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Analysis of interfacial crack propagation under asymmetric loading in anisotropic materials

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Abstract. This paper considers a steady-state crack propagating along an interface between dissimilar orthotropic materials under an asymmetric load. Although most of the known results so far deal with symmetric loading, it has been shown recently that a significant asymmetry in the applied loading may lead to a pronounced effect in terms of the values of the SIFs. The aim of the paper is to extend these results from the static case to a moving crack. In particular, we show the significance of the asymmetry of the loading for computing the energy release rate.

1. Problem formulation and preliminary results

As a model, a semi-infinite crack propagating at a constant, subsonic speed, v, along a perfect interface between two semi-infinite anisotropic materials is considered. In our model the crack is said to occupy left hand portion of the interface whereas the materials are bonded along the remaining region of the interface. The separation point (crack tip) is moving from left to right. The material occupying the region above the interface will be referred to as material I and the material occupying the region below the interface as material II. When applying the theory to orthotropic materials the axes of orthotropy are assumed to be oriented in such a way that one axis is parallel and another perpendicular to the interface between the two materials. The results shown in this paper also hold for monoclinic materials if the plane of symmetry is the plane spanned by the x_1 and x_2 coordinates. In the absence of body forces, the applied tractions on the crack faces are self-balanced but distributed arbitrarily (continuously or as point forces) and move with the same speed, v, in the direction of the crack propagation. An example of this system for a given loading is shown in Fig. 1. For the purpose of this paper it is not necessary for $\sigma^{\pm}$ to vanish at infinity but we do have the condition that $\sigma^{\pm}$ has to vanish in a region of the crack tip.

To begin with we consider the static case, which has the governing equation defined by Hooke's law

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} = C_{ijkl}\frac{\partial u_k}{\partial x_l}, \qquad \text{for } i,j,k,l = 1,2, \tag{1}$$

where the tensors of stress, σ, and strain, ϵ, are related via the stiffness tensor, C, for each

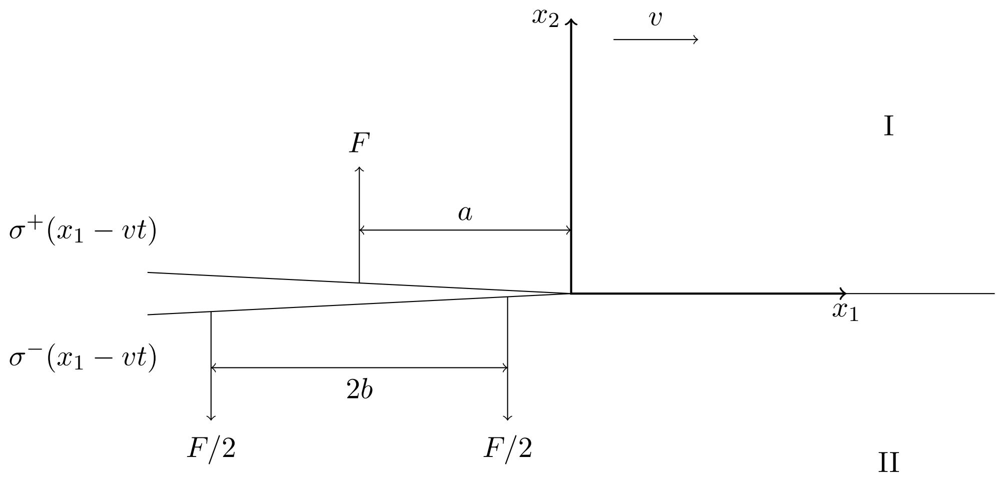

Figure 1. Geometry

material. In the absence of body forces, the balance equation reads

$$\sum_{j,k,l=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j} = 0. \tag{2}$$

Combining (1) and (2) gives the Navier-Stokes equation for an arbitrary anisotropic material

$$C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} = 0. \tag{3}$$

Following [3], a solution for the displacement field is sought in the form $u_i = A_i f(x_1 + px_2)$ and this, in conjunction with (3), gives the following eigenvalue problem to find the respective eigenvalues p:

$$(\mathbf{Q} + p(\mathbf{R} + \mathbf{R}^T) + p^2\mathbf{T})\mathbf{A} = 0, \tag{4}$$

where $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{T}$ all depend on elastic constants for the given material. The matrix $\mathbf{A}$ consists of the two eigenvectors A_i. Knowing this result, Suo in [11] constructed the solution for an interface crack problem and found expressions for the traction along the interface ahead of the crack tip and the jump in displacement over the crack. These results were found under the assumption that the load applied on the crack faces was symmetric.

A similar method has been used to study the steady-state problem. Now the governing equation also contains an inertial term:

$$C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{5}$$

where ρ is the material density. Introducing the respective moving coordinate system: ($\tilde{x}_1 = x_1 - vt, \tilde{x}_2 = x_2$), this can be rewritten as

$$\tilde{C}_{ijkl} \frac{\partial^2 u_k}{\partial \tilde{x}_j \partial \tilde{x}_l} = 0, \tag{6}$$

in the new frame of reference, where $\tilde{C}_{ijkl} = C_{ijkl} - \rho v^2 \delta_{ik}\delta_{1j}\delta_{1l}$. This allowed Yang [14] to solve the interface crack problem in an anisotropic bimaterial under symmetrical load prescribed along the crack surfaces.

From this stage on, for convenience, the moving coordinate system will be written as $\tilde{x}_1 = x$ and $\tilde{x}_2 = y$. Searching for a solution in the form $u_i = A_i f(x + py)$, Yang obtain the eigenvalue problem (4), where the matrix $\mathbf{Q}$ now relies on both the material constants and the crack velocity, while matrices $\mathbf{R}$ and $\mathbf{T}$ are identical to those in (4). Using the same method as seen in [11], it was possible to find the physical solution of the problem under the assumption that the loading on the crack faces was symmetric.

The theory has recently been developed further in [4] where the results were obtained for asymmetric loading on the crack faces, for a static crack. The method used in this analysis originates from the approach used in [12] where special weight functions were incorporated into the fundamental Betti formula. These results have been then extended to a moving crack under asymmetric loading in [9] and it is the application of these results that we will discuss in the remainder of this paper.

In order to perform further work, certain information regarding the asymptotic expansions of the traction, $\mathbf{t}(x)$, and displacement jump, $\mathbf{d}(x)$, at the crack tip is required. The expansions, as $x \to 0$, are given by

$$\mathbf{t}(x) = \frac{x^{-\frac{1}{2}}}{2\sqrt{2\pi}}\mathbf{T}(x)\mathbf{K} + \frac{x^{\frac{1}{2}}}{2\sqrt{2\pi}}\mathbf{T}(x)\mathbf{Y} + \mathcal{O}((x)^{\frac{3}{2}}), \tag{7}$$

$$\mathbf{d}(x) = \frac{(-x)^{\frac{1}{2}}}{\sqrt{2\pi}}\mathcal{U}(x)\mathbf{K} + \frac{(-x)^{\frac{3}{2}}}{\sqrt{2\pi}}\mathcal{U}(x)\mathbf{Y} + \mathcal{O}((-x)^{\frac{5}{2}}), \tag{8}$$

where $\mathbf{K} = [K, \bar{K}]$ and $\mathbf{Y} = [Y, \bar{Y}]$. Here, $K = K_1 + iK_2$ is the complex stress intensity factor of the system [1, 13]. The matrices $\mathcal{U}(x)$ and $T(x)$ are given by the following equations

$$\mathcal{U}(x) = \frac{2(H + \bar{H})}{\cosh \pi\epsilon}\left[\frac{\mathbf{w}(-x)^{i\epsilon}}{1 + 2i\epsilon}, \frac{\bar{\mathbf{w}}(-x)^{-i\epsilon}}{1 - 2i\epsilon}\right], \qquad \mathbf{T}(x) = 2\left[\mathbf{w}x^{i\epsilon}, \bar{\mathbf{w}}x^{-i\epsilon}\right].$$

The matrix $\mathbf{H}$ is a bimaterial matrix defined as $\mathbf{B}_I + \bar{\mathbf{B}}_{II}$, where $\mathbf{B}$ is the surface admittance tensor, $\mathbf{B} = i\mathbf{A}\mathbf{L}^{-1}$, for each material. The vector $\mathbf{w}$ and Dundurs paramater ϵ are found from the eigenvalue problem [11]

$$\bar{\mathbf{H}}\mathbf{w} = e^{2\pi\epsilon}\mathbf{H}\mathbf{w}. \tag{9}$$

Follwing the work of [2] an expression was also found for the energy release rate at the crack tip, G, for the static case in [11] in the form

$$G = \frac{\bar{\mathbf{w}}^T(\mathbf{H} + \bar{\mathbf{H}})\mathbf{w}|K|^2}{4\cosh^2(\pi\epsilon)}. \tag{10}$$

It was noted in [15] that the same method can be used to find the energy release rate for a moving crack as long as it moves at subsonic speeds. As this paper only considers cracks with sub-Rayleigh velocities this expression can also be used in our analysis. For orthotropic materials equation (10) can be simplified further

$$G = \frac{H_{11}(1 - \beta^2)|K|^2}{4}. \tag{11}$$

The Dundurs parameter, β, and H_{11} are both obtained from the bimaterial matrix $\mathbf{H}$ (see [9]).

The following section of the paper considers a specific example of loading and materials to calculate energy release rates using the methods developed in [9]. We concentrate on the consequence of the asymmetry in the applied load for the major fracture mechanics paramaters, that is the stress intensity factors and energy release rate, highlighting some effects which cannot be observable in a dissimilar structure when only symmetric loading is applied.

2. Specific example

The loading on the crack faces is given by a point force of magnitude F acting on the upper crack face a distance a behind the crack tip and two point forces, both of magnitude $F/2$, acting on the lower crack face a distance b away from the point force acting upon the upper crack face. All forces move at the same speed and in the same direction as the crack propagation. This system is shown in Fig. 1.

In order to compare the contribution of the different parts of the loading to the overall energy release rate, we decompose the stress intensity factor, K, into its symmetric and asymmetric parts, denoted K^S and K^A respectively. Both components are complex numbers.

We now compute the energy release rate for two given materials. The piezoceramic Barium Titanate is set as material I, with material properties $C_{11} = 120.3$GPa, $C_{22} = 120.3$GPa, $C_{12} = 75.2$GPa, $C_{66} = 21$GPa and $\rho = 6,020$kgm^{-3}. Material II is set as the Aluminium which has material properties $C_{11} = 107.3$GPa, $C_{22} = 107.3$GPa, $C_{12} = 60.9$GPa, $C_{66} = 28.3$GPa and $\rho = 2,700$kgm^{-3}. For the purpose of the calculations in this paper we set $a = 1$. The velocity has been normalised by dividing by c_R, the lowest Rayleigh wave speed for either material, which in this case is the Rayleigh wave speed of Barium Titanate. We present a dimensionless form of the ERR in our results, where G is normalised in the following manner: $GC_{66}^{(1)}/F^2$. For this normalisation we take our value of $C_{66}^{(1)}$ as that of the material above the crack.

Fig. 2 shows the normalised ERR and Fig. 3 indicates the normalised energy release rates, G^S and G^A, corresponding to K^S and K^A, respectively. Both components are normalised by the total energy release rate corresponding to $K = K^S + K^A$, given by G.

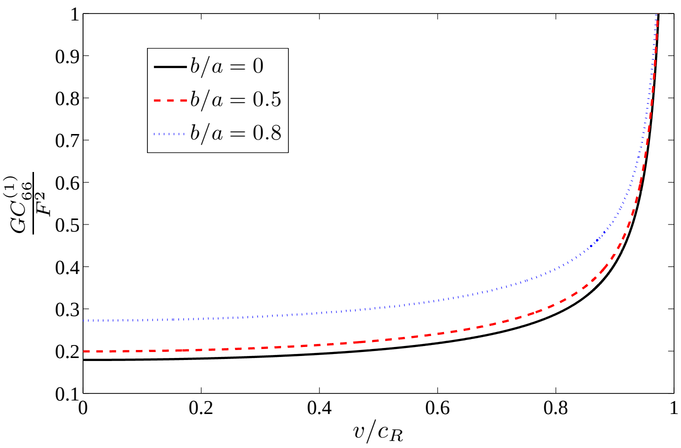

Figure 2. The normalised ERR, as a function of the velocity, for different positions of the self-balanced point forces applied to the crack surfaces, described by the ratio b/a in Fig. 1.

The graph in Fig. 2 clearly shows that ERR increases as the velocity increases for all values of b/a and approaches infinity as the velocity approaches the Rayleigh wave speed , as expected. Moreover, the energy release rate is higher for larger values of b/a, that is, as the asymmetry of the loading increases. Therefore, symmetric loading is energetically more beneficial than any choice of asymmetric one.

Fig. 3 show that the contribution of the asymmetric part of the loading to the overall energy release rate is small compared to the contribution of the symmetric part for the lower crack velocities. However, when the crack is moving at velocities close to the Rayleigh wave speed the asymmetric nature of the loading begins to play a more crucial role in the calculations. This fact is also highlighted in the left hand graph of Fig. 4. Interestingly, the energy release rate corresponding to the symmetric part of the loading is higher than the overall ERR for the physical loading, apart from when the crack velocity is very close to the Rayleigh wave speed. The behaviour of the symmetric part of G near the Rayleigh wave speed is particularly noteworthy as it can be seen that for asymmetric loading (when $b/a > 0$) the ERR stops increasing and starts decreasing. Note that, for the symmetrical load ($b/a = 0$), the energy release does not depend on the crack speed. This was previously observed for isotropic and anisotropic bimaterials in [13, 14].

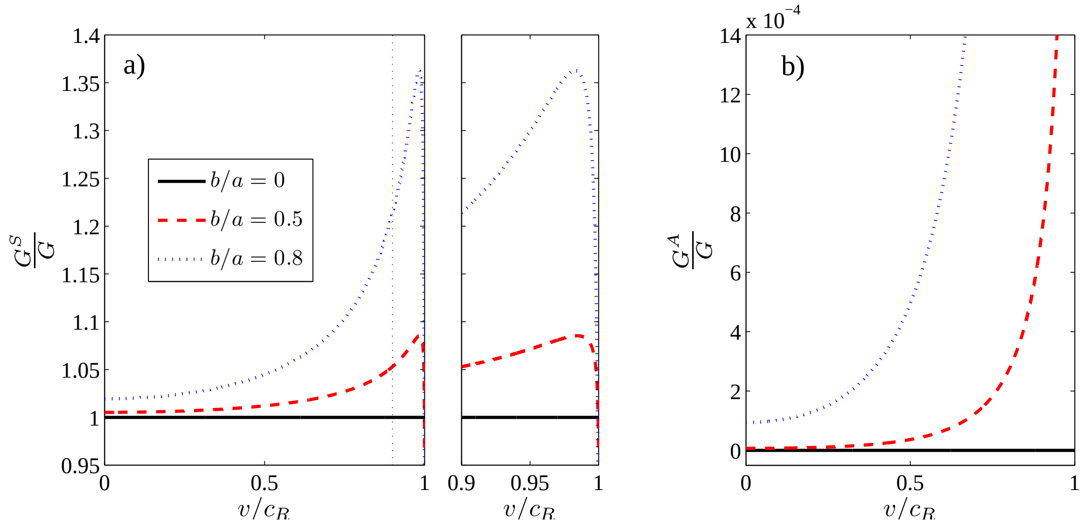

Figure 3. a) The ratio G^S/G, as a function of velocity, for different values of b/a, with particular attention being given to the behaviour near c_R. b) The ratio G^A/G with a different scale on the vertical axis.

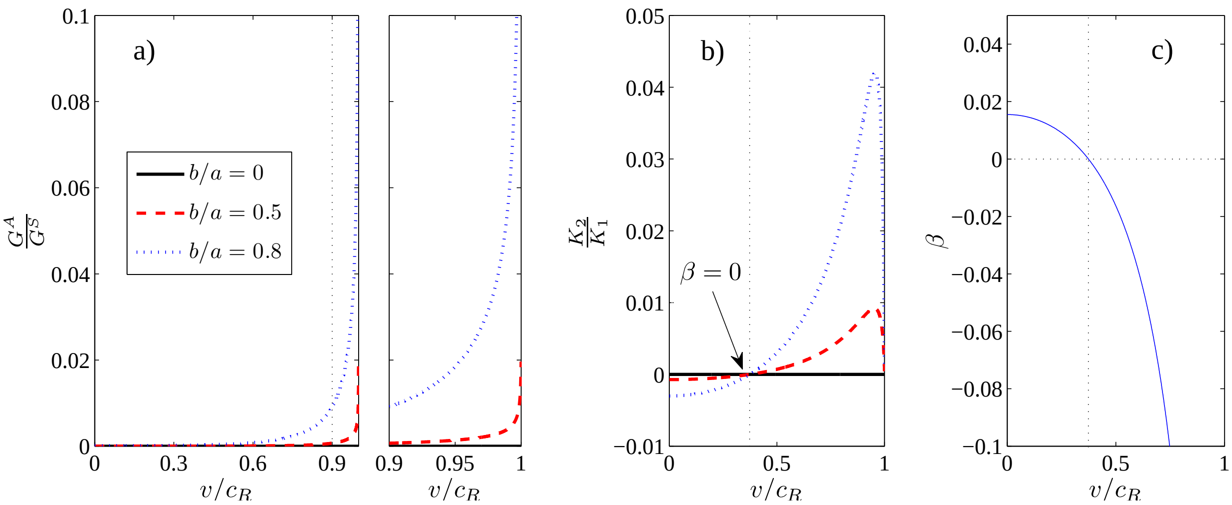

Figure 4. a) The ratio G^A/G^S, as a function of the velocity, for different values of b/a with particular attention being given to the behaviour near c_R. b) The ratio K_2/K_1, as a function of thevelocity, for different values of b/a. c) The value of β as a function of the velocity.

Fig. 4a shows the ratio of the mode 2 contribution of the stress intensity factor, K, to the mode 1 component. For the symmetric case the mode 2 component is 0 for all values of the velocity, as one would expect, whereas for asymmetric symmetry the ratio is initially negative and then at a certain velocity the sign of the ratio changes. This is connected to the fact that for a determinate value of the propagation speed there is a change in the sign of the Dundurs parameter, β (which only depends on the elastic constants of the materials). It is therefore possible to obtain a characteristic value for v, depending only on the material properties. This may lead to a possible change in the fracture mechanism or indicate a possible redirection of the interface crack propagation, for example kinking or branching. Note that there are other investigations, both theoretical and experimental, demonstrating an existence of a specific sub-Rayleigh velocity which is related to the stability of the crack propagation [5, 6].

3. Conclusions

In this paper we have analysed some results concerning the ERR and SIF for a crack propagating along the interface between anisotropic materials with a constant speed, under asymmetric, self-balanced loading acting upon the crack faces. For small velocities the results agree with those obtained for the stress intensity factors for the static crack in [4]. In particular, the load asymmetry influences the critical loading magnitude. Moreover, it has been shown that the asymmetry may play an important role for a moving crack, where the SIF changes its sign at a certain velocity depending on the material parameters but independent of the asymmetry of the load. However,this effect could only be observered in the presence of the asymmetry and may be important when analysing possible crack kinking or branching.

Acknowledgments

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