Sustainable Energy, Grids and Networks 38 (2024) 101391

Contents lists available at ScienceDirect

Sustainable Energy, Grids and Networks

journal homepage: www.elsevier.com/locate/segan

A cooperative game-theory approach for incentive systems in local energy communities

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ARTICLE INFO

Keywords:
Renewable energies incentive systems
Smart grid
Cooperative game theory
Peer-to-peer energy trading

ABSTRACT

Prosumers have a central role in the context of smart grids, and in particular within local energy communities (LECs), as they are capable of being both energy producers and consumers. In a scenario where peer-to-peer (P2P) energy trading is allowed, prosumers can exchange the energy they produce with other prosumers: the primary outcome of this is the improvement of energy self-consumption across the grid, which leads to decreased transmission losses, as well as lower energy costs and diminished long-term damage to the grid itself. Previous work proposed a mechanism to achieve multiple objectives for a cooperative game theory perspective for small coalitions, but its behavior for coalitions of arbitrary size remains unexplored, and it does not consider the objective of peak shaving. This paper aims to (i) design an algorithm for calculating schedules for coalitions of arbitrary size, (ii) analyze the behavior of this mechanism for large coalitions, (iii) create a new incentive mechanism by proposing new selling functions that ensure that the resulting mechanism would optimize for the objective of peak shaving when all the prosumers work together in one large coalition, and (iv) demonstrate the performance of the existing mechanism in terms of peak shaving, by comparing against the mechanism specifically optimized for this objective. Simulations conducted on data from a grid in Cardiff, UK, reveal that the existing mechanism works particularly well for the non-cooperative game, achieving results for cost reduction and self-consumption almost identical to the cooperative game, no matter the size of the coalitions. More precisely, although all mechanisms achieve optimal peak shaving for the grand coalition, the existing mechanism achieves this objective even within the framework of the selfish game, resulting in a reduction of the peak by approximately 29 compared to alternative methods. Furthermore, the mechanism is proven to optimally achieve peak shaving in both cooperative and non-cooperative cases.

1. Introduction

Exploiting renewable energies has become a crucial focus in recent decades, addressing both environmental concerns and the quest for alternative energy sources. Numerous policies have been implemented in order to encourage grid users to join renewable production, and become therefore *prosumers* - i.e., users who are producers in addition to being consumers. The most common incentives for prosumers are financial, such as payment for the energy prosumers produce and inject into the grid, or provisioning of free energy at a later point in time. The introduction of those policies came along with some issues that were not present before: renewable energy generation is often unpredictable, and this brings new challenges to grids in terms of stability [1,2], as well as having to align consumption with production through the grid as closely as possible.

A key concept for addressing those challenges is *energy flexibility*: that is, the ability to adjust energy loads with respect to their time allocation and quantity of consumed energy [3]. Certain incentive mechanisms leverage energy flexibility by introducing payment systems that encourage users to align their consumption with the energy production peaks through the grid [4–8]. In particular, a notably interesting mechanism is NRG-X-Change [5]: the idea behind it is that prosumers can export their energy production to the grid and be rewarded for that, while consumers may draw energy from the grid and have to pay accordingly. The rewards for production and costs for consumption are determined by two functions, and the behavior of those functions influences the behavior of the users from the grid. Also, the mechanism originally supported a virtual currency called NRGcoin, although its underlying principle remains effective even with fiat currency. The mechanism revolves around prosumers producing energy and sharing it with the community, while consumers purchase energy: prices for selling and buying energy are regulated by two functions that depend on the amount of energy consumed and produced

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https://doi.org/10.1016/j.segan.2024.101391
Received 16 January 2024; Received in revised form 20 March 2024; Accepted 17 April 2024
Available online 25 April 2024
2352-4677/

within the grid. There have been several studies with the aim of improving this mechanism, focusing on user behavior both from a single user perspective [9] and the whole grid's perspective [10,11]. For the specific case of the whole grid, a game theory approach has been chosen.

Concerning the topic of game theory, numerous studies have delved into the application of game theoretical methods in smart grids, specifically focusing on enhancing self-consumption through the grid and improving economic benefits by utilizing energy flexibility [12]. However, none of them have addressed the case of pricing schemes tailored in order to prevent curtailment and user self-consumption. The work performed by authors in [10] has done this from a perspective where users adopt a selfish behavior: this behavior is modeled by defining a game over the grid, demonstrating that exploiting flexibility can lead to improvements in energy costs and self-consumption. Building upon this, another research effort [11] has extended the previous work from a coalition game theory perspective, analyzing the consequences of the possibility of forming small coalitions. It has been shown that small coalitions (i.e., with 4 or less players in each) do not yield significant advantages compared to the selfish game. However, it is still unknown what happens with games with bigger coalitions since, in this case, it is difficult to compute the optimal schedules for the coalitions. Moreover, until now, the effect of incentive mechanisms has been studied mainly in terms of energy consumption [9–11]; however, there are other aspects, such as peak shaving [13], that would be important to consider. Therefore, this work has the following main objectives:

- Enhance the current incentive mechanism by devising a methodology that efficiently computes optimization for energy flexibility within coalitions of arbitrary sizes, accomplishing this task within a practical timeframe.
- Analyze the impact of coalitions of arbitrary size for flexibility exploitation in a local grid.
- Introduce two novel incentive mechanisms by devising two distinct selling functions, explicitly aimed at achieving peak shaving, while coalitions optimize for profitability.
- Evaluate the existing mechanism's performance in terms of peaks, both for the cooperative (of any size) and non-cooperative cases, and determine if optimal peak shaving is achieved.

It is crucial to emphasize that this paper does not encompass general applications of game theory to local energy communities (LECs); rather, it is specifically centered on incentive mechanisms. Consequently, the primary focus is on investigating the impact of these mechanisms on user behavior through the lens of game theory. Furthermore, the analysis of peak shaving in this study aims to determine whether the proposed incentive mechanism actively promotes its attainment or not. The development of a computation method for large coalitions is essential for observing the behavior of the grand coalition in this context.

This work is organized as follows. Section 2 contains the related work, Section 3 describes the game theoretical model and the existing frameworks, while Section 4 contains the new optimization techniques and functions. Furthermore, Section 5 shows the experiments we have carried out to validate our results, and Section 6 discusses the results found. Finally, Section 7 describes our conclusions and draws the future directions where we are headed.

2. Related work

In this section, we describe the works present in the literature that are relevant to our research.

This paper builds upon prior research endeavors. The original incentive mechanism, NRG-X-Change [5], conceptualized a LEC wherein energy buying and selling prices were contingent upon the aggregate energy production and consumption within the community. Subsequent work [9] refined this mechanism by mitigating prosumers' tendencies to curtail their energy production or excessively consume energy simultaneously, thereby minimizing congestion risks. Instead, prosumers were incentivized to prioritize the consumption of self-generated energy before engaging in excess buying or selling activities. This refinement was further enhanced in [10] through the introduction of a selfish game theory framework to scrutinize user behavior. Additionally, a new mechanism was devised to preserve the advantageous properties of previous iterations while ensuring the existence of a Nash Equilibrium. Furthermore, [11] extended this line of inquiry by examining cooperative games of small scale and proposing more versatile selling and buying functions.

For what incentive mechanisms are concerned, the work conducted by authors in [14] describes the status of traditional incentive mechanisms in southeast Europe, giving an overview of the electricity production in those countries with a focus on renewable energy, and offers an economic insight on feed-in tariffs and incentives on renewable energy. On the other hand, the work in [15] provides an analysis of how incentive mechanisms are evolving, comparing old mechanisms such as net metering and feed-in tariffs with more recent ones, demonstrating the superior performance of the latter. In a more specific and localized context, the work described in [16] is a review of the current legislation and incentive mechanisms in Italy for LECs, with a focus on the currently existing projects and the challenges they are facing. Moreover, [17] illustrates the application of incentive mechanisms in a specific scenario in Italy, highlighting weaknesses and improvements in local self-consumption for the case of a desalination unit. In a different context, researchers in [18] presented a case study in Nepal for peer-to-peer energy trading, showcasing and comparing different approaches, and emphasizing the main challenges for isolated energy systems and microgrids. Regarding incentive mechanisms that propose payment schemes, the work conducted by authors in [19] proposes two payment schemes to allow distribution companies to promote energy efficiency: one considers uncertainty, while the other does not. This work compares performance-based mechanisms against fixed prices and also proposes a hybrid model incorporating both those components. In [20], a pricing method is introduced to discourage grid users from cheating when reporting their energy consumption. This is based on the principle of an auction game, ensuring that the users will be truthful about their energy consumption, which will encourage a better usage of energy through the grid. The work [21] proposes a two-stage mechanism that operates on the demand response market, whose objective is to address the issues of the uncertainty of consumption baselines and curtailment costs by requesting for day-ahead probabilistic estimates, and the actual consumption baseline when called for demand response.

Concerning the application of game theory in the context of smart grids, various examples exist. The study referenced in [22] effectively illustrates the dynamics of the game within a LEC, considering flexible loads and the formation of coalitions. However, a key distinction lies in the focus of our work, which centers on investigating incentive mechanisms and their interplay within the cooperative game framework. In contrast, [22] primarily examines coalition formation within a context where coalitions are dynamic. Also, [23] studies cooperative game theory effects on LECs, with a game formulation similar to ours: here the main difference is that the strategies are to join or not a coalition, while this paper focuses on exploiting flexibility for achieving profit maximization, and therefore taking part in coalitions means to unite flexibility with the other coalition members. A similar approach has been taken from [24], which uses cooperative game theory and energy storage to improve global self-consumption. However, this work has its main focus on single buildings, and its exploitation of cooperative game theory refers again to taking part in a coalition or not as a strategy, rather than exploiting flexibility. Regarding other game theory works authors in [25] provided a survey paper that describes in general how game theory is employed in relation to smart grid issues, describing, in particular, both the selfish and cooperative cases. The same has been done in [26], but with an emphasis on cooperative games: the three main points of this work are to emphasize why cooperation is important, in which ways it may occur, and how game theory can model it. Another approach [27] introduces a 3-level game theoretical approach for managing electric vehicles and charging stations. The work considers three different approaches from a system operator perspective, comparing voltage magnitude for each load. In another work [28], game theory is employed for selling stored energy by exploiting competition between aggregators. The effectiveness of the approach is demonstrated on four different use cases. Meanwhile, [29] proposes a simple game theoretical approach for selling and buying energy, and compares it against some single-behavior baselines, demonstrating its benefits in terms of economic profit. One more work described in [30] provides a simple example of coalition formation between different grids, showcasing how cooperative game theory can minimize energy losses between different smart grids when allowed to trade energy between each other. Game theory has in particular been applied to the NRG-X-Change mechanism itself. The work illustrated in [31] builds and simulates the functioning of NRG-X-Change, although it uses the original functions and only hints at the idea of exploiting cooperative game theory in future work. However, none of these works explicitly integrates game theory with multi-objective incentive mechanisms while defining the impact of flexibility in the game. The work we propose in this paper aims to fill this gap, allowing the possibility of forming coalitions of arbitrary size.

Finally, peak shaving is a widely discussed problem in literature. The work illustrated in [13] describes a use case in Switzerland, where battery storage is employed for peak shaving within a deep learning load forecasting model, demonstrating the effectiveness of the approach and the economic savings obtained. In [32] it is shown how sharing distributed energy resources can be used by the aggregator to achieve peak shaving and load balance: the strategy is based on an asymmetric Nash bargaining model [33], and it is implemented in a decentralized manner by using an Alternating Direction Method of Multipliers (ADMM) algorithm [34]. Finally, the work performed in [35] optimizes demand-side management for a building to improve peak shaving, by defining a multi-energy flexibility measure and utilizing it for building operation optimization, reducing the peak and associated costs in the process.

Following the same objective, in this paper we introduce two new functions that optimize peak shaving for big coalitions.

3. Preliminaries

This section outlines the model employed to represent our problem. We will show how the user behavior is modeled in terms of game theory, and how our approach interacts with the users' behavior. Our model draws inspiration by [36] and [37].

Table 1 provides a nomenclature for all the symbols we have used throughout the paper.

3.1. Game theory approach to our smart grid problems

We start by defining what a game is. We define a game as a triple $G = U\ S\ Q)$ as follows:

- $U = \{U_1\ \dots\ U_N\}$ are the players.
- $S = \{S_1\ \dots\ S_N\}$ contains all the strategies. More specifically, for every $i \in \{1\ \dots\ N\}$, S_i is the set of strategies for the player U_i.
- $Q = \{q_1\ \dots\ q_N\}$ is the set of payoff functions. More precisely, for each $i \in \{1\ \dots\ N\}$, $q_i : \bigtimes_{j=1}^{N} S_j$ is U_i's payoff function.

It is important to notice that, throughout this paper, S_i indicates the set of strategies of U_i, while the notation s_j indicates an element of S_i, i.e., one single strategy for the user U_i.

The objective of each user U_i is to maximize the function q_i: this will depend on the choices of the strategies from U_i and all other users.

Table 1
Table of symbols used throughout this paper.

Notation	Description
U_i	Player of a given game.
S_i	Set of strategies of U_i.
q_i	Payoff function for player U_i.
N	Number of players in a given game.
T	Time horizon.
$\mathbf{c_i}$	Consumption vector of U_i.
$\mathbf{p_i}$	Production vector of U_i.
$\mathbf{n_i}$	Net consumption vector of U_i.
t_c	Total consumption among all users at a given time.
t_p	Total production among all users at a given time.
t_c^{-i}	Total consumption among all users except u_i at a given time.
t_p^{-i}	Total production among all users except u_i at a given time.
$\mathbf{f_i}$	Fixed load vector of U_i.
$\mathbf{h_i^j}$	jth flexible load vector of U_i.
r_k	Shift operation on vector by k places forward.
B	Congestion threshold.
ec_t	Effective global consumption at time t.
TC	Total effective global consumption through the day.

We define a game on a grid with N users as follows. The players are the grid users, which we will designate as U_i, for $i \in 1\ \dots\ N$. Following that, we will define the payoff functions for each player, and finally their respective set of strategies.

We take into consideration two vectors for each user U_i: $\mathbf{c_i}$, which represents the energy *consumption* of U_i, and $\mathbf{p_i}$, which represents the energy *production* of U_i. These vectors consist of non-negative real numbers, representing the amount of energy consumed and produced during the considered time unit. In particular, a user who does not produce energy will have the $\mathbf{p_i}$ vector equal to the zero vector. The number of time intervals that make up the temporal horizon under consideration — which we shall refer to as T — determines the length of these vectors. For instance, in our experiments, we will use a time horizon of 24 h, and the time intervals we consider are each 15 min long. As a result, there are 96 total time intervals, and both $\mathbf{c_i}$ and $\mathbf{p_i}$ will be vectors of length 96.

We also need to define the vector

$$\mathbf{n_i} = \mathbf{c_i} - \mathbf{p_i} \tag{3.1}$$

This vector represents the difference between consumption and production of the user U_i, and is positive in case the consumption is higher than the production, negative otherwise. We denote by $\mathbf{n_i^+}$ the vector obtained by replacing all the negative components of $\mathbf{n_i}$ with zero, and by $-\mathbf{n_i^-}$ the vector obtained by replacing all the positive components of $\mathbf{n_i}$ with zero (we used the minus sign so that all the components of $\mathbf{n_i^-}$ are non-negative). The vectors $\mathbf{n_i^+}$ and $\mathbf{n_i^-}$ represent respectively the net consumption and net production of U_i.

We now define each payoff function q_i as the combined utility over each time unit for the user U_i. In more precise terms,

$$q_i = \sum_{t=1}^{T} q_i\ t) \tag{3.2}$$

where the term $q_i\ t)$ represents the utility of U_i at time $t \in T$, and can be written as

$$q_i\ t) = g\ \mathbf{n_i^-}\ t)\ t_p\ t_c) - h\ \mathbf{n_i^+}\ t)\ t_p\ t_c). \tag{3.3}$$

In this case, g is a function representing the reward for the user for producing energy, while h represents the cost for the user for consuming energy. Those functions are defined in such a way that, for every t_p $t_c \in$,

$$g\ 0\ t_p\ t_c) = h\ 0\ t_p\ t_c) = 0 \tag{3.4}$$

which implies that users will not receive rewards if they do not produce energy, and they will not incur charges if they do not consume energy. The functions g and h define the mechanism, since the game changes depending on their definition: we will show in the following sections how they can be defined. For every time t, t_p and t_c are defined as:

$$\begin{aligned} t_p\ t) &= \sum_{k=1}^{N} \mathbf{n_k^-}\ t) \\ t_c\ t) &= \sum_{k=1}^{N} \mathbf{n_k^+}\ t). \end{aligned} \tag{3.5}$$

t_p and t_c represent, respectively, the total amount of energy produced in the grid at the considered time, and the total amount of energy consumed in the grid at the considered time. At each time t, by definition, the values of t_p and t_c depend respectively on the values of $\mathbf{n_i^-}\ t)$ and $\mathbf{n_i^+}\ t)$, since they are included in their respective sum. This is important to consider in Eq. (3.3).

Another work in literature [5] inspired us to create cost and reward functions based on the overall quantity of energy generated and consumed. The proposed mechanism in that work serves as the foundational element upon which our work has been constructed.

The players of the game and the payoff functions have been previously described: the only missing element is the set of strategies. To achieve this, it is necessary to specify the types of energy loads available and their degrees of flexibility. A similar approach has been taken in [37], where this modeling has been employed for solving demand-side management problems. Three main types of energy loads will be considered in this paper.

Production: It is denoted as $\mathbf{p_i}$, and describes how much energy has been produced by U_i. In this paper, it will always be treated as a fixed vector; however, in general, this vector may possess some degree of flexibility.

Fixed consumption: It is denoted as $\mathbf{f_i}$. It is a fixed vector, and it describes the part of energy consumption for the user U_i that does not have any flexibility. In this paper, we assume that this value is known in advance; however, generally speaking, it is not always possible to know it beforehand, as it may be subject to uncertainty. This problem is addressed in [38].

Shiftable load: If U_i has several shiftable loads, its j−th shiftable load will be denoted as $\mathbf{h_i^j}$. A *shiftable* load is an energy load whose energy profile cannot be changed in amount, but whose starting point in time can be chosen. Suppose we have the load $\mathbf{h_i^j}$, and we want to start it k time units later: we will denote this new energy profile as $r_k\ \mathbf{h_i^j})$. We can describe r_k as a vector operation that rotates forward (from left to right) the components of the vector by k places. Of course, k can also have negative values, corresponding to a backward rotation (from right to left). In our case, we assume the loads can only be operated within a 24-hour time interval: for this reason, the rotation of the load cannot move a nonzero element from last to first place, or from first to last place if backward.

It is important to notice that the vector $\mathbf{c_i}$, representing the consumption from U_i, is obtained by the sum of the fixed consumption f_i and the shiftable loads $\mathbf{h_i^j}$. In formal terms, this is expressed as follows. If, for each j, the jth shiftable load $\mathbf{h_i^j}$ is shifted forward by k_j time units, $\mathbf{c_i}$ can be written as

$$\mathbf{c_i} = \ \mathbf{f_i} + \sum_{j=1}^{n} r_{k_j}\ \mathbf{h_i^j})). \tag{3.6}$$

Note that $\mathbf{c_i}$ depends on the choice of each k_j, and shifting the loads changes the user's energy profile accordingly.

For each user U_i, the set of strategies can be therefore defined as all the possible values of $\mathbf{c_i}$ that can be obtained by shifting the loads.

3.2. Cooperative game

So far we described a game in which every player aims to maximize her[1] payoff function and, consequently, her benefit. In this section, we want to explore what would happen if players were allowed to form coalitions. We define a *coalition* as the union of one or more players, who choose their strategy collectively to maximize their combined payoff function. Formally, if the users $U_1\ \dots\ U_i$ form a coalition, they will choose the strategies $s_1\ \dots\ s_i$ respectively so that the function

$$q_1\ s_1) + \cdots + q_i\ s_i)$$

is maximized among all the possible choices of $s_1 \in S_1\ \dots\ s_i \in S_i$. Note that in general, it is not possible to maximize each q_k one by one, as every function q_k depends on the strategies of all the users. For example, the choice that maximizes q_1 might greatly reduce every other q_k, or vice versa.

We want to formally introduce the existence of coalitions in a game $G = \ U\ S\ Q)$. We do so by creating a new game, which we will call $G_{\overline{U}} = \ \overline{U}\ \overline{S}\ \overline{Q})$: it is a game that behaves similarly to the one defined in Section 3.1, except the players are not the single users, but the coalitions. We now explain more in detail how this game works, using the notation $G = \ U\ S\ Q)$ for the game defined in Section 3.1.

Let us call $\overline{U}_1\ \dots\ \overline{U}_M$ the coalitions: the set of players of $G_{\overline{U}}$ is then defined as $\overline{U} = \{\overline{U}_1\ \dots\ \overline{U}_M\}$. Every coalition has at least one user, and every user belongs to one and only one coalition: in other words, $\overline{U}$ is a partition of U.

We define now the set of strategies $\overline{S} = \{\overline{S}_1\ \dots\ \overline{S}_M\}$. Consider the coalition $\overline{U}_k$: we define $\overline{S}_k$ as the set of all possible allocations of every consumption load belonging to the coalition (i.e., every $\mathbf{c_i}$ such that $U_i \in \overline{U}_k$). This set is isomorphic to $\bigtimes_{U_j \in \overline{U}_k} S_j$. This is because the coalition behaves like a bigger user that can allocate every load to maximize its interest, so it can choose every possible combination of allocations for the users inside. Finally, we have to define $\overline{Q} = \{\overline{q_1}\ \dots\ \overline{q_M}\}$. Considering the coalition $\overline{U_k}$, the payoff function

$$\overline{q_k} : \bigtimes_{j=1}^{M} \overline{S_j} \tag{3.7}$$

for the coalition is defined as the combined payoff functions of each user belonging to the coalition $\overline{U_k}$. In other words,

$$\overline{q_k} = \sum_{U_j \in \overline{U_k}} q_j. \tag{3.8}$$

This is intuitive, as the coalition represents the interests of all the users belonging to it, and therefore its utility corresponds to the combined utility of the users within the coalition.

This is the generic formulation of a cooperative game. In particular, in this paper, we will focus on games with large coalitions, i.e., coalitions with many members. It is not easy to compute the optimal strategy for such coalitions, since by definition of $\overline{S_k}$ the number of possible strategies grows exponentially with respect to the number of users inside the coalition. In Section 5 we will show how we addressed this problem.

[1] from now on, this will be read as his/her.

3.3. Incentive mechanisms

This section will describe existing incentive mechanisms for the game outlined in Section 3.1. As mentioned earlier, the game relies entirely on the definition of the selling function g and the buying function h: this is because the payoff functions are entirely defined by g and h. Therefore, we will see how those functions have been defined in existing mechanisms, and how they affect users' choices of strategies.

We start from the NRG-X-Change mechanism [5]. Here, g is defined as

$$g\ x\ t_p\ t_c) = x\ \ \frac{q}{e^{\frac{t_p-t_c)^2}{a}}} \tag{3.9}$$

where q and a are two positive real numbers. The number q corresponds to the maximum possible reward per unit of energy, while a determines how the reward changes depending on how different t_p and t_c are. The rationale behind this choice for g is to encourage self-consumption at grid level: the reward is higher when t_p and t_c are close, i.e., when grid consumption matches grid production, and lower when the difference between t_p and t_c is high.

The function h is defined as

$$h\ y\ t_p\ t_c) = y\ \ \frac{r\ \ t_c}{t_c + t_p}. \tag{3.10}$$

This function depends on a parameter r, corresponding to the maximum possible unitary cost for energy. The function h has been designed so that the cost becomes higher when there is overconsumption in the local grid, that is, when t_c is higher than t_p: the purpose is to discourage consumption during such occurrences and encourage consumption when production is higher.

There are some issues in this mechanism, that have been outlined in [9]. Specifically, the mechanism may encourage production curtailment, may discourage self-consumption, and does not take congestion into account. In [9], a new selling function and a new buying function have been proposed so that the resulting game would encourage some specific behavior from the users that solve the aforementioned problems.

Given a user U_i, we will denote by ${t_p}^{-i}$ and ${t_c}^{-i}$ the values of t_p and t_c when x and y, i.e., respectively the amount of energy produced and consumed by the user, are both zero. In general,

$$\begin{aligned} t_p^{-i}\ t) &= \sum_{j\ \ i} \mathbf{p_j}\ t) \\ t_c^{-i}\ t) &= \sum_{j\ \ i} \mathbf{c_j}\ t). \end{aligned} \tag{3.11}$$

In other words, t_p^{-i} and t_c^{-i} are respectively the total values of energy production and consumption around the grid, counting all the users except U_i.

The function g proposed in [9] is therefore defined as

$$\begin{aligned} g\ x\ t_p\ t_c) =& P_{\max}\ \ \big(\ g_a\ t\ x\ t_p^{-i}\ t_c^{-i}))- \\ & g_a\ t\ 0\ t_p^{-i}\ t_c^{-i}))\ \big) - P\ x\ t_p^{-i}\ t_c^{-i}) \end{aligned} \tag{3.12}$$

Here, P is a penalty function that assumes values above zero only if t_p and t_c would cause congestion for overproduction in the local grid. $P_{\max}$ determines the theoretical maximum unitary cost for energy. The function g_a is defined as

$$g_a\ u) = \begin{cases} 0 & if\ u \leq 0 \\ \frac{1}{1+e^{\frac{2u-1}{u^2-u}}} & if\ u \in\ \ 0\ 1) \\ 1 & if\ u \geq 1 \end{cases} \tag{3.13}$$

and defines the behavior of g. The function t is defined as

$$t\ x\ t_p^{-i}\ t_c^{-i}) = \frac{t_p^{-i} + x - t_c^{-i}}{2B} + \frac{1}{2} \tag{3.14}$$

where B is a number indicating the congestion threshold. The idea behind this is that if $|t_p^{-i} - t_c^{-i}| < B$, a congestion will not happen.

Regarding the buying function h, it is defined as

$$\begin{aligned} &h\ y\ t_c^{-i}\ t_p^{-i}) = \\ &Q_{\max}\ \ h_a\ \ \frac{t_c^{-i} + y - t_p^{-i}}{B} + 1)\ \ y + P\ y\ t_c^{-i}\ t_p^{-i}). \end{aligned} \tag{3.15}$$

The parameter $Q_{\max}$ corresponds to half of the maximum possible tariff, and P is a function that assumes values higher than zero if and only if t_p and t_c would cause congestion for overconsumption in the grid. The function h_a is defined as

$$h_a\ u) = \begin{cases} 0 & if\ u \leq 0 \\ \sqrt{u} & if\ u \in\ \ 0\ 1] \\ 2 - \sqrt{2-u} & if\ u \in [1\ 2) \\ 2 & if u \geq 2. \end{cases} \tag{3.16}$$

The purpose of this mechanism was to fulfill three main properties: (i) discourage energy production curtailment from the prosumers, (ii) actively discourage consumption/production if it would cause congestion to the grid, and (iii) encourage energy self-consumption for the prosumers. The rationale behind proposing these specific functions is their monotonicity concerning x or y, ensuring property (i); their inclusion of a penalty term and specific convexity properties, ensuring property (ii); and their consistent guarantee of $g < h$ with appropriate parameters, thereby ensuring property (iii).

The two mechanisms described above have been defined outside of a game-theory context and then analyzed in that context in the following work [10]. However, it has been proved that neither of them guarantees the existence of a Nash Equilibrium (NE) for the game. Therefore, the next step has been to create new selling and buying functions that would fulfill the same properties as the ones above but, in addition to that, would also guarantee the existence of a Nash equilibrium.

Several functions have been created for this purpose [34]. We report them below. The proposed selling functions are

$$g_1\ x\ t_p\ t_c) = k_1\ \ \left(ln\frac{x + Z + a_1}{Z + a_1}\right). \tag{3.17}$$

and

$$g_2\ x\ t_p\ t_c) = k_1\ \ \ \ x + Z + a_1)^{\frac{1}{n}} -\ \ Z + a_1)^{\frac{1}{n}}) \tag{3.18}$$

while the proposed reward functions are

$$h_1\ y\ t_p\ t_c) = k_2\ \ \ \ y - Z + a_2)^n -\ \ a_2 - Z)^n) \tag{3.19}$$

and

$$h_2\ y\ t_p\ t_c) = k_2\ \ \ \ Z + a_2)^{\frac{1}{n}} -\ \ Z + a_2 - y)^{\frac{1}{n}}) \tag{3.20}$$

.

It has been proved that they guarantee the existence of an NE in the game described at the beginning of the section because of their concavity/convexity. However, they have been analyzed for cooperative games with small coalitions (4 members or less per coalition) [11], and they did not encourage cooperation between users. In this paper, we will examine the cooperative game with the possibility of forming arbitrarily big coalitions.

3.4. Game mechanics

The game described in this section is performed as follows. For simplicity, we will use the notation of Section 3.1.

We start from a configuration $\ s_1\ \dots\ s_N)$, where each $s_k \in S_k$ is the strategy chosen by U_k. U_1 is the first player to choose the strategy and will update s_1 to the strategy that maximizes q_1 in the current configuration. In other words,

$$s_1 = \arg\max_{x \in S_1} q_1\ x\ s_2\ \dots\ s_N) \tag{3.21}$$

which, generalizing, becomes:

$$s_k = \arg\max_{x \in S_k} q_k(s_1, \dots, s_k, \dots, s_N). \tag{3.22}$$

After this, we do the same for U_2 by updating s_2, and so on until we do the same for U_n. We call the process just described an *iteration* of the game. Now, we check if the current configuration $(s_1, \dots, s_N)$ is the same as it was at the beginning of the iteration. If it is, the game is over and an NE has been found since the strategy was already optimal for all the players. Otherwise, we start a new iteration and continue doing so until an NE is found.

If the model guarantees the existence of an NE, the game will conclude in a finite number of iterations. Otherwise, this scenario is not guaranteed, and it may happen that a sequence (greater than 1) of configurations will repeat over and over from a certain point onward. The algorithm's computational complexity is $O(T \cdot N)$ for each iteration. As demonstrated in [11], the average number of iterations required for game convergence is typically fewer than 5, with no instances exceeding 15. Nonetheless, for practical application, a termination threshold N_{it} can be defined to halt the game after a set number of iterations. It is noteworthy that employing nonlinear functions for energy selling or buying does not affect the model's implementation. These functions are solely utilized to compute potential values of the payoff function for each choice, while user optimization is executed by the $\arg\max$ function.

4. Methods and techniques

This section highlights the key contributions of the paper. Specifically, Section 4.1 outlines the algorithm employed for load optimization, while Section 4.2 describes the newly proposed functions designed to maximize peak shaving.

4.1. Game optimization algorithm

In this section, we describe the algorithm that we use for optimizing load allocations within a coalition. It is a diagonalization algorithm, similar to the one from [39].

The algorithm works as follows. Suppose that we have some users $U_1, \dots, U_k$ forming a coalition. We call q the combined utility of the users, that is,

$$q = q_1 + \dots + q_k.$$

We will allocate the loads as follows. First, we re-arrange the indexes of the users, so that U_1 has the biggest load, U_2 the second biggest, and so on. Now, we consider an initial configuration $(s_1, \dots, s_k)$. From here, we choose a new strategy for U_1 that maximizes the combined profit of all the users of the coalition: in other words,

$$s_1 = \arg\max_{x \in S_1} q(x, s_2, \dots, s_k).$$

We then do the same for U_2 by updating s_2 and so on, in such a way that

$$s_j = \arg\max_{x \in S_j} q(s_1, \dots, s_j, \dots, s_k). \tag{4.1}$$

Once we have done this for all the users, we have performed an *iteration* of the optimization. If the current configuration remains unchanged from the iteration's start, the optimization concludes. Otherwise, we start a new iteration and continue until the configuration remains stable. It is important to notice that, under certain conditions, we can be sure that the optimization ends: in particular, this happens if each payoff function q_j is convex. This is because in that case the function q is convex too, and the optimization is equivalent to a game that guarantees the existence of an NE, which in our case means that the optimization reaches an end.

4.2. Proposed functions

The functions proposed so far, i.e., the ones described in Eq. (3.17), Eq. (3.18), Eq. (3.19) and Eq. (3.20), comply with a series of requirements, which encourage the behavior of the users to work towards specific goals (e.g., self-consumption, congestion avoidance) or ensure the existence of an NE for the game. However, these functions were not explicitly designed for maximizing peak shaving. For this reason, we propose two novel buying functions h_{new1} and h_{new2} for the users. These functions have been designed with the specific purpose of maximizing peak shaving within the described game when the users cooperate all together (i.e., there is only one coalition with all the users inside it, called *grand coalition*). The two functions are defined as follows:

$$h_{new1}(x, t_p^{-i}, t_c^{-i}) = k_1 \cdot \frac{x}{t_c^{-i} + x} \cdot \left(\frac{t_c^{-i} - t_p^{-i} + x + k_2}{k_2}\right)^2 \tag{4.2}$$

$$h_{new2}(x, t_p, t_c) = k_1 \cdot \frac{x}{t_c^{-i} + x} \cdot \left(1 - \sqrt{\frac{k_2 - t_c^{-i} + t_p^{-i} - x}{k_2}}\right) \tag{4.3}$$

We aim to explain how these functions incentive user behavior to maximize peak shaving. We will denote from now on the *effective global consumption*, i.e., the difference between total consumption and total production, as

$$ec_t = t_c - t_p \tag{4.4}$$

where t is the time unit we are operating in. If we are referring to selling functions, i.e., x refers to consumption, we can also use the equivalent formulation:

$$ec_t = t_c^{-i} - t_p^{-i} + x \tag{4.5}$$

which is valid for every user U_i, where of course x refers to U_i's consumption. It is also helpful to remember that, in this case,

$$t_c = t_c^{-i} + x. \tag{4.6}$$

Proposition 1. *Consider the game described in Section 3.2 with a selling function of either g_1 or g_2 and buying function h_{new1}. If the only coalition is the grand coalition, then the functions encourage the users to behave to decrease the highest values for ec_t and increase the lowest values for ec_t.*

Proof. We will start from a generic time unit t. We know that Eq. (4.2) describes the cost for prosumer i depending on the consumed amount x. By replacing Eqs. (4.5) and (4.6) inside, it becomes

$$k_1 \cdot \frac{x}{t_c} \cdot \left(\frac{ec_t + k_2}{k_2}\right)^2. \tag{4.7}$$

Now, the function that each user is trying to optimize at each step of the coalition optimization is the sum of all the h_{new1} for each user inside the coalition. Since this is the grand coalition, the objective function at time t becomes

$$\begin{aligned} &\sum_{i=1}^{N} k_1 \cdot \frac{x_i}{t_c} \cdot \left(\frac{ec_t + k_2}{k_2}\right)^2 = \\ &k_1 \cdot \frac{\sum_{i=1}^{N} x_i}{t_c} \cdot \left(\frac{ec_t}{k_2} + 1\right)^2. \end{aligned} \tag{4.8}$$

Since by definition $\sum_{i=1}^{N} x_i = t_c$, we have that the terms of the first fraction cancel out, and the function becomes

$$k_1 \cdot \left(\frac{ec_t}{k_2} + 1\right)^2. \tag{4.9}$$

The objective function is the sum of Eq. (4.9) over time, i.e.,

$$\sum_{t=1}^{T} k_1 \cdot \left(\frac{ec_t}{k_2} + 1\right)^2. \tag{4.10}$$

The objective is to minimize this sum, within the constraint that

$$\sum_{t=1}^{T} ec_t = TC \tag{4.11}$$

for a certain number TC that represents the total effective consumption across the grid through the whole day. Now, the minimum for Eq. (4.10) is reached when all the terms of the sum have the same value, i.e., when all the ec_t have the same value. This can be easily proven, for example by comparing the quadratic and arithmetic mean of the terms in Eq. (4.9). For this reason, the shiftable loads will be moved from the time units with the highest values for ec_t to the time units with the lowest values for ec_t, which is what we wanted to prove. □

Proposition 2. *Consider the game described in Section 3.2 with a selling function of either g_1 or g_2 and buying function h_{new2}. If the only coalition is the grand coalition then the functions encourage the users to behave to decrease the highest values for ec_t and increase the lowest values for ec_t.*

Proof. Again, we start from a generic time unit t. Like before, by replacing Eqs. (4.5) and (4.6) inside Eq. (4.3), we obtain

$$k_1 \quad \frac{x}{t_c} \quad \left(1 - \sqrt{\frac{k_2 - ec_t}{k_2}}\right) \tag{4.12}$$

Similar to the previous proof, each user aims to optimize the function given by the sum of all the h_{new2} for each user inside the coalition, which means that the objective function has the following component at time t:

$$\sum_{i=1}^{N} k_1 \quad \frac{x}{t_c} \quad \left(1 - \sqrt{\frac{k_2 - ec_t}{k_2}}\right) = \\ k_1 \quad \frac{x}{t_c} \quad \left(1 - \sqrt{1 - \frac{ec_t}{k_2}}\right). \tag{4.13}$$

We use the fact that $\ {}_{i=1}^{N} x_i = t_c$ to cancel out the first fraction, so that we have

$$k_1 \quad \left(1 - \sqrt{1 - \frac{ec_t}{k_2}}\right). \tag{4.14}$$

Once again, the objective function is the sum of Eq. (4.14) over time, i.e.,

$$\sum_{t=1}^{T} k_1 \quad \left(1 - \sqrt{1 - \frac{ec_t}{k_2}}\right). \tag{4.15}$$

The objective is to minimize this sum, under the constraint

$$\sum_{t=1}^{T} ec_t = TC. \tag{4.16}$$

In this case, the minimum for Eq. (4.15) is reached when all the terms of the sum have the same value, i.e., when all the ec_t have the same value. This can be proven by exploiting the fact that the function is convex, and therefore we can compare our function with

$$T \quad k_1 \quad \left(1 - \sqrt{1 - \frac{TC}{T \quad k_2}}\right). \tag{4.17}$$

According to Jensen inequality, our function will always have a higher value than the one in Eq. (4.17), with equality occurring if the variables have all the same value. Consequently, shiftable loads will be moved from the time units with the highest values for ec_t to the time units with the lowest values for ec_t which is, again, what we wanted to prove. □

5. Simulations and results

This section outlines the simulations conducted to validate our approaches. Section 5.1 describes the dataset we have used for our simulations, and Section 5.2 assesses the efficiency of the cooperative game optimization algorithm within a single coalition, while Section 5.3 shows the effect of many coalitions in the whole grid in terms of self-consumption and cost savings. Finally, Section 5.4 showcases the performances of the mechanism in terms of peak shaving.

5.1. Dataset

Data for the experiments has been taken from a grid in Cardiff, UK. The number of grid users is 184, and 40 of them can also produce energy. We extracted the following information from the dataset:

Non-flexible consumption data for all the 184 grid users.
Type and consumption pattern for every flexible device owned by a grid user. Specifically, the available devices are:

- Dishwashers
- Electric overs
- Electric vehicles
- Tumble dryers
- Washing machines

Energy production data for all the 40 prosumers.

Consumption and production information depends on five settings: smart technology usage (it can be *low*, *medium* or *high*), day of the week (it can be *weekday*, *Saturday* or *Sunday*), season (it can be *winter*, *spring*, *autumn*, *summer* or *high summer*), and two settings related to specific policies adopted in the UK, one of which can be either *2020* or *2030*, the other *green* or *business as usual*: those last settings are described better in [40]. For simplicity, we have chosen the settings that enable the most users to use flexible loads: *high*, *weekdays*, *summer*, *2030*, *green*. Further information on the dataset can be found in the deliverables of the MAS^{2}TERING project [41–43].

5.2. Coalition optimization results

In this paragraph, we will describe the results of the experiment we performed for validating our cooperative game optimization, described in Section 4.1. More precisely, we show how the algorithm from Section 4.1 performs for one single coalition, in terms of accuracy and scalability with respect to a baseline [11]. The baseline calculates the best possible allocation for every grid user by trying all the available combinations and choosing the best one. In more formal terms, suppose that the coalition contains k users $U_1 \ \dots \ U_k$, and that the loads $s_1 \ \dots \ s_k$ belong to the users $U_1 \ \dots \ U_k$ respectively. This algorithm evaluates

$$q_1 \;\; s_1) + \cdots + q_k \;\; s_k) \tag{5.1}$$

for every possible choice of

$$s_1 \;\; \dots \;\; s_k) \in S_1 \times \cdots \times S_k \tag{5.2}$$

and then chooses the combination of allocations that minimizes the value in Eq. (5.1). The algorithm from [11] is the same proposed for load optimization in [38]; the one from [38] is a specific case that targets multiple loads from the same user but does the same computation, and scales in the same way. Therefore, for simplicity, we compared our algorithm with the one from [38] for single users with multiple loads, since for this purpose a grid user with k loads behaves in the same way as a coalition of k users with one load each, as described in Section 3.2. The experiment was conducted by performing optimization across the entire grid: there were 81 cases of users with two loads, 79 cases of users with three loads, and 19 cases of users with four loads. Our objective is to demonstrate that the newly proposed algorithm scales more efficiently than the baseline while maintaining similar accuracy, as presented in Tables 2 and 3. Table 2 showcases the results for optimization time: the **All cases** column represents the average time (in seconds) required to optimize loads for users with the algorithm from [38], while the column **New** illustrates the average time needed with the algorithm from Section 4.1. As we can see, the newly proposed method outperforms the previous one, and, in particular, scales significantly better with respect to the number of loads. To be more precise, the **New** algorithm does not take more than 0.1305 s

Table 2
Average time (in seconds) needed for optimization.

Users	All cases	New
2	0.1300	0.1141
3	0.4013	0.1152
4	7.5985	0.1305

Table 3
Average costs (in GBP) with different optimization types.

Users	All Cases	New	Increase	
2	9.091	9.091	$-3.5\ \ 10^{-15}$	$-3.9\ \ 10^{-14}$
3	9.474	9.474	$5.6\ \ 10^{-5}$	$6\ \ 10^{-4}$
4	27.609	27.610	$1\ \ 10^{-3}$	$4.3\ \ 10^{-3}$

Table 4
Effect of coalitions in terms of cost reduction (in GBP) and self-consumption (in kWh), with functions g_1 and h_1.

Size	Costs	Consumption
1	353.545	2380.436
2	353.541	2380.439
5	353.536	2380.472
10	353.524	2380.481
20	353.514	2380.527
30	353.514	2380.516
40	353.505	2380.554

Table 5
Effect of coalitions in terms of cost reduction (in GBP) and self-consumption (in kWh), with functions g_1 and h_2.

Size	Costs	Consumption
1	349.986	2605.241
2	349.984	2605.247
5	349.981	2605.261
10	349.974	2605.269
20	349.965	2605.277
30	349.963	2605.281
40	349.959	2605.283

for all the cases, and the optimization time grows linearly with the number of loads: this was expected, as in Section 3.4 we showed that the computational complexity is linear with respect to N. Conversely, the **All cases** algorithm requires more than 0.4 s for three loads and more than 7.59 s for four loads, and the optimization time grows exponentially with the number of loads. Regarding accuracy, the results can be seen in Table 3. This table displays the total average costs for optimization of 2, 3, and 4 loads, respectively. The columns **All cases** and **New** describe the results after applying the respective algorithm. The column **Increase** describes how higher is the **New** cost on average, while the last column describes the percent increase. We can see that the increase for 3 and 4 loads is minimal, with the **New** optimization proving highly accurate, as the increase in cost is less than 0.01 of the total. For 2 users the increase results are negative due to numerical approximation but can be considered negligible.

5.3. Coalitions profits

This paragraph outlines the results for the entire game, illustrating the impact of coalition formation against the selfish game. The algorithm from Section 4.1 is used to perform load optimization for each coalition. These simulations were conducted on grids of size 40 users, 20 of them being prosumers. For each possible choice of the old selling (g_1 and g_2) and buying (h_1 and h_2) functions described at the end of Section 3.3, 25 different grids have been randomly selected and simulated, with the reported results being averaged across them.

Table 6
Effect of coalitions in terms of cost reduction (in GBP) and self-consumption (in kWh), with functions g_2 and h_1.

Size	Costs	Consumption
1	381.481	2532.020
2	381.479	2532.027
5	381.475	2532.048
10	381.462	2532.069
20	381.448	2532.097
30	381.446	2532.091
40	381.440	2532.117

Table 7
Effect of coalitions in terms of cost reduction (in GBP) and self-consumption (in kWh), with functions g_2 and h_2.

Size	Costs	Consumption
1	304.713	2344.661
2	304.711	2344.661
5	304.705	2344.670
10	304.699	2344.680
20	304.691	2344.682
30	304.690	2344.684
40	304.685	2344.697

Table 8
Results for peak shaving for every couple of functions tested.

Size	$g_1\ h_1$	$g_1\ h_2$	$g_2\ h_1$	$g_2\ h_2$	$g_2\ h_3$	$g_2\ h_4$
1	75.06	75.06	75.06	75.06	106.16	104.58
2	75.06	75.06	75.06	75.06	106.16	103.92
5	75.06	75.06	75.06	75.06	103.20	102.61
10	75.06	75.06	75.06	75.06	102.79	97.02
20	75.06	75.06	75.06	75.06	92.74	91.36
30	75.06	75.06	75.06	75.06	78.63	84.94
40	75.06	75.06	75.06	75.06	75.06	75.06

Tables 4–7 present the results depending on the coalition size, which is described in the **Size** column. For each of those values, the column **Costs** describes the global energy costs, while the column **Consump tion** shows the energy consumption through the grid. Lower values in this column correspond to higher self-consumption. Regarding the **Size** column, the value 1 means that coalitions have size 1, i.e., the game is non-cooperative.

As can be seen from the tables, regardless of the chosen combination of buying and selling functions, forming larger coalitions aids in cost reduction, albeit minimally. More specifically, the game with the global coalition is just 0.01 cheaper compared to the selfish game. Additionally, we observe a marginal increase in consumption through the grid with larger coalitions, although the increase is minimal and the global coalition game consumes 0.001 more energy compared to the selfish game.

5.4. Peak shaving

In this section, we present the results for peak shaving. Our simulations were conducted for the game with each pair of selling and buying functions described in Section 5.3, and also with the function couples $g_2\ h_3$) and $g_2\ h_4$) respectively, and the average consumption peak across the entire grid was measured for each coalition size. In the table, **Size** represents the coalition size, while $g_i\ h_j$ denotes the pair of, respectively, selling and buying functions that are being considered.

Table 8 displays the results for the specified couples of functions. The table shows, for each coalition size, the result for peak shaving for each tested combination of the functions. More precisely, the table describes the highest value of energy consumption across the grid through the whole day, measured in kWh — in other words, the peak consumption. We have chosen 40 grid users, more precisely 20 prosumers and 20 consumers, and we have performed the game on those users for every couple of functions: we have done this for 5 different combinations of users, took away the one that yields the highest average peak values and the one that yields the lowest average peak values respectively, and averaged the 3 remaining ones. As we can see, with the couples $g_2 \ h_3$) and $g_2 \ h_4$), the peak gradually decreases as the coalition size increases, reaching the highest value for the selfish game at 106.16 kWh and 104.58 kWh respectively reaching the lowest value, i.e., 75.06 kWh, for the grand coalition. Nevertheless, the remaining sets of functions already yield identical results for peak shaving in the non-cooperative game (i.e., the game where coalitions have a size of 1). This outcome remains consistent as the coalition size increases, indicating that the current incentive mechanism effectively achieves optimal peak shaving for both the cooperative and non-cooperative games. Specifically, for the non-cooperative game, the current incentive mechanism obtains a peak reduction above 29 compared to the couple $g_2 \ h_3$), and above 28 compared to the couple $g_2 \ h_4$).

6. Discussion

This section presents the results obtained from the experiments and outlines the key findings from our paper. The experiments yielded valuable insights into arbitrary size optimization, cost reduction, and peak shaving.

Regarding coalition optimization, we have introduced an algorithm that enables the generation of schedules for coalitions of arbitrary size. The idea was to slightly sacrifice accuracy for the schedule, in exchange for much better scalability in comparison to the algorithm used in [11] for small coalitions. The results demonstrate that, for larger coalitions, our algorithm is notably faster (Table 2) than the baseline from [11], consistently staying below 0.14s for coalitions with four members, while the baseline algorithm takes almost 7.6 s in this scenario and, more crucially, exhibits exponential growth with the number of users, by a factor bigger than 15 for each additional user. This was expected because each new user added to the coalition increases multiplicatively the number of cases that need to be tried. Therefore, the computational complexity is exponential, leading to a corresponding increase in computational time. Conversely, the time needed for performing our proposed algorithm scales linearly with respect to the number of users, as discussed in Section 3.4. As for accuracy, it becomes lower with more users; however, the sub-optimality of the solution found by our proposed algorithm is below 0.01 for coalitions of four users (Table 3). These results affirm that our proposed algorithm is much more scalable than the baseline, while the loss in accuracy compared to the baseline algorithm is minimal.

We also sought to analyze the impact on profits and energy self-consumption as coalitions increase in size. In [11] we found out that small coalitions give minimal benefits in terms of both profits and self-consumption. Tables 4–7 illustrate that the same is true even for bigger coalitions. Even for coalitions of size 40, the cost reduction achieved relative to the selfish game is marginal, amounting to only 0.01 of the total cost. The same is true for energy consumption, which gets worse (although negligibly) for bigger coalitions. The mechanism with the old selling (g_1 and g_2) and buying (h_1 and h_2) functions was originally designed for performing well in the case of the selfish game, and these results show that, indeed, the results for the selfish game are almost as good as the grand coalition case.

Lastly, we sought to verify whether our mechanism also optimally performs peak shaving. To do so, we created two baseline functions h_3 and h_4 that ensure optimal peak shaving for the grand coalition game. Results in Table 8 indicate that our mechanism indeed achieves optimal peak shaving regardless of the coalition size in the game, as $g_2 \ h_3$) and $g_2 \ h_4$) achieve optimal peak shaving with 40 users, and the functions used for our mechanism obtain the same result for any coalition size. In particular, as the functions were designed for the selfish game, we showed that even in scenarios where coalitions are not allowed, optimal peak shaving is achieved, showcasing the effectiveness of our mechanism. Precisely, the average peak consistently remains at 75.06 kWh for the existing mechanisms. In contrast, $g_2 \ h_3$) and $g_2 \ h_4$) attain this level exclusively within the grand coalition, while yielding peaks of 106.16 kWh and 104.58 kWh, respectively, in the selfish game scenario. This signifies a reduction of over 29 in the former case and 28 in the latter.

7. Conclusions and future work

The incentive mechanism proposed in [9] and improved in [10, 11] promotes good behavior for grid users in terms of load shifting, enabling increased self-consumption, avoiding unnecessary production curtailment and congestion, and achieving all these objectives for the whole community whether the users behave selfishly or form small coalitions. The objective of this paper was to analyze the behavior of this mechanism for coalitions of arbitrary size and explore its efficiency in terms of peak shaving. To achieve this, we proposed an algorithm capable of computing load shifting for coalitions of any size. Additionally, we designed two energy-selling functions with the specific objective of maximizing peak shaving when users form the grand coalition. Our simulations show that the formation of coalitions has minimal influence in terms of cost reduction and energy self-consumption across the grid, showing that the mechanism achieves good results even when grid users are not permitted to form coalitions. Furthermore, we have demonstrated that the current mechanism achieves optimal peak shaving in scenarios where coalitions are both allowed and not allowed, respectively. This finding underscores the effectiveness of the existing mechanism in optimizing peak shaving outcomes across varying conditions. Whether coalitions are permitted or not, the established mechanism consistently delivers optimal results, highlighting their robust performance in diverse settings. More precisely, the current mechanism is capable of attaining optimal peak shaving even within the context of the selfish game, leading to a reduction of approximately 29 compared to alternative approaches. Future work will investigate further improvements on this mechanism, in terms of objectives and game theory formulation. Moreover, we will also consider multi-objective optimization for aspects like user comfort and CO2 emissions.

CRediT authorship contribution statement

Fabio Lilliu: Writing – original draft, Validation, Software, Methodology, Formal analysis, Data curation. **Diego Reforgiato Recupero:** Writing – review & editing, Supervision, Project administration, Investigation.

Declaration of competing interest

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

Data availability

Data will be made available on request.

Acknowledgments

Funded by the European Union's Horizon Europe programme under Grant Agreement n° 101096453 (PARMENIDES).

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