previous section, in order to form a VC convoy, the follower needs to reach the distance to the desired distance as close as possible while strictly adhering to the safety distance, meanwhile the two distance depend on the velocities of the follower and the leader. Thus, the spacing principle is described with respect to the model (14) as

$$x_1 \to d_0 + \frac{kx_2^2}{2}, \tag{CO1}$$

$$x_1 \geq d_0 + \frac{x_2^2}{2a_{sb}} - \frac{v_l^2}{2a_{eb}}. \tag{CO2}$$

Note that (CO1) is a soft constraint that aims to stabilizes the system (which can be accomplished by a CLF), whereas (CO2) is a hard constraint that assures the system's safety (can be achieved by a CBF).

Meanwhile, the control input u, i.e. the T/B force, is naturally constrained by the train mechanics defined as

$$-a_{mb}\cdot m \leq u \leq a_{mt}\cdot m, \tag{CC}$$

where $a_{mb} > 0$ is the maximum braking rate and $a_{mt} > 0$ is the maximum traction rate.

C. The CBF-CLF-QP for VC convoy formation control

We employ CLF constraints satisfying (4) to achieve the stability objective (CO1), which can be written as $x_1 - D_{de}(t) \to 0$. The Lyapunov function $V(\boldsymbol{x}) = (x_1 - D_{de}(t))^2$ is chosen, then

$$\begin{aligned}\dot{V}(\boldsymbol{x}) &= L_fV(\boldsymbol{x}) + L_gV(\boldsymbol{x})u\\ &= \frac{\partial V}{\partial \boldsymbol{x}}\cdot f(\boldsymbol{x}) + \frac{\partial V}{\partial \boldsymbol{x}}\cdot g(\boldsymbol{x})u\end{aligned} \tag{15}$$

where

$$\begin{aligned}\frac{\partial V}{\partial \boldsymbol{x}} &= \left[\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}\right]\\ &= \left[2\left(x_1 - \frac{kx_2^2}{2} - d_0\right),\ -2kx_2\left(x_1 - \frac{kx_2^2}{2} - d_0\right)\right].\end{aligned}$$

The CLF constraint is

$$\begin{aligned}&L_fV(\boldsymbol{x}) + L_gV(\boldsymbol{x})u + \lambda V(\boldsymbol{x})\\ &= \left(x_1 - \frac{kx_2^2}{2} - d_0\right)\cdot\\ &\left[2(-x_2+v_l) + \frac{2k(F_r-u)x_2}{m} + \lambda\left(x_1 - \frac{kx_2^2}{2} - d_0\right)\right] \leq \delta.\end{aligned} \tag{16}$$

TABLE II

THE SIMULATION PARAMETERS

Meaning	**Symbol**	**Value**
Train dynamic parameters		
Weight of the train	m $[kg]$	383000
Resistance coefficient	A $[N/kg]$	0.73
Resistance coefficient	B $[Ns/m\cdot kg]$	0.008
Resistance coefficient	C $[Ns^2/m^2\cdot kg]$	0.000148
Emergency braking rate	a_{eb} $[m/s^2]$	1.8
Serving braking rate	a_{sb} $[m/s^2]$	1.5
Maximum braking rate	a_{mb} $[m/s^2]$	3
Maximum traction rate	a_{mt} $[m/s^2]$	3
Default margin	d_0 $[m]$	5
Distance threshold	θ_d $[m/s]$	10
Velocity threshold	θ_v $[m/s]$	0.2
Control parameters		
CLF constraint coefficient	λ	0.1
CBF constraint coefficient	γ	5
QP penalty function	p	$2e-2$
QP slack variable	δ	$2e-2$

To construct the inequality constraints that enforce the safety control objective (CO2), we consider the CBF derived directly from (CO2), i.e. $b(\boldsymbol{x}) = x_1 - D_{rb}(t) - d_0 \geq 0$. According to (6), the control input should satisfy

$$\begin{aligned}&L_fb(\boldsymbol{x}) + L_gb(\boldsymbol{x})u + \gamma b(\boldsymbol{x})\\ &= -x_2 + v_l + \frac{F_rx_2}{ma_{sb}} + \frac{-x_2}{ma_{sb}}\cdot u\\ &+\gamma(x_1 - \frac{x_2}{2a_{sb}} + \frac{v_l^2}{2a_{eb}} - d_0) \geq 0\end{aligned} \tag{17}$$

Applying the designed CBF in (17) and CLF in (16), the QP for the control of the follower is constructed by combining the above constraints as

$$\begin{aligned}u^*(x) = \arg\min&\left((u-u_{\text{ref}})^TH(u-u_{\text{ref}}) + p\delta^2\right)\\ \text{s.t. }&(16), (17), \text{and (CC)},\end{aligned} \tag{18}$$

where u_{ref} is the reference value of the control output u, and H is the quadratic objective term.

V. SIMULATION AND RESULTS

The effectiveness of the proposed control method is verified by numerical simulations. Parameters used for the simulations are from the Train a Grande Vitesse (TGV) France, as listed in Table II.

We categorize two types of scenarios: 1) Static formation (SF), during which the leader's velocity stays at zero during the formation, and 2) Dynamic formation (DF), which refers that the leader cruises at a constant speed during the formation. For each type, two cases (SF-1, SF-2 for the static formation, and DF-1, DF-2 for the dynamic formation) with varied initial conditions are simulated[1].

Table III presents the initial conditions and the corresponding formation time t_C. Let $u_{\text{ref}} = F_r$, and the cost function

[1]The QPs are solved using Matlab toolbox `quadprog`. The obtained closed loop system is simulated using function `ode45`.

TABLE III
DIFFERENT INITIAL CONDITIONS AND SIMULATION RESULTS IN THE FOUR CASES

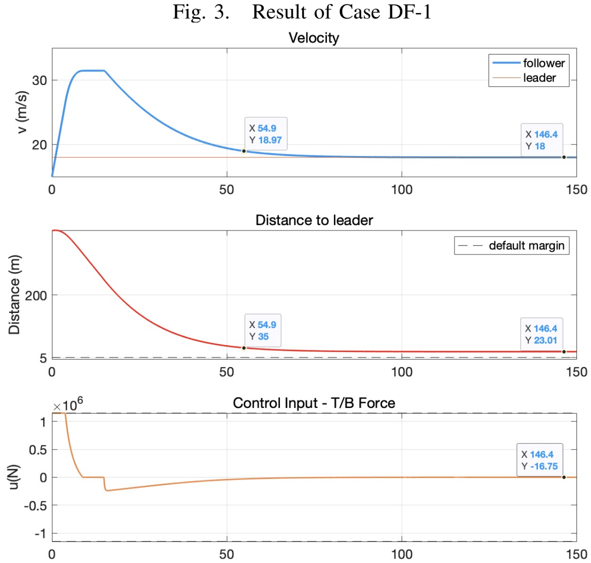
Case	v_l	$x_1(0)$	$x_2(0)$	t_C
SF-1	0m	400m	0m/s	28.96s
SF-2	0m	400m	15m/s	24.98s
DF-1	18m	400m	15m/s	54.9s
DF-2	18m	800m	20m/s	69.24s

$H = 2/m^2$. Performing a simulation of $150s$ with the step size $0.02s$, the results of the velocity, distance and control input in the case DF-1 are plotted in Fig. 3. The follower undergoes acceleration, idling and deceleration to form a formation with the preceding train at $54.9s$.

After that, the distance continues to narrow down to $23m$, which equals the desired distance by (10). And the difference in velocity between the follower and the leader stabilizes to zero.

The total time for running the simulation was around $18.1s$, and the average time for implementing one loop (7500 loops in total) was around $0.0024s$, which meets the real-time requirement of automatic operation module in the train onboard system [17].

Fig. 3. Result of Case DF-1



VI. CONCLUSIONS

This paper focuses on the following train's speed control problem in the Virtual Coupling. We develop the spacing principle for the train convoy formation, and describe the control objectives on spacing utilizing the CLF (realizes the desired distance) and CBF (maintains the safety distance). The QPs are formulated to compute the traction and braking force that satisfy all the control objectives and constraints. Finally, the simulation results show that the proposed method is able to reach the desired distance that enhancing capacity while ensuring the safety.

Future work will extend the proposed control method to more complex scenarios in the VC, such as the convoy entering or leaving the station, unexpected braking by the leader, as well as other realistic line profiles. We are also interested in designing a robust convoy controller that is resilient to communication delays or losses. Additionally, we aim to apply the method to the convoy with N trains among which heterogeneity, i.e., different weights, length, and braking performance, exists.

REFERENCES

[1] C. Di Meo, M. Di Vaio, F. Flammini, R. Nardone, S. Santini, and V. Vittorini, "ERTMS/ETCS Virtual Coupling: Proof of Concept and Numerical Analysis," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, pp. 2545–2556, June 2020.

[2] U. Bock and G. Bikker, "Design and Development of a Future Freight Train Concept – "Virtually Coupled Train Formations"," *IFAC Proceedings Volumes*, vol. 33, no. 9, pp. 395–400, 2000.

[3] J. Aoun, E. Quaglietta, and R. M. P. Goverde, "Roadmap Development for the Deployment of Virtual Coupling in Railway Signalling," *Technological Forecasting and Social Change*, vol. 189, p. 122263, Apr. 2023.

[4] Y. Liu, R. Liu, C. Wei, J. Xun, and T. Tang, "Distributed Model Predictive Control Strategy for Constrained High-Speed Virtually Coupled Train Set," *IEEE Transactions on Vehicular Technology*, vol. 71, pp. 171–183, Jan. 2022.

[5] J. Park, B.-H. Lee, and Y. Eun, "Virtual Coupling of Railway Vehicles: Gap Reference for Merge and Separation, Robust Control, and Position Measurement," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, pp. 1085–1096, Feb. 2022.

[6] E. Quaglietta, M. Wang, and R. M. P. Goverde, "A Multi-State Train-Following Model for the Analysis of Virtual Coupling Railway Operations," *Journal of Rail Transport Planning & Management*, vol. 15, p. 100195, Sept. 2020.

[7] J. Xun, Y. Li, R. Liu, Y. Li, and Y. Liu, "A Survey on Control Methods for Virtual Coupling in Railway Operation," *IEEE Open Journal of Intelligent Transportation Systems*, vol. 3, pp. 838–855, 2022.

[8] J. Felez and M. A. Vaquero-Serrano, "Virtual Coupling in Railways: A Comprehensive Review," *Machines*, vol. 11, p. 521, May 2023.

[9] Q. Wu, X. Ge, Q. Han, and Y. Liu, "Railway Virtual Coupling: A Survey of Emerging Control Techniques," *IEEE Transactions on Intelligent Vehicles*, vol. 8, pp. 3239–3255, May 2023.

[10] S. Su, J. She, K. Li, X. Wang, and Y. Zhou, "A Nonlinear Safety Equilibrium Spacing-Based Model Predictive Control for Virtually Coupled Train Set Over Gradient Terrains," *IEEE Transactions on Transportation Electrification*, vol. 8, pp. 2810–2824, June 2022.

[11] A. D. Ames, J. W. Grizzle, and P. Tabuada, "Control Barrier Function Based Quadratic Programs with Application to Adaptive Cruise Control," in *53rd IEEE Conference on Decision and Control*, pp. 6271–6278, Dec. 2014.

[12] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, and P. Tabuada, "Control Barrier Functions: Theory and Applications," in *2019 18th European Control Conference (ECC)*, pp. 3420–3431, June 2019.

[13] W. Xiao, C. G. Cassandras, and C. Belta, *Safe Autonomy with Control Barrier Functions: Theory and Applications*. Synthesis Lectures on Computer Science, Cham: Springer International Publishing, 2023.

[14] D. Emery, "Enhanced ETCS L2/L3 train control system," in *WIT Transactions on State of the Art in Science and Engineering* (B. Ning, ed.), vol. 1, pp. 113–122, WIT Press, 1 ed., June 2010.

[15] J. Wang and H. A. Rakha, "Longitudinal Train Dynamics Model for A Rail Transit Simulation System," *Transportation Research Part C: Emerging Technologies*, vol. 86, pp. 111–123, Jan. 2018.

[16] B. P. Rochard and F. Schmid, "A Review of Methods to Measure and Calculate Train Resistances," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 214, no. 4, pp. 185–199, 2000.

[17] X2Rail-1, "Deliverable D3.1. User & System Requirements (Telecommunications)," tech. rep., EU-Rail, 2016. https://ec.europa.eu/research/participants/documents/downloadPublic?documentIds=080166e5c0facc5d&appId=PPGMS.