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Distributed Potential-Based Coordination within Open Multi-Agent Systems

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***Abstract*— Potential-based coordination frameworks represent a fundamental paradigm for distributed coordination in multi-agent systems, enabling stable collective behaviors through gradient-based local updates. However, their conventional formulations typically assume a fixed agent set, limiting applicability in dynamic settings where agents may enter or exit the network. This paper introduces a generalized potential-based coordination framework tailored for open multi-agent systems (OMASs) in discrete time. Fundamental properties that such potential-based open systems must satisfy are formally established, and a distributed protocol is presented to ensure their fulfillment. Specifically, the proposed approach extends classical gradient-based laws by incorporating cumulative dissipation mechanisms to handle time-varying network cardinality. Numerical simulations validate the effectiveness of the proposed framework compared to different baselines.**

***Index Terms*— Potential-based coordination, open multi-agent systems, distributed control**

I. Introduction

THE potential-based coordination framework in multi-agent systems (MAS) is a well-established approach for designing distributed control laws that yield desired collective behaviors [1]. In this framework, the overall potential function is constructed from pairwise potential functions defined between each pair of neighboring agents. These functions encode the desired collective behavior of the system, such as maintaining inter-agent distances, achieving formation structures, or avoiding collisions [2]. Each agent then moves according to the negative gradient of the global potential, driving the system toward configurations that minimize the overall potential. This paradigm has been applied for a multitude of applications in MASs, including aggregation [2], coverage [3], flocking [4], and data-driven coordination on learned potentials [5]. This popularity arises from its generality, intuitive design, ease of distributed implementation, and inherent stability properties: the gradient-based motion laws induce stable behaviors and ensure convergence to critical points of the potential.

Some efforts have been made in the literature to tackle different real-world challenges. For instance, the study in [6] combines potential-based theory and distributed model predictive control to handle *uncertainties* in the target reference, while the work in [7] integrates radial basis function neural networks with potential fields, handling *approximation errors* and external *disturbances* through a robustness term. However, most existing approaches on this topic assume a *fixed number of agents* within the system. While this assumption simplifies the analysis and control design, it limits the applicability of such methods to realistic scenarios. Indeed, in many practical MASs, the set of active agents is inherently time-varying, leading to open MASs (OMASs): agents may join or leave the network due to operational factors such as the addition of new resources, battery recharging cycles, maintenance activities, or unexpected faults. Consequently, designing coordination strategies that can handle dynamic sets of agents is crucial for effective deployment in real-world settings [8]. OMASs have been attracting increasing attention in recent years due to their practical relevance, including applications such as mode computation in [9], optimization and learning in [10], common reference frame estimation in [11], Nash equilibrium seeking [12], output consensus tracking under actuator attacks in [13], consensus [14], dynamic tracking of graph parameters [15], and connectivity maintenance in [16].

To bridge this gap, this work introduces a general potential-based framework for OMAS operating in discrete time. While our previous work [17] established a framework for discrete-time MAS under actuation constraints, its applicability was limited to *closed* systems, *convex* potential functions, and *asymptotic* step size computation. We advance the state of the art by proposing a new solution for potential-based coordination under OMAS, a formulation currently absent in the literature. Specifically, our main contributions are: *(i)* we formalize the properties that a potential-based OMAS framework must satisfy; *(ii)* we propose a distributed coordination protocol leveraging cumulative dissipation states to manage the *open* nature of the system and accommodate heterogeneous actuation limitations, operating with *non-convex* potential functions under differentiability and Lipschitz assumptions; *(iii)* we develop a novel distributed algorithm for the computation of the step size that guarantees *finite-time* convergence. Numerical results confirm the effectiveness of the method.

II. Preliminaries and Problem Formulation

We denote the set of natural and real numbers by $\mathbb{N}$ and $\mathbb{R}$, with positive subsets $\mathbb{N}_+$ and $\mathbb{R}_+$, respectively.

A. Preliminaries

1) Open Multi-Agent Systems: An *open multi-agent system* consists of a time-varying number $n_k \in \mathbb{N}_+$ of agents over discrete times $k \in \mathbb{N}$, assumed strictly positive and not *a priori* bounded. The interactions among the agents are modeled by an

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undirected time-varying graph $\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k)$, where $\mathcal{V}_k \subset \mathbb{N}$ is a finite node set with $n_k = |\mathcal{V}_k|$ and $\mathcal{E}_k \subseteq \mathcal{V}_k \times \mathcal{V}_k$ is the edge set. A path between nodes i and j in $\mathcal{G}_k$ is a sequence of consecutive and distinct edges $(i,p),(p,q),\dots,(s,j)$ belonging to $\mathcal{E}_k$. The diameter $\rho_k \in \mathbb{N}_+$ of $\mathcal{G}_k$ at time k is defined as the length of the longest shortest path between any pair of nodes in $\mathcal{V}_k$. The following assumption is made on $\mathcal{G}_k$.

Assumption 1. *$\mathcal{G}_k$ is connected for each $k \in \mathbb{N}$, i.e., there always exists a path between any two nodes in the graph, and its diameter is uniformly upper bounded by $\bar{\rho} \in [1, \infty)$.*

Both conditions are standard in the MAS literature: connectivity ensures network-wide information propagation, while bounded diameter is weaker than assuming known network size, e.g., [18], [19], and can be relaxed via distributed estimation [15]. Agents i and j are neighbors at time k if $(i,j) \in \mathcal{E}_k$. The neighbor set is $\mathcal{N}_k^i := \{j \in \mathcal{V}_k \setminus \{i\} : (i,j) \in \mathcal{E}_k\}$. We introduce the sets [9]: *i)* $\mathcal{R}_k = \mathcal{V}_k \cap \mathcal{V}_{k+1}$, denoting agents remaining active from k to $k+1$; *ii)* $\mathcal{A}_k = \mathcal{V}_{k+1} \setminus \mathcal{V}_k$, denoting agents joining the network at $k+1$; *iii)* $\mathcal{D}_k = \mathcal{V}_k \setminus \mathcal{V}_{k+1}$, denoting agents leaving the network at $k+1$.

Each agent $i \in \mathcal{V}_k$ is associated with a physical vector state $x_k^i \in \mathbb{R}^m$ (e.g., its position) and a vector control input $u_k^i \in \mathbb{R}^p$ at time $k \in \mathbb{N}$; the vectors defined by stacking these variables are denoted by $x_k \in \mathbb{R}^{mn_k}$ and $u_k \in \mathbb{R}^{pn_k}$, respectively. The state of a remaining agent $i \in \mathcal{R}_k$ is updated according to a single integrator dynamics, while the state of an arriving agent $i \in \mathcal{A}_k$ is initialized to some value $x_{\tau_i}^i$ with $\tau_i \in \mathbb{N}_+$ the time step at which the i-th agent joined the network, yielding [8]

$$x_{k+1}^i = \begin{cases} x_k^i + \Delta T u_k^i & \text{if } i \in \mathcal{R}_k, \\ x_{\tau_i}^i & \text{if } i \in \mathcal{A}_k, \end{cases} \quad k \in \mathbb{N}_+, \tag{1}$$

with $\Delta T \in \mathbb{R}_+$ the sampling time. We next define the concept of eventually closed OMASs, as considered in [9], [14].

Definition 1 (Eventually closed OMAS). *An OMAS with network $\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k)$ is eventually closed if there is a finite time $k_{cl} \in \mathbb{N}$ such that for all subsequent times no agent joins or leaves the network, i.e., $\mathcal{V}_k = \mathcal{V}_{k_{cl}}$ for all $k \geq k_{cl}$.*

The eventually closed setting is interpreted over a sufficiently large operational window, rather than the entire system lifetime. In continuous applications, short join or departure transients occur due to reconfigurations, faults, or recharging, after which the agent set remains fixed over long intervals. In these cases, the proposed framework characterizes the post-transient steady-state behavior without centralized re-initialization. The convergence of eventually closed OMASs is stated as

$$\exists x_{eq} \in \mathbb{R}^{mn_{cl}} : \quad \lim_{k\to\infty} x_k = x_{eq}, \qquad \forall x_0 \in \mathbb{R}^{mn_0},$$

with $n_{cl} \in \mathbb{N}_+$ the network size once it becomes closed.

2) Discrete-Time Potential-Based Coordination Framework with Actuator Saturations: We review the discrete-time potential-based formulation introduced in our prior work [17] for static networks with actuator-saturated agents. We omit the subscript k for static quantities, e.g., n denotes a static number of agents. Let $P : \Omega \subseteq \mathbb{R}^{mn} \to \mathbb{R}$, with $\Omega \neq \emptyset$, be an aggregate scalar potential function such that its critical points $\nabla P(x_k^\star) = 0_{mn}$, with $x_k^\star \in \mathbb{R}^{mn}$, and 0_{mn} the $mn-$dimensional zero vector, represent desired network configurations. The potential is defined as the sum of pairwise interaction terms $P^{ij} : \Omega^{ij} \subseteq \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$, i.e.,

$$P(x_k) = \tfrac{1}{2} \textstyle\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}^i} P^{ij}(x_k^i, x_k^j),$$

fulfilling the following assumption as in [20].

Assumption 2. *The pairwise potential functions $P^{ij} : \Omega^{ij} \subseteq \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ encode the local objectives, such as aggregation or distance maintenance, and satisfy the anti-symmetry property, i.e.,*

$$\begin{cases} \nabla_y P^{ij}(y,z) = -\nabla_z P^{ij}(y,z), \\ \nabla_y P^{ij}(y,z) = \nabla_z P^{ji}(z,y), \end{cases} \qquad \forall y, z \in \mathbb{R}^m.$$

The objective of potential-based control is to design the local inputs u_k^i ensuring the following fundamental properties [17]: *(P1-D) Invariance of the sum of the states*: the state sum is preserved over time, i.e., $\sum_{i\in\mathcal{V}} x_{k+1}^i = \sum_{i\in\mathcal{V}} x_k^i$, preventing global drift due to internal interactions.
(P2-D) Convergence to critical points: the system state evolves towards a critical point of the potential, i.e., $x_k \to x^*$ as $k \to \infty$, with $\nabla_x P(x^*) = 0_{mn}$.

Additionally, for each agent i, the control input must satisfy

$$\|u_k^i\| \leq U^i, \quad \forall k \geq 0, \tag{2}$$

where $U^i \in \mathbb{R}_+$ is the maximum actuation capability of agent i. Under the assumption of connected MASs and convex potentials, each agent i follows the potential negative gradient direction, scaled by a *common* time-varying gain α_k^* [17], i.e.,

$$u_k^i = -\alpha_k^* \nabla_{x^i} P(x_k),$$

where α_k^* is obtained by solving the following local problem:

$$\begin{aligned} &\arg\min_{\alpha^i \in A_k^i} \sum_{j\in\mathcal{N}_k^i} P^{ij}(x_k^i - \alpha^i \Delta T \nabla_{x^i} P(x_k), x_k^j - \alpha^i \Delta T \nabla_{x^j} P(x_k)), \\ &\text{s.t.} \quad \alpha^i = \alpha^j, \end{aligned} \tag{3}$$

with $A_k^i = \{\alpha \in \mathbb{R}_+ : \alpha \|\nabla_{x^i} P(x(k))\| \leq U^i\}$ the gain feasibility set of agent i. Notably, using a common gain α_k^* across the network preserves symmetric pairwise interactions, so property (P1-D) holds structurally.

B. Problem Formulation

Our goal is to develop a potential-based control framework for OMAS, handling time-varying agents and actuator saturation constraints. As no existing potential-based formulations address open networks, we first extend properties (P1-D) and (P2-D) to this setting. Indeed, (P1-D) no longer holds when agents enter or leave the system. We introduce the *open operator* $P_k : \Omega_k \subseteq \mathbb{R}^{mn_k} \to \mathbb{R}$ representing an aggregate *time-varying* and *size-varying* scalar potential function defined as

$$P_k(x_k) = \tfrac{1}{2} \textstyle\sum_{i\in\mathcal{V}_k} \sum_{j\in\mathcal{N}_k^i} P^{ij}(x_k^i, x_k^j),$$

and reformulate the properties for eventually closed OMASs: *(P1-O) Convergence of the sum of the states*: the sum of the

states of the active agents asymptotically converges to the sum of their entry states, i.e.,

$$\lim_{k\to\infty} \sum_{i\in\mathcal{V}_k} x_k^i = \sum_{i\in\mathcal{V}_{cl}} x_{\tau_i}^i.$$

This implies that the asymptotic sum of the states of the system depends exclusively on the initial states of the agents in the closed network, excluding contributions from departed agents. *(P2-O) Convergence to critical points*: let P_{cl} denote the potential once the network is closed, the active agents converge towards a critical point of the potential, $\lim_{k\to\infty} x_k = x_{cl}^\star$, such that $\nabla P_{cl}(x_{cl}^\star) = 0_{mn_{cl}}$, $x_{cl}^\star \in \mathbb{R}^{mn_{cl}}$.

We can now formally state the problem we aim to address.

Problem 1. *Consider a discrete-time eventually closed OMAS with agents evolving according to* (1) *and subject to the saturation constraints in* (2)*. The goal is to design a potential-based distributed control input u_k^i such that:*

i) Properties (P1-O) and (P2-O) are fulfilled.

ii) The heterogeneous actuator saturation constraints (2) *are satisfied for all active agents $i \in \mathcal{V}_k$.*

We make the following assumption on the potentials, which is standard in distributed optimization, e.g., [12], [20].

Assumption 3. *The pairwise functions $P^{ij} : \Omega^{ij} \to \mathbb{R}$, for all $i, j \in \mathcal{V}_k$, are of class C^2, bounded from below, L-smooth in any compact set $\bar{\Omega}^{ij} \subset \Omega^{ij}$, and it holds $\lim_{(x_i,x_j)\to\partial\Omega^{ij}} P^{ij} = \infty$, with $\partial\Omega^{ij}$ the boundary set of Ω^{ij}.*

III. OMAS Potential-based Coordination

In this section, we derive a distributed strategy for time-varying OMAS that incorporates arriving agents into the potential while removing the influence of departed ones.

To this aim, we first introduce additional *virtual* state variables. For each agent $i \in \mathcal{V}_k$, we define a cumulative dissipation state $\zeta_k^i \in \mathbb{R}^m$ which acts as an energy memory: it stores the residual energy from terminated interactions so that the influence of departed neighbors can be progressively dissipated. Moreover, for each agent i, we introduce transient variables $z_k^{ij} \in \mathbb{R}^m, \forall j \in \mathcal{N}_k^i$, which track the cumulative interaction history between i and each of its current neighbors. When an edge (i, j) is removed, the value stored in z_k^{ij} is transferred to ζ_k^i to dissipate the respective residual energy. Note that the variable z_k^{ij} only exists as long as the edge (i, j) is active. At each time step $k \in \mathbb{N}$, each agent $i \in \mathcal{V}_k$ operates differently depending on its status in the network.

1) Arriving agents $i \in \mathcal{A}_k$: initialize the physical state according to the configuration at arrival, enforcing a zero-energy condition on the dissipation states and transient variables:

$$x_{k+1}^i = x_{\tau_i}^i, \quad \zeta_{k+1}^i = 0, \quad z_{k+1}^{ij} = 0, \quad \forall j \in \mathcal{N}_{k+1}^i. \tag{4}$$

2) Remaining agents $i \in \mathcal{R}_k$: update their variables according to the rules given below, parameterized by the step size $\alpha \in \mathbb{R}_+$; then, a distributed algorithm is presented to allow the agents to agree, at each time step $k \in \mathbb{N}$, on the step size $\alpha_k^\star$ satisfying both feasibility and stability bounds.

Transient variables update: These variables track the interaction history between i and each neighbor $j \in \mathcal{N}_k^i$ as follows

$$z_{k+1}^{ij}(\alpha) = z_k^{ij} - \alpha\Delta T\, \nabla_{x^i} P^{ij}(x_k^i, x_k^j). \tag{5}$$

Dissipation state update: The dissipation state minimizes the internal energy in the absence of departures, and compensates for leaving agents upon a departure event:

$$\zeta_{k+1}^i(\alpha) = \zeta_k^i + \alpha\Delta T\delta_k^i + \sum_{j\in\mathcal{N}_k^i\cap\mathcal{D}_k} z_k^{ij}, \tag{6}$$

where $\delta_k^i \in \mathbb{R}^m$ is the dissipation direction given by

$$\delta_k^i = -\gamma\zeta_k^i, \tag{7}$$

with $\gamma \in \mathbb{R}_+$. We later prove (Theorem 2) that, with the gain $\alpha_k^\star$ defined below, ζ_k^i decays exponentially in the absence of departing neighbors; instead, if neighbor j leaves at step k, i.e., $j \in \mathcal{N}_k^i \cap \mathcal{D}_k$, the corresponding transient variable z_k^{ij} acts as a "phantom" of the lost interaction and is injected into the dissipation state. Clearly, this contribution is only considered at departure, as departed agents are removed from $\mathcal{D}_k$. Thus, the dimension of ζ_k^i remains constant, while the transient variables allow the system to adapt to the time-varying set of vertices.

Physical state update: the physical motion is driven by the potential gradient, corrected by a compensation term from the dissipation state. We define the control direction $d_k^i \in \mathbb{R}^m$ as

$$d_k^i = -\sum_{j\in\mathcal{N}_k^i} \nabla_{x^i} P^{ij}(x_k^i, x_k^j) - \gamma\zeta_k^i, \tag{8}$$

yielding the physical state update

$$x_{k+1}^i(\alpha) = x_k^i + \alpha\Delta T\, d_k^i. \tag{9}$$

Intuitively, the term $-\gamma\zeta_k^i$ acts as an interconnection force: the memory releases its stored energy by performing work on the agent, compensating for missing gradients. When the residual memory term vanishes, the state update coincides, as expected, with the classical potential-based coordination strategy.

Step size computation: The step size $\alpha_k^\star$ for updating the variables is computed in a distributed manner via a min-consensus algorithm operating at a faster timescale with sampling time $\Delta T_l \ll \Delta T$. This is justified by the fact that ΔT_l governs pure communication rounds for gain computation, which can occur at much higher rates than the physical dynamics evolution. At the start of the consensus process, each agent i initializes:

- *Feasibility bound $\bar{\alpha}^i$,* representing the maximum step size allowed by its actuator saturation U^i:

$$\bar{\alpha}^i = \begin{cases} \frac{U^i}{\|d_k^i\|} & \text{if } \|d_k^i\| \neq 0, \\ \infty & \text{otherwise.} \end{cases} \tag{10}$$

- *Stability-based step $\hat{\alpha}^i$,* representing the step size ensuring local stability as proven later in Theorem 2:

$$\hat{\alpha}^i = 1/\left(\Delta T\, L_k^i + \varepsilon\right), \tag{11}$$

with $\varepsilon > 0$ and $L_k^i = \gamma + 2\sum_{j\in\mathcal{N}_k^i} \left\|\nabla_{x^i}^2 P^{ij}\right\|$, where $\left\|\nabla_{x^i}^2 P^{ij}\right\|$ is the spectral norm of the Hessian of P^{ij}.

Based on these quantities, the min-consensus process is executed. Let $v_i(l)$ be the consensus state for agent i at the (consensus) iteration l, initialized as $v_i(0) = \min\{\bar{\alpha}^i, \hat{\alpha}^i\}$. This evolves as

$$v_i(l+1) = \min_{j\in\mathcal{N}_k^i\cup\{i\}}\{v_j(l)\}. \tag{12}$$

Convergence to the global minimum $v_i^\star$, representing the tightest and safest stability step, is guaranteed in a finite

number of iterations upper bounded by the network diameter ρ_k [21]. Note that, as the network diameter is typically limited and the communication bandwidth allows for high-frequency exchange (compared to the physical sampling time ΔT), the min-consensus process can be completed within a single control interval [22]. Thus, each agent can locally determine the common feasible step size $\alpha_k^\star$ by using $v_i^\star$, i.e., $\alpha_k^\star = v_i^\star$.

Remark 1. *The consensus process depends only on the neighbors at step k since agents joining or leaving the network between steps k and $k+1$ affect only the potential at $k+1$, making their inclusion at time k inconsistent.*

Remark 2. *The proposed gain consensus strategy can be interpreted as computing a suboptimal solution to the following extension of* (3) *via Lipschitz continuity analysis [23]*

$$\begin{aligned} \alpha_k^\star = \underset{\alpha^i \in A_k^i, \forall i}{\arg\min} \quad & \textstyle\sum_{i\in\mathcal{V}_k} V^i\left(x_{k+1}(\alpha^i), \zeta_{k+1}^i(\alpha^i)\right), \\ \text{s.t.} \quad & \alpha^i = \alpha^j, \end{aligned} \tag{13}$$

with $A_k^i = \{\alpha > 0 : \alpha\|d_k^i\| \le U^i\}$, and V^i represents the total potential function including both the physical configuration and the residual energy stored in the state ζ, i.e.,

$$V^i(x_k, \zeta_k^i) = \textstyle\sum_{j\in\mathcal{N}_k^i} P^{ij}(x_k^i, x_k^j) + \frac{\gamma}{2}\|\zeta_k^i\|^2. \tag{14}$$

Overall algorithm: The procedure is detailed in Algorithm 1. At every step k, arriving agents ($i \in \mathcal{A}_k$) initialize their variables (lines 2–3), while remaining agents ($i \in \mathcal{R}_k$) proceed as follows. Each agent $i \in \mathcal{R}_k$ computes its local feasibility bound $\bar{\alpha}^i$ and stability-based step $\hat{\alpha}^i$ (line 5), and uses them to initialize $v_i(0)$ for the distributed min-consensus process (line 6). The consensus runs for T_c iterations (lines 7–9), during which agents exchange and update local estimates with their active neighbors. The resulting step size $\alpha_k^\star$ is then applied to update x_{k+1}^i, ζ_{k+1}^i, and z_{k+1}^{ij} (lines 10–11).

Algorithm 1 Potential-Based Coordination for OMASs

Input: Initial entry state $x_{\tau_i}^i$
Output: State trajectories x_k^i satisfying (P1-O) and (P2-O)
1: **for** $k = 0, 1, 2, \ldots$ **each agent** i **does:**
2: **if** $i \in \mathcal{A}_k$ **is an arriving agent:**
3: Initialize variables as in (4)
4: **else if** $i \in \mathcal{R}_k$ **is a remaining agent:**
5: Compute $\bar{\alpha}^i$ and $\hat{\alpha}^i$ using (10)-(11)
6: Initialize $v_i(0) \leftarrow \min\{\bar{\alpha}^i, \hat{\alpha}^i\}$
7: **for** $l = 0, \ldots, T_c - 1$
8: Send and receive $v_j(l)$ from $j \in \mathcal{N}_k^i$
9: Update $v_i(l+1)$ using (12)
10: $\alpha_k^\star \leftarrow v_i$
11: Update x_{k+1}^i, ζ_{k+1}^i, z_{k+1}^{ij} as in (5)-(9) with $\alpha = \alpha_k^\star$

IV. Theoretical Analysis

A. Analysis of Convergence of the Sum of the States

In this section, we analyze the evolution of the states' sum.

Theorem 1. *Let Assumptions 1-2 hold and consider an OMAS with agents executing Algorithm 1. If $T_c \ge \bar{\rho}$, then:*

$$\textstyle\sum_{i\in\mathcal{V}_k}(x_k^i - \zeta_k^i) = \sum_{i\in\mathcal{V}_k} x_{\tau_i}^i.$$

Proof: Consider the decomposition of the agent set at time $k+1$ into remaining agents and arriving agents, i.e., $\mathcal{R}_k \cup \mathcal{A}_k = \mathcal{V}_{k+1}$. It follows that

$$\sum_{i\in\mathcal{V}_{k+1}}(x_{k+1}^i - \zeta_{k+1}^i) = \sum_{i\in\mathcal{R}_k}(x_{k+1}^i - \zeta_{k+1}^i) + \sum_{i\in\mathcal{A}_k}(x_{k+1}^i - \zeta_{k+1}^i).$$

For arriving agents $i \in \mathcal{A}_k$, the initialization (4) yields $\sum_{i\in\mathcal{A}_k} x_{k+1}^i = \sum_{i\in\mathcal{A}_k} x_{\tau_i}^i$. For remaining agents $i \in \mathcal{R}_k$, under the assumption $T_c \ge \bar{\rho}$, variables v_i converge in finite time, allowing the agents to compute the common step size $\alpha_k^\star = \min_i\{\bar{\alpha}^i, \hat{\alpha}^i\}$. Using (8)-(9), yields

$$\sum_{i\in\mathcal{R}_k} x_{k+1}^i = \sum_{i\in\mathcal{R}_k} x_k^i - \alpha_k^\star \Delta T\Big(\sum_{i\in\mathcal{R}_k}\sum_{j\in\mathcal{N}_k^i} \nabla_{x^i} P^{ij} + \gamma \sum_{i\in\mathcal{R}_k} \zeta_k^i\Big).$$

Since the gradients of the potentials satisfy the antisymmetric property, it implies $\sum_{i\in\mathcal{R}_k}\sum_{j\in\mathcal{N}_k^i} \nabla_{x^i} P^{ij} = 0$. Thus, the drift is governed by the dissipation term

$$\textstyle\sum_{i\in\mathcal{R}_k} x_{k+1}^i = \sum_{i\in\mathcal{R}_k} x_k^i - \alpha_k^\star \gamma \Delta T \sum_{i\in\mathcal{R}_k} \zeta_k^i. \tag{15}$$

Using (7)-(6), the aggregate dissipative state evolves as

$$\begin{aligned} \sum_{i\in\mathcal{R}_k} \zeta_{k+1}^i &= \sum_{i\in\mathcal{R}_k}(1 - \alpha_k^\star\gamma\Delta T)\zeta_k^i + \sum_{i\in\mathcal{R}_k}\sum_{j\in\mathcal{N}_k^i\cap\mathcal{D}_k} z_k^{ij} \\ &= \textstyle\sum_{i\in\mathcal{R}_k}\zeta_k^i - \alpha_k^\star\gamma\Delta T\sum_{i\in\mathcal{R}_k}\zeta_k^i, \end{aligned} \tag{16}$$

where, due to the dynamics in (5) and the antisymmetry of the potential gradients, it holds $\sum_{i\in\mathcal{R}_k}\sum_{j\in\mathcal{N}_k^i\cap\mathcal{D}_k} z_k^{ij} = 0$. Subtracting (16) from (15), it follows

$$\textstyle\sum_{i\in\mathcal{R}_k}(x_{k+1}^i - \zeta_{k+1}^i) = \sum_{i\in\mathcal{R}_k}(x_k^i - \zeta_k^i). \tag{17}$$

By recursively applying (17) backwards to the entry time τ_i of each active agent, and recalling the initialization condition, i.e., $\zeta_{\tau_i}^i = 0$, the proof is concluded by $\sum_{i\in\mathcal{R}_k}(x_{k+1}^i - \zeta_{k+1}^i) = \sum_{i\in\mathcal{R}_k}(x_{\tau_i}^i - \zeta_{\tau_i}^i) = \sum_{i\in\mathcal{R}_k} x_{\tau_i}^i$. ■

This invariance highlights the role of the memory states as storage variables that accumulate the virtual displacement induced by topological changes, making any drift in the sum of states fully reversible through the dissipation process.

Corollary 1. *Consider an eventually closed OMAS. If the system converges to a configuration with fully dissipated residual memory, i.e., $\lim_{k\to\infty}\zeta_k^i = 0$ for all $i \in \mathcal{V}_{cl}$, then, property (P1-O) is satisfied, i.e., $\lim_{k\to\infty}\sum_{i\in\mathcal{V}_k} x_k^i = \sum_{i\in\mathcal{V}_{cl}} x_{\tau_i}^i$.*

B. Analysis of Convergence to Critical Points

In this section, we demonstrate that the eventually closed OMAS converges to a stable configuration in Theorem 2, which builds upon the following technical lemma.

Lemma 1. *Let Assumptions 1-3 hold. Let H_k be the Hessian matrix of the global augmented potential $V_k = \sum_{i\in\mathcal{V}_k} V^i(x_k, \zeta_k^i)$ with $V^i(x_k, \zeta_k^i)$ in* (14)*. The global minimum Lipschitz constant L_k is bounded by the maximum local variable L_k^i in* (11)*, i.e., $L_k \le \max_{i\in\mathcal{V}_k} L_k^i$.*

Proof: The global Hessian H_k is a block matrix with diagonal blocks $H_k^{ii} \in \mathbb{R}^{2m\times 2m}$ accounting for the agent's

internal dissipation and the sum of local curvatures, and off-diagonal blocks $H_k^{ij} \in \mathbb{R}^{2m\times 2m}$, $\forall j \in \mathcal{N}_k^i$, representing the coupling terms, i.e.,

$$H_k^{ii} = \begin{bmatrix} \sum_{j\in\mathcal{N}_k^i} \nabla^2_{x^i} P^{ij} & O_m \\ O_m & \gamma I_m \end{bmatrix}, H_k^{ij} = \begin{bmatrix} \nabla^2_{x^i x^j} P^{ij} & O_m \\ O_m & O_m \end{bmatrix}, \tag{18}$$

with $O_m, I_m \in \mathbb{R}^{m\times m}$ being zero and identity matrices, respectively. From the antisymmetric property, it follows that

$$\nabla^2_{x^i} P^{ij} = -\nabla_{x^i}(\nabla_{x^j} P^{ij}) = -\nabla^2_{x^i x^j} P^{ij}, \tag{19}$$

and $\nabla^2_{x^i x^j} P^{ij} = \nabla^2_{x^j x^i} P^{ji}$, thus H_k is symmetric. Given its symmetry, the spectral norm is bounded by its block infinity norm, defined as the maximum sum of the spectral norms of the blocks along a block row

$$\|H_k\| \le \max_{i\in\mathcal{V}_k} \textstyle\sum_{j\in\mathcal{N}_k^i\cup\{i\}} \|H_k^{ij}\|. \tag{20}$$

Substituting (18) into (20), applying the triangle inequality to the diagonal block, and using (19), we obtain for each i

$$\begin{aligned}\sum_{j\in\mathcal{N}_k^i\cup\{i\}} \|H_k^{ij}\| &\le \gamma + \sum_{j\in\mathcal{N}_k^i} \|\nabla^2_{x^i} P^{ij}\| + \sum_{j\in\mathcal{N}_k^i} \|\nabla^2_{x^i x^j} P^{ij}\| \\ &\le \gamma + 2\textstyle\sum_{j\in\mathcal{N}_k^i} \|\nabla^2_{x^i} P^{ij}\| = L_k^i.\end{aligned}$$

Since $L_k = \|H_k\|$ follows from the symmetry of H_k, it holds that $L_k \le \max_{i\in\mathcal{V}_k} L_k^i$. ∎

Theorem 2. *Let Assumptions 1-3 hold and consider an eventually closed OMAS with agents executing Algorithm 1. If $T_c \ge \bar{\rho}$, then the system state converges to a stable configuration with properties (P1-O) and (P2-O) satisfied.*

Proof: Let us first observe that, since $\mathcal{D}_k = \emptyset$ for all $k \ge k_{cl}$, the injection of the transient variables z_k^{ij} into ζ_k^i in (6) is null. Consequently, the system dynamics for each agent $i \in \mathcal{V}_{cl}$ reduces to the evolution of the augmented state $s_k^i = [(x_k^i)^\top, (\zeta_k^i)^\top]^\top \in \mathbb{R}^{2m}$ updating along the joint descent direction $D_k^i = [(d_k^i)^\top, (\delta_k^i)^\top]^\top \in \mathbb{R}^{2m}$, yielding $s_{k+1}^i(\alpha) = s_k^i + \alpha\Delta T D_k^i$. Let $V_k = \sum_{i\in\mathcal{V}_k} V^i(x_k, \zeta_k^i)$ be the Lyapunov candidate. Since its gradient is Lipschitz continuous with a constant bounded by $L_k = \max_{i\in\mathcal{V}_k} L_k^i$ as for Lemma 1, we can apply the descent lemma [23], i.e.,

$$\Delta V_k \le \alpha_k^\star \Delta T \nabla_s V_k^\top D_k + L_k(\alpha_k^\star \Delta T)^2 \|D_k\|^2/2, \tag{21}$$

with $\Delta V_k = V_{k+1} - V_k$, $s_k = [(s_k^1)^\top, \dots, (s_k^{n_{cl}})^\top]^\top$ and $D_k = [(D_k^1)^\top, \dots, (D_k^{n_{cl}})^\top]^\top$, and with $\alpha_k^\star$ common to all agents. We now analyze the scalar product $\nabla_s V_k^\top D_k$. Using (6)-(8), the product for each agent i is

$$\begin{aligned}(\nabla_{s^i} V_k)^\top D_k^i &= -\begin{bmatrix}(\nabla_{x^i} P_{cl})^\top & (\gamma\zeta_k^i)^\top\end{bmatrix}\begin{bmatrix}\nabla_{x^i} P_{cl} + \gamma\zeta_k^i \\ \gamma\zeta_k^i\end{bmatrix} \\ &= -\left((\nabla_{x^i} P_{cl})^\top(\nabla_{x^i} P_{cl} + \gamma\zeta_k^i) + (\gamma\zeta_k^i)^\top(\gamma\zeta_k^i)\right) \\ &= -\left(\|\nabla_{x^i} P_{cl}\|^2 + (\nabla_{x^i} P_{cl})^\top(\gamma\zeta_k^i) + \|\gamma\zeta_k^i\|^2\right).\end{aligned} \tag{22}$$

By computing $\|D_k^i\|$, we observe that

$$\begin{aligned}\|D_k^i\|^2 &= \left\|\begin{bmatrix}\nabla_{x^i} P_{cl} + \gamma\zeta_k^i \\ \gamma\zeta_k^i\end{bmatrix}\right\|^2 = \|\nabla_{x^i} P_{cl} + \gamma\zeta_k^i\|^2 + \|\gamma\zeta_k^i\|^2 \\ &= \|\nabla_{x^i} P_{cl}\|^2 + 2(\nabla_{x^i} P_{cl})^\top(\gamma\zeta_k^i) + 2\|\gamma\zeta_k^i\|^2.\end{aligned} \tag{23}$$

Considering (22) and (23), it follows

$$\|\nabla_{x^i} P_{cl}\|^2 + (\nabla_{x^i} P_{cl})^\top(\gamma\zeta_k^i) + \|\gamma\zeta_k^i\|^2 = \tfrac{1}{2}(\|D_k^i\|^2 + \|\nabla_{x^i} P_{cl}\|^2).$$

Since $\|\nabla_{x^i} P_{cl}\|^2 \ge 0$, from (22) it holds

$$(\nabla_{s^i} V_k)^\top D_k^i \le -\|D_k^i\|^2/2. \tag{24}$$

Substituting (24) into (21), it follows that

$$\begin{aligned}\Delta V_k &\le -\alpha_k^\star \Delta T \|D_k\|^2/2 + L_k(\alpha_k^\star \Delta T)^2 \|D_k\|^2/2 &(25)\\ &\le -\alpha_k^\star \Delta T \|D_k\|^2 \left(1 - \alpha_k^\star \Delta T L_k\right)/2. &(26)\end{aligned}$$

Since the min-consensus protocol ensures $\alpha_k^\star < \frac{1}{\Delta T L_k}$, the term in parentheses is non-negative, ensuring $\Delta V_k < 0$. Moreover, since V_k is bounded from below by construction, according to the monotone convergence theorem, the sequence $\{V_k\}$ converges, resulting in $\Delta V_k \to 0$ and thus $\|D_k\| \to 0$. This implies that $\lim_{k\to\infty} \nabla_x P_{cl} = 0$ and $\lim_{k\to\infty} \zeta_k^i = 0$, yielding (P2-O) and, by Corollary 1, (P1-O). ∎

V. Numerical Validation

To validate the proposed strategy, we consider a discrete-time OMAS, starting with 50 agents. The physical sampling time is $\Delta T = 0.1$s, while the communication sampling time is set to $\Delta T_l = 0.005$s, leading to $T_c = 20$ communication rounds. The pairwise potential is $P^{ij}(x_k^i, x_k^j) = \frac{1}{2}\|x_k^i - x_k^j\|^2 - \ln(\|x_k^i - x_k^j\|^2)$, with $\Omega^{ij} = \{(x^i, x^j) : x^i, x^j \in \mathbb{R}^2, x^i \ne x^j\}$. Actuator bounds U_i are uniformly sampled in $[2, 3]$, while the dissipation gain is set to $\gamma = 2$. The topology $\mathcal{G}_k$ is an undirected ErdsRényi graph with link probability $p = 0.5$, constrained to be connected. Join events $\mathcal{A}_k$ occur at $k \in \{1100, 1450, 1550\}$, and leave events $\mathcal{D}_k$ occur at $k \in \{200, 500, 600, 700, 1500, 1570\}$.

We compare the proposed min-consensus strategy against two baselines: Consensus ADMM (C-ADMM) [17] and a Distributed First-Order (DFO) algorithm based on the distributed gradient method [24], both employed to solve the step size optimization problem (13). Unlike the proposed method, which computes a suboptimal step size in finite steps, C-ADMM and DFO are asymptotic algorithms. Both are here truncated at T_c iterations for a fair comparison.

Figure 1 shows the global augmented potential V_k, which exhibits strictly non-increasing behavior during intervals of fixed topology for all approaches. This confirms that the consensus ensures a stable descent. Discontinuities in V_k appear at topological changes (green and red dashed lines), where new agents enter the system or energy is transferred to the dissipation states ζ_k^i after departures. However, the difference between the proposed method and the baselines mainly emerges in the capability of preserving the system's structural properties. Figure 2 shows the evolution of the sum of the states, including the potential-based protocol without compensation mechanism (referred to as the closed-system baseline). For all methods, when agents leave the network, a transient drift occurs between the sum of states and the target $\sum_{i\in\mathcal{V}_k} x_{\tau_i}^i$. To preserve this sum, agents must correctly dissipate the residual interaction energy upon departures and agree on an identical step-size. The proposed min-consensus strategy guarantees this agreement in finite time, at the cost of

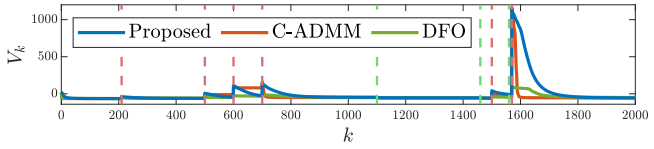

Fig. 1. Evolution of the global augmented potential V_k.

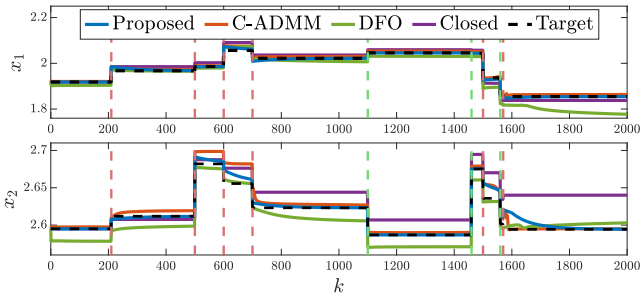

Fig. 2. Evolution of the sum of the states (target value is dashed).

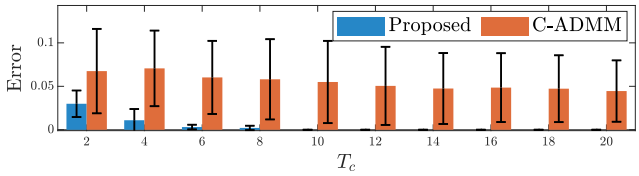

Fig. 3. Centroid error as a function of the communication rounds.

a slightly more conservative step-size, while the dissipation mechanism ensures convergence to the correct target after each topological change. In contrast, C-ADMM and DFO, being asymptotic methods, fail to reach exact consensus when truncated at T_c rounds, resulting in a persistent drift. Finally, the closed-system baseline consistently exhibits a bias in the sum of the states as it is not able to compensate for the residual energy of departed agents.

Comparison with varying communication rounds. To further support the previous observations, Figure 3 reports the centroid error achieved by the proposed method and the best-performing baseline, i.e., C-ADMM, when varying the number of communication rounds (2 to 20) over a network of $n = 20$ agents with random topologies of diameter $\rho_k = 10$ and random arrival/departure events. The plot highlights that the centroid error reaches zero as soon as $T_c \geq \bar{\rho}$ with our method. Moreover, when $T_c < \bar{\rho}$, i.e., only a few communication rounds are made, the min-consensus does not fully converge, yet it remains robust, resulting in significantly lower centroid errors than C-ADMM on all tested values of T_c.

VI. Conclusion

This letter proposed a discrete-time potential-based coordination framework for OMASs with heterogeneous actuator saturation. Cumulative dissipation states handle network changes by dissipating residual energy from departing agents, while a distributed min-consensus strategy computes the step size, guaranteeing descent of an augmented global potential. Theoretical analysis and numerical validations show that the active agents asymptotically converge to a stable configuration, in which the sum of their states equals the sum of the initial states. Future work will address jointly connected networks and robustness to packet losses and delays.

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