Discussion on "Influence of incoming wave conditions on the hysteretic behavior of an oscillating water column system for wave energy conversion" by J. Peng, C. Hu and C. Yang DOI: 10.1016/j.oceaneng.2023.116207

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Abstract

Recently, Peng, Hu and Yang presented a lumped parameter model to quantify the hysteresis in Oscillating Water Column systems. We noticed that the model they presented is remarkably similar to the one we introduced in some of our previously published works. The similarity extends not only to the assumptions, derivation and methodology used to obtain an analytical solution, but even to the almost totality of the symbols chosen for the many model variables. None of the papers where we introduced the model and its solution were referenced by Peng and his coauthors, who therefore claimed for themselves the credit due to the original authors of the model. Peng and his coauthors have then applied the lumped parameter model to a test case different from the one that we had validated it on. This gives further confirmation of the validity of the model, which we feel the responsibility to reestablish the scientific property of.

Introduction

In their paper [1], Peng, Hu and Yang presented a lumped parameter model (LPM) to quantify the hysteresis in Oscillating Water Column (OWC) systems. We could not avoid noticing that the assumptions at its origin, the derivation of the LPM and the methodology used to obtain an analytical solution (after linearization) are remarkably similar to the ones presented in some of our published works [2, 3, 4], none of which were referenced in [1]. The similarities between the model presented by Peng and his coauthors and the model published by ourselves are significant, to the point that we find hard to believe that they could have derived it authonomously. Our conviction comes from the many similarities between the two models, even down to the symbols used by Peng and his coauthors for the model variables, the majority of which are the same as the ones used in our published works. In this brief discussion, we aim to explain our reasons, in order to reestablish the credit we deserve as the original authors of the model and methodology used by Peng, Hu and Yan in [1].

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Similarities between the LPM Model Presented by Peng, Hu and Yang and Our Published Work

The assumptions, derivation and analytical solution of the LPM by Peng and his coauthors is presented in Section 2 of [1]. The authors proposed an LPM to describe the behavior of a generic OWC system, by applying the conservation of mass to the OWC air chamber and the conservation of momentum inside the turbine duct, following the same methodology previously introduced by ourselves in [2, 3]. In fact, in addition to using a schematic for the OWC system (figure 2b in [1]) that is remarkably similar to the one shown in figure 1 of [2], the model equations 6, 7 and 8 in [1] are the same as equations 4, 5 and 6 in [2]. A side-by-side comparison of these equations, extracted from the original papers without any modifications, is reported in Fig. 1.

After proposing the LPM model, analogously to what done in our previous works, Peng and his coauthors defined six non-dimensional parameters (coincidentally, the same we had defined in our previouly published papers). Together with a simplification made possible by the assumption of negligible temperature variation (equation 9 in [1] equal to equation 7 in [2]), these non-dimensional parameters were used to define a non-dimensional form for the equations of conservation of mass and momentum. It is hard to find any differences between the two non-dimensional equations derived by Peng and his coauthors (equation 10 in [1]) and the one presented in our work (equation 8 in [2]). After this, Peng, Hu and Yang obtained a single non-dimensional second-order differential equation by differentiating the momentum equation and then replacing the derivative of the non-dimensional pressure drop from the continuity equation (equation 11 in [1], which reads the same as equation 11 in [2]). Again, a side-by-side comparison between the original equations from our model [2] and the ones presented by Peng, Hu and Yang in [1] is presented in Fig. 1.

In order to obtain an analytical solution for the above non-linear equation, the latter was linearized. To this purpose, Peng, Hu and Yang made use of the assumptions in equation 12 of [1], which are the same that we proposed in [2] (equation 12 and following). The resulting linearized second-order differential equation (equation 13 in [1]) is the same as equation 14 in [2]. Then, Peng and his coauthors derived the transfer function for the second-order linear system (equation 14 in [1] is the same as equation 15 in [2]). From the transfer function, they could then obtain the angular frequency and damping of the system (equation 15 in [1] which reads the same as equations 18 and 19 in [2]). All the above information allowed Peng and his coauthors to write the solution of the system (equations 18 and 19 in [1], which are the same as equations 22 and 23 in [2]) in terms of the module and phase of the transfer function (equations 16 and 17 in [1], which are the same as equations 20, 21 and 25 [2]). These are the last equations pertaining to the development and solution of the LPM both in [2] and in [1]. The existence of a non-zero phase in the transfer function demonstrates and quantifies hysteretic effects in OWC systems due to the delay between the movement of the water column in the chamber and the flow passing through the turbine. This was then an original and important result, that gave a new explanation to a phenomenon was not clearly understood at the time [5, 6]. A side-by-side comparison between the original equations from our model [2] and the ones presented by Peng, Hu and Yang in [1] is presented in Fig. 2.

Original Contributions of the Paper by Peng, Hu and Yang

In our previous publications [2, 3], the lumped parameter model was solved both analytically (with linearization) and numerically (without linearization), and validated by replicating the conditions in the experimental tests conducted on an OWC simulator by Setoguchi *et al.* [5]. Analytical and numerical results from the LPM were compared to the experimental data and to more computationally intensive CFD simulations, demonstrating for the first time how even this comparatively simple model we had devised was able to capture and quantify the hysteresis in OWC systems due to the capacitive effect of the chamber.

In [1], Peng, Hu and Yang used the LPM embezzled from our work to replicate the experimental tests on a different OWC, accidentally the one we built and conducted experimental work on at the University of Cagliari more than a decade ago [7]. We had already published some CFD analyses based on these tests [8, 9]. These CFD results were also reproduced by Peng and his coauthors in their paper, with an approach very similar to the one presented in our paper.

In our opinion, the only original contributions presented by the Peng and his coauthors in [1] are therefore the application of the LPM introduced in [2, 3] to the experimental tests in [7], and the study of the influence of compression ratio and rotational speed on the OWC hysteresis for the specific geometry. While it is not our task to judge on whether this is a sufficient contribution for a new publication, we firmly believe that Peng and his coauthors should have clearly referenced the papers that first presented the model they made use of.

Conclusion

Despite being pleased to see that our work has inspired other authors, and that the results from Peng, Hu and Yang give a further confirmation of the validity and usefulness of our model and approach [2, 3], we firmly believe on the value of scientific publication, which dictates that authors, especially when reusing a model without any original contribution to its development as shown in this case, should give the deserved credit to the authors who first introduced it.

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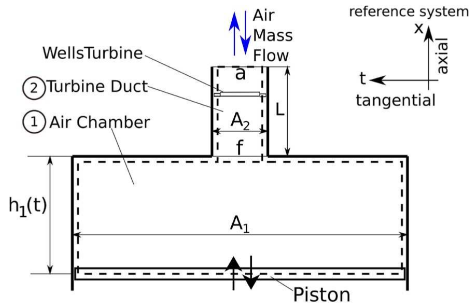
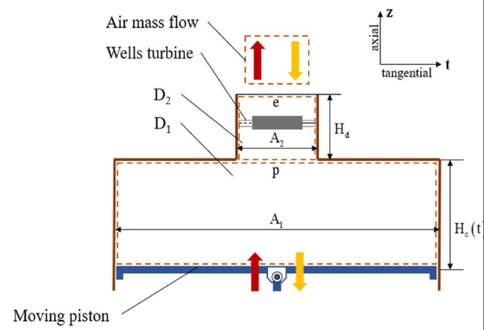
1. Definition of the lumped parameter model in dimensional by applying the conservation of mass to the air chamber in the OWC, and the conservation of momentum to the turbine duct. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].



$$\begin{cases} \dfrac{\mathrm{d}M_1}{\mathrm{d}t} = -\rho_f V_f A_f \\ \dfrac{\mathrm{d}(\rho_2 V_{2,x} A_2 L)}{\mathrm{d}t} = (p_f - p_a)A_f + F_x \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{\mathrm{d}M_1}{\mathrm{d}t} = h_1 A_1 \dfrac{\mathrm{d}\rho_1}{\mathrm{d}t} + \rho_1 A_1 \dfrac{\mathrm{d}h_1}{\mathrm{d}t} = -\rho_a V_{2,x} A_2 \\ \dfrac{\mathrm{d}(\rho_2 V_{2,x} A_2 L)}{\mathrm{d}t} = (p_1 - p_a)A_2 + F_x \end{cases} \tag{5}$$

$$\begin{cases} \dfrac{h_1}{r_m}\dfrac{A_1}{A_2}\dfrac{\mathrm{d}(\rho_1/\rho_a)}{\mathrm{d}(t\omega)} + \dfrac{\rho_1}{\rho_a}\dfrac{A_1}{A_2}\dfrac{\mathrm{d}(h_1/r_m)}{\mathrm{d}(t\omega)} = -\dfrac{V_{2,x}}{\omega r_m} \\ \dfrac{L}{r_m}\dfrac{\mathrm{d}(V_{2,x}/(\omega r_m))}{\mathrm{d}(t\omega)} = \dfrac{(p_1 - p_a)}{\rho_a(\omega r_m)^2} + \dfrac{F_x}{\rho_a(\omega r_m)^2 A_2} \end{cases} \tag{6}$$



$$\begin{cases} \dfrac{\mathrm{d}M_1}{\mathrm{d}t} = -\rho_p V_p A_p \\ \dfrac{\mathrm{d}(\rho_2 V_z A_2 H_d)}{\mathrm{d}t} = (p_p - p_e)A_2 + F_z \end{cases}, \tag{6}$$

$$\begin{cases} \dfrac{\mathrm{d}M_1}{\mathrm{d}t} = H_c A_1 \dfrac{\mathrm{d}\rho_1}{\mathrm{d}t} + \rho_1 A_1 \dfrac{\mathrm{d}H_c}{\mathrm{d}t} = -\rho_p V_z A_2 \\ \dfrac{\mathrm{d}(\rho_2 V_z A_2 H_d)}{\mathrm{d}t} = (p_1 - p_e)A_2 + F_z \end{cases}. \tag{7}$$

$$\begin{cases} \dfrac{H_c}{r_t}\dfrac{A_1}{A_2}\dfrac{\mathrm{d}(\rho_1/\rho_e)}{\mathrm{d}(t\omega)} + \dfrac{\rho_1}{\rho_e}\dfrac{A_1}{A_2}\dfrac{\mathrm{d}(H_c/r_t)}{\mathrm{d}(t\omega)} = -\dfrac{V_z}{\omega r_t} \\ \dfrac{H_d}{r_t}\dfrac{\mathrm{d}(V_z/(\omega r_t))}{\mathrm{d}(t\omega)} = \dfrac{(p_1 - p_e)}{\rho_e(\omega r_t)^2} + \dfrac{F_z}{\rho_e(\omega r_t)^2 A_2} \end{cases}. \tag{8}$$

2. Choice of non-dimensional parameters. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

$$\frac{A_1}{A_2}\frac{\mathrm{d}(h_1/r_m)}{\mathrm{d}(t\omega)} = -\phi_p \quad \text{(piston – based flow coefficient)}$$

$$\frac{V_{2,x}}{\omega r_m} = \phi_l \quad \text{(local flow coefficient)}$$

$$\frac{p_1 - p_a}{\rho_a(\omega r_m)^2} = P^* \quad \text{(non – dimensional pressure – drop)}$$

$$\frac{\rho_1}{\rho_a} = \rho^* \quad \text{(non – dimensional flow density)}$$

$$\frac{F_x}{\rho_a(\omega r_m)^2 A_2} = c_x \quad \text{(turbine axial force coefficient)}$$

$$t\,\Omega = \frac{t\omega}{\omega/\Omega} = t^* \quad \text{(non – dimensional time, based on piston period)}$$

$$\frac{A_1}{A_2}\frac{\mathrm{d}(H_c/r_t)}{\mathrm{d}(t\omega)} = -\varphi_p, \frac{V_z}{\omega r_t} = \varphi_l, \frac{p_1 - p_e}{\rho_e(\omega r_t)^2} = P^*,$$

$$\frac{\rho_1}{\rho_e} = \rho^* = \frac{\gamma(\omega r_t)^2}{a^2}P^* + 1, \frac{F_z}{\rho_e(\omega r_t)^2 A_2} = c_z, t\Omega = \frac{t\omega}{\omega/\Omega} = t^*,$$

3. Assumption of negligible temperature variation in the control volume. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

$$P^* = \frac{p_1 - p_a}{\rho_a(\omega r_m)^2} = \frac{(\rho_1 - \rho_a)RT}{\rho_a(\omega r_m)^2} = (\rho^* - 1)\frac{a^2}{\gamma(\omega r_m)^2} \tag{7}$$

$$P^* = \frac{p_1 - p_e}{\rho_e(\omega r_t)^2} = \frac{(\rho_1 - \rho_e)RT}{\rho_e(\omega r_t)^2} = (\rho^* - 1)\frac{a^2}{\gamma(\omega r_t)^2}. \tag{9}$$

4. Derivation of the LPM equations in non-dimensional form. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

$$\begin{cases} \dfrac{h_1 A_1 \Omega}{A_2}\dfrac{\gamma(\omega r_m)}{a^2}\dfrac{\mathrm{d}P^*}{\mathrm{d}t^*} - \left(\dfrac{\gamma(\omega r_m)^2}{a^2}P^* + 1\right)\phi_p = -\phi_l \\ \dfrac{L}{r_m}\dfrac{\Omega}{\omega}\dfrac{\mathrm{d}\phi_l}{\mathrm{d}t^*} = P^* + c_x \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{H_c A_1 \Omega}{A_2}\dfrac{\gamma(\omega r_t)}{a^2}\dfrac{\mathrm{d}P^*}{\mathrm{d}t^*} - \left(\dfrac{\gamma(\omega r_t)^2}{a^2}P^* + 1\right)\varphi_p = -\varphi_l \\ \dfrac{H_d}{r_t}\dfrac{\Omega}{\omega}\dfrac{\mathrm{d}\varphi_l}{\mathrm{d}t^*} = P^* + c_z \end{cases}, \tag{10}$$

5. Derivation of a single second order non-dimensional equation for the OWC system. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

$$\frac{L}{r_m}\frac{\Omega}{\omega}\frac{\mathrm{d}^2\phi_l}{\mathrm{d}t^{*2}} = \frac{\mathrm{d}P^*}{\mathrm{d}t^*} + \frac{\mathrm{d}c_x}{\mathrm{d}t^*} = = \frac{a^2}{\gamma(\omega r_m)(h_1\Omega)}\frac{A_2}{A_1}\left[\left(\frac{\gamma(\omega r_m)^2}{a^2}P^* + 1\right)\phi_p - \phi_l\right] + \frac{\mathrm{d}c_x}{\mathrm{d}t^*} \tag{11}$$

$$\frac{H_d}{r_t}\frac{\Omega}{\omega}\frac{\mathrm{d}^2\varphi_l}{\mathrm{d}t^{*2}} = \frac{\mathrm{d}P^*}{\mathrm{d}t^*} + \frac{\mathrm{d}c_z}{\mathrm{d}t^*} = \frac{a^2}{\gamma(\omega r_t)(H_c\Omega)}\frac{A_2}{A_1}\left[\left(\frac{\gamma(\omega r_t)^2}{a^2}P^* + 1\right)\varphi_p - \varphi_l\right] + \frac{\mathrm{d}c_z}{\mathrm{d}t^*}. \tag{11}$$

Figure 1: Side by side comparison of the LPM model assumptions, derivation and linearized solution originally published by Cambuli *et al.* in 2019 [2], and the one recently presented by Peng, Hu and Yang [1]. Equations as reported as they appear in the papers, without modifications. (part 1 of 2)

6. Assumptions for linearizing the non-linear second-order LPM equation in non-dimensional form. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

   1. The dependence between local flow coefficient ϕ_l and turbine axial force coefficient c_x can be approximated with a straight line (see Fig. 7)

   $$c_x = -c_{x,\phi}\phi_l \qquad (12)$$

   2. The height of the air chamber h_1 can be approximated with its value at rest, h_{10}
   3. $\left(\frac{(\omega r_m)^2}{RT}P^*+1\right)$ can be approximated with its value at rest (= 1).

   $$c_z = c_{z,\varphi}\varphi_l,\, H_c(t)\approx H_c(0),\, \left(\frac{\gamma(\omega r_t)^2}{a^2}P^*+1\right)\approx 1, \qquad (12)$$

7. Linearized non-dimensional LPM equation. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

   $$A\frac{\mathrm{d}^2\phi_l}{\mathrm{d}t^{*2}} + B\frac{\mathrm{d}\phi_l}{\mathrm{d}t^*} + C\phi_l = D\phi_p \qquad (14)$$

   where:

   $$\begin{cases} A = \dfrac{L}{r_m}\dfrac{\Omega}{\omega} \\ B = c_{x,\phi} \\ C = D = \dfrac{a^2}{\gamma(\omega r_m)(h_{10}\Omega)}\dfrac{A_2}{A_1} \end{cases}$$

   $$\underbrace{\frac{H_d}{r_t}\frac{\Omega}{\omega}}_{A}\frac{\mathrm{d}^2\varphi_l}{\mathrm{d}t^{*2}} + \underbrace{c_{z,\varphi}}_{B}\frac{\mathrm{d}\varphi_l}{\mathrm{d}t^*} + \underbrace{\frac{a^2}{\gamma(\omega r_t)(H_c(0)\Omega)}\frac{A_2}{A_1}}_{C}\varphi_l = \underbrace{\frac{a^2}{\gamma(\omega r_t)(H_c(0)\Omega)}\frac{A_2}{A_1}}_{D}\varphi_p. \qquad (13)$$

8. Transfer function for the linearized non-dimensional LPM equation. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

   $$G(s^*) = \frac{D}{As^{*2}+Bs^*+C} = \frac{\frac{D}{C}}{\frac{A}{C}s^{*2}+\frac{B}{C}s^*+1} = \frac{\frac{D}{C}}{\left(\frac{\Omega}{\Omega_n}\right)^2 s^{*2}+1+2\zeta\left(\frac{\Omega}{\Omega_n}\right)s^*} \qquad (15)$$

   $$G(s^*) = \frac{D}{As^{*2}+Bs^*+C} = \frac{\frac{D}{C}}{\frac{A}{C}s^{*2}+\frac{B}{C}s^*+1} = \frac{\frac{D}{C}}{\left(\frac{\Omega}{\Omega_n}\right)^2 s^{*2}+1+2\zeta\left(\frac{\Omega}{\Omega_n}\right)s^*}. \qquad (14)$$

9. Derivation of angular frequency and damping for the linearized non-dimensional LPM equation. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

   $$\Omega_n = \sqrt{\frac{C}{A}}\Omega = \sqrt{\frac{a^2}{\gamma h_{10}L}\frac{A_2}{A_1}} \qquad (18)$$

   $$2\zeta = \frac{B}{C}\frac{\Omega_n}{\Omega} = \frac{B}{C}\sqrt{\frac{C}{A}} = \frac{B}{\sqrt{AC}} = \frac{c_{x,\phi}}{\sqrt{\frac{a^2}{\gamma}\frac{L}{h_{10}}\frac{A_2}{A_1}\frac{1}{\omega r_m}}} \qquad (19)$$

   $$\begin{cases} \Omega_n = \sqrt{\dfrac{C}{A}}\Omega = \sqrt{\dfrac{a^2}{\gamma H_c(0)H_d}\dfrac{A_2}{A_1}} \\ 2\zeta = \dfrac{B}{C}\dfrac{\Omega_n}{\Omega} = \dfrac{B}{C}\sqrt{\dfrac{C}{A}} = \dfrac{B}{\sqrt{AC}} = \dfrac{c_{z,\varphi}}{\sqrt{\dfrac{a^2}{\gamma}\dfrac{L}{H_c(0)}\dfrac{A_2}{A_1}\dfrac{1}{\omega r_t}}} \end{cases}. \qquad (15)$$

10. Derivation of module and phase of the transfer function. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

    $$|G(s^*)| = \frac{D}{\sqrt{(C-A)^2+B^2}} = \frac{\frac{D}{C}}{\sqrt{\left[-\left(\frac{\Omega}{\Omega_n}\right)^2+1\right]^2+\left[2\zeta\left(\frac{\Omega}{\Omega_n}\right)\right]^2}} \qquad (20)$$

    $$\xi = \tan^{-1}\left(\frac{-B}{-A+C}\right) = \tan^{-1}\left(\frac{-2\zeta\frac{\Omega}{\Omega_n}}{-\left(\frac{\Omega}{\Omega_n}\right)^2+1}\right) \qquad (21)$$

    $$\xi = \tan^{-1}\frac{c_{x,\phi}}{\frac{L}{r_m}\frac{\Omega}{\omega}-\frac{a^2}{\gamma(\omega r_m)(h_{10}\Omega)}\frac{A_2}{A_1}} \qquad (25)$$

    $$|G(s^*)| = \frac{D}{\sqrt{(C-A)^2+B^2}} = \frac{\frac{D}{C}}{\sqrt{\left[-\left(\frac{\Omega}{\Omega_n}\right)^2+1\right]^2+\left[2\zeta\left(\frac{\Omega}{\Omega_n}\right)\right]^2}}, \qquad (16)$$

    $$\xi = \tan^{-1}\left(\frac{-B}{-A+C}\right) = \tan^{-1}\left(\frac{-2\zeta\frac{\Omega}{\Omega_n}}{-\left(\frac{\Omega}{\Omega_n}\right)^2+1}\right) = \tan^{-1}\left(\frac{c_{z,\varphi}}{\frac{H_d}{r_t}\frac{\Omega}{\omega}-\frac{a^2}{(\omega r_t)(H_c(0)\Omega)}\frac{A_2}{A_1}}\right) \qquad (17)$$

11. Solution of the linearized non-dimensional LPM equation in terms of module and phase of the transfer function. On the left, the orginal derivation from Cambuli *et al.* [2], on the right the one recently proposed by Peng, Hu and Yang [1].

    $$\phi_l = \phi_{l0}\,e^{it^*+\xi} \qquad (22)$$

    where:

    $$|\phi_l| = \phi_{l0} = |\phi_p||G(s^*)| = \phi_{p0}|G(s^*)| \qquad (23)$$

    $$\varphi_l = \varphi_{l0}e^{it^*+\xi}, \qquad (18)$$

    where

    $$|\varphi_l| = \varphi_{l0} = |\varphi_p||G(s^*)| = \varphi_{p0}|G(s^*)|. \qquad (19)$$

Figure 2: Side by side comparison of the LPM model assumptions, derivation and linearized solution originally published by Cambuli *et al.* in 2019 [2], and the one recently presented by Peng, Hu and Yang [1]. Equations as reported as they appear in the papers, without modifications. (part 2 of 2)