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Gradient-based adaptive twisting-like algorithm with asymptotic convergence

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Abstract—This paper addresses asymptotic stabilization of a perturbed double-integrator affected by a bounded perturbation featuring a constant upperbound which is *a priori* unknown. We propose an adaptive feedback law, composed by the interconnection between a linear state-feedback controller and the twisting controller with two automatically tuned non-decreasing time-varying gains adjusted by gradient-based laws. Lyapunov analysis, complemented by Barbalat-based argument, is developed to certify the asymptotical stability of the closed loop system. The analysis provides a closed-form bound for the adaptive gains and it constructively yields simple design conditions for the controller parameters. We prove boundedness of all closed-loop signals and establish asymptotic convergence of both state variables to the origin without requiring prior knowledge of the disturbance bound. Simulation results are presented to corroborate the theoretical findings.

I. INTRODUCTION

A central challenge in sliding mode control is the robust stabilization of uncertain systems in the absence of *a priori* knowledge about the bounds of the uncertainties affecting the plant. This issue has motivated the development of a wide class of adaptive sliding mode control (ASMC) techniques, where the switching gain is adjusted online so as to compensate for matched perturbations with unknown bounds. A common approach in this framework consists in enforcing a monotonic increase of the switching gain until the sliding manifold is approached [12], [2], [3], [7], [10].

Bidirectional adaptation algorithms have been also developed, where the adaptive gain can increase and decrease [1], [5], [13], [4], [11]. This is beneficial for implementation purposes and is less susceptible to over-estimation, which in turn helps alleviating the chattering phenomenon. However, as it is typical with algorithms of this type, only practical convergence towards a vicinity of zero can be guaranteed.

Higher-order sliding mode (HOSM) algorithms have been proposed to alleviate the chattering phenomenon, among which the twisting algorithm plays a prominent role due to its finite-time convergence properties and robustness features. The combination of twisting control with adaptive mechanisms has therefore attracted increasing attention in recent years. [8], [11]

The primary objective of the present paper is to design and analyze an adaptive twisting-based controller for a class of uncertain nonlinear systems, guaranteeing asymptotic stabilization of the state variables while ensuring the boundedness of the adaptive gains. The proposed strategy relies on a Lyapunov-based design that allows for a transparent tuning

procedure by employing a Lyapunov function containing simple and relatively standard quadratic and absolute-value terms.

It is worth stressing that an adaptive twisting controller capable of guaranteeing *finite-time* convergence of the sliding variables, together with uniform boundedness of the adaptive gain, has already been proposed in [8]. As discussed in Remark 2, the control law introduced therein outperforms the algorithm proposed in the present paper in terms of convergence speed. However, the Lyapunov analysis supporting the finite-time results in [8] turns out to be highly tailored to the finite-dimensional setting and mathematically involved, making difficult to extend it to more general and challenging scenarios.

The main novelty of this work lies not in improving the convergence rate of adaptive twisting control in finite-dimensional systems, but in the development of an adaptive second-order sliding mode framework whose Lyapunov analysis is sufficiently simple and modular to allow extensions to infinite-dimensional dynamics. A similar research direction has recently been pursued in [6], where an adaptive boundary control strategy for partial differential equations was derived starting from the adaptive first-order sliding mode controller proposed in [10]. Inspired by this successful generalization from ordinary differential equations to partial differential equations, the present paper aims at laying the theoretical foundations for an adaptive version of the second-order sliding mode boundary controller introduced in [9] for uncertain heat processes with matched perturbations having uncertain bounds.

In this perspective, the adaptive twisting algorithm proposed here is conceived as an intermediate and necessary step towards the extension of adaptive second-order sliding mode techniques to PDE control. To this end, priority is given to analytical tractability and transparency of the Lyapunov-based analysis rather than to finite-time convergence properties, which, although achievable in finite-dimensional settings, are considerably more challenging to preserve in the infinite-dimensional case, especially under the boundary control and sensing restrictions.

The remainder of the paper is organized as follows. Section II formulates the control problem and introduces the proposed adaptive twisting controller along with the associated tuning conditions. The stability analysis is carried out in Section III. Simulation results are reported in Section IV, and concluding remarks are drawn in Section V.

II. PROBLEM FORMULATION AND MAIN RESULT

Consider the perturbed double integrator

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = f(t) + u(t). \end{cases} \quad (1)$$

with manipulable control $u(t)$ and uncertain disturbance $f(t)$. The initial conditions on the state variables are

$$x_1(0) = x_{10}, \quad x_2(0) = x_{20}. \quad (2)$$

The uncertain drift disturbance $f(t)$ is supposed to satisfy the following assumption

Assumption 1: There exist a constant $F \geq 0$, a priori unknown, such that

$$|f(t)| \leq F, \quad \forall t \geq 0 \quad (3)$$

The discontinuous adaptive feedback law

$$\begin{aligned} u(t) = & -k_1(t) \operatorname{sign}(x_1(t)) - k_2(t) \operatorname{sign}(x_2(t)) \\ & - c_1 x_1(t) - c_2 x_2(t), \end{aligned} \quad (4)$$

is under investigation along with the gradient-based adaptation laws

$$\dot{k}_1(t) = \gamma_1 |x_1(t)|, \quad \gamma_1 > 0, \quad k_1(0) = k_{10} \geq 0, \quad (5)$$

$$\dot{k}_2(t) = \gamma_2 |x_2(t)|, \quad \gamma_2 > 0, \quad k_2(0) = k_{20} \geq 0, \quad (6)$$

with the positive tuning constant c_1, c_2, γ_1 and γ_2 subject to design inequalities to be specified later on,

The main result of the present paper is established in the following Theorem

Theorem 1: Consider system (1) subject to Assumption 1. Let the discontinuous feedback control (4)-(6) be implemented in the closed loop, where the strictly positive tuning constants c_1, c_2, γ_1 and γ_2 are chosen according to the design inequalities

$$0 < \varepsilon < \min \left(\sqrt{c_1}, c_2, \frac{\gamma_1}{\gamma_2} \right) \quad (7)$$

$$\varepsilon c_1 > \gamma_1 \quad (8)$$

where ε is an arbitrary positive constant. Then, $x_1(t)$ and $x_2(t)$ approach the origin asymptotically whereas $k_1(t)$ and $k_2(t)$ are uniformly bounded for all $t \geq 0$.

Remark 1: The tuning conditions (7)-(8) are feasible and yield a variety of options for selecting the control parameters. For instance, one can arbitrarily set c_2 to any positive value and then, by relying on the choice $\varepsilon = \eta c_2$ for an arbitrary

$$0 < \eta < 1 \quad (9)$$

selecting the remaining parameters according to

$$c_1 \geq c_2^2 \quad (10)$$

$$0 < \gamma_1 < \eta c_1 c_2 \quad (11)$$

$$0 < \gamma_2 \leq \frac{\gamma_1}{c_2} \quad (12)$$

Remark 2: In [8] the adaptive twisting controller

$$u(t) = -L(t) [W_1 \operatorname{sign}(x_1(t)) + W_2 \operatorname{sign}(x_2(t))] \quad (13)$$

$$\dot{L}(t) = \begin{cases} \rho & \|x\|_2 = 0 \\ 0 & \|x\|_2 > 0 \end{cases} \quad (14)$$

where $\|x\|_2 = \sqrt{x_1^2 + x_2^2}$ was proposed to address the same control problem deal with in the present paper, and under the tuning conditions $W_1 > W_2 > 0$ and $\rho > 0$ it was shown to provide for the finite-time, rather than just asymptotic, convergence of both x_1 and x_2 to zero, along with the guarantee that the adaptive gain $L(t)$ is uniformly bounded. Thus, the algorithm proposed in the present paper is overperformed, in terms of convergence features, by the control law (13)-(14) proposed in [8]. However, the Lyapunov analysis supporting the finite-time convergence features of the control algorithm (13)-(14) proves to be hard to be generalized to more challenging scenarios such as the twisting-based boundary stabilization of heat and wave PDEs. The present treatment, therefore, can be considered as a starting point for future investigations towards the design of adaptive boundary controllers for some classes of PDEs where the present Lyapunov analysis, suitably reshaped and generalized, could be effective, as it was the case in [6] where an adaptive relay control

III. PROOF OF THEOREM 1

Define the candidate Lyapunov function

$$\begin{aligned} V(t) = & \frac{1}{2} x_2^2(t) + \frac{1}{2} (c_1 + \varepsilon c_2) x_1^2(t) + \varepsilon x_1(t) x_2(t) \\ & + k_1(t) |x_1(t)| + \frac{1}{2\gamma_2} (k_2(t) - \Phi_2)^2 \\ & + \frac{\varepsilon}{2\gamma_1} (k_1(t) - \Phi_1)^2. \end{aligned} \quad (15)$$

where $\Phi_1 > 0, \Phi_2 > 0$ and $\varepsilon > 0$ are analysis constants.

A. Positivity of $V(t)$

The quadratic form in (x_1, x_2) in (15), namely the first three elements in the right-hand side of (15), reads as

$$\frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} M \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (16)$$

where matrix

$$M = \begin{bmatrix} c_1 + \varepsilon c_2 & \varepsilon \\ \varepsilon & 1 \end{bmatrix} \quad (17)$$

whose diagonal entries are positive since c_1, c_2 and ε are all positive by construction, is positive definite provided that its determinant is positive, i.e.

$$(c_1 + \varepsilon c_2) - \varepsilon^2 > 0. \quad (18)$$

By virtue of conditions $\varepsilon > 0$ and $c_2 > 0$, the next estimation can be made

$$(c_1 + \varepsilon c_2) - \varepsilon^2 \geq c_1 - \varepsilon^2,$$

implying that condition $c_1 - \varepsilon^2 > 0$ is sufficient to enforce the relation (18). Therefore, a sufficient condition for the positive definiteness of M is

$$0 < \varepsilon < \sqrt{c_1}. \quad (19)$$

Under (19), since all remaining terms in (15) are nonnegative, it follows that $V(t) \geq 0$.

B. Computation of $\dot{V}(t)$

Differentiating (15) one obtains

$$\begin{aligned} \dot{V} = & x_2 \dot{x}_2 + (c_1 + \varepsilon c_2) x_1 \dot{x}_1 + \varepsilon (\dot{x}_1 x_2 + x_1 \dot{x}_2) \\ & + \dot{k}_1 |x_1| + k_1 \dot{x}_1 \text{sign}(x_1) + \frac{\dot{k}_2}{\gamma_2} (k_2 - \Phi_2) \\ & + \varepsilon \frac{\dot{k}_1}{\gamma_1} (k_1 - \Phi_1). \end{aligned} \quad (20)$$

Evaluating (20) along the closed-loop system's trajectories of (1), (4)-(6) yields

$$\begin{aligned} \dot{V} = & x_2(f + u) + (c_1 + \varepsilon c_2) x_1 x_2 + \varepsilon (x_2^2 + x_1(f + u)) \\ & + \gamma_1 x_1^2 + k_1 x_2 \text{sign}(x_1) + |x_2|(k_2 - \Phi_2) \\ & + \varepsilon |x_1|(k_1 - \Phi_1). \end{aligned} \quad (21)$$

The input-dependent terms $x_2(f + u)$ and $x_1(f + u)$ in the right hand side of (21) read as follows in light of the control law formulation (4)

$$\begin{aligned} x_2(f + u) = & x_2 f - k_1 x_2 \text{sign}(x_1) - k_2 |x_2| \\ & - c_1 x_1 x_2 - c_2 x_2^2. \end{aligned} \quad (22)$$

$$\begin{aligned} x_1(f + u) = & x_1 f - k_1 |x_1| - k_2 x_1 \text{sign}(x_2) \\ & - c_1 x_1^2 - c_2 x_1 x_2. \end{aligned} \quad (23)$$

Substituting (22) and (23) into (21) and performing straightforward simplifications and reordering yields the final expression

$$\begin{aligned} \dot{V} = & x_2 f - (c_2 - \varepsilon) x_2^2 - \Phi_2 |x_2| - (\varepsilon c_1 - \gamma_1) x_1^2 + \varepsilon x_1 f \\ & - \varepsilon \Phi_1 |x_1| - \varepsilon k_2 x_1 \text{sign}(x_2) \end{aligned} \quad (24)$$

C. Negativity of $\dot{V}(t)$

Using (3), the estimations

$$x_2 f \leq F |x_2| \quad (25)$$

$$\varepsilon x_1 f \leq \varepsilon F |x_1| \quad (26)$$

of the disturbance-dependent terms in (24) can be made. Furthermore, the sign-indefinite term $-\varepsilon k_2 x_1 \text{sign}(x_2)$ can also be bounded as follows

$$-\varepsilon k_2 x_1 \text{sign}(x_2) \leq \varepsilon k_2 |x_1| \quad (27)$$

Substituting (25)-(27) into (24) and grouping together the homogeneous terms yields

$$\begin{aligned} \dot{V} \leq & - (c_2 - \varepsilon) x_2^2 - (\Phi_2 - F) |x_2| - (\varepsilon c_1 - \gamma_1) x_1^2 \\ & - \varepsilon (\Phi_1 - (F + k_2)) |x_1|. \end{aligned} \quad (28)$$

Choosing the analysis and design parameters as follows

$$0 < \varepsilon < \min(\sqrt{c_1}, c_2) \quad (29)$$

$$\varepsilon c_1 > \gamma_1 \quad (30)$$

$$\Phi_2 > F \quad (31)$$

the quadratic and $|x_2|$ terms in (28) are strictly dissipative.

Assume that the nonnegative gain $k_2(t)$ is uniformly bounded, i.e. that there exists a finite $k_{2M} > 0$ such that

$$0 \leq k_2(t) < k_{2M} \quad (32)$$

Hypothesis (32) will be rigorously validated later on. Using (32), the explicit requirement

$$\Phi_1 > F + k_{2M} \quad (33)$$

can be formulated on the analysis parameter Φ_1 . By considering (29)-(31) and (33), all terms in the right-hand side of (28) are strictly dissipative and the relation

$$\begin{aligned} \dot{V} \leq & - (c_2 - \varepsilon) x_2^2 - (\Phi_2 - F) |x_2| - (\varepsilon c_1 - \gamma_1) x_1^2 \\ & - \varepsilon (\Phi_1 - (F + k_{2M})) |x_1| \leq 0 \end{aligned} \quad (34)$$

holds.

D. Boundedness of the system variables

Relation (34) implies that

$$0 \leq V(t) \leq V(0) := V_0 \quad \forall t \geq 0. \quad (35)$$

One concludes from (35) and the definition (15) of the Lyapunov function V , whose first three terms can be compactly represented as in (16), that

$$\frac{1}{2} \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix} M \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \leq V_0 \quad \forall t \geq 0. \quad (36)$$

$$\frac{1}{2\gamma_1} (k_1(t) - \Phi_1)^2 \leq V_0 \quad \forall t \geq 0. \quad (37)$$

$$\frac{1}{2\gamma_2} (k_2(t) - \Phi_2)^2 \leq V_0 \quad \forall t \geq 0. \quad (38)$$

From relation (36) and the positive definiteness of matrix M (whose smallest eigenvalue is denoted as $\lambda_{\min}(M)$), by exploiting standard properties of positive-definite quadratic forms one concludes that

$$\frac{1}{2} \lambda_{\min}(M) (x_1^2(t) + x_2^2(t)) \leq V_0 \quad \forall t \geq 0. \quad (39)$$

implying that

$$|x_1(t)| \leq x_{1M} := \sqrt{\frac{2}{\lambda_{\min}(M)}} V_0 \quad \forall t \geq 0. \quad (40)$$

$$|x_2(t)| \leq x_{2M} := \sqrt{\frac{2}{\lambda_{\min}(M)}} V_0 \quad \forall t \geq 0. \quad (41)$$

Similarly, relations (37)-(38) imply that

$$|k_1(t) - \Phi_1| \leq \sqrt{2\gamma_1 V_0} \quad \forall t \geq 0, \quad (42)$$

$$|k_2(t) - \Phi_2| \leq \sqrt{2\gamma_2 V_0} \quad \forall t \geq 0, \quad (43)$$

and therefore, by exploiting the fact that $k_1(t)$ and $k_2(t)$ are non-negative by construction, one concludes that

$$0 \leq k_1(t) \leq k_{1M} := \Phi_1 + \sqrt{2\gamma_1 V_0} \quad \forall t \geq 0. \quad (44)$$

$$0 \leq k_2(t) \leq k_{2M} := \Phi_2 + \sqrt{2\gamma_2 V_0} \quad \forall t \geq 0. \quad (45)$$

E. Feasibility of (33)

The actual verification of (45) is conditioned to the fact that parameter Φ_1 can be chosen in accordance with (33), which is rewritten as follows using (45)

$$\Phi_1 > F + k_{2M} = F + \Phi_2 + \sqrt{2\gamma_2 V_0}. \quad (46)$$

The feasibility of (46) is nontrivial since $V_0 = V_0(\Phi_1)$, hence considerations ensuring that the inequality (46) is feasible are needed. From (15), the initial condition V_0 takes the explicit expression

$$V_0 = \frac{1}{2}x_{20}^2 + \frac{1}{2}(c_1 + \varepsilon c_2)x_{10}^2 + \varepsilon x_{10}x_{20} + k_{10}|x_{10}| + \frac{1}{2\gamma_2}(k_{20} - \Phi_2)^2 + \frac{\varepsilon}{2\gamma_1}(k_{10} - \Phi_1)^2. \quad (47)$$

Substituting (47) into (46) the latter is rewritten as

$$\Phi_1 > F + \Phi_2 + \sqrt{a_2\Phi_1^2 + a_1\Phi_1 + a_0}. \quad (48)$$

where

$$a_2 = \varepsilon \frac{\gamma_2}{\gamma_1} \quad (49)$$

$$a_1 = -2k_{10}\varepsilon \frac{\gamma_2}{\gamma_1} \quad (50)$$

$$a_0 = \varepsilon \frac{\gamma_2}{\gamma_1} k_{10}^2 + 2\gamma_2 \left[\frac{1}{2}x_{20}^2 + \frac{1}{2}(c_1 + \varepsilon c_2)x_{10}^2 + \varepsilon x_{10}x_{20} + k_{10}|x_{10}| + \frac{1}{2\gamma_2}(k_{20} - \Phi_2)^2 \right] \quad (51)$$

Since for large enough values of Φ_1 the right-hand side of (48) asymptotically grows proportionally to $\sqrt{a_2}\Phi_1$, provided that

$$a_2 < 1, \quad (52)$$

i.e. that

$$\varepsilon < \frac{\gamma_1}{\gamma_2}, \quad (53)$$

there exists Φ_1^* such that inequality (48) holds for all $\Phi_1 > \Phi_1^*$. Actually, Φ_1^* turns out to be the unique solution of the equation

$$\Phi_1^* = F + \Phi_2 + \sqrt{a_2(\Phi_1^*)^2 + a_1\Phi_1^* + a_0}. \quad (54)$$

Thus, provided that (29) is replaced by the more restrictive condition

$$0 < \varepsilon < \min\left(\sqrt{c_1}, c_2, \frac{\gamma_1}{\gamma_2}\right) \quad (55)$$

and that

$$\Phi_1 > \Phi_1^* \quad (56)$$

relation (45), with V_0 defined in (47), is actually in force. Both the adaptive gains $k_1(t)$ and $k_2(t)$ are then proven to be uniformly bounded.

F. Asymptotic stability

We are now in position to exploit the Barbalat Lemma to establish the asymptotic convergence to zero of the state variables x_1 and x_2 . From (28) it is obtained that

$$\dot{V} \leq -\beta(x_1^2 + x_2^2) \quad (57)$$

where

$$\beta = \min(\varepsilon c_1 - \gamma_1, c_2 - \varepsilon) > 0 \quad (58)$$

Integrating both sides of (57) from $t = 0$ to infinity yields

$$\lim_{t \rightarrow \infty} \{V(0) - V(t)\} \geq \beta \int_0^\infty (x_1^2(\tau) + x_2^2(\tau)) d\tau. \quad (59)$$

By virtue of (35) it follows that

$$\lim_{t \rightarrow \infty} \{V(0) - V(t)\} \in [0, V(0)] \quad (60)$$

which, considered together with (59), shows that the integral term in the right-hand side of (59) exists. To apply Barbalat's Lemma to derive that $(x_1^2(t) + x_2^2(t))$ asymptotically converges to zero, it remains to show that function

$$H(t) = x_1^2(t) + x_2^2(t) \quad (61)$$

is uniformly continuous. The uniform boundedness of $\dot{H}(t)$ is sufficient for this. The expression of $\dot{H}(t)$ is

$$\dot{H}(t) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 = 2x_2^2 + 2x_2(f + u) \quad (62)$$

Substituting in (62) the control law (4), and simplifying the resulting expression, yields

$$\begin{aligned} \dot{H}(t) = & 2x_2^2 + 2x_2f - 2x_2k_1 \operatorname{sign}(x_1) - 2k_2|x_2| \\ & - 2c_1x_1x_2 - 2c_2x_2^2 \end{aligned} \quad (63)$$

By virtue of (3), (40)-(41), and (44)-(45), it can be concluded that all terms appearing in the right-hand side of (63) are uniformly bounded. This implies, according to Barbalat's Lemma, that

$$\lim_{t \rightarrow \infty} H(t) = \lim_{t \rightarrow \infty} (x_1^2(t) + x_2^2(t)) = 0 \quad (64)$$

Under the design conditions (7) and (8), the closed-loop trajectories thus satisfy

$$\lim_{t \rightarrow \infty} x_1(t) = 0, \quad \lim_{t \rightarrow \infty} x_2(t) = 0 \quad (65)$$

and the adaptive gains $k_1(t)$ and $k_2(t)$ are bounded, which concludes the proof.

IV. SIMULATION RESULTS

This section illustrates the behavior of the proposed adaptive twisting controller through numerical simulations. The results are used to highlight the convergence of the state variables, the boundedness of the adaptive gains, and to compare the closed-loop performance with the finite-time adaptive twisting controller proposed in [8].

System (1) is affected by an unknown matched perturbation given by

$$f(t) = 3 + 6 \sin(2\pi t), \quad (66)$$

whose bound

$$|f(t)| \leq F = 9 \quad (67)$$

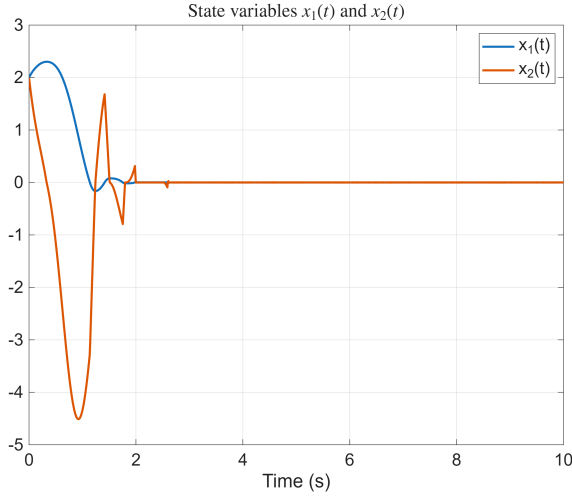


Fig. 1. Time evolution of the state variables x_1 and x_2 under the proposed adaptive twisting controller.

is not assumed to be known. The initial conditions for the states x_1 , x_2 and for the adaptive gains k_1 , k_2 are selected as

$$x_{10} = 2, \quad x_{20} = 2, \quad k_{10} = 0, \quad k_{20} = 0, \quad (68)$$

away from equilibrium in order to emphasize the transient response. The controller parameters are chosen as follows in accordance with the conditions (7)-(8)

$$c_1 = 4, \quad c_2 = 2, \quad \gamma_1 = 6, \quad \gamma_2 = 3. \quad (69)$$

where the value $\varepsilon = 1$ is considered.

Note that, due to implementation aspects such as measurement noise, but also temporal discretization and finite-precision mathematics, the exact convergence of x_1 and x_2 towards zero is no longer achieved neither in actual nor in simulated virtual implementation. With the adaptation laws (5) and (6) this would cause an unbounded drift of the switching gains $k_1(t)$ and $k_2(t)$. To circumvent this effect, the adaptation laws are subject to a standard dead-zone-based boundary-layer redesign according to

$$\dot{k}_i(t) = \begin{cases} \gamma_i |x_i| & \text{if } |x_i| > \delta \\ 0 & \text{else} \end{cases}, \quad i = 1, 2. \quad (70)$$

That is, the adaptation is paused when the state magnitude lies within a band around zero of width δ . The value $\delta = 0.01$ is used in the presented simulations.

The time evolution of the state variables x_1 and x_2 under the proposed adaptive twisting controller (4), with modified dead-zone adaptations as in (70), is displayed in Fig. 1. Both states converge to a close vicinity of zero despite the presence of the unknown perturbation. Although asymptotic convergence is only guaranteed to occur according to the presented Lyapunov analysis, the behaviour of Fig. 1 appears to suggest that finite-time convergence is achieved.

Figure 2 depicts the corresponding evolution of the adaptive gains k_1 and k_2 . Both gains increase only as much as required to counteract the disturbance and then settle to finite steady-state values.

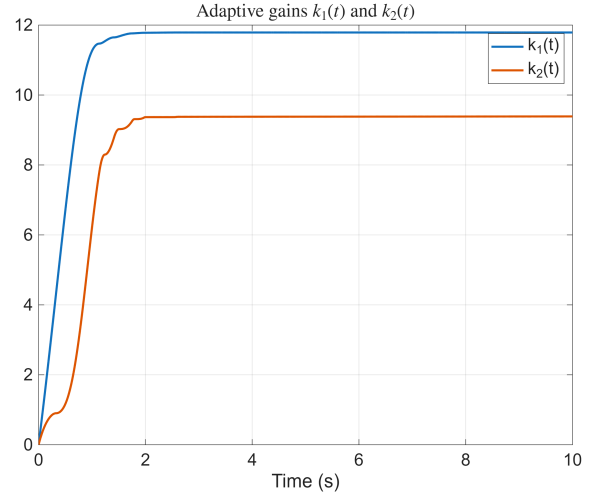


Fig. 2. Time evolution of the adaptive gains k_1 and k_2 under the proposed control scheme.

A performance comparison with the finite-time adaptive twisting controller (13) proposed in [8] is made. Due to analogous reasons as those that motivated the adoption of the thresholded adaptation (70), the adaptive law (14) is replaced by the approximation

$$\dot{L}(t) = \begin{cases} \rho & \text{if } \|x\|_2 > \delta \\ 0 & \text{else} \end{cases}, \quad i = 1, 2. \quad (71)$$

To get a comparison as fair as possible, the values $W_1 = 2$ and $W_2 = 1$ are considered, and the parameter ρ , defining the slope at which $L(t)$ is increased during the reaching transient, has been set as $\rho = 12$ in such a way that the transient duration is approximately the same as that achieved in Fig. (1). Fig. 3 displays the state trajectories obtained with the controller (13), (71) under identical initial conditions and disturbance. Correspondingly, the transient behaviour of x_1 and x_2 are similar to those observed in in Fig. (1), with the only difference that using the finite time controller x_2 exhibits a slightly large transient overshoot. The time evolution of the adaptive gain $L(t)$ is displayed in Fig. 4. Figure 5 compares the control input $u(t)$ obtained using the two controllers. It is apparent that the discontinuous control authority is higher using the finite-time adaptive twisting controller (13), which implies chattering attenuation. We, however, do not claim any guaranteed chattering reduction performance of our proposal against the finite-time adaptive twisting controller of [8].

V. CONCLUSIONS

This paper has proposed an adaptive twisting-based control strategy for a class of uncertain nonlinear systems affected by unknown matched perturbations. The proposed design guarantees asymptotic convergence of the sliding variables while ensuring boundedness of the adaptive gain, without requiring *a priori* knowledge of the uncertainty bounds. The analysis relies on a simple Lyapunov-based framework that enables a transparent tuning procedure.

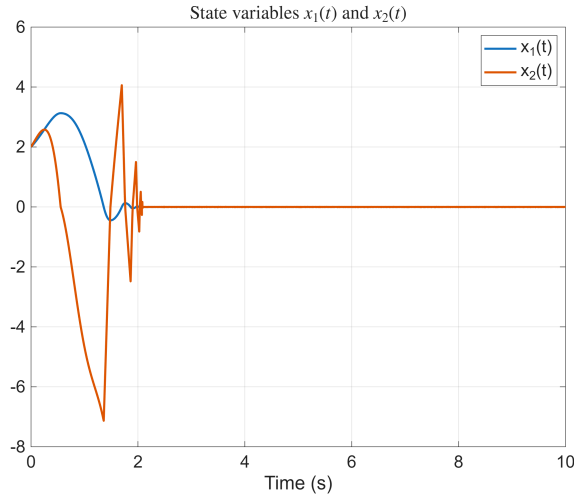


Fig. 3. Time evolution of the state variables x_1 and x_2 under the finite-time adaptive twisting controller of [8].

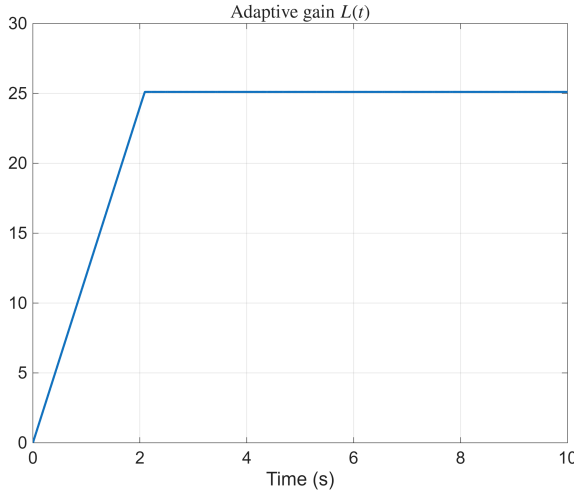


Fig. 4. Time evolution of the adaptive gain $L(t)$ of the finite-time adaptive twisting controller of [8].

Although finite-time convergence can be achieved by more involved adaptive twisting schemes, the proposed approach prioritizes simplicity and analytical tractability of the underlying Lyapunov analysis

This choice is motivated by the long-term objective of extending adaptive second-order sliding mode techniques to infinite-dimensional systems. In this respect, the present results are intended as a foundational step towards the development of adaptive boundary controllers for partial differential equations, such as heat processes subject to matched perturbations with unknown bounds. Future work will focus on the extension of the proposed adaptive twisting framework to PDE control problems and on its experimental validation in thermal distributed-parameter systems.

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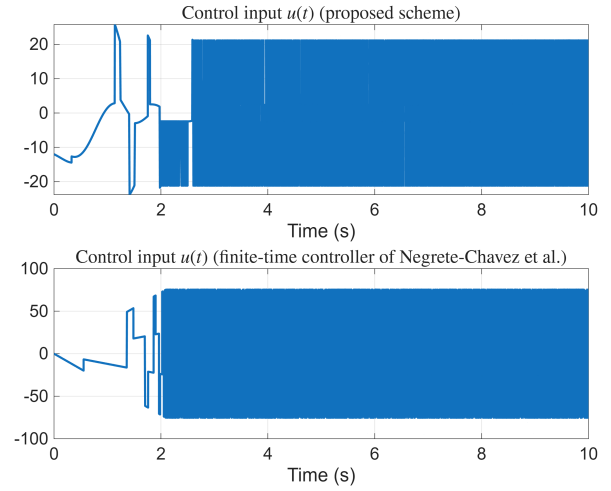


Fig. 5. Time evolution of the control input $u(t)$ with the proposed scheme (top) and the finite-time adaptive twisting controller of [8] (bottom).

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