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The route to chaos under stabilizing passive monetary policy

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Abstract

Can a locally stable equilibrium undergo a global indeterminate solution? This paper investigates the conditions for the emergence of homoclinic chaos in the neighborhood area of local stability, within a generalized monetary model, as developed in Dupor (2001). Specifically, under the same parameter configuration that ensures uniqueness, and along with a passive monetary policy rule followed by the monetary authority, this analysis demonstrates that a stable limit cycle may bifurcate from a homoclinic orbit connecting the equilibrium to itself. The resulting complex dynamics reveals a new attracting set in the global structure of the model, where equilibrium trajectories implied by the eigenvalues of the system are eventually trapped, thus constraining the dynamics in an outer region away from the desired steady state. Consequently, a passive monetary policy action, that is traditionally expected to stabilize the economy, may instead generate an undesired and not predetermined macroeconomic outcome.

Keywords Passive monetary policy · Homoclinic orbit · Global indeterminacy · Long-term unpredictability

JEL classification C61 · C62 · E12 · E52 · E63

1 Introduction

The stabilization of a macroeconomic equilibrium through monetary policy actions has been receiving growing attention in the literature. As emphasized by Christiano and Takahashi (2018), the standard New Keynesian monetary framework can exhibit

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multiple equilibria when either fiscal or monetary authorities intervene in the economy to counteract the emerging downward economic cycles with active stimuli. Such irregular dynamics may prevent the convergence towards a long-run stable equilibrium and lead instead the economic system to a low-growth trapping region.

Pioneering contributions by Benhabib et al. (2001a, b) investigated the implications of interest rate feedback rules in the presence of sticky prices. They established that uniqueness of equilibrium prevails when an active rule is followed, meaning that the monetary authority increases the real interest rate by more than 1% point in response to a one-point rise in current inflation. Alternatively, a passive stance, which calibrates the interest rate increase to prevent possible crowding-out effects on private investments, can result in equilibrium indeterminacy. More recently, Barnett et al. (2022) demonstrated that even under an active monetary policy, complex and chaotic dynamics can emerge within the predicted area of local stability implied by the New Keynesian framework.

Dupor (2001) found that the results are reversed when the firms' decision regarding physical capital investment is incorporated into the model. In this case, the strategy to follow a passive monetary policy may ensure uniqueness and determinacy of the equilibrium, whereas an active monetary policy can trigger indeterminacy of equilibria. The underlying intuition is straightforward: if households expect high current inflation, the monetary authority responds by raising the real interest rate. But with investment in the model, to avoid arbitrage between private stocks and government bonds, the capital rental rate moves upwards too, increasing the marginal productivity of capital and thus reinforcing inflationary expectations. Hence, Dupor (2001) interestingly speculates that the possible source of indeterminacy emerging in New Keynesian models might stem from the interactions within the different sectors of the economy and not from monetary policy mismanagement.

Traditionally, as it is common in the New Keynesian macroeconomic setup, the analyses of monetary policies mainly focus on achieving a unique and locally stable equilibrium – often interpreted as the "desired" steady state – by valuing its adherence to a standard Taylor-type rule or a money growth rule. However, when the monetary authority opts for a regime switch, due to unexpected shocks in current inflation or economic recessions, coupled with shifts in agents' expectations of future economic outcomes, the economy can fall into unwanted high-inflationary and low-output poverty traps.

The novelty of the analysis proposed in this paper lies in demonstrating the possibility of homoclinic chaos, giving rise to periodic oscillations in the vicinity of the area of local determinacy found in the three-dimensional extended system proposed by Dupor (2001). Specifically, we show that the equilibrium trajectories, when studied in their global evolution, may start to cycle around the steady state, or move chaotically within the tubular neighbourhood of an outer bounded region described by a homoclinic orbit that results in a stationary trapping region (see, Bella et al. 2021). This finding challenges the conventional wisdom about the inherent effectiveness of a passive monetary policy to stabilize the economy. Additionally, this apparent stability may conceal a deep structural instability once a global dynamic analysis is investigated.

The fact is that the standard analysis of the local linearization of a nonlinear macroeconomic system provides only partial information of its dynamic behaviour near the equilibrium. In order to get the full complexity of the global dynamics one must preserve the nonlinearity of the model. This approach is able to reveal the possible transitions from local stability to global oscillations or chaotic patterns, including periodic orbits, strange attractors, and homoclinic or heteroclinic connections (see, Brito and Venditti 2010; Antoci et al. 2022). A rule-of-thumb follows consequently in terms of monetary policy decisions, should crucial baseline parameters escape the range that permit a self-fulfilling globally determinate equilibrium, and drive to indeterminate or volatile outcomes, potentially requiring adjustments in the followed monetary regime, and perhaps change the commitment from an inflation-targeting Taylor rule to a more restrictive Volcker-type rule to limit banks' investments, or rather set a money-growth-based policy to prevent explosive dynamics.

Understanding this global dynamic behaviour becomes therefore essential to prevent the economy from drifting towards undesired long-run indeterminate equilibria, even though local analysis would predict stability. To this end, the results presented in this paper reconcile Dupor's (2001) findings with the broader literature, suggesting that a passive monetary policy may appear stabilizing locally, whereas it can expose the system to global indeterminacy. This also aligns with the conventional view that a weak monetary response to inflation will tend to foster instability, while stronger and active policies may prevent the emergence of multiple equilibria.

Numerous studies have documented so far global indeterminacy in the presence of multiple steady states – typically involving a low-growth stable equilibrium and a high-growth unstable one. The former represents an undesirable trap, while the latter is the desired (targeted) equilibrium that the economy cannot reach without a policy intervention by governments or central banks (see, for example, Gaspar et al. 2023; Antoci et al. 2022; Ilabaca and Milani 2021; Cornaro and Agliari 2011).

The novelty of the type of indeterminacy analyzed here arises even in the presence of a unique (locally stable) equilibrium, when the rupture of a Shilnikov-type homoclinic orbit – connecting the equilibrium to itself – generates a stable attracting limit cycle. In this setting, trajectories originating near the homoclinic path will remain confined within its vicinity, producing persistent oscillations and long-run unpredictability. From an economic standpoint, this would also support the economic definition of indeterminacy, since two different economies, or monetary systems in this analysis, with very small differences in the initial setting will begin at some point to behave differently over time in the transition towards the long run equilibrium.

The paper proceeds as follows. In Sect. 2, we first describe the economy proposed by Dupor (2001), in its generalized version, and derive the associated three-dimensional system of first-order differential equations. Sect. 3 investigates the parametric configuration for the emergence of a determinate equilibrium. In Sect. 4, we establish the conditions for global indeterminacy of the equilibrium through homoclinic chaos and infer the related policy implications. The conclusion reassesses the main findings of the paper and the final Appendix provides all necessary proofs.

2 The model

We begin by providing a full derivation of the extended monetary model described by Dupor (2001).[1] The general equilibrium of our economy consists of identical household-firms (henceforth, called $\mathcal{R}$), whose optimization strategy is to maximize the following current flow of utility in separable terms:

$$U = \int_{0}^{\infty} \left[\log(c) + \log(m) - \tfrac{1}{1+\xi} n^{1+\xi} - \tfrac{\gamma}{2} \left(\tfrac{\dot{P}}{P} - \pi^{*} \right)^{2} \right] e^{-\rho t} dt \qquad (1)$$

over consumption, c, real money balances, m, and the measure of labor supply, n, which enters negatively to capture the disutility of labor, where $\xi \geq 0$ denotes the labor supply elasticity. Moreover, assuming the current price level, P, with its time-variation $\dot{P}$, the deviation of price variation from the target steady-state inflation rate, π^{*}, will also negatively enter the utility function, being $\gamma > 0$ a parameter measuring aversion to inflation. Finally, $\rho \in [0,1]$ is the social discount rate.

On the production side, $\mathcal{R}$ produces according to a standard Cobb-Douglas function:

$$y = \widehat{k}^{\alpha}\, \widehat{n}^{1-\alpha} \qquad (2.1)$$

which depends on the units of physical capital, $\widehat{k}$, and units of labor, $\widehat{n}$, employed in the producing sector, and where $\alpha \in [0,1]$ is the share of physical capital in production. The function in (2.1) is subject to a standard Dixit-Stiglitz constraint on the demand function, Y^{d}, given the economy's aggregate price level, $\bar{P}$:

$$y = Y^{d} d \left(\frac{P}{\bar{P}} \right) \qquad (2.2)$$

where d satisfies $d(1) = 1,\ d'(1) = \phi < -1$. In detail, ϕ can be economically interpreted as the elasticity of substitution across varieties of produced (i.e., differentiated) goods. In this general setting, the representative household-firm accumulates both physical capital and financial assets. Capital accumulates at the pace of planned investment, i, net of depreciation of physical capital, *k*, at the rate $\delta \in [0,1)$:

$$\dot{k} = i - \delta k \qquad (3)$$

Alternatively, non-capital (i.e., financial) wealth s evolves according to the law of motion:

[1] We refer to the extensions of the basic model outlined in Dupor (2001), Sect. 4, pp. 103–105. Since it has no impact on the results of the dynamic stability of the model, we assume directly that the parameter v (weighting the labor force in the utility function) be set to one, which is also considered in Dupor (2001) when the numerical example is presented.

$$\dot{s} = (R - \pi)\, s - Rm + \frac{P}{\bar{P}} y - w\widehat{n} - r\widehat{k} + rk + wn - i - c \tag{4}$$

where r is the rental rate of capital, and R is the nominal interest rate.[2] Additionally, the current inflation rate can be defined by the standard formula

$$\frac{\dot{P}}{P} = \pi \tag{5}$$

The formal structure of the maximization problem faced by $\mathcal{R}$ implies the optimization of the objective utility function in (1), subject to the constraints in (3), (2.2) with (2.1), (5) and (4), and given the initial endowments:

$$s(0) = s_0; \quad k(0) = k_0; \quad P(0) = P_0$$

The associated Hamiltonian function is therefore:

$$\mathcal{H} = \log(c) + \log(m) - \tfrac{1}{1+\xi} n^{1+\xi} - \tfrac{\gamma}{2}\left(\tfrac{\dot{P}}{P} - \pi^*\right)^2 + \eta\,(i - \delta\, k) + \mu \left[Y^d d\left(\tfrac{P}{\bar{P}}\right) - \widehat{k}^{\alpha}\,\widehat{n}^{1-\alpha}\right]$$

$$+\lambda\left[(R - \pi)\, s - Rm + \frac{P}{\bar{P}}\widehat{k}^{\alpha}\,\widehat{n}^{1-\alpha} - w\widehat{n} - r\widehat{k} + rk + wn - i - c\right] + \theta\,\left(\pi\, P - \dot{P}\right)$$

with the following set of first-order necessary conditions:

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{1}{c} - \lambda = 0 \tag{6}$$

$$\frac{\partial \mathcal{H}}{\partial m} = \frac{1}{m} - \lambda\, R = 0 \tag{7}$$

$$\frac{\partial \mathcal{H}}{\partial n} = -n^{\xi} + \lambda\, w = 0 \tag{8}$$

$$\frac{\partial \mathcal{H}}{\partial \widehat{n}} = -\mu\,(1 - \alpha)\,\widehat{k}^{\alpha}\,\widehat{n}^{-\alpha} + \lambda\,(1 - \alpha)\,\frac{P}{\bar{P}}\widehat{k}^{\alpha}\,\widehat{n}^{-\alpha} - \lambda\, w = 0 \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial \widehat{k}} = -\mu\,\alpha\,\widehat{k}^{\alpha-1}\widehat{n}^{1-\alpha} + \lambda\,\alpha\,\frac{P}{\bar{P}}\widehat{k}^{\alpha-1}\widehat{n}^{1-\alpha} - \lambda\, r = 0 \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial i} = \eta - \lambda = 0 \tag{11}$$

[2] Without any loss of generality, and because of no impact in terms of the mathematical result, we can easily assume herein the absence of any lump sum tax, originally assumed by Dupor (2001) to negatively influence Eq. (4) in the original setting of the model.

$$\frac{\partial \mathcal{H}}{\partial \pi} = -\gamma \ (\pi \ - \pi^*) \ \frac{1}{P} + \theta \ = 0 \tag{12}$$

$$\dot{\lambda} \ = \lambda \rho \ - \frac{\partial \mathcal{H}}{\partial s} = \lambda \rho \ - \lambda \ (R - \pi) \tag{13}$$

$$\dot{\eta} \ = \eta \rho \ - \frac{\partial \mathcal{H}}{\partial k} = \eta \rho \ + \eta \delta \ - \ \lambda r \tag{14}$$

and

$$\dot{\theta} \ = \rho \theta \ - \tfrac{\partial \mathcal{H}}{\partial P} = \rho \theta \ - \left[\gamma \ \left(\tfrac{\dot{P}}{P} - \pi^* \right) \tfrac{1}{P^2} + \mu \ \tfrac{1}{\overline{P}} Y^d d' \ \left(\tfrac{P}{\overline{P}} \right) + \lambda \ \tfrac{1}{\overline{P}} \widehat{k}^{\alpha} \ \widehat{n}^{1-\alpha} \right] \tag{15}$$

Taking the time derivatives of (12) we obtain that:

$$\begin{aligned} \dot{\theta} \ &= \tfrac{d}{dt} \left[\tfrac{\gamma}{P} \left(\tfrac{\dot{P}}{P} - \pi^* \right) \right] = \gamma \ \left[-\tfrac{\dot{P}}{P^2} \left(\tfrac{\dot{P}}{P} - \pi^* \right) + \tfrac{1}{P} \tfrac{d \left(\frac{\dot{P}}{P} - \pi^* \right)}{dt} \right] \\ &= \gamma \ \left[-\tfrac{\dot{P}}{P^2} \left(\tfrac{\dot{P}}{P} - \pi^* \right) + \tfrac{1}{P} \tfrac{d(\pi - \pi^*)}{dt} \right] = \gamma \ \left[-\tfrac{\dot{P}}{P^2} \left(\tfrac{\dot{P}}{P} - \pi^* \right) + \tfrac{1}{P} \dot{\pi} \right] \end{aligned} \tag{16}$$

Substituting and equating the expression (16) into (15), we obtain:

$$\gamma \ \left[-\tfrac{\dot{P}}{P} \left(\tfrac{\dot{P}}{P} - \pi^* \right) + \dot{\pi} \right] = \rho \gamma \ \left(\tfrac{\dot{P}}{P} - \pi^* \right) - \gamma \ \left(\tfrac{\dot{P}}{P} - \pi^* \right) \tfrac{\dot{P}}{P} - \ \mu Y^d d' \ \left(\tfrac{P}{\overline{P}} \right) - \lambda \widehat{k}^{\alpha} \widehat{n}^{1-\alpha}$$

which becomes

$$\gamma \dot{\pi} \ = \ \rho \gamma \ (\pi \ - \pi^*) - \ \mu Y^d d' \ \left(\frac{P}{\overline{P}} \right) - \lambda \widehat{k}^{\alpha} \widehat{n}^{1-\alpha} \tag{17}$$

We impose now the consistency conditions for a symmetric equilibrium: $\widehat{n} = n$, $\widehat{k} = k$, $\overline{P} = P$.[3] Taking the log-derivatives of (6), we derive:[4]

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda}$$

where we can substitute Eq. (13), and obtain:

[3] Under the symmetric equilibrium condition, and the fact that in equilibrium $\mu \neq 0$, the problem is quasi-concave since the Hamiltonian $\mathcal{H}$ is clearly the sum of a series of quasi-concave functions.

[4] We can easily observe that Eq. (7) becomes redundant in the derivation of the autonomous system of differential equations implied by the maximization problem, since when moving to the growth rates we have identically that $\frac{\dot{c}}{c} = \frac{\dot{m}}{m} = -\frac{\dot{\lambda}}{\lambda}$.

$$\frac{\dot{c}}{c} = R - \rho - \pi \tag{18}$$

Additionally, recalling (2.1) and considering the expressions in (6) and (8), we can obtain from (9) that:

$$(1-\alpha)\mu \frac{y}{n} + n^{\xi} = (1-\alpha)\frac{y}{cn}$$

and therefore:

$$\mu = \frac{1}{c} - \frac{n^{1+\xi}}{(1-\alpha)y} = \frac{(1-\alpha)y - cn^{1+\xi}}{(1-\alpha)yc} \tag{19}$$

which can be substituted into (10) to obtain:

$$n = \left[\frac{(1-\alpha)rk}{\alpha c}\right]^{\frac{1}{1+\xi}} \tag{20}$$

Moreover, given the identity $\eta = \lambda$ implied by (11), from (13) and (14) we can observe that:

$$r = R - \pi + \delta \tag{21}$$

which links the rental rate of capital, r, and the nominal interest rate, R.

Finally, substituting (19) into (17), and recalling the assumptions governing the demand function in (2.2), then:

$$\dot{\pi} = \rho(\pi - \pi^{*}) - \frac{(1+\phi)}{\gamma}\frac{k^{\alpha} n^{1-\alpha}}{c} + \frac{\phi n^{1+\xi}}{(1-\alpha)\gamma} \tag{22}$$

Hence, substituting the obtained expressions for R, r and n, we get the three-dimensional system of differential equations described in Dupor (2001), which introduces physical capital investment in the New Keynesian monetary framework. The optimization strategy of the representative household-firm, in presence of a symmetric equilibrium, allows us to derive the following forward-backward three-dimensional system of differential equations, henceforth *M*:

$$\dot{\pi} = \rho(\pi - \pi^{*}) - frac1 + \phi\gamma \left(\frac{1-\alpha}{\alpha}\right)^{b}\left(\frac{k^{\alpha+b}}{c^{1+b}}\right)(R+\delta-\pi)^{b} + \frac{\phi k}{\alpha\gamma c}(R+\delta-\pi)$$

$$\dot{c} = c(R - \rho - \pi) \tag{M}$$

$$\dot{k} = k^{\alpha+b}\left(\frac{1-\alpha}{\alpha c}\right)^{b}(R+\delta-\pi)^{b} - \delta k - c$$

where k is the predetermined variable, since $k(0) = k_0$ is set exogenously, and c and π are the non-predetermined (endogenous) variables. Lastly, $b = (1-\alpha)/(1+\xi)$ simplifies the notation.

The nominal interest rate is set by the central bank according to the following rule:

$$R = R^* + \psi(\pi - \pi^*) + \omega\left(\frac{y - y^*}{y^*}\right) \tag{23}$$

as a function of both current inflation gap relative to the target, π^*, and the output deviation from its long run desired level, y^*. More crucially, for the specification of the model outlined hereafter, the parameter $\omega \geq 0$ represents the impact of the output gap on the nominal interest rate setting. Using the specifications given in (2.1), (19) and (20), we can now transform (23) into the following expanded Taylor rule:

$$R = R^* + \psi(\pi - \pi^*) + \tfrac{\omega}{y^*}k^{\alpha+b}\left(\tfrac{1-\alpha}{\alpha c}\right)^b (R + \delta - \pi)^b - \omega \tag{24}$$

which defines the nominal interest rate as an implicit function of the variables of the model, $R = R(\pi, c, k)$.[5]

In equilibrium, at $\dot{c} = \dot{\pi} = \dot{k} = 0$, the steady state is represented by the triplet $P^* \equiv (c^*, \pi^*, k^*)$, where:

$$c^* = \left(\tfrac{\alpha}{z}\right)^{\frac{1}{b}}\left(\tfrac{1-\alpha}{\alpha}\right)(\rho + \delta)^{\frac{b-1}{b}}(k^*)^{(\alpha+b-1)/b} \tag{25}$$

$$k^* = \left[\frac{\left(\frac{\alpha}{z}\right)^{\frac{1}{b}}(1-\alpha)(\rho+\delta)^{\frac{b-1}{b}}}{z(\rho+\delta) - \alpha\delta}\right]^{b/\beta} \tag{26}$$

given $z = \phi/(1+\phi)$. Additionally, π^* is a solution of the Eq. (24), with $R = \rho + \pi$, at $\pi = \pi^*$. Hence:

$$\pi^* = R^* + \tfrac{\omega}{y^*}k^{*\alpha+b}\left(\tfrac{1-\alpha}{\alpha c^*}\right)^b(\rho+\delta)^b - \omega - \rho \tag{27}$$

System M can be approximated around the stationary equilibrium $P^* \equiv (c^*, \pi^*, k^*)$ and put in the following convenient matrix notation:

$$\begin{pmatrix}\dot{\pi}\\ \dot{c}\\ \dot{k}\end{pmatrix} = \mathrm{J}\begin{pmatrix}\pi - \pi^*\\ c - c^*\\ k - k^*\end{pmatrix} \tag{28}$$

[5] Notice that, when $\omega = 0$, the nominal interest rate will solely depend on the inflation rate. In that case, we move back to the standard representation where its derivative, $\partial R/\partial\pi = \psi \lesseqgtr 1$, represents the standard monetary policy rule that measures the reaction of nominal interest rate to raising inflation (see, Benhabib et al. 2001a, b).

where $\mathbf{J}$ is the Jacobian matrix associated with system *M*, whose elements are computed in the Appendix.

Studying the stability properties of system M requires the computation of the structure of eigenvalues, Λ_i, associated with $\mathbf{J}$ in (28), that are the solutions of the following characteristic equation:

$$Det\left(\lambda\mathbf{I}-\mathbf{J}\right)=\Lambda^3-Tr\left(\mathbf{J}\right)\Lambda^2+B\left(\mathbf{J}\right)\Lambda\ -Det\left(\mathbf{J}\right) \tag{29}$$

given the identity matrix, $\mathbf{I}$, and where $Tr(\mathbf{J})$, $Det(\mathbf{J})$ and $B(\mathbf{J})$ are the trace, the determinant, and the sum of principal minors of order 2 of $\mathbf{J}$, respectively.

Since (29) is a standard cubic equation, it can be solved with the Cardano formula which provides the following solutions:

$$\Lambda_1=\frac{Tr\left(\mathbf{J}\right)}{3}+\left(u+v\right)\equiv\ \varrho \tag{30.a}$$

$$\Lambda_{2,3}=\tfrac{Tr(\mathbf{J})}{3}-\tfrac{(u+v)}{2}\pm\sqrt{3}\tfrac{(u-v)}{2}i\equiv\ \kappa\ \pm\ \vartheta\, i \tag{30.b}$$

with $u=\sqrt[3]{-\frac{q}{2}+\sqrt{\varDelta}}$ and $v=\sqrt[3]{-\frac{q}{2}-\sqrt{\varDelta}}$, where $\varDelta\ =\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2$ is the discriminant. Furthermore, $p=\frac{3\mathrm{B}(\mathbf{J})-\mathrm{Tr}(\mathbf{J})^2}{3}$, $q=-\ Det\left(\mathbf{J}\right)-2\frac{Tr(\mathbf{J})^3}{27}+\frac{Tr(\mathbf{J})B(\mathbf{J})}{3}$, and $\ i=\sqrt{-1}$ is the imaginary unit. Crucially, the sign of $\ \varDelta\ $ discriminates the area with three real eigenvalues ($\varDelta\ <0$), from the area with one real and a pair of complex conjugate eigenvalues ($\varDelta\ >0$).

As explained in the next section, for the scope of this paper, we are first interested in the study of the area where the equilibrium is locally determinate (i.e., of local uniqueness), with a structure of three real eigenvalues ($\varDelta\ <0$), and in presence of a passive monetary policy. This aligns with Dupor's (2001) parametrization. We aim then to investigate the possible deviation from the implied transitional dynamics via a bifurcation analysis that produces an unexpected chaotic limit set.

Remark 1 Dupor (2001*) shows that*, *under a passive monetary policy rule*, $\frac{\partial\, R}{\partial\, \pi}=\psi\ <1$, *the equilibrium associated with the generalized model version is determinate for a labor supply elasticity* $\xi\ =2$, *and a weight of output gap on the nominal interest rate*, $\ 0<\omega\ <0.5$.

Figure 1 clearly shows that when we calibrate the model at a labor supply elasticity $\xi\ =2$, and under a passive monetary policy (i.e., $\psi\ <1$), the eigenvalues are all of real type ($\varDelta\ <0$) if $\ 0.1\leq\ \omega\ <0.5$. Alternatively, when $\ 0\leq\ \omega\ <0.1$ the eigenvalues move to a structure with one real and a pair of complex conjugate ones ($\varDelta\ >0$).

Additionally. fixing ψ=0.5 as in Dupor (2001), we can show in Fig. 2 the two-dimensional relationship between Δ and ω, which confirms the fact that, for any ω>0.1, the eigenvalues remain all real, until ω≳0.5, where a pair of complex conju-

Fig. 1 Structural properties of the eigenvalues

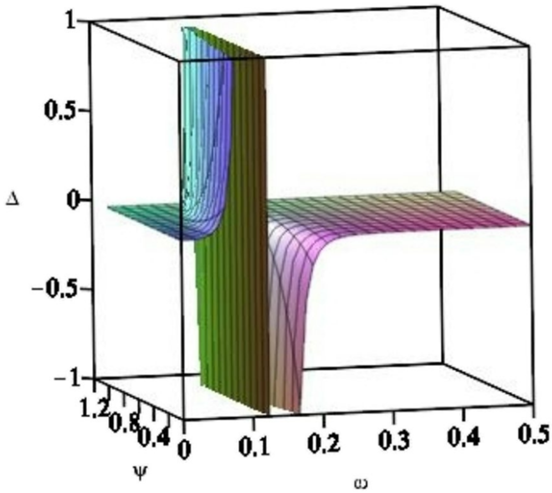
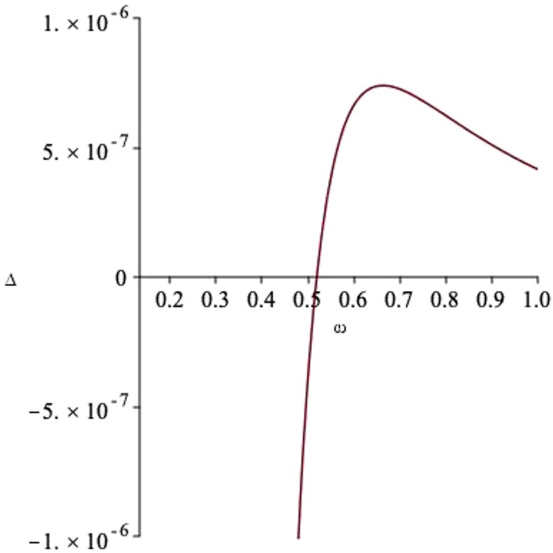


Fig. 2 Properties of the eigenvalues under a passive monetary policy



gate eigenvalues emerges, and the equilibrium moves from real saddle configuration to a saddle-focus type

It is crucial to understand the sign sequence of the eigenvalues, that will be necessary for the application of the bifurcation theorem that we are going to apply in Sect. 3. Hence, we will be interested in the following scenarios:

Remark 2 *The equilibrium is unique (i.e., determinate) if the Jacobian matrix*, $\mathbf{J}$, *has one negative eigenvalue and two eigenvalues with positive real parts.*[6] *Alternatively, the equilibrium is indeterminate if* $\mathbf{J}$ *has one positive eigenvalue and two eigenvalues with negative real parts.*[7]

For this purpose, the well-established Routh-Hurwitz criterion can be applied. Specifically, since the eigenvalues of $\mathbf{J}$ are the solutions of the characteristic Eq. (29), the number of roots with positive real parts is equal to the number of sign variations in the scheme:

$$-1 \quad Tr\left(\mathbf{J}\right) \quad G\left(\mathbf{J}\right) \quad Det\left(\mathbf{J}\right) \tag{31}$$

where $G\left(\mathbf{J}\right) = -B\left(\mathbf{J}\right) + \frac{Det(\mathbf{J})}{Tr(\mathbf{J})}$.

[6] Interestingly, in this latter case the structure of eigenvalues exhibits a time-reversal asymmetry, since if we repeat the analysis in backward time (reverting time to $\tau = -t$) the eigenvalues change to one positive and two with negative real parts, which predicts indeterminacy of the equilibrium instead, breaking the stability condition previously observed. This hints complexity in the dynamics of the model and that the possible chaotic dynamics, not observed in the forward time computation, might exist in the reversal-time system (see, Jiang et al. 2013).

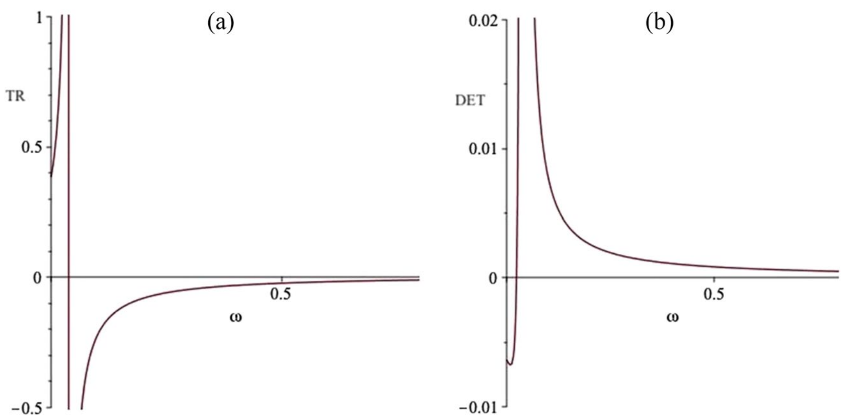


Fig. 3 Sign of trace and determinant

[7] Indeterminacy can also occur when the eigenvalues have three negative real parts, but the case can be excluded in the present analysis, since in our parametric configuration, as shown in Fig. 3, the trace and determinant are always of opposite sign, which prevents from the permanence of sign in the sequence in (31), as required by the Routh-Hurwitz criterion.

In our case, setting the model parameters $\{(\alpha, \gamma, \delta, \rho, \pi^{*}, \phi, \xi)\} \equiv (0.3, 350, 0.025, 0.0045, 0, -21, 2)$ as in Dupor (2001), and assuming a passive monetary policy rule, $\psi = 0.5$, we want to characterize the sign sequence for the elements in the scheme (31). Figure 3 reports the evolution of the trace and the determinant as functions of ω. Numerical results shows that a sign inversion of the two elements occurs at $\bar{\omega} = 0.1190854338$.

It is straightforward to derive that, when $\omega < \bar{\omega}$, $Tr(J) > 0$ and $Det(J) < 0$. Hence, independently of $G(\mathbf{J})$, we have the following sequence of signs:

$$- \quad + \quad \pm \quad -$$

Observing that there can only be one permanence of sign and two variations, then the characteristic equation in (29) must exhibit one negative eigenvalue and two eigenvalues with positive real parts: P^{*} is therefore a saddle of index 1, and the equilibrium is locally unique.

On the contrary, when $\omega > \bar{\omega}$, $Tr(\mathbf{J}) < 0$ and $Det(\mathbf{J}) > 0$. Again, independently of $G(J)$, the sequence becomes:

$$- \quad - \quad \pm \; +$$

where now only one variation and two permanence of sign can be found, meaning that then the characteristic equation in (29) exhibits one positive eigenvalue and two eigenvalues with negative real parts: P^{*} is in this case a saddle of index 2, and the equilibrium is indeterminate.

We can therefore choose ω, all other parameters fixed, as a convenient bifurcation parameter for the analysis of possible global indeterminacy through a homoclinic bifurcation of the equilibrium that switches from a real saddle to a saddle-focus dynamics, when ω approaches its critical value around 0 .5 shown in Fig. 2.

We shall now proceed to prove the conditions for the emergence of global indeterminacy through the rupture of a homoclinic orbit giving rise to an attracting three-dimensional limit cycle (i.e., a chaotic attractor) that may trap the equilibrium trajectories in a basin located off the long run steady state, thus undermining the possible policy actions adopted to stabilize the economy.

3 The Shilnikov homoclinic loop

This section investigates the possible emergence of a Shilnikov-type homoclinic orbit and the resulting complex dynamics that may characterize the global behavior of system M (see, Chua et al. 2001; ch. 12–13). Specifically, we examine the conditions under which the system transitions from a locally stable equilibrium to a globally oscillatory or chaotic dynamics through the rupture of a homoclinic orbit connecting the equilibrium to itself.

We rely on the following theorem, which provides the theoretical foundation for identifying Shilnikov chaos (see, Theorem 6.6 in Kuznetsov 2004, p. 220).

Theorem 1 *Consider a generic system of differential equations* $\dot{y} = f(y, \tau)$, *with the set of variables* $y \in \mathbb{R}^3$ *and the bifurcation parameter* $\tau = \omega - \widehat{\omega} \in R^+$, *such that at* $\tau = 0$ *there exists an equilibrium point* $y_0 = 0$, *with eigenvalues* $\Lambda_1 > 0 > Re\Lambda_{2,3}$, *and a homoclinic orbit* Γ_0. *Assume that the following genericity conditions hold*:

1. $\sigma = \Lambda_1 + Re\Lambda_{2,3} > 0$;
2. $\Lambda_2 \neq \Lambda_3$;

Then, system M exhibits an infinite number of saddle limit cycles $\mathcal{L}$ *in the neighborhood of* Γ_0 *for all sufficiently small* $|\varphi|$, *where* $\varphi(\tau)$ *is the split function associated with* Γ_0.

We aim to characterize the specific region of the parameter space where the above conditions in Theorem 1 are met in system *M*. As previously anticipated, under a passive monetary policy, the generalized economy described in Dupor (2001) can exhibit the structure of eigenvalues requested by Theorem 1 (i.e., one positive real eigenvalue and a pair of complex conjugate eigenvalues with negative real parts) if $\omega \gtrsim 0.5$. This drives the dynamics around the equilibrium to the possible emergence of indeterminacy associated with a chaotic scenario. Then, we want to check the way the passive monetary response to inflation has to be tuned in order to avoid the attracting limit cycle trapping region, and the implied global indeterminacy of the solution, due to a possible homoclinic loop rupture.

As demonstrated in Bella et al. (2017), the first genericity condition in Theorem 1 can be verified by using the following expression:

$$\sigma \equiv \frac{2}{3}Tr(\mathbf{J}) + \frac{(u+v)}{2} \tag{32}$$

which represents the saddle quantity associated to the application of the Cardano formula to the characteristic polynomial in (29). Then, Fig. 4 illustrates (both in

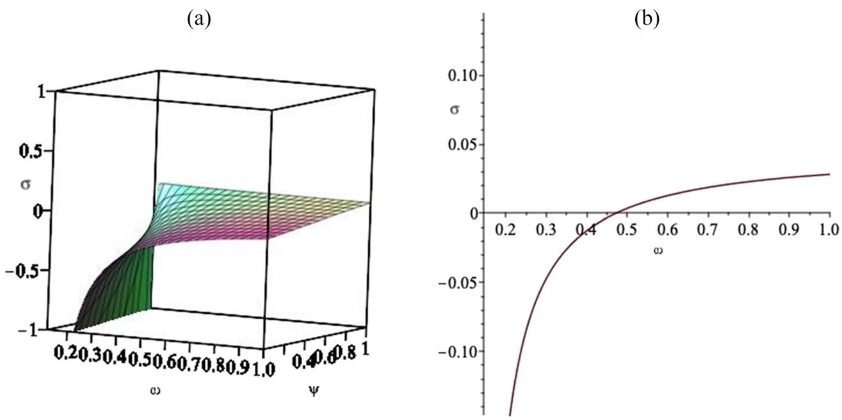


Fig. 4 Saddle quantity properties

three- and two-dimensional space) the combinations of σ for different values of the bifurcation parameter, ω, and the monetary policy parameter, ψ.[8]

In the region of interest, with $\omega > 0.5$, and under a passive monetary policy ($\psi < 1$), the system exhibits a saddle-focus configuration with $\sigma > 0$. Then a sequence of infinite limit cycles may emerge in the vicinity of the homoclinic orbit connecting the equilibrium to itself, giving rise to a Shilnikov chaotic attractor.

Moreover, Theorem 1 requires the existence of a homoclinic orbit, Γ_0, that connects the equilibrium point to itself. By applying the method developed by Shang and Han (2005), we show that Γ_0 emerges if the following condition holds:

$$\varphi \equiv \zeta - \frac{\bar{F}_{1d}}{\varrho}\zeta^2 - \zeta^2\left(\frac{(\kappa - 2\varrho)\bar{F}_{2d}}{4\varrho^2 - 4\varrho\kappa + \kappa^2 + \vartheta^2} + \frac{\vartheta \bar{F}_{3d}}{4\varrho^2 - 4\varrho\kappa + \kappa^2 + \vartheta^2}\right) = 0 \quad (33)$$

which represents the so-called split function[9], depending on the arbitrary constant

$\zeta \in (0,1)$, the nonlinear term coefficients, $\bar{F}$, of the Taylor-expanded system *M*, and the structure of eigenvalues of the associated linearization matrix, $\mathbf{J}$.[10]

We are now ready to prove the following statement.

Proposition 1 *Consider Theorem* 1 *and assume all genericity conditions hold. Take* $\omega = \widehat{\omega} \cong 0.5$ *as the bifurcation value, which is a solution of the split function in (33),* $\varphi(\widehat{\omega}) = 0$. *Then, the homoclinic orbit* Γ_0 *is an equilibrium trajectory of system M.*

Proof Since all genericity conditions in Theorem 1 are satisfied, there exists a region in the parameters' space such that P^* is a saddle-focus equilibrium with $\sigma > 0$, and there is a critical value of the parameter $\omega = \widehat{\omega}$ at which an orbit connecting the equilibrium to itself in backward and forward time emerges as a solution trajectory. Additionally, since the split function in (33) is smooth in ω, and vanishes at $\omega \to \widehat{\omega}$, Theorem 1 can be applied to system *M* ∎

The next example confirms the statement numerically.

Example 1 *Consider the set* $S \equiv \{(\alpha, \gamma, \delta, \rho, \pi^*, \phi, \xi)\} \equiv (0.3, 350, 0.025, 0.0045, 0, -21{,}2)$ *as in* Dupor (2001*).* Figure 5 *represents in panel (a) the three-dimensional split function in (33) as* $\varphi = (\omega, \psi)$. *Additionally, since at the bifurcation value,* $\omega = \widehat{\omega}$, *the split func-*

[8] Condition $\lambda_2 \neq \lambda_3$ is automatically verified by definition of Cardano formula in (30.b).

[9] In the Shilnikov saddle-focus setting, the split function measures the distance between the unstable and the stable manifold, which means that it measures how far manifolds of the saddle-focus equilibrium are from connecting each other. Hence, when it vanishes, the distance is zero, the two branches intersect, therefore producing the emergence of a homoclinic orbit. As detailed in Kuznetsov (2004), for the Theorem 6.6 case with positive saddle quantity (pp. 220–224), as the split function approaches zero, an infinite number of limit cycles may emerge making a global turn along the homoclinic orbit. These cycles are differentiated by the number of rotations that they make near the saddle-focus equilibrium.

[10] All detailed computations are reported in Appendix.

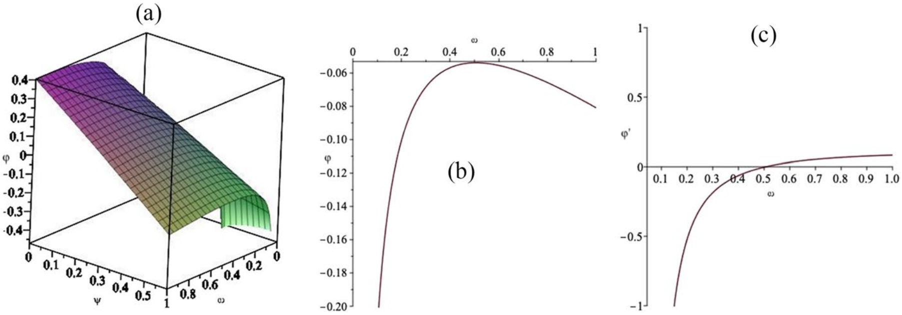


Fig. 5 The parametric surface of the split function

tion vanishes, $\varphi(\widehat{\omega}) = 0$, *we represent in panel (b) the two-dimensional evolution of* $\varphi(\omega)$, *in the presence of a passive monetary policy (*$\psi \cong 0.5$*), whereas panel (c) shows the pattern of the first derivative,* $\varphi\prime(\omega)$.[11] *It can be observed that there is a very narrow value of the bifurcation parameter,* $\widehat{\omega} = 0.5142$, *at which the homoclinic orbit appears as a solution of system M. In this parametric configuration,* $P^* \equiv (c^*, \pi^*, k^*) = (1.921546876, 0, 24.55650957)$, $Tr(\mathbf{J}) = -0.2902489178$, $Det(\mathbf{J}) = 0.004688867929$, *and the structure of eigenvalues is as expected* $\Lambda_1 = 0.1055938202$ *and* $\Lambda_{2,3} = -0.097921369 \pm 0.07233179589i$.

4 Homoclinic rupture, route to chaos and global indeterminacy

The results derived in Proposition 1 allow us to establish the conditions under which the rupture of the homoclinic orbit leads to the emergence of chaotic dynamics and global indeterminacy in system *M*.

Proposition 2 *Assume the bifurcation parameter* $\omega = \widehat{\omega}$, *at which simultaneously: the split function in (33) vanishes,* $\varphi(\widehat{\omega}) = 0$, *and the homoclinic orbit* Γ_0 *emerges as an equilibrium trajectory of system M. Then, for values of* $\widetilde{\omega}$ *sufficiently close to* $\widehat{\omega}$, *such that* $\varphi\left(\widetilde{\omega}\right) \approx 0$ *and* $\varphi\prime\left(\widetilde{\omega}\right) \neq 0$, *an infinite number of saddle limit cycles around* Γ_0 *emerge.*

Proof As shown in Example 1, when the output gap response parameter approaches its critical bifurcation value ($\omega = \widehat{\omega}$) under a passive monetary policy ($\psi < 1$), the system develops a sequence of stable limit cycles forming a homoclinic loop. Hence, each orbit in the vicinity of the loop oscillates around the equilibrium point, generating a series of recurrent orbits, for any sufficiently small deviation of $\omega \approx \widetilde{\omega}$ from $\widehat{\omega}$. The emerging structure defines the basin of a chaotic attractor, within which the equilibrium trajectories remain confined ∎

[11] As detailed in Kuznetsov (2004), Sect. 6.3, p. 213, we assume that $\varphi\prime \neq 0$ to avoid possible degeneracies when $\varphi = 0$.

We can verify our results by constructing the following example.

Example 2 *Consider again the set of parameters in Example 1. Assume a passive monetary policy by fixing* $\psi = 0.5$ *as in* Dupor (2001)*, and* $\widetilde{\omega} = 0.51$. *Since* $\varphi\left(\widetilde{\omega}\right) \approx 0$ *and* $\varphi'\left(\widetilde{\omega}\right) \neq 0$, *the requirements in Proposition 2 are satisfied. In this parametric configuration,* $P^* \equiv \left(c^*, \pi^*, k^*\right) = (1.921546876, 0, 24.55650957)$, $Tr\left(\mathbf{J}\right) = -0.2934640575$, $Det\left(\mathbf{J}\right) = 0.004752165135$, *and the structure of eigenvalues is as expected* $\Lambda_1 = 0.1057739173$ *and* $\Lambda_{2,3} = -0.0996189874 \pm 0.07127292913i$. *Hence, we can proceed with the numerical computation of the emerging chaotic attractor, by using the normal form structural transformation explained in equation (*B.2*) in the Appendix.* Figure 6 *outlines the evolution of this set of attracting limit cycles.*[12]

As is clear, the evolutionary dynamics along the spiraling trajectories of the attractor suggest that the economy may exhibit a pattern of irregular and aperiodic oscillations around the equilibrium, confirming the presence of chaotic dynamics, when the equilibrium trajectories start to diverge from the saddle-focus equilibrium point. This is further verified by the positiveness of the Lyapunov exponent computation ($\ell = 0.00141652182839505$), which provides evidence of deterministic chaos.[13] The economic implication of this finding suggests that any small change in the initial conditions of the model variables can generate a substantially different transitional dynamics towards the long run intended equilibrium. We show this sensitivity by

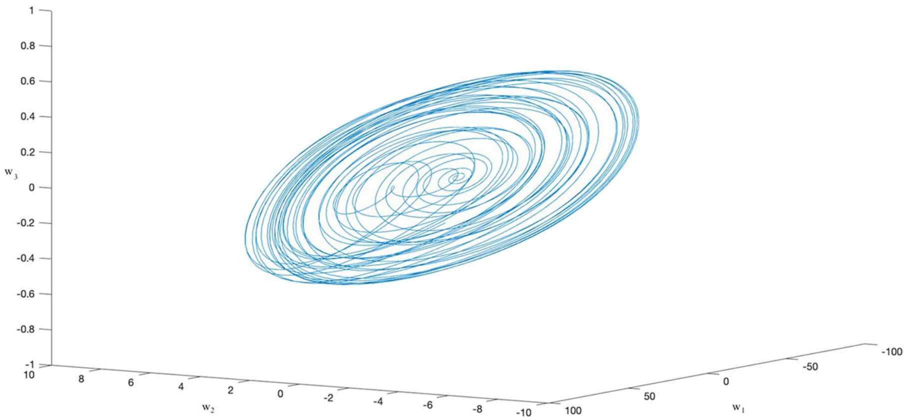


Fig. 6 The chaotic attractor

[12] The application of the normal form theory, with the definition of a triplet of auxiliary variables, w_i, is fundamental to numerically compute, in a standard Matlab algorithm, the trajectories of the chaotic attractor. In details, the simplification of the higher order terms implied by the normalization procedure allows to save the memory which is necessary in the full step-by-step computation of the attractor.

[13] We use a standard routine in R software to compute the Lyapunov exponent.

plotting in Fig. 7 the evolution of the equilibrium trajectories for different initial levels of the inflation variable, $\pi(0)$.[14]

The behavior outlined in Fig. 7 indicates that the chaotic attractor is robust to small variations in the bifurcation parameter, showing that, within the interval $(0.44{,}0.57)$, as ω approaches the bifurcation value 0.5, solution trajectories of inflation, starting at different values of π will begin to exhibit a different and deviating oscillating behavior, which remains nonetheless confined in an attracting region in the vicinity of the inflationary (targeted) steady state level, $\pi^* = 0$, giving rise to unpredictable equilibrium paths, which therefore imply the possibility of global indeterminacy of the equilibrium. This also confirms the sensitive dependence of the equilibrium trajectories on initial states of the economy, which is a defining feature of chaotic systems. Additionally, all trajectories remain confined within the bounded region surrounding the saddle-focus equilibrium, indicating that the chaotic attractor acts as a global trapping set. This oscillatory behavior is illustrated in Fig. 8, which shows the time profile of the inflation rate, $\pi(t)$, along the emerging Shilnikov chaotic scenario.

[14] The time series of inflation is obtained by means of the transformation matrix outlined in Appendix B, that links the original variables to the auxiliary variable adopted for the software computation of the system in normal form.

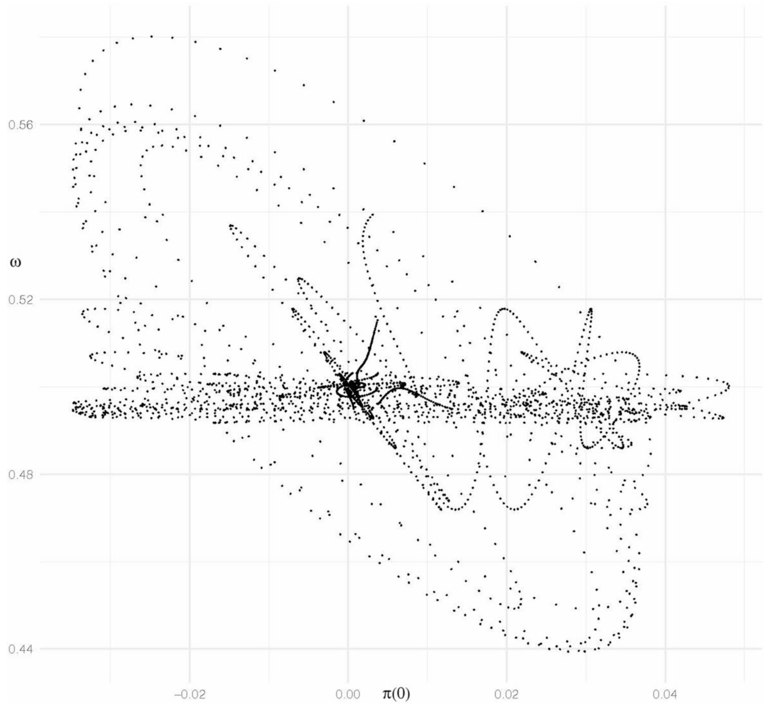


Fig. 7 Sensibility of the initial condition $\pi(0)$ along the chaotic attractor

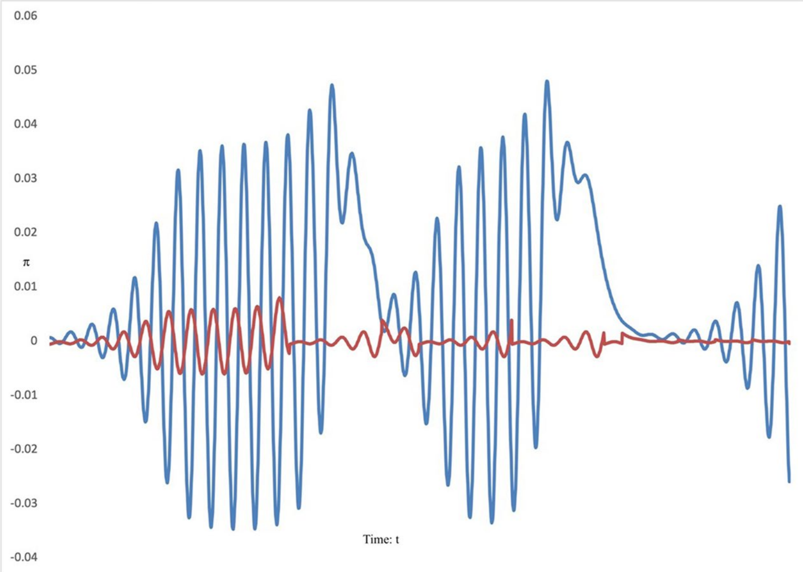


Fig. 8 Time profile of the chaotic inflation rate

The oscillations depicted in Fig. 8 reveal the presence of alternating phases of convergence towards the targeted inflation, followed by abrupt spikes of expansionary dynamics. These cycles could be interpreted as endogenous responses to expectations-driven fluctuations in the economic activity. Periods of rising inflation prompt tighter monetary responses, while subsequent contractions may induce deflationary pressures, forcing the central bank authority to reverse its policy stance repeatedly. Specifically, the blue waves generated by the spiraling structure of the attractor clearly represent the possible emergence of periods in which the inflation rate declines towards the targeted rate, but then a burst due to sudden oscillatory activity follows, possibly due to an expected expansionary phase of the economy. This would force the monetary authority to aggressively intervene by raising the interest rates, which may eventually depress the economy and lead to a deflationary state, that will need a change in the monetary policy action to push back the dynamics towards a growing equilibrium pattern. Therefore, the monetary policy actions adopted in the presence of economic uncertainty could interfere in the way of achieving a stable and intended equilibrium if the parameter ω, that measures the impact of the output gap on the setting of the nominal interest rate, exhibits a range that may trap the economy in a chaotic attractor. As reported in the red curve in Fig. 8, if we instead reduce the values of ω within the range that implies local determinacy of the equilibrium ($\omega < 0.1$) the spikes in the inflationary time series are reduced, and the associated equilibrium trajectories present a pattern where stability towards the intended null-inflation equilibrium is restored.

We can summarize our findings as follows. The emergence of a Shilnikov (chaotic) dynamics is governed by the parameter that captures the response of the monetary authority to the output gap, rather than by the degree of responsiveness to the inflation deviations from its target, as it is common in the Taylor rule setting. This mechanism suggests that the route to chaos is not primarily associated with the anchoring of inflation expectations, which is standard in any typical New Keynesian setting. Instead, the arising instability may find its basis in the real-side transmission channel of monetary policy, operating through the interaction between interest rates (savings), capital accumulation (investment), and production (consumption) decisions. In other words, any variation in the output-gap response parameter can finally modify the resulting feedback between the rental rate of capital in the financial market and its expected returns, thereby affecting the transitional dynamics of the model and enabling the conditions for the emergence of the (Shilnikov) homoclinic bifurcation. This implies also that the global instability in the model will be due to the policy reaction to real economic fluctuations rather than to nominal rigidities of the interest rates alone. Consequently, the analysis developed in this paper suggests that even when inflation appears locally anchored, the monetary policy rules that place significant weight on the stabilization of long run output may inadvertently create the conditions for complex (chaotic) global dynamics. More broadly, these findings highlight the crucial importance of considering the full nonlinear transmission mechanism of monetary policy interventions when assessing equilibrium stability, because the (in) determinacy properties may depend critically on the feedback coming from real sector of the economy rather than solely on the anchoring of inflationary expectations, by tightening the policy stance to commit to the decided inflation target.

5 Conclusions and policy implications

In this paper, we show that global indeterminacy may result in the presence of locally unique equilibrium dynamics, through the rupture of a homoclinic orbit that produces an attracting region outside the intended steady state implied by the local dynamic analysis. The emergence of these periodic oscillations, forming a three-dimensional attracting limit cycle, is of particular interest in the monetary model economy proposed by Dupor (2001), where passive monetary policy, producing local stability, is suggested as a stabilizing instrument to achieve the zero inflationary equilibrium by the monetary authority. We find instead that if the homoclinic bifurcation occurs, the equilibrium trajectories, when studied in their global evolution, may start to cycle around the intended steady state, and perhaps move chaotically within the tubular neighbourhood of an outer long run stationary trapping region, that exhibits higher than expected inflationary rates. Therefore, this cyclical instability highlights a key policy implication: even when a passive monetary policy ensures local determinacy, it may lead to global indeterminacy, once the full nonlinear dynamics of the model are considered. We observe that when the output-gap response parameter lies near its bifurcation threshold, the monetary authority's stabilizing policy actions can inadvertently trap the economy in a chaotic regime characterized by persistent and unpredictable equilibrium paths. Hence, reducing the parameter to values consistent with the region of local determinacy might dampen the inflationary spikes and restore the convergence towards the intended steady state. This finding underscores the delicate balance between policy responsiveness and systemic stability in nonlinear macroeconomic environments. An overly accommodative (passive) monetary stance may fail to anchor expectations globally, while excessively aggressive interventions may amplify the volatility through nonlinear feedback channels. An approach that calibrates and incorporates adaptive or state-dependent policy rules could succeed in mitigating the risk of these unwanted chaotic transitions.

Therefore, from a policy standpoint, the effectiveness of the stabilizing passive monetary policy suggested by the local analysis can be undermined if the global analysis is considered. Caution is therefore warranted against relying exclusively on local stability analysis when designing monetary policy actions, because the study of the global nonlinear structure of the economy may generate dynamics that could invalidate the conclusions indicated by the local analysis. It becomes thus important to identify the exact parametric configuration that allows to locate the economy onto the equilibrium path that prevents undesired indeterminate solutions and ensure instead global stability.

As a possible step forward, it could be interesting to extend the analysis in this framework by introducing heterogeneous agents with different expectations regarding their decisions on consumption, investment and the level of inflation. Such extensions may help to explain how the role of agents' interactions with different beliefs can amplify or dampen global indeterminacy, offering deeper insights into the design of effective and resilient monetary regimes. We leave this extension to future research.

Appendix

The transversality condition

To prove that the transversality condition in the maximization problem is satisfied we need to demonstrate that.

$$\lim_{t \to \infty} e^{-\rho t} \left[\lambda \ (t)\, s\,(t) + \eta \ (t)\, k\,(t) + \mu \ (t)\, P\,(t) \right] = 0$$

holds. We can thus proceed to analyze each part of the condition separately and show whether the asymptotic properties of the solutions hold and represent a maximum for problem (see, Benhabib and Perli 1994). Basically, studying the behavior of the optimal paths as time tends to infinity, (i.e., equilibrium reaches the steady state solution), ensures that the intertemporal problem is optimized and rules out explosive or non-optimal trajectories.

For the first part, we need then to prove that:

$$\lim_{t \to \infty} e^{-\rho t} \lambda \ (t)\, s\,(t) = 0$$

which in growth rates terms needs that in the limit must satisfy:

$$\lim_{t\to\infty}\left(-\rho+\frac{\dot{\lambda}}{\lambda}+\frac{\dot{s}}{s}\right)<0$$

which, substituting (13) and the condition for $\dot{s}$ in (4), given in respect of the national identity $y=c+i$, reduces to $-\frac{Rm}{s}<0$.

As for the second part:

$$\lim_{t\to\infty} e^{-\rho t}\eta\ (t)\,k\,(t)=0$$

that is similarly:

$$\lim_{t\to\infty}\left(-\rho+\frac{\dot{\eta}}{\eta}+\frac{\dot{k}}{k}\right)<0$$

considering (11) and (14), and the condition for $\dot{k}$ in (3), we obtain that $\frac{-rk+i}{k}<0$, which is satisfied since the gain from capital must be higher than the cost of investment.

Finally, to prove that

$$\lim_{t\to\infty} e^{-\rho t}\mu\ (t)\,P\,(t)=0$$

given (20), and that $r=\alpha\,\frac{y}{k}$, then (19) implies that $\mu\ (t)=0$.

The Jacobian matrix

First order conditions of the maximization problem allow us to derive the following three-dimensional system of differential equations:

$$\dot{\pi}\ =\rho\ (\pi\ -\pi^{\ *})-\frac{1+\phi}{\gamma}\left(\frac{1-\alpha}{\alpha}\right)^b\left(\frac{k^{\alpha\ +b}}{c^{1+b}}\right)(R+\delta\ -\pi\ )^b+\frac{\phi\,k}{\alpha\,\gamma\,c}(R+\delta\ -\pi\ )$$

$$\dot{c}=c\,(R-\rho\ -\pi\ ) \tag{M}$$

$$\dot{k}=k^{\alpha\ +b}\left(\frac{1-\alpha}{\alpha\ c}\right)^b$$
$$(R+\delta\ -\pi\ )^b-\delta\,k-c$$

which can be put in the convenient matrix notation:

$$\begin{pmatrix}\dot{\pi}\\ \dot{c}\\ \dot{k}\end{pmatrix}=\mathbf{J}\begin{pmatrix}\pi\\ c\\ k\end{pmatrix}$$

Being J the Jacobian matrix associated to system *M*, whose elements are:

$$\mathbf{J} = \begin{bmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{bmatrix}$$

where each element reads as:

$$f_1 = \frac{\partial \dot{\pi}}{\partial \pi} = \rho + \frac{1+\phi}{\gamma} \left(\frac{1-\alpha}{\alpha}\right)^b \left(\frac{k^{\alpha+b}}{c^{1+b}}\right) b \left(R + \delta - \pi\right)^{b-1} (R_\pi - 1) + \frac{\phi\, k}{\alpha\, \gamma\, c} (R_\pi - 1)$$

$$f_2 = \frac{\partial \dot{\pi}}{\partial c} = -\frac{1+\phi}{\gamma} \left(\frac{1-\alpha}{\alpha}\right)^b \left(\frac{k^{\alpha+b}}{c^{2(1+b)}}\right) \left[b \left(R + \delta - \pi\right)^{b-1} R_c c^{1+b} - (1+b)\, c^b \left(R + \delta - \pi\right)^b\right] + \frac{\phi\, k}{\alpha\, \gamma\, c^2} (R_c c - R)$$

$$f_3 = \frac{\partial \dot{\pi}}{\partial k} = -\frac{1+\phi}{\gamma} \left(\frac{1-\alpha}{\alpha}\right)^b \left(\frac{k^{\alpha+b-1}}{c^{1+b}}\right) \left[(\alpha + b) \left(R + \delta - \pi\right)^b + kbR_k \left(R + \delta - \pi\right)^{b-1}\right] + \frac{\phi}{\alpha\, \gamma\, c} (R + \delta - \pi + kR_k)$$

$$g_1 = \frac{\partial \dot{c}}{\partial \pi} = c(R_\pi - 1) \; g_2 = \frac{\partial \dot{c}}{\partial c} = cR_c \; g_3 = \frac{\partial \dot{c}}{\partial k} = cR_k$$

$$h_1 = \frac{\partial \dot{k}}{\partial \pi} = k^{\alpha+b} \left(\frac{1-\alpha}{\alpha\, c}\right)^b b \left(R + \delta - \pi\right)^{b-1} (R_\pi - 1)$$

$$h_2 = \frac{\partial \dot{k}}{\partial c} = k^{\alpha+b} \left(\frac{1-\alpha}{\alpha\, c^2}\right)^b \left[b \left(R + \delta - \pi\right)^{b-1} R_c c^b - bc^{b-1} \left(R + \delta - \pi\right)^b\right] - 1$$

$$h_3 = \frac{\partial \dot{k}}{\partial k} = \left(\frac{1-\alpha}{\alpha\, c}\right)^b \left[(\alpha + b)\, k^{\alpha+b-1} \left(R + \delta - \pi\right)^b + k^{\alpha+b} b \left(R + \delta - \pi\right)^{b-1} R_k\right] - \delta$$

given:

$$R_\pi = \frac{\psi' - \frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha\, c}\right)^b k^{\alpha+b} b(R+\delta-\pi)^{b-1}}{1 - \frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha\, c}\right)^b k^{\alpha+b} b(R+\delta-\pi)^{b-1}}$$

$$R_c = -\frac{\frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha}\right)^b k^{\alpha+b} (R+\delta-\pi)^b bc^{b-1}}{1 - \frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha\, c}\right)^b k^{\alpha+b} b(R+\delta-\pi)^{b-1}}$$

$$R_k = \frac{\frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha\, c}\right)^b (\alpha+b) k^{\alpha+b-1} (R+\delta-\pi)^b}{1 - \frac{\omega}{y^*}\left(\frac{1-\alpha}{\alpha\, c}\right)^b k^{\alpha+b} b(R+\delta-\pi)^{b-1}}$$

The reduced model computation

When $\omega = 0$, Being $\mathbf{J}_R$ the Jacobian matrix associated to the reduced model computation, whose elements are:

$$\mathbf{J}_R = \begin{bmatrix} f_1 & f_2 & 0 \\ g_1 & 0 & 0 \\ h_1 & h_2 & h_3 \end{bmatrix}$$

where:

$$f_1 = \frac{\partial \dot{\pi}}{\partial \pi} = \rho + (1+b)\frac{\phi k}{\alpha \gamma c}(\psi' - 1)$$

$$f_2 = \frac{\partial \dot{\pi}}{\partial c} = \frac{b\phi k}{\alpha \gamma c^2}(R + \delta - \pi)$$

$$g_1 = \frac{\partial \dot{c}}{\partial \pi} = c(\psi\prime - 1)$$

$$h_3 = \frac{\partial \dot{k}}{\partial k} = \left(\frac{1-\alpha}{\alpha c}\right)^b \left[(\alpha + b)\, k^{\alpha + b - 1}(R + \delta - \pi)^b\right] - \delta$$

given the steady state values:

$$c^* = \left(\frac{\alpha}{z}\right)^{\frac{1}{b}} \left(\frac{1-\alpha}{\alpha}\right)(\rho + \delta)^{\frac{b-1}{b}} (k^*)^{(\alpha + b - 1)/b}$$

$$k^* = \left[\frac{\left(\frac{\alpha}{z}\right)^{\frac{1}{b}}(1-\alpha)(\rho + \delta)^{\frac{b-1}{b}}}{z(\rho + \delta) - \alpha \delta}\right]^{b/\beta}$$

Given the structure of $\mathbf{J}_R$, and the guarantee for a real eigenvalue at $\widetilde{\Lambda}_1 = h_3$, the dynamics around the equilibrium are fully determined by the submatrix:

$$\mathbf{A} = \begin{bmatrix} f_1 & f_2 \\ g_1 & 0 \end{bmatrix}$$

whose eigenvalues will be given by $\widetilde{\Lambda}_{2,3} = \frac{f_1}{2} \pm \sqrt{\Delta}$, given that $\Delta = \left(\frac{f_1}{2}\right)^2 + f_2 g_1 > 0$.

Appendix B

The homoclinic orbit computation

Let us first consider:

$$\dot{\mathbf{x}} = \mathbf{J}\mathbf{x} + \mathbf{G} \tag{B.1}$$

as a second order Taylor expansion of system M, with the vector of variables, $\mathbf{x} = (\pi, c, k)^T$, and the nonlinear terms $\mathbf{G}$ (see, for example, Bella et al. 2017). Consider that $\mathbf{J}$ has one positive real eigenvalue, $\Lambda_1 = \varrho$, and two complex conjugate eigenvalues with negative real parts, $\Lambda_{2,3} = \kappa \pm \theta i$, with $\kappa < 0$, and proceed to compute the following relationships:

$$\begin{array}{c}\mathbf{J}\mathbf{u} = \kappa\,\mathbf{u} - \theta\,\mathbf{v}\\ \mathbf{J}\mathbf{v} = \kappa\,\mathbf{v} + \theta\,\mathbf{u}\\ \mathbf{J}\mathbf{z} = \varrho\,\mathbf{z}\end{array}$$

where $\mathbf{u} = (u_1, u_2, u_3)^{\mathrm{T}}$, $\mathbf{v} = (v_1, v_2, v_3)^{\mathrm{T}}$, and $\mathbf{z} = (z_1, z_2, z_3)^{\mathrm{T}}$ are three $(3\times\ 1)$ vectors.

It follows that a possible structure of eigenvectors be given by:

$$\mathbf{u} = \begin{pmatrix} u_1\\ u_2\\ u_3\end{pmatrix} = \begin{pmatrix} \frac{f_1+f_3v_3-\kappa}{\theta}\\ \frac{f_2+g_3v_3}{\theta}\\ 1\end{pmatrix};$$

$$\mathbf{v} = \begin{pmatrix} v_1\\ v_2\\ v_3\end{pmatrix} = \begin{pmatrix} 1\\ 0\\ \frac{\theta\ -h_1}{h_3-\kappa}\end{pmatrix};$$

$$\mathbf{z} = \begin{pmatrix} z_1\\ z_2\\ z_3\end{pmatrix} = \begin{pmatrix} z_1\\ 1\\ \frac{h_1z_1+h_2}{\varrho\ -h_3}\end{pmatrix}$$

where $z_1 = \frac{f_3g_2-f_2(\varrho\ -h_3)}{(f_1-\varrho\ )(\varrho\ -h_3)-f_3h_1}$.

Hence, take the transformation matrix $\mathbf{T} = (z, u, v)$ to put (B.1) into the standard normal form:

$$\begin{pmatrix} \dot{w}_1\\ \dot{w}_2\\ \dot{w}_3\end{pmatrix} = \begin{bmatrix} \varrho & 0 & 0\\ 0 & \kappa & -\theta\\ 0 & \theta & \kappa\end{bmatrix}\begin{pmatrix} w_1\\ w_2\\ w_3\end{pmatrix} + \begin{pmatrix} F_1\\ F_2\\ F_3\end{pmatrix} \tag{B.2}$$

in the new coordinates via the transformation $\mathbf{x} = \mathbf{T}\mathbf{w}$, and the transformed nonlinear terms $F\,(\mathbf{w}) = \mathbf{T}^{-1}G\,(\mathbf{T}\mathbf{x})$.

We can now proceed to compute the stable and unstable manifolds of the saddle equilibrium point and derive the homoclinic loop condition by using the method of undetermined coefficients developed by Shang and Han (2005). A generic equation of the equilibrium manifold associated to system (B.2) can be described as:

$$w\,(t) = d_0 + \sum_{k=1}^{\infty} d_k e^{\Lambda\,kt} \tag{B.3}$$

where $d_k = (a_k, b_k, c_k)$, with a time derivative given by:

$$\dot{w}\,(t) = \sum_{k=1}^{\infty} d_k\Lambda\,ke^{\Lambda\,kt} \tag{B.4}$$

Substituting (B.3) and (B.4) into (B.2) we have:

$$\begin{pmatrix} \sum_{k=1}^{\infty} a_k \Lambda\, k e^{\Lambda\, kt} \\ \sum_{k=1}^{\infty} b_k \Lambda\, k e^{\Lambda\, kt} \\ \sum_{k=1}^{\infty} c_k \Lambda\, k e^{\Lambda\, kt} \end{pmatrix} = \begin{bmatrix} \varrho & 0 & 0 \\ 0 & \kappa & -\theta \\ 0 & \theta & \kappa \end{bmatrix} \begin{pmatrix} a_0 + \sum_{k=1}^{\infty} a_k e^{\Lambda\, kt} \\ b_0 + \sum_{k=1}^{\infty} b_k e^{\Lambda\, kt} \\ c_0 + \sum_{k=1}^{\infty} c_k e^{\Lambda\, kt} \end{pmatrix} + \begin{pmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \bar{F}_3 \end{pmatrix} \tag{B.5}$$

and equating the corresponding coefficients up to the second iteration, we derive:

$$\begin{gathered} \begin{pmatrix} a_0 \\ b_0 \\ c_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \begin{pmatrix} \zeta \\ 0 \\ 0 \end{pmatrix}; \\ \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \zeta^{\,2} \\ \begin{pmatrix} -\frac{\bar{F}_{1d}}{\varrho} \\ \frac{(\kappa-2\varrho)\bar{F}_{2d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} + \frac{\theta\,\bar{F}_{3d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} \\ \frac{(\kappa-2\varrho)\bar{F}_{3d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} - \frac{\theta\,\bar{F}_{2d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} \end{pmatrix} \end{gathered} \tag{B.6}$$

Therefore, the series coordinates, w_i, can be approximated at $t = 0$ as:

$$w_1(0) = \zeta - \frac{\bar{F}_{1d}}{\varrho}\zeta^{\,2}$$

$$w_2(0) = \zeta^{\,2}\left(\frac{(\kappa-2\varrho)\bar{F}_{2d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} + \frac{\theta\,\bar{F}_{3d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}}\right);$$

$$w_3(0) = \zeta^{\,2}\left(\frac{(\kappa-2\varrho)\bar{F}_{3d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}} - \frac{\theta\,\bar{F}_{2d}}{4\varrho^{\,2}-4\varrho\,\kappa+\kappa^{\,2}+\theta^{\,2}}\right)$$

so we can let that the orbit, $w_1(0)$, associated with the positive real eigenvalue, and one of the orbits associated with the complex conjugate eigenvalues, for example $w_2(0)$, connect to determine the parametric condition for the existence of an orbit homoclinic to P^*.

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