

## Dynamic simulator and model predictive control of a milk pasteurizer

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**Abstract:** In this study, the design, optimization and dynamic modelling of a milk pasteurization unit have been developed, using the pseudo-component approach for describing milk properties. The fluid has been regarded as a mixture of five major categories, namely water, fats, proteins, carbohydrates, and minerals. Exploiting the optimal pasteurizer configuration, selected based on the total annualized cost, a dynamic model of the process has been also derived. The simulation of the system is then used as a virtual plant to develop a nonlinear model predictive control (NMPC) designed for rejecting the more important disturbances that can enter the system. The predicted trajectories have been calculated with a simplified version of the dynamic model, obtained by neglecting parameters temperature dependence. The NMPC performance has been compared with a PI controller in terms of set-point tracking and disturbance rejection. Similar results have been obtained when using the different control algorithms for the output responses, but the NMPC showed better behaviour of the manipulated variables.

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### 1. INTRODUCTION

Pasteurisation is a very common process in the food industry. It consists of a heat treatment applied to some food products (milk and milk-based products, fruit juices, eggs, etc.) by exposing them to a certain minimum temperature for a certain minimum time. The aim of this process is to reduce pathogenic micro-organisms and thereby extend the shelf-life of the food minimizing the hazards for the public health. The temperature/time combination that will sufficiently reduce a given bacteria population varies according to the nature and conditions (viscosity, percentage of fat, solid contents, etc.) of the product to be pasteurised.

Due to the required holding time, the process dynamics of the pasteurization process contains an inherent time delay. Design of controllers for systems with a dominant time delay is notoriously difficult. Furthermore, a pasteurization system includes typical behaviours of many industrial processes, such as complex dynamical models with non-linearities, which imply important challenges when a suitable controller needs to be designed. The availability of a mathematical model capable of describing the process in a detailed way is important in order to carry out analyses on the system and for the implementation of control strategies. Several works dealing with process optimization, safety and control design have been proposed for pasteurization plants. In many cases, the development of mathematical models or digital twins to simulate the process has been reported (e.g. Niamsuwan et al. 2014; Kayalvizhi et al., 2017; Bottani et al 2020).

In this paper a pasteurization system based on the production rate of Arla's Danish plant located in Christiansfeld has been

investigated, with the following main goals: (i) design and optimisation of the unit, (ii) obtainment of a dynamical model that acts as simulator of the plant and (iii) design of the control system.

A three-section High Temperature Short Time (HTST) pasteurization unit is here considered and the pseudo-component approach is used to characterize the milk and evaluate the property models. The optimal design from the economical point of view is then used to develop a dynamic simulator of the system. The novelty, with respect to previous investigations (Rasmussen et al., 2020; Niamsuwan et al. 2014), is that the temperature dependence of milk properties is embedded in the design and mathematical model of the process, assuring a more detailed description of the pasteurization unit. The virtual plant is then exploited for designing two different control systems: (i) PI feedback control and (ii) nonlinear model predictive control (NMPC), which uses a simplified model of the process for the evaluation of the output prediction. The former control algorithm is indeed widely used in industry, while the latter is suitable for systems with time delays and nonlinear behaviour (Khadir, 2020). The performance of the two controllers has been compared for set-point tracking and disturbance rejection.

### 2. Milk production and pasteurization

According to OECD-FAO, the world milk production is distributed as 81% cow milk, 15% buffalo milk, and 4% of combined goat, sheep, and camel milk with an overall expected growth of 1.7% in the next decade (OECD-FAO, 2019). This growth corresponds to a production of 981 Mt by 2028. Considering that milk consumption represents a routine for many people around the world, a safe treatment is

mandatory to preserve the public health. The huge consumption and the number of people involved acquires a high relevance especially in connection to the fact that unpasteurized dairy products are recognized as the most common cause of dairy-associated foodborne illness (Barreto et al., 2019).

Different thermal treatments have been proposed over the years, with a remarkable process evolution that goes from simple holding methods to continuous processes in heat exchangers (Rankin et al., 2017). A pasteurization temperature of 72°C maintained for 15 seconds is recommended by the European legislation (Commission Regulation EU).

## 2.1 Process description

HTST pasteurization processes in gasketed plate heat exchangers (PHE) are one of the widely used methods that ensures a safe milk treatment (Carola et al., 2014).

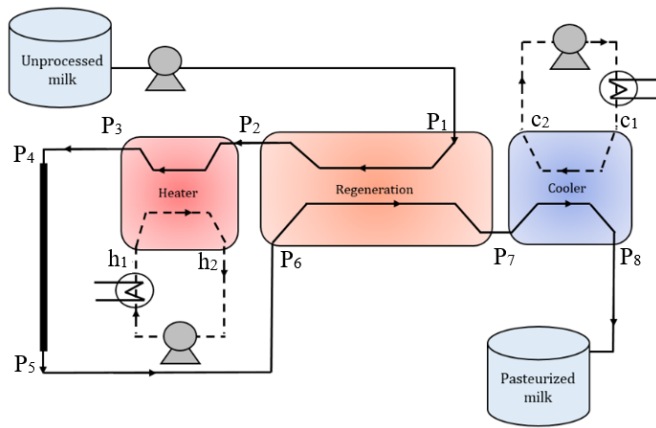


Fig. 1. HTST pasteurization process

According to the configuration reported in Fig. 1, the unprocessed milk is fed to the regeneration section where it is pre-heated by the pasteurized milk. The pre-heated milk is transferred to the heating section to reach the pasteurization temperature. The milk then reaches the holding tube that assures that the pasteurization temperature is kept for 15 seconds. At the exit of the holding tube the pasteurized milk passes through the regeneration section and the cooler.

## 2.2 Milk characterization

The development of a mathematical model for design and control purposes requires the description of milk properties. Because the milk is a complex mixture consisting of more than 2,000 molecules (Madoumier, 2015), it is close to impossible to obtain a component-based characterization. The pseudo-component approach proposed by Madoumier et al. (2015) has been followed in the present work. According to this method, milk has been divided into 5 classes of components, named water, fat, protein, carbohydrate, and mineral. The property of each class is temperature dependent and contributes to the whole based on its composition in the milk. As an example, the heat capacity of the milk is reported in Eq. (1).

$$C_p = \sum C_{p,i} \cdot X_i \quad (1)$$

where  $X_i$  is the mass fraction of the component and  $C_{p,i} \left[ \frac{J}{g \cdot ^\circ C} \right]$  is the category contribution as reported in Eqs. (2-6).

$$C_{p,fat} = 1.9842 + 1.4733 \cdot 10^{-3} \cdot T - 4.8008 \cdot 10^{-6} \cdot T^2 \quad (2)$$

$$C_{p,protein} = 2.0082 + 1.2089 \cdot 10^{-3} \cdot T - 1.3129 \cdot 10^{-6} \cdot T^2 \quad (3)$$

$$C_{p,carbohydrate} = 1.5488 + 1.9625 \cdot 10^{-3} \cdot T - 5.9399 \cdot 10^{-6} \cdot T^2 \quad (4)$$

$$C_{p,mineral} = 1.0926 + 1.8896 \cdot 10^{-3} \cdot T - 3.6817 \cdot 10^{-6} \cdot T^2 \quad (5)$$

$$C_{p,water} = 4.1711 + 5.51 \cdot 10^{-4} \cdot T - 0.83 \cdot 10^{-7} \cdot T^2 \quad (6)$$

## 2.3 Design and optimization

The design of the pasteurization unit is based on a flowrate of 55,000 L h<sup>-1</sup>, corresponding to the amount treated by the Danish Arla's plant located in Christiansfeld. The steady state design was performed starting from the well-known Eq. (7):

$$A_e = \frac{\dot{Q}}{U \cdot \Delta T_{lm} \cdot F_g} \quad (7)$$

where  $A_e$  is the effective heat transfer area,  $\dot{Q}$  the heat exchanger duty,  $U$  the overall heat transfer coefficient,  $\Delta T_{lm}$  the logarithmic mean temperature difference, and  $F_g$  the corrective factor that takes into account deviations from a pure counter-current arrangement. This factor is a function of the stream temperatures, their heat capacities, and the flow arrangement.

In order to initialize the design procedure, the following variables have been fixed using the same notation as the scheme presented in Fig. 1:

The temperature of the milk at the entrance of the regeneration ( $P_1$  in Fig. 1) and at the exit of the cooling section ( $P_8$ ) is equal to 5 °C.

Thermal losses are neglected due to the assumption of well isolated pipes, resulting in the temperatures  $P_3$ ,  $P_4$ ,  $P_5$ , and  $P_6$  in Fig. 1 being equal to 72 °C.

Heating water is available at 75 °C for the heater ( $h_1$ ), while the exit temperature ( $h_2$ ) is fixed as 3 °C higher than the milk at the entrance of the heater ( $P_2$ ).

Cooling water ( $c_1$ ) is available at 2.5 °C, while the exit temperature is fixed as 5 °C lower than the milk fed to the cooler ( $P_7$ ).

The fouling resistance for milk streams was fixed to  $2.16 \cdot 10^{-5} \text{ m}^2 \text{ } ^\circ\text{C W}^{-1}$  that corresponds to 25% of a shell and tube heat exchanger resistance (Budiati and Biyanto, 2018). The value of  $9.00 \cdot 10^{-6} \text{ m}^2 \text{ } ^\circ\text{C W}^{-1}$  was assumed for the utilities side (Wang et al., 2007).

The number of passes was assumed equal to 3 for the regeneration section and 2 for the heater and cooler. The value was defined according to the maximum value of pressure drop of 100 kPa per meter channel length (Wang et al., 2007).

Based on the milk flowrate defined, Alfa Laval Front 10 GHE was selected with a chevron angle of  $45^\circ$  and a distance between plates of 4 mm.

Details on the equations used for the heat transfer coefficient evaluation can be found in Wang et al. (2007). The design procedure is iterative and it was initialized with a guess value for the overall heat transfer coefficient of  $5000 \text{ W m}^{-2} \text{ K}^{-1}$  for the heating, cooling, and regeneration section, and a value of  $F_g$  equal to 1. The design was considered satisfactory when the difference between the guessed and the calculated values was lower or equal to 0.01.

Different designs are possible depending on the extend of the pre-heating achieved in the regeneration section. According to Wang et al. (2007), this temperature can vary in the range of  $57\text{--}68^\circ\text{C}$ . In order to compare the designs corresponding to the different temperatures, the annualized cost has been evaluated. For the operative cost evaluation,  $5.00 \text{ \$ GJ}^{-1}$  and  $0.07 \text{ \$ (kWh)}^{-1}$  for the cold and hot utility have been considered, respectively. The operative time has been estimated at 8212 hours per year in order to take into account the cleaning cycles. The evaluation of the capital costs considers stainless steel as construction material and an operational time of 10 years. The best design resulted in a pre-heated milk leaving the regeneration section at  $64^\circ\text{C}$ , corresponding to a temperature of milk entering the cooling section at  $13^\circ\text{C}$ , a total number of plates of 436, 53, and 54 for the regeneration, heating, and cooling sections, respectively.

### 3. DYNAMIC MODEL

Based on the system depicted in Fig. 1 and the design described in Section 2, the mathematical model used to dynamically describe the pasteurization system is reported in Eqs. (8-13). Perfectly mixed conditions and well insulated piping have been considered. Physical and chemical properties of liquids, such as density ( $\rho_m$ ), heat capacity, thermal conductivity, and viscosity have been evaluated as a function of composition and temperature. The overall heat transfer coefficient in the regeneration, heating and cooling sections  $U_R$ ,  $U_H$ ,  $U_C$ , has been considered dependent on process conditions. The corresponding volume of the heat exchanger ( $V_{R,H,C}$ ) was estimated based on the exchanger area and exchanger geometry.

$$\frac{dT_{P2}}{dt} = \frac{F_m(T_{P1}-T_{P2})}{V_R} + \frac{U_R A_R \Delta T_{lmR}}{\rho_m c_{pm} V_R} \quad (8)$$

$$\frac{dT_{P7}}{dt} = \frac{F_m(T_{P6}-T_{P7})}{V_R} - \frac{U_R A_R \Delta T_{lmR}}{\rho_m c_{pm} V_R} \quad (9)$$

$$\frac{dT_{P3}}{dt} = \frac{F_m(T_{P2}-T_{P3})}{V_H} + \frac{U_H A_H \Delta T_{lmH}}{\rho_m c_{pm} V_H} \quad (10)$$

$$\frac{dT_{h2}}{dt} = \frac{F_{wh}(T_{h1}-T_{h2})}{V_H} - \frac{U_H A_H \Delta T_{lmH}}{\rho_w c_{pw} V_H} \quad (11)$$

$$\frac{dT_{P8}}{dt} = \frac{F_m(T_{P7}-T_{P8})}{V_C} - \frac{U_C A_C \Delta T_{lmC}}{\rho_m c_{pm} V_C} \quad (12)$$

$$\frac{dT_{c2}}{dt} = \frac{F_{wc}(T_{c1}-T_{c2})}{V_C} + \frac{U_C A_C \Delta T_{lmC}}{\rho_w c_{pw} V_H} \quad (13)$$

The holding tube has been modelled as a pure delay system, with a time delay equal to 15 s.

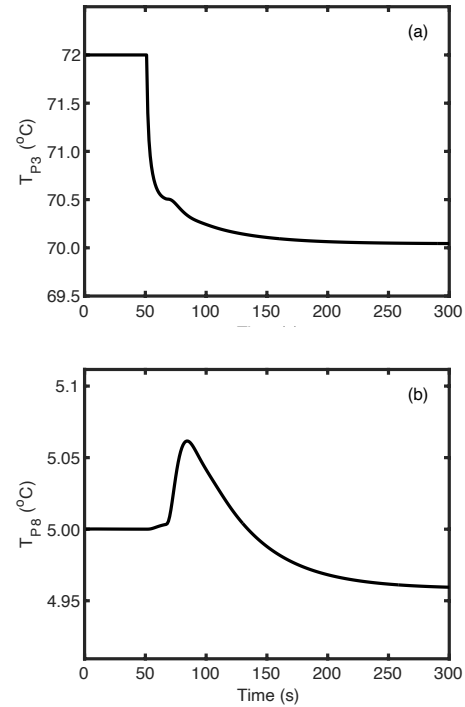


Figure 2. Milk temperature at the exit of the heater (a) and at the exit of cooler (b) after a step change of the hot water temperature equal to  $-2^\circ\text{C}$  from the nominal set-point.

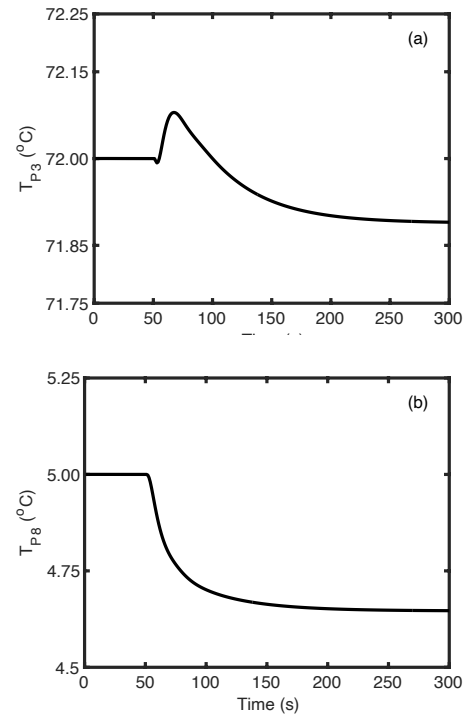


Figure 3. Milk temperature at the exit of the heater (a) and at the exit of cooler (b) after a step change of the inlet milk temperature equal to  $-2^\circ\text{C}$  from the nominal set-point.

Hot and cold water flowrates,  $F_{wh}$  and  $F_{wc}$  respectively, have been considered as manipulated inputs, while inlet temperatures ( $T_{P1}$ ,  $T_{h1}$ ,  $T_{c1}$ ) are considered as disturbances. Dynamic simulations of the open loop system are reported in

Figures 2-4, where the effects of disturbances have been represented. It is evident that the highest gains are obtained for changes of the hot ( $T_{h1}$ ) water temperature on the milk temperature after the heater and for changes of the cold water temperature ( $T_{c1}$ ) on the outlet milk temperature.

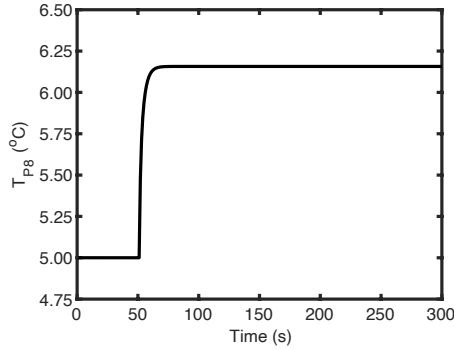


Figure 4. Milk temperature at the exit of the cooler after a step change of the cold water equal to  $+1.5^{\circ}\text{C}$  from the nominal set-point.

#### 4. NONLINEAR MPC

The use of a proper control strategy is required in the plant for rejecting the disturbances effects and avoiding a deterioration of milk quality and safety. Excessive heat exposure of proteins and enzymes, for example, can lead to a product of poorer quality, therefore variations of no more than about  $+0.5^{\circ}\text{C}$  are allowed for the milk in the holding tube (Niamsuwan et al., 2014; Rasmussen et al., 2020). Furthermore, if the temperature does not reach  $72^{\circ}\text{C}$ , the milk will be rejected and must be treated once more. Care must be taken also for the cold milk leaving the pasteurizer, because a deviation from the reference condition can be detrimental for the product quality and the process economy. The results reported in the previous section proved how disturbances can offset the system from the design targets leading to the necessity to design an efficient automatic control scheme.

For the problem at hand a nonlinear control strategy has been used, based on a mechanistic model of the process to control of the form:

$$\frac{dx}{dt} = f(x, u, d) \quad (14)$$

$$y = h(x) \quad (15)$$

where  $x \in \mathcal{R}^{n_s}$  is the state vector whose dynamics is described by the vector field  $f(x, u, d)$ ,  $u \in \mathcal{R}^{n_m}$  is the control vector,  $d \in \mathcal{R}^{n_d}$  is the disturbance vector, and  $y \in \mathcal{R}^{n_o}$  is the measured output vector which is related to the states through the nonlinear functions  $h(x)$ . The NMPC control configuration used as output variables  $y = [T_{p2}, T_{p8}]$  and controls  $u = [F_{wh}, F_{wc}]$ . The model (14-15) used to predict the outputs is the model of the pasteurization (8-13), but considering the physical and chemical properties of liquids constant and evaluated at the nominal conditions. In this way, modeling errors are introduced in the prediction, simulating a more realistic condition.

Denoting with  $p$  the prediction horizon and  $m$  the control horizon, the following control problem is solved

$$\min_{\Delta u(k), \dots, \Delta u(k+m-1)} \sum_{i=1}^p \varepsilon_r(k+i)^T \Gamma \varepsilon_r(k+i) + \sum_{i=1}^m \Delta u(k+i)^T R \Delta u(k+i) \quad (16)$$

where  $\varepsilon_r$  is the difference between the desired output and the predicted value. The prediction error is corrected by using the available measured outputs. The controller finds the vector of the future control moves that minimizes the sum of squared error while constraining the magnitude  $\Delta u$  for a prediction horizon  $p$  and a control horizon  $m$  (Mei et al., 2018). The Nelder-Mead simplex algorithm (Lagarias et al., 1998) has been used to solve the optimization problem.

The selection of the tuning parameters is of critical importance for the control performance. Based upon simulation tests, the prediction horizon  $p$  has been set equal to 21, with a sampling time equal to 1s, the control horizon  $m$  was set equal to 4, the identity matrix was used for  $\Gamma$ ,  $10^{-6}$  is the value of the coefficients of the matrix  $R$  related to  $F_{wh}$  and  $10^{-7}$  for  $F_{wc}$ . Only the first control move is applied at each sampling time.

#### 5. RESULTS AND DISCUSSION

The capability of the control strategy to reject the disturbances has been assessed by simulating the process when varying inlet temperatures  $T_{h1}$  and  $T_{c1}$ .

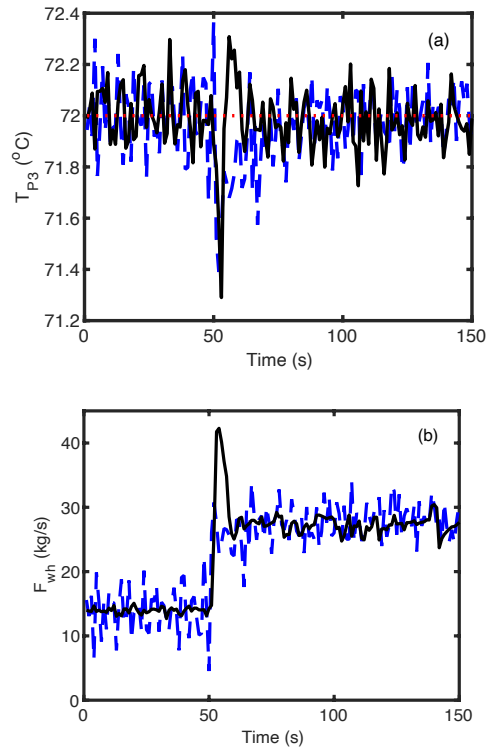


Figure 5. Temperature  $T_{p3}$  (a) and flowrate  $F_{wh}$  (b) responses when  $T_{h1}$  varies from  $75^{\circ}\text{C}$  to  $73^{\circ}\text{C}$ ,  $T_{c1}$  from  $2.5^{\circ}\text{C}$  to  $4^{\circ}\text{C}$ . NMPC (black continuous line), PI (dashed blue line) and set-point (red dotted line).

Noise with zero-mean and normally distributed has been added in order to mimic the behavior of the system in a more realistic fashion. A maximum value equal to three times the nominal one has been used for the manipulated inputs.

Figures 5 and 6 show the response of the controlled system for a step variation from  $75^{\circ}\text{C}$  to  $73^{\circ}\text{C}$  of the hot water entering the heater ( $T_{h1}$ ) and from  $2.5^{\circ}\text{C}$  to  $4^{\circ}\text{C}$  of the cold water entering the cooling section ( $T_{c1}$ ). The two inputs have been changed simultaneously so that a critical situation in the plant has been simulated. The decreasing of the heating medium temperature implies that pasteurization temperature is not reached at nominal hot water flowrate, therefore a fast rejection is required. The increase of the cooling water temperature implies that the product is not properly refrigerated, implying an increase of the microbial population growth rate. The response of the controller outputs and manipulated variables has been compared with a PI feedback controller, which has been tuned using the IMC approximate model tuning rules (Ogunnaike and Ray, 1994).

Results (Fig. 6-7) show similar behavior for NMPC and PI, being both able to efficiently reject the disturbances. NMPC leads generally to a smoother behaviour on the manipulated input if compared to PI controller.

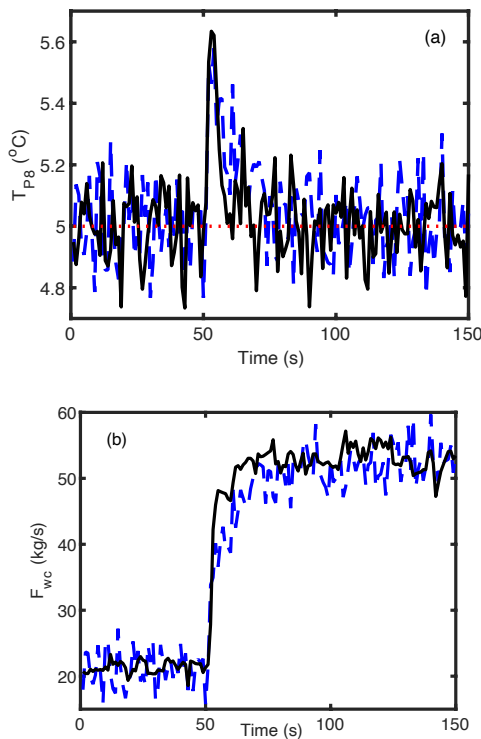


Figure 6. Temperature  $T_{PG}$  (a) and flowrate  $F_{wc}$  (b) responses when  $T_{h1}$  varies from  $75^{\circ}\text{C}$  to  $73^{\circ}\text{C}$ ,  $T_{c1}$  from  $2.5^{\circ}\text{C}$  to  $4^{\circ}\text{C}$ . NMPC (black continuous line), PI (dashed blue line) and set-point (red dotted line).

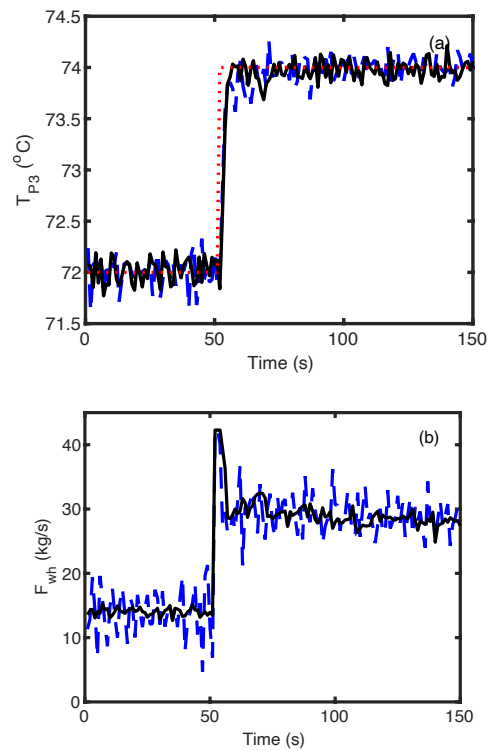


Figure 7. Temperature  $T_{P3}$  (a) and flowrate  $F_{wh}$  (b) responses for set-point change from  $72^{\circ}\text{C}$  to  $74^{\circ}\text{C}$ . NMPC (black continuous line), PI (dashed blue line) and set-point (red dotted line).

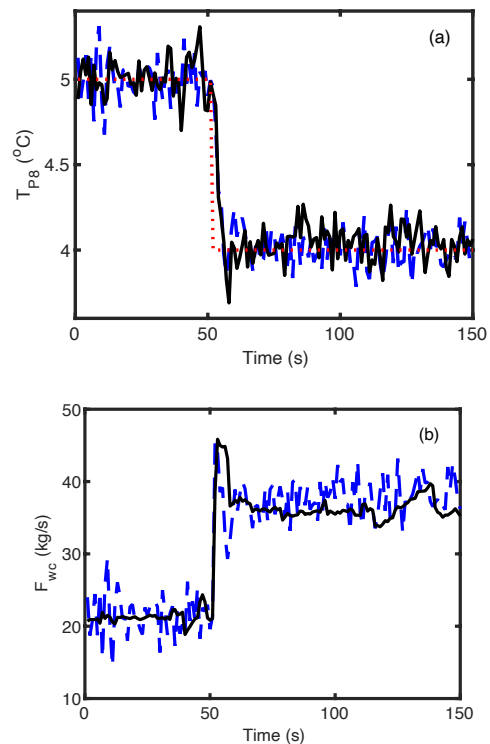


Figure 8. Temperature  $T_{PG}$  (a) and flowrate  $F_{wc}$  (b) responses for set-point change of  $T_{PG}$  from  $5^{\circ}\text{C}$  to  $4^{\circ}\text{C}$ . NMPC (black continuous line), PI (dashed blue line) and set-point (red dotted line).

Even if it is more likely that the control system is used to reject disturbances entering the pasteurization unit, set-point tracking can be required in the plant. For example, based on the type of product, a different temperature of pasteurization, or a lower refrigeration temperature may be required. To show the behaviour for set-point tracking, the reference value of the temperature of the milk leaving the heater has been varied from 72°C to 74°C, while the set-point of the milk leaving the pasteurizer has been decreased from 5 to 4°C. Results are reported in Figures 7 and 8, showing again comparable responses for PI and NMPC, but the optimal controller show again a smoother response of the manipulated inputs.

## 6. CONCLUSION

The use of an accurate design of a pasteurization system led to the development of a dynamic model that takes into account the dependence of the model parameters from the operating conditions (temperature, milk composition, flows). The dynamic simulation showed to be a valid tool for understanding the process, evaluating the impact of possible disturbances entering the system, and designing the control system.

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