PROCEEDINGS OF SCIENCE

Emergence of resummation scales in the evolution of the QCD strong coupling and PDFs

Valerio Bertone,[a] **Giuseppe Bozzi**[b,c,*] **and Francesco Hautmann**[d,e,f]

[a] *IRFU, CEA, Université Paris-Saclay, F-91191 Gif-sur-Yvette, France*
[b] *Dipartimento di Fisica, Università di Cagliari, Cittadella Universitaria, I-09042 Monserrato, Italy*
[c] *INFN, Sezione di Cagliari, Cittadella Universitaria, I-09042 Monserrato, Italy*
[d] *CERN, Theoretical Physics Department, CH 1211, Geneva, Switzerland*
[e] *Elementaire Deeltjes Fysica, Universiteit Antwerpen, B 2020 Antwerpen, Belgium*
[f] *Theoretical Physics Department, University of Oxford, Oxford OX1 3PU, United Kingdom*
E-mail: valerio.bertone@cea.fr, giuseppe.bozzi@unica.it, hautmann@thphys.ox.ac.uk

The QCD strong coupling (α_s) and the parton distribution functions (PDFs) of the proton are fundamental ingredients for phenomenology at high-energy facilities such as the Large Hadron Collider (LHC). It is therefore of crucial importance to estimate any theoretical uncertainties associated to them. Both α_s and PDFs obey their own renormalisation-group equations (RGEs) whose solution determines their scale evolution. Although the kernels that govern these RGEs have been computed to very high perturbative precision, they are not exactly known. In this contribution, we outline a procedure that allows us to assess the uncertainty on the evolution of α_s and PDFs due to our imperfect knowledge of their respective evolution kernels. Inspired by transverse-momentum and threshold resummation, we introduce additional scales, that we dub resummation scales, that can be varied to estimate the uncertainty on the evolution of α_s and PDFs at any scale. As a test case, we consider the deep-inelastic-scattering structure function F_2 in a region relevant for the extraction of PDFs. We study the effect of varying these resummation scales and compare it to the usual renormalisation and factorisation scale variations.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

*Speaker

PoS(ICHEP2022)816

Precision calculations for high-energy collider physics often involve perturbative solutions of renormalisation group equations (RGEs). When trying to estimate theoretical uncertainties on predictions for physical observables, a significant role is often played by differences among RGE solutions. It is well known that, at a definite perturbative order, these differences are originated by solution methods which are equivalent only up to subleading orders.

In the context of QCD resummation, this kind of uncertainty is usually taken into account through the introduction of a so-called resummation scale: its central value is arbitrary, but is customarily chosen of the order of the hard momentum-transfer scale of the process, and it is allowed to vary in a suitably chosen range to get an estimate of the corresponding shift induced in theoretical predictions.

In other common applications of QCD to collider physics, on the other hand, the estimate of theoretical uncertainties stemming from resummation scale variations is not performed. An important example is, for instance, the determination of parton distribution functions (PDFs) from global fits to experimental data, [1–7], an essential ingredient for particle phenomenology at hadron colliders. In this case, perturbative solutions of renormalisation group equations are used for the evolution of the PDFs themselves and for the evolution of the running coupling α_s. However, resummation scale variation is not performed, and thus RGE theory uncertainties are not taken into account: this leads to an underestimate of the theoretical systematics associated with the extraction of PDFs.

We illustrated this point in detail in a recent work [8]: we discussed several aspects of theory uncertainties arising from RGE perturbative solutions and suggested to use resummation scale methods to estimate their impact on physical observables. Our recipe is analogous to what is usually done in Sudakov resummation: namely, we propose an extension of the formalism of the so-called g functions to the resummation of single logs. An application to deep-inelastic scattering (DIS) structure functions revealed that resummation scale effects are numerically significant in kinematical regions relevant to PDF determinations. In this contribution we summarize some of the results from Ref. [8], mainly focusing on the F_2 structure function.

We consider a RGE, for a general renormalised quantity R, of the form

$$\frac{d\ln R}{d\ln\mu}(\mu,\alpha_s(\mu)) = \gamma(\alpha_s(\mu))\,, \tag{1}$$

where R is a function of the strong coupling α_s and the renormalisation scale μ, with the anomalous dimension γ computable as a power series expansion in the coupling α_s.

In the case of the running coupling, R is proportional to α_s and γ to the QCD β function. In the case of PDF evolution, R has to be identified with the Mellin transform of a non singlet parton distribution and γ with the Mellin transform of DGLAP splitting functions. In Ref. [8] we studied the evolution of the structure function F_2 at next-to-leading (NLO) and next-to-next-to-leading (NNLO) order, and analysed RGE solutions obtained by methods which differ by subleading-order terms.[1] The differences between such solutions are manifest when considering the evolution operator G which connects R at two scales μ and μ_0: $R_\mu = G(\mu,\mu_0)R_{\mu_0}$. For a given μ', in general one has $G(\mu,\mu_0) \neq G(\mu,\mu')G(\mu',\mu_0)$. In the paper, we dubbed this effect *perturbative hysteresis*.

[1]All the examples considered in Ref. [8] deal with resummation of single logs. For a discussion of similar effects in Sudakov (double logs) problems, see e.g. in Refs. [9–11].

The extraction of PDFs $f_j(x,\mu)$ is made possible by factorisation formulas relating f_j to physical observables Σ, e.g. DIS structure functions, allowing for a direct comparison of theoretical predictions with collider data. The factorisation is written in the following (schematic) form:

$$\Sigma(x,Q) = \sum_j \int dz H_j(z,Q,\alpha_s(\mu_R),\mu_F) f_j(x/z,\mu_F), \tag{2}$$

where Q is the hard momentum-transfer scale of the process, H_j are hard-scattering functions computable order by order in perturbation theory, and μ_F, μ_R are the factorisation and renormalisation scale, respectively. The standard estimate of theoretical uncertainties associated with unknown higher orders in Eq. (2) normally proceeds by setting criteria for variations of the scales μ_F and μ_R.

Nonetheless, theoretical uncertainties on the extraction of PDFs arise both from μ_F and μ_R variations in Eq. (2) *and* from unknown higher orders in the kernels γ in Eq. (1): Ref. [8] proposes to take the latter into account by resummation scale techniques through two different but essentially equivalent methods.

The first method is based on expressing the evolution operator G via the analytic formalism of g-functions, which is frequently used in soft-gluon resummation calculations. Schematically, at the k-th order of logarithmic accuracy N^{k}LL, the evolution operator reads

$$G^{\mathrm{N}^k\mathrm{LL}} \sim g_0^{\mathrm{N}^k\mathrm{LL}}(\lambda) \exp\left[\sum_{l=0}^{k} \alpha_s^l(\mu) g_{l+1}(\lambda)\right], \tag{3}$$

where the g's are functions of the scaling variable $\lambda = \alpha_s(\mu)\beta_0 \ln(\mu^{(\mathrm{Res})}/\mu)$ and admit a perturbative expansion, and μ_{Res} is the resummation scale introduced in analogy with what is customarily done in the soft-gluon resummation framework. Explicit expressions for the g-functions up to $k = 3$ will be given in Ref. [12].

A second method relies on displacing the argument of the coupling appearing in the perturbative expansion of the anomalous dimension γ: in this way, one effectively defines a new anomalous dimension differing from the original one by subleading terms. This method is particularly useful for numerical solutions of the evolution equations.

In either method, one ends up introducing resummation scales: $\mu_{\mathrm{PDF}}^{(\mathrm{Res})}$ for the PDF evolution and $\mu_{\alpha_s}^{(\mathrm{Res})}$ for the coupling evolution.

As an application, we take Σ in Eq. (2) to be the DIS structure function $F_2(x,Q)$. We perform variations of the renormalisation (μ_R), factorisation (μ_F) and resummation scales, which we parameterise as $\mu_{\alpha_s}^{(\mathrm{Res})} = \xi_{\alpha_s} Q$ and $\mu_{\mathrm{PDF}}^{(\mathrm{Res})} = \xi_{\mathrm{PDF}} Q$.

Fig. 1 shows results for F_2 at $Q = 10$ GeV (using MSHT20 PDFs at the initial scale $Q_0 = 2$ GeV) at NLO (left panel) and NNLO (right panel), as a function of x. Resummation scale uncertainties are observed to be generally non-negligible with respect to the renormalisation and factorisation scale uncertainties in the kinematic region considered. From the left panel, we clearly see that the effect of the resummation scale parameter ξ_{PDF} dominates in the low-x region, while the effect of the factorization scale μ_F dominates at the largest x. The right panel shows that the uncertainty band is reduced when going to NNLO. The behaviour shown above reflects the fact that higher-order corrections to F_2 at small x are dominated by flavor-singlet quark anomalous dimensions [13, 14].

The importance of resummation scale effects, relative to that of renormalisation and factorization scale effects, increases with Q. The behaviour of the uncertainty bands with varying Q is

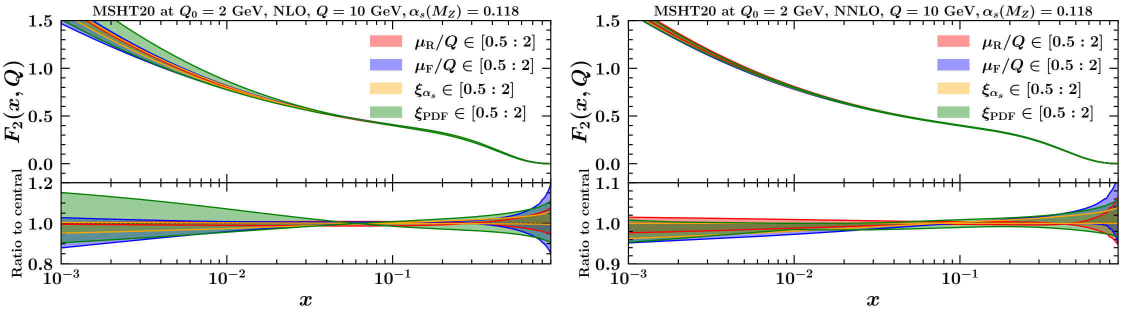

Figure 1: The structure function F_2 from Ref. [8] versus x, at NLO and NNLO in perturbation theory, with uncertainty bands associated with variations of renormalisation and factorization scales, μ_R and μ_F, and resummation scale parameters ξ_{α_s} and ξ_{PDF}. We use MSHT20 PDFs at $Q_0 = 2$ GeV and $\alpha_s(M_Z) = 0.118$ as RGE inputs.

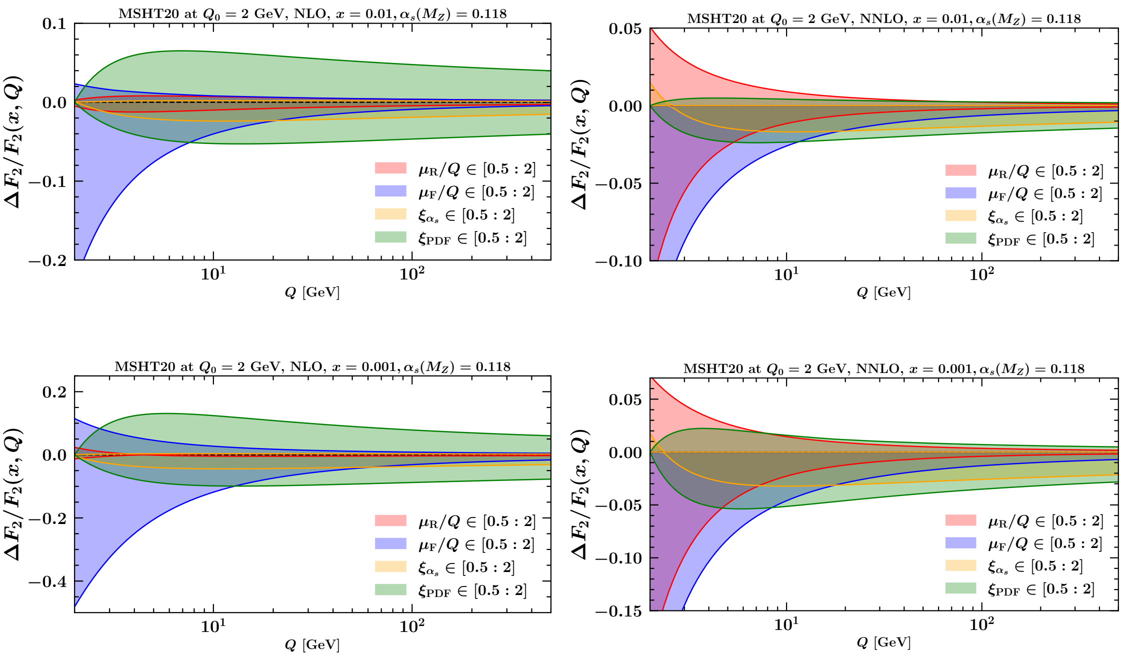

Figure 2: Q-dependence of the relative variation $\Delta F_2/F_2$ for $x = 10^{-2}$ (top) and $x = 10^{-3}$ (bottom), at NLO (left) and NNLO (right), associated with variations of renormalisation and factorization scales, μ_R and μ_F, and resummation scale parameters ξ_{α_s} and ξ_{PDF}.

clearly visible in Fig. 2, where we plot the Q-dependence of the relative variation $\Delta F_2/F_2$. The ξ_{PDF} contribution starts from zero and grows rapidly with Q, remaining sizeable even at large Q, while the μ_F contribution is largest at low Q and decreases with increasing Q. Analogously, the μ_R contribution is important at low Q and decreases with Q, while the ξ_{α_s} is subdominant at low Q but becomes relevant at high Q. The comparison of the upper plots ($x = 10^{-2}$) with the lower plots ($x = 10^{-3}$) shows that the size of the ξ_{PDF} band increases with decreasing x.

The above numerical results demonstrate that RGE uncertainties are relevant in kinematical regions which influence current and forthcoming PDF extractions, and that yhey will also be relevant for future lepton-hadron colliders [15, 16]. In addition, as the $\xi_{\rm PDF}$ contribution remains significant at large Q, RGE uncertainties are expected to be sizeable also for high-scale PDF probes, such as very energetic jets and top quarks.

Finally, we note that the methodology described in this contribution can be used in processes sensitive to collinear as well as transverse momentum dependent (TMD) [17] distributions. It will be thus applicable in the extraction of both PDFs and TMDs.

Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation program under Grant Agreement STRONG 2020 - No.824093. The authors warmly thank the organizers of ICHEP 2022 for putting up a very stimulating conference and for allowing us to present these results.

References

[1] R. D. Ball, J. Butterworth, A. M. Cooper-Sarkar, A. Courtoy, T. Cridge, A. De Roeck, J. Feltesse, S. Forte, F. Giuli and C. Gwenlan, *et al. The PDF4LHC21 combination of global PDF fits for the LHC Run III*, arXiv:2203.05506 [hep-ph].

[2] R. D. Ball, S. Carrazza, J. Cruz-Martinez, L. Del Debbio, S. Forte, T. Giani, S. Iranipour, Z. Kassabov, J. I. Latorre and E. R. Nocera, *et al. The path to proton structure at 1% accuracy: NNPDF Collaboration*, *Eur. Phys. J. C* **82** (2022) 428 [arXiv:2109.02653 [hep-ph]].

[3] R. Abdul Khalek *et al.* [NNPDF], *Parton Distributions with Theory Uncertainties: General Formalism and First Phenomenological Studies*, *Eur. Phys. J. C* **79** (2019) 931 [arXiv:1906.10698 [hep-ph]].

[4] S. Bailey, T. Cridge, L. A. Harland-Lang, A. D. Martin and R. S. Thorne, *Parton distributions from LHC, HERA, Tevatron and fixed target data: MSHT20 PDFs*, *Eur. Phys. J. C* **81** (2021) 341 [arXiv:2012.04684 [hep-ph]].

[5] T. J. Hou, J. Gao, T. J. Hobbs, K. Xie, S. Dulat, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin and C. Schmidt, *et al. New CTEQ global analysis of quantum chromodynamics with high-precision data from the LHC*, *Phys. Rev. D* **103** (2021) 014013 [arXiv:1912.10053 [hep-ph]].

[6] S. Alekhin, J. Bluemlein, S. Moch and R. Placakyte, *Parton distribution functions, α_s, and heavy-quark masses for LHC Run II*, *Phys. Rev. D* **96** (2017) 014011 [arXiv:1701.05838 [hep-ph]].

[7] H. Abramowicz *et al.* [H1 and ZEUS], *Combination of measurements of inclusive deep inelastic $e^{\pm}p$ scattering cross sections and QCD analysis of HERA data*, *Eur. Phys. J. C* **75** (2015) 580 [arXiv:1506.06042 [hep-ex]].

[8] V. Bertone, G. Bozzi and F. Hautmann, *Perturbative hysteresis and emergent resummation scales*, *Phys. Rev. D* **105** (2022) 096003 [arXiv:2202.03380 [hep-ph]].

[9] M. A. Ebert, *Analytic results for Sudakov form factors in QCD*, *JHEP* **02** (2022) 136 [arXiv:2110.11360 [hep-ph]].

[10] G. Billis, F. J. Tackmann and J. Talbert, *Higher-Order Sudakov Resummation in Coupled Gauge Theories*, *JHEP* **03** (2020) 182 [arXiv:1907.02971 [hep-ph]].

[11] F. Hautmann, L. Keersmaekers, A. Lelek and A.M. Van Kampen, *Dynamical resolution scale in transverse momentum distributions at the LHC*, *Nucl. Phys. B* **949** (2019) 114795 [arXiv:1908.08524 [hep-ph]].

[12] V. Bertone *et al.*, in preparation.

[13] S. Catani and F. Hautmann, *Quark anomalous dimensions at small x*, *Phys. Lett. B* **315** (1993) 157; *Nucl. Phys. B* **427** (1994) 475; [arXiv:hep-ph/9408251].

[14] R. K. Ellis, F. Hautmann and B. R. Webber, *QCD scaling violation at small x*, *Phys. Lett. B* **348** (1995) 582 [arXiv:hep-ph/9506303].

[15] P. Agostini *et al.* [LHeC and FCC-he Study Group], *The Large Hadron-Electron Collider at the HL-LHC*, *J. Phys. G* **48** (2021) 110501 [arXiv:2007.14491 [hep-ex]].

[16] Y. Hatta *et al.*, *Proceedings, Probing Nucleons and Nuclei in High Energy Collisions: Dedicated to the Physics of the Electron Ion Collider: Seattle (WA), United States, October 1 - November 16, 2018*, arXiv:2002.12333 [hep-ph].

[17] R. Angeles-Martinez *et al.*, *Transverse Momentum Dependent (TMD) parton distribution functions: status and prospects*, *Acta Phys. Polon. B* **46** (2015) 2501 [arXiv:1507.05267 [hep-ph]].