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Contagion and Interdependencies between BRICS-plus Countries on the Markets of Commodities and Derivative Financial Instruments

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Abstract

Purpose: This study was started on the main macroeconomic data (2010-2019), coming from the countries of the BRICS-plus group. While a significant contribution of these emerging economies to global economic output has generally been observed, this has also been accompanied by persistent domestic imbalance.

Design/Methodology/Approach: Regarding the model, we state and contextualize the (long-term) risk co-incidence of the set of macroeconomic variables for this coalition of countries. Particular attention was paid to a valuation, description and forecasting model based on the calculation of the Dynamic Conditional Correlation (DCC) in a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process on the time series of credit default swaps (CDS).

Findings: The empirical analysis and calculations carried out have verified the significance of the parameters and confirmed the conditional dynamic correlation between the economies of the Brics "expanded", especially in the presence of shocks, which also involve mutual contagion (temporary increase in DCC) and even interdependence (increasing DCC leads to new, less unstable levels).

Implications/Originality/Value: The research offers a look at the most current trends in terms of international economic balances and the entire global system, focusing on commodity markets, financial derivatives and the impacts of trade at the dawn of the new BRICS-plus coalition.

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Introduction

The scientific issue underlying this contribution concerns the relations between the international commodities market and the BRICS countries, especially in the aftermath of the summit held in

South Africa at the end of August 2023, during which it was announced that the aforementioned emerging market economic coalition would be extended to include six more nations: Argentina, Egypt, Ethiopia, Iran, Saudi Arabia and the United Arab Emirates, as of 1 January 2024 (Ciuriak, 2023). Thus, BRICS-plus will become a de facto commodity cartel (Gouvea and Gutierrez, 2023). As far as fossil fuels are concerned, this enlarged partnership will essentially hold 40% of all oil and gas production, and 70% of coal production, whilst on the metals front, the new BRICS-plus will produce 80% of platinum, 70% of palladium, 80% of aluminium and 50% of copper (Devonshire-Ellis, 2022). If, therefore, there is an element of coordination between these nations, it will be the fact that they represent a prevailing player in the production of raw materials in the world (Sawal and Anjum, 2023). Consolidation of this enlargement could also lead to an economic-political bipolarity of the world order, with the establishment of an alternative governance to that of the G7. In addition, the incumbent Brazilian President, Luiz Inácio da Silva (known mononymously as 'Lula'), has made some statements regarding a BRICS-plus monetary unification (Iglesias, 2023), which could lead to a de-dollarisation of certain assets (Liu and Papa, 2022), including related financial derivatives trading (Coquidé *et al*., 2023).

In the financial sector (Amoako *et al.*, 2022), one ought to wonder about how the above circumstances could impact markets and returns in the future (Creti and Joëts, 2013). In a period still and increasingly characterised by a commodity price super-cycle (Boako *et al*., 2020), could it be time for an ETF or an ETC on one of these countries, or would the same opportunities for investing in the futures of the segment remain? Or, should new risks of greater currency volatility, related to the stability of the underlying assets, etc., be considered and appreciated, even though there would be no need for arbitrage or hedging operations?

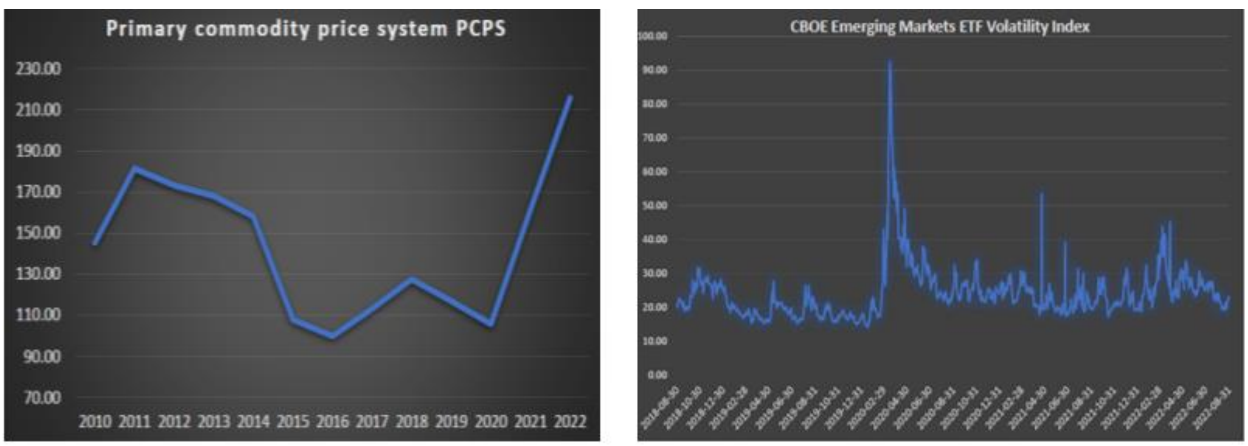


Figure 1: Commodity price trends and CBOE emerging markets ETF volatility index

Source: Our own elaborations based on IMF – International Monetary Fund (2023) [PCPS], and Federal Reserve Bank of St. Louis (2023a) [CBOE] data

To be able to specifically answer this last question, we have to first, following the work of Borovkova and Geman (2006), discuss a general calculation model of the expected value of a future with a generic commodity or a basket of commodities as its underlying standard (however, as far as seasonal fluctuations are concerned, obviously, energy commodities should be considered among the most susceptible). We then go on to think about the model, affirming and contextualizing the (long-term) impact on the risk of the series of variables that describe the macroeconomic trends of the coalition of countries we are discussing in this paper. This is accomplished through an evaluation, description, and prediction model based first on the calculation of the dynamic conditional correlation (DCC) in a Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) process on the time series of Credit Default Swaps (CDS) - which have been regarded as sensitive indicators of the macroeconomic stability of the countries (Bragoudakis and Voulgarakis, 2019; Akyüz and Bekar, 2021) involved in the extended BRICS-plus partnership - and subsequently, through the synthetic outcome of the model, an attempt is made to forecast and analyze the trajectories of the correlated dynamics of the group's macroeconomic variables (Bouri *et al*., 2023). Also considered would be the relevance of investing in derivatives with underlying commodities during times of high inflation or when the future expectation over the short-term is such, and in stock market financial products, should we witness or expect a low or negative inflation rate, or a trend of inflation remission. As is well known and as mentioned in the premise for the case at hand, commodity futures are financial instruments that allow investors to speculate on the future price of certain commodities, predominantly on the holdings of developing economies or emerging countries in the BRICS-plus partnership. Often, these instruments, which are traded on commodity markets, or commodity exchanges, allow investors to try to profit from commodity price movements without obviously having to physically own those commodities. These futures can cover a wide range of commodities, such as crude oil, precious metals (gold, silver), agriculture (wheat, corn, coffee), energy resources and more. The origin and supply of these underlying commodities depend on the natural resources and industries of the individual emerging countries or groups of countries. It should also be noted that commodity futures can themselves shape emerging countries' economies in several ways. For example, they can help create a benchmark price for such commodities, which would then influence domestic and international prices. Furthermore, if managed effectively, they can attract foreign investment and contribute to economic growth. These assessments of the "autopoietic" capacity of futures contracts on the outcome of individual subscriptions themselves are difficult to appreciate and will not be considered in the model discussed herein. Nevertheless, they will be briefly taken up and explored in our conclusions.

Furthermore, all this is based on the assumption that commodity futures markets in the countries under investigation are regulated and transparent, and provide a fair and reliable trading environment (Mehrotra and Carbonnier, 2021). Based on the fact that commodity prices can be extremely volatile, investing in futures involves a certain degree of risk. Emerging countries can be especially susceptible to economic, political and environmental factors, which might affect commodity prices. Whilst futures can generally offer some risk mitigation related to individual country economies, since the underlying commodities would be diversified in their origin, and price estimates tend to be quantitative and globalist, if the new 11-country BRICS-plus group does indeed form a cartel, it would constitute an interconnected and correlated macroeconomic dynamic capable of affecting the entire commodities and financial derivatives market. In other words, there is still, at the current state – lacking the degree of concentration in the detention and production of commodities that will instead need to be addressed with the formation of BRICS-plus – a substantial control, within the same mechanism of futures or ETC issuance and the associated appreciation of the underlying assets, aimed at maintaining an adequate level of non-specificity regarding the country of origin.

Macroeconomic Data of the BRICS-plus Countries

Observations of emerging economy countries (BRICS-plus 11) have shown – against their remarkable contribution to the world economy in the interim – evidence of a predatory character in investments, technologies and market positioning, as well as a counter to the continuity of significant internal imbalances. Monetary policies tend to be loose, with little action on inflation and general disorder in the management of transitions. There also appears to be a sustained dependence on foreign trade and foreign capital and financing. Technological availability is centred on a small number of economic players, as is the location of *commodity* sectors, whilst accelerating GDP growth has led to granular wealth flows (Desogus and Casu, 2022), resulting in increased domestic inequality. The economic fundamentals show structural imbalances, which create risks that can undermine social and political stability. There are also a trends towards progressive reductions in infrastructure investments. The internal transactions among agents do not often bring about substantial ameliorative changes in the distribution of wealth, on the other hand, macroeconomic policy interventions aimed at endeavours to rebalance macroeconomic variables – taking the current circumstances of the economic systems considered here into account – could lead to a reduction in production driven growth (Refakar and Ravaonorohanta, 2020).

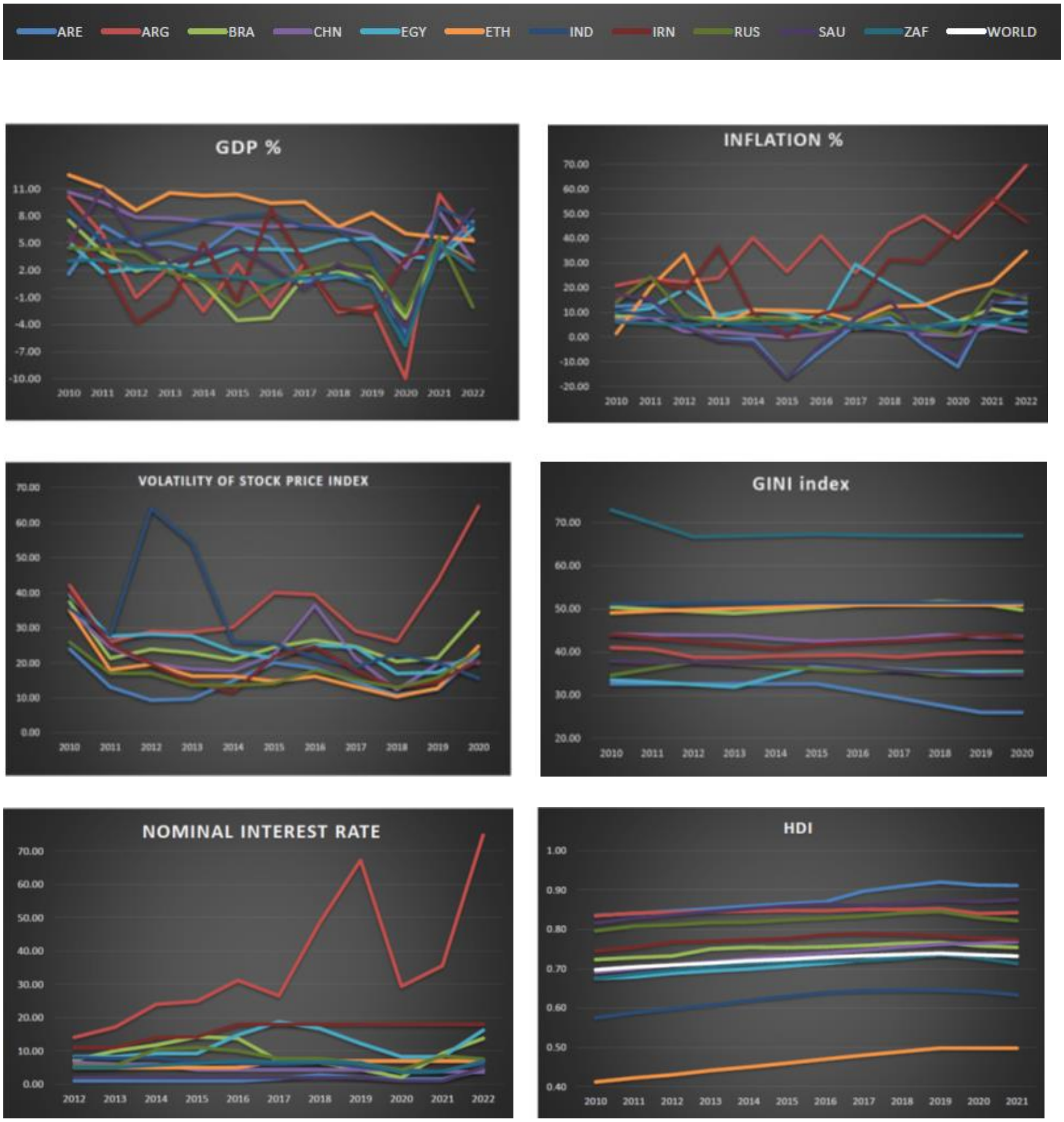


Figure 2: BRICS-plus economic data [GDP, inflation, volatility of stock price, inequality index (Gini), nominal interest rate, human development index (HDI)]

Sources: Our own elaborations based on World Bank (2023a and 2023b) [GDP and Inflation], Federal Reserve Bank of St. Louis (2023b) [Volatility of stock price], UNU-WIDER – United Nations University: World Institute for Development Economics Research (2023) [Gini], FX Empire (2023) and National Central Banks [Interest rate], and UNDP – United Nations Development Programme (2023) [HDI] data.

The data and analysis alluded to here could very well be used as an assessment tool for investors, and more in general as a means to try to seize and interpret the scenario of near-future markets for commodities and derivative financial products. Actually, this effort is being made to offer a renewed and organised focus on the correlated trends of this new coalition's economic systems, and on their expected trajectories with respect to points of stability, as it zeros in on gaining additional understanding of the new risks whilst trying to forecast any significant shifts in performance and positioning on the global chessboard.

Volatility Analysis in the Commodity Derivatives Market

Futures show the outcome of a higher or lower demand for the underlying commodities in advance, forming certain forward curves over specific time intervals. If a spot price is lower than the future prices implied by the corresponding futures contracts there will be what is known as "contango". In contrast, when the value of the futures contracts is lower than the spot price there will be what is known as "backwardation". The dynamics governing the alternation of contango and backwardation depend on the nature of the underlying commodity. For commodities, backwardation is essentially attributable to strong demand for futures contracts with closer maturities. For financial instruments such as equity indices and their futures contracts, the different prices are instead attributable to the anticipation of expected dividends.

In the absence of arbitrage, the prices of the futures at time t and the delivery of the underlying commodity at time T will be determined by:

$$F(t,T) = S(t)e^{(r(t)-y(t,T)(T-t)} \qquad (1)$$

Where $S(t)$ represents the spot price of the underlying commodity at time t, and its drift is $r(t) - y(t,T) = (r + c - \hat{y}) = \gamma(t)$, which describes its periodicity: r is the interest rate, c stands for the fixed costs (for example, storage), and $\bar{y}$ is the convenience yield. Hence, $y = \hat{y} - c$, which is the convenience yield net of costs. The forward curve on the date t is an increasing or decreasing function towards maturity T, depending on the sign of $(r - y)$: as shown above, contango or backwardation respectively. Since $S(t)$ is associated with the valuation time t, it obviously does not carry information on periodicity fluctuations (whether rhythmic – for example, seasonality – or related to macroeconomic contextual dynamics), since these are characteristics of maturity in T.

The formula:

$$\log \bar{F}(t) = \frac{1}{N}\sum_{T=1}^{N} \log F(t,T) \qquad (2)$$

represents the (geometric) average of forward prices, with N as the last maturity. The equation is written on the logarithm for convenience and elegance of formalism.

And still:

$$F(t,T) = \bar{F}(t)e^{(s(T)-\gamma(t,T-t)(T-t))} \qquad (3)$$

Where $s(T)$ indicates the final – deterministic – premium given by the succession of periodicity values (independent from valuation time) and $\gamma(t,\tau)$, with $\tau = T - t$, which represents the stochastic convenience yield. The formula shows how the average of the forward prices can be the proxy for the spot price.

We can define $K(t) = \log \bar{F}(t)$:

$$dK(t) = \vartheta\big(l - K(t)\big)dt + \sigma_i dW_i(t) \qquad (4)$$

(4) is an expression of the constant mean-reverting Wiener process of the mean of forward prices, with the parameters

- ϑ: measure of the mean-reverting speed;
- l: mean value of the long-term average of forward prices;
- σ_i: measure of the volatility of the mean of forward prices, which we may assume to be constant.

It should be noted that, over the long-term, K tends towards the value l.

Furthermore, although with independent Brownian motion, we are also going to describe the dynamics of $\gamma(t,\tau)$, which detains and expresses the stochastic factors (also including financing, transport, and storage costs) capable of determining the premium or discount $F(t,T)$ with respect to $\bar{F}(t)$. The stochastic convenience yield, still in agreement with Borovkova and Geman (2006), is considered applicable to commodities, whether storable or non-storable, since we are going to examine the attitudes of the forward curve, with spot prices having little relevance.

$$d\gamma^{\tau}(t) = -\zeta^{\tau}\gamma^{\tau}(t)dt + \sigma_j^{\tau}dW_j(t) \qquad (5)$$

Which we can rewrite as:

$$d\gamma^{\tau}(t) = -\zeta^{\tau}\gamma^{\tau}(t)dt + \frac{\mathrm{Cov}(\gamma^{\tau}(t),Y)}{\beta_j}\,dW_j(t) \qquad (6)$$

The process is still mean-reverting: therefore, the parameter ζ measures the speed of return to the mean, and $\sigma_j = \frac{\mathrm{Cov}(\gamma^{\tau}(t),Y)}{\beta_j}$ is volatility, with Y being the country's macroeconomic change index, representable by the parameter – in basis points (bpt) – of the corresponding Credit Default Swap (CDS) market on sovereign bonds.

Therefore, the randomness of the stochastic convenience yield is linked to Brownian motion W_j. The – not correlated – process W_i also determines the forward price averages.

Assuming $J(t,T) = \log F(t,T)$, we can rewrite (3) on the basis of equations (4) and (5):

$$d\big(J(t,T)\big) = (\vartheta(m - K(t)) + \gamma^{\tau}(t)(\zeta^{\tau}\tau + 1))dt + \sigma_i dW_i(t) - \sigma_j^{\tau}dW_j(t) \qquad (7)$$

From which we return to $J(t,T)$ by performing the integral of (7) with the initial conditions: $F(0,T) = \bar{F}(0)e^{s(T)-\gamma^{\tau}(0)T}$. Then, $F(t,T)$ follows a log-normal distribution with a price volatility structure consistent with some extensions of Vasicek's model (Desogus and Venturi, 2019): in fact, the variance is $\varphi^2(t,\tau) = \sigma_i^2 + \left(\sigma_j^2\tau\right)^2$, or, even better, see the role of Y:

$$\varphi^2(t,\tau) = \sigma_i^2 + \left(\left(\frac{\mathrm{Cov}(\gamma^{\tau}(t),Y)}{\beta_j}\right)^2\tau\right)^2 \qquad (8)$$

And the overall volatility is therefore manifested by $\varphi(t,\tau)$ with periodicity τ from 1 to N.

The Role of the Random Vector of Macroeconomic Context Variables

Going back to (5), it can be seen that $dW_j = Y\sqrt{dt}$.

From the amount Y as already mentioned above – which represents the macroeconomic context variable of the (BRICS-plus) group capable of influencing and characterising the markets of the underlying reference commodities – there is derived the random element G whose variation dG in an interval dt is the result of the sum of the variations of the elementary states of G independent among themselves. The variance is similarly the sum of the elementary variances: if $dG(dt)$ has a variance σ^2, over the entire interval dt, we can observe a total variance of dG equal to σ^2dt and, with the same probability distribution of the elementary variations, and a corresponding standard deviation of $\sigma\sqrt{dt}$. Therefore: $dG = Y\sigma\sqrt{dt}$ (Desogus and Casu, 2020).

Therefore, we can infer that the randomness of the process referred to in (5) (and thus, affecting (3)) is strongly determined by Y. In essence, if the result of the tensor product of the respective multivariate random variables remains below a certain dimension, the system "holds" and no peaks of volatility and instability are observed. $Y = \bigotimes^{n} Y_j$, where Y_j represents the macroeconomic context variable attributable to each individual country, and $n = 11$ is the number of countries participating in BRICS-plus. We can decompose Y_j as follows:

$$Y_j = ax_j + b\jmath_j \qquad (9)$$

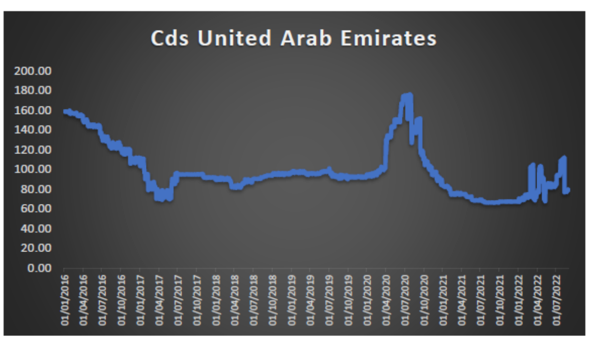
Where x_j is the random vector of the macroeconomic context variables proper (considered upon the formation of the multivariate random variable x_j: inflation level, volatility of stock price, and inequality index) related to the countries within the coalition formed by the new BRICS-plus. This vector influences, through a mechanism of mutual contagion to be demonstrated later in this work, the combined arrangement of k_j returns for each financial market, particularly in the commodity sector. On the other hand, $\jmath_j$ represents the idiosyncratic random factor (primarily qualitative) of individual countries, and can be, with a good approximation, represented by their respective Human Development Index (HDI). It should be noted that x_j can still influence a certain portion of $\jmath_j$.

Study of the Existence of a Dynamic Conditional Correlation: The Model and Model Estimation

To better understand the contribution that will be expressed by this variable x in the determination of the values of futures, or in general of the ETCs or ETFs on commodities following the implementation of the 11-country BRICS-plus partnership, which would represent the (*de facto*) cartel on commodities discussed in the introduction, we now move on to investigate the reciprocal interplay among the several national markets that are part of BRICS-plus. This can be more closely studied by analysing and verifying the correlation between the series of the BPTs through which the Credit Default Swaps are traded on the sovereign bonds of the individual countries belonging to the group (Billio and Caporin, 2005; Sabkha *et al*., 2019).

The CDS spreads appear to be the optimal tool – as has emerged from the literature (*ex multis*: Papaioannou, 2011; Baltaci and Akyol, 2016) – for assigning a reference for the macroeconomic trends of the countries under investigation. This is based on the dynamics of greater or lesser susceptibility of the premiums to be paid against the financial risks assumed by investors in sovereign bonds, and the conditional correlation observable between the individual series – and predictions – within the new 11-country BRICS-plus partnership. Methodologically, the analysis will be performed through using a multivariate Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) with Dynamic Correlation (DCC) model.

The historical time series of CDS (Credit Default Swap) spread levels (expressed in basis points) for the countries under investigation in this study have been considered from January 2016 to September 2023. Data for Ethiopia and Iran are not available, and for Argentina, there are discontinuous data points. Nevertheless, the sample of 8 out of 11 countries and its extensive geographical coverage have proven to be sufficient and reliable. For Russia, the findings from March 2022 were excluded: since in the first months of the Ukrainian conflict, there was a sudden peak (up to 13,775.17 bpt at the end of May) which compromised the presentability of the overall results on the group of countries observed.



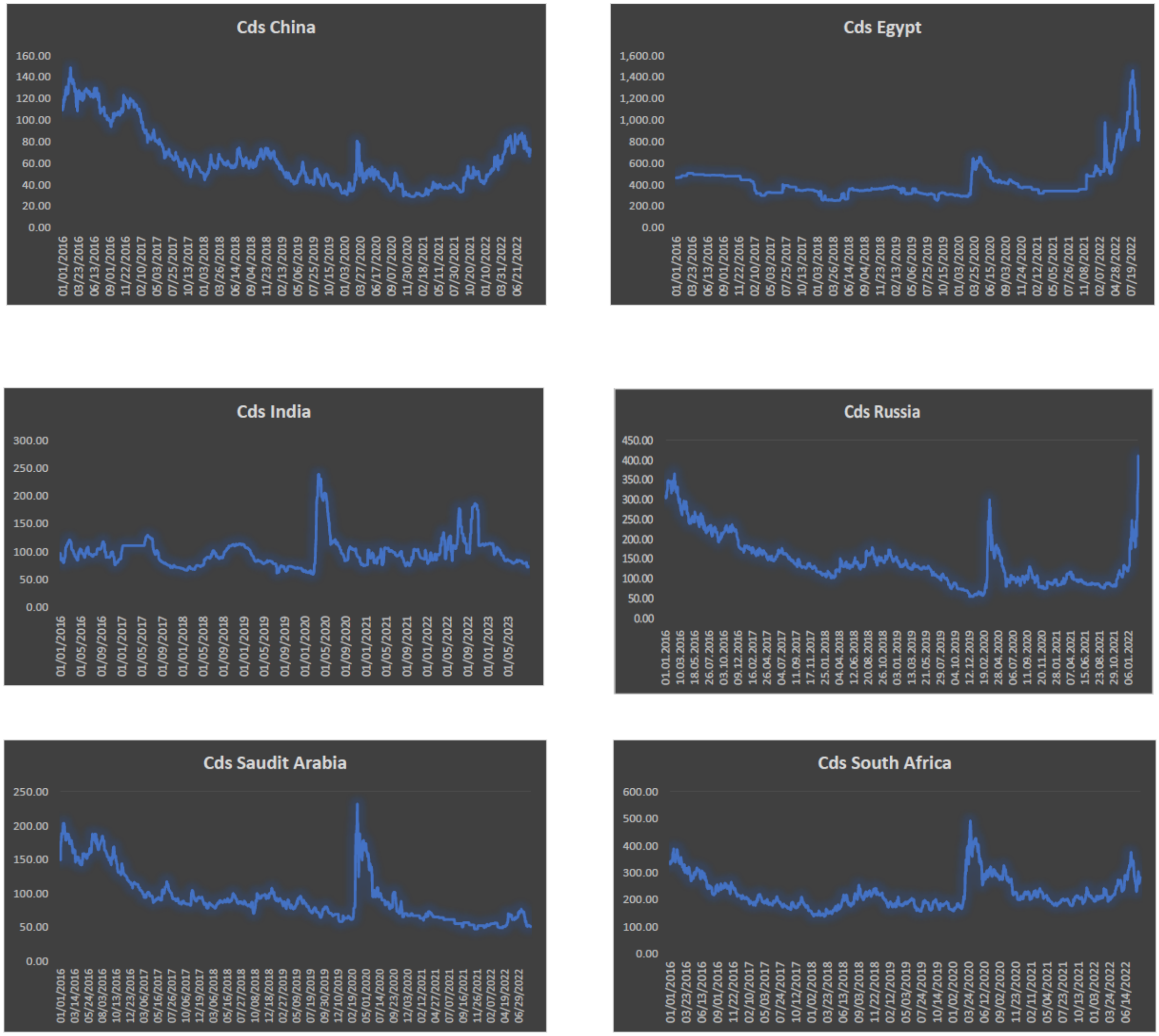


Figure 3: Credit Defaul Swaps spread levels (in bpt)

Source: Our own elaborations based on Investing.com (2023), and National Central Banks data.

Table 1: Test KPSS (Kwiatkowski, Phillips, Schmitdt e Shin)

Null Hypothesis: CDS "Country" is trend stationary

Lags	Include Trend	Significance Level
0	true	0.05

Country	Null Rejected	P-Value	Test Statistic	Critical Value
United Arab Emirates	true	0.01	9.1925	0.146
Brazil	true	0.01	20.9448	0.146
China	true	0.01	36.3596	0.146
Egypt	true	0.01	30.9993	0.146
India	true	0.01	2.5673	0.146

Russia	true	0.01	14.7567	0.146
Saudi Arabia	true	0.01	10.6823	0.146
South Africa	true	0.01	11.8200	0.146

Before applying the assessments using the aforesaid procedures, we filtered the abovementioned series using a Vector Autoregression model (VAR(p)), which is presented here in the Wold representation form (Miranda-Agrippino and Ricco, 2023) for methodological completeness:

$$\mathcal{Y}_{jt} = \left(I - \Xi(\mathfrak{L}_j)\mathfrak{L}_j\right)^{-1} c_j + \left(I - \Xi(\mathfrak{L}_j)\mathfrak{L}_j\right)^{-1} \mathcal{A}_{j0}^{-1} \nu_{jt} = \mathcal{E}_j + \Upsilon(\mathfrak{L}_j)\nu_{jt} \qquad (10)$$

Where $\Xi(\mathfrak{L}_j) = \sum_{\mathrm{i}=1}^{p-1} \Xi_{\mathrm{i}} \mathfrak{L}_j^{\mathrm{i}}$ is a matrix polynomial of order p in the lag operator $\mathfrak{L}_j$, such that $\mathfrak{L}_j^{\mathrm{i}} \mathcal{Y}_{jt} = \mathcal{Y}_{jt-1}$ and $\mathcal{Y}_{jt} = \begin{vmatrix} \mathscr{Y}_{1jt} \\ \vdots \\ \mathscr{Y}_{njt} \end{vmatrix}$. c_j is a vector of constants. $\mathcal{A}_{j0}^{-1}$ represents the synchronous relationships between the components of $\mathcal{Y}_{jt}$; ν_{jt} is the disturbance vector, denoting white noise with uncorrelated elements $E[\nu_{\mathscr{g}jt}\nu_{\mathscr{h}jt}] = 0$ for $\mathscr{g} \neq \mathscr{h}$.

$\Upsilon(\mathfrak{L}_j)$ is another matrix polynomial in the lag operator $\mathfrak{L}_j$, but of infinite order; $\mathcal{E}_j$ is the unconditional expected value. In this configuration, it is worth noting the equivalence with a moving average process (below an ARIMA - AutoRegressive Integrated Moving Average process will be applied).

In proceeding whilst using a GARCH (ϵ_t) process – with $\epsilon_t = (\epsilon_{1t}, \dots, \epsilon_{tm})'$ – we can assume that the distribution of the k conditional returns will be multivariate Gaussians with zero mean and the covariance matrix $H_{xt} = \mathrm{Var}(\epsilon_t | \epsilon_u, u < t) = E(\epsilon_t \epsilon_t' | \epsilon_u, u < t)$.

$$r_{xt} | \mathcal{F}_{t-1} \sim \mathcal{N}(0, H_{xt}) \qquad (11)$$

$H_{xt} \equiv D_{xt} R_{xt} D_{xt}$, with:

D_{xt} the diagonal matrix $k \times k$ containing the standard deviation values derived from the univariate GARCH model ($\sqrt{h_{x\iota t}}$ is the element on the diagonal ι):

$$h_{x\iota t} = \omega_\iota + \sum_{p=1}^{P_\iota} \alpha_{\iota p} r_{x\iota(t-p)}^2 + \sum_{q=1}^{Q_\iota} \beta_{\iota q} r_{x\iota(t-q)}^2 \qquad (12)$$

for $\iota = 1, 2, \dots, k$, with $\sum_{p=1}^{P_\iota} \alpha_{\iota p} + \sum_{q=1}^{Q_\iota} \beta_{\iota q} < 1$ and subject to verification of stationarity and nonnegativity constraints. The parameters α denote the conditional volatility; β the persistence of the volatility. ω is a calibration constant.

R_{xt} is the time-variable correlation matrix. The $\epsilon_t \sim N(0, R_{xt})$ are standardised residuals.

The log-likelihood is:

$$\mathcal{L} = -\frac{1}{2} \sum_{t=1}^{T} k \log(2\pi) + 2\log(|D_{xt}|) + \log(|R_{xt}|) + \epsilon_t' R_{xt}^{-1} \epsilon_t \qquad (13)$$

The dynamic correlation structure is:

$$\mathcal{Q}_{xt} = \left(1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n\right) \bar{\mathcal{Q}}_x + \sum_{m=1}^{M} \alpha_m \left(\epsilon_{(t-m)} \epsilon_{(t-m)}'\right) - \sum_{n=1}^{N} \beta_n \mathcal{Q}_{x(t-n)} \qquad (14)$$

With $\bar{\mathcal{Q}}_x$ being the unconditional covariance matrix of the standardised residuals. We can rewrite $R_{xt} = \mathcal{Q}_{xt}^{*-1} \mathcal{Q}_{xt} \mathcal{Q}_{xt}^{*-1}$, where $\mathcal{Q}_{xt}^{*} = \begin{bmatrix} \sqrt{\mathscr{q}_{11}} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{\mathscr{q}_{22}} & 0 & \dots & 0 \\ 0 & 0 & \sqrt{\mathscr{q}_{33}} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sqrt{\mathscr{q}_{kk}} \end{bmatrix}$; and, therefore: $\rho_{x\iota\xi t} = \frac{\mathscr{q}_{\iota\xi t}}{\sqrt{\mathscr{q}_{\iota\iota}\mathscr{q}_{\xi\xi}}}$ constitutes the typical element of R_{xt}. So that the correlation matrix R_{xt} can be positively defined, we have to make sure that Q_{xt} is positively defined: this can be found if H_{xt} is uniformly defined as positive; here are sufficient – though not necessary – conditions for H_{xt} to be defined positive for each t:

- $\omega_\iota > 0$;
- $h_{x\iota 0} > 0$;
- $\alpha_{\iota p} \forall p \in (1, \dots, P_\iota)$ and $\beta_{\iota q} \forall q \in (1, \dots, Q_\iota)$ are such that $h_{x\iota t}$ is positive with probability of 1;
- The roots of $1 - \sum_{p=1}^{P_\iota} \alpha_{\iota p} Z^p + \sum_{q=1}^{Q_\iota} \beta_{\iota q} Z^q$ are external to the unit circle;
- $\alpha_m \geq 0 \ \forall m \in (1, \dots, M_\iota)$;
- $\beta_n \geq 0 \ \forall n \in (1, \dots, N_\iota)$;
- $\sum_{m=1}^{M} \alpha_m + \sum_{n=1}^{N} \beta_n < 1$;
- the minimum eigenvalue $\lambda_{R_x \min} > 0 \wedge \lambda_{R_x \min} < \bar{R}_x$

We have estimated the DCC model (θ) through a two-stage procedure. This model θ has parameters $(\phi, \Psi) = (\phi_1, \phi_2, \dots, \phi_k, \Psi)$, with $\phi_\iota = (\omega, \alpha_{1\iota}, \dots, \alpha_{P_\iota \iota}, \beta_{1\iota}, \dots, \beta_{Q_\iota \iota})$ of the univariate GARCH from (12).

1) Sum of the log-likelihoods of the univariate GARCH for each series of residuals on each asset: use of the (quasi)-likelihood function from (13) with the substitution of R_{xt} by an identity matrix with the dimension k (I_k), in the form:

$$
\begin{aligned}
q\mathcal{L}_1(\phi|r_{xt}) &= -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_{xt}|) + \log(|I_k|) + r_x{}' R_{xt}^{-1} D_{xt}^{-1} r_{xt}) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi) + \sum_{n=1}^{k}\left(\log(h_{x\iota t}) + \frac{r_{x\iota t}^2}{h_{x\iota t}}\right)\right) \qquad (15)
\end{aligned}
$$

2) Model estimation – going back to R_t – still adopting the likelihood formula, also conditioned by the parameters in (15):

$$
\begin{aligned}
q\mathcal{L}_2(\Psi|\hat{\phi}, r_t) &= -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_{xt}|) + \log(|R_{xt}|) + r_x{}' D_{xt}^{-1} R_{xt}^{-1} D_{xt}^{-1} r_{xt}) \\
&= -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_{xt}|) + \log(|R_{xt}|) + \epsilon_t' R_{xt}^{-1}\epsilon_t) \qquad [16]
\end{aligned}
$$

Where the first addends of the summation of $k\log(2\pi)$ and $2\log(|D_t|)$ are excludable because they are irrelevant in the selection of the parameters, since one is conditioning $\hat{\phi}$:

$$q\mathcal{L}_2(\Psi|\hat{\phi}, r_{xt}) = (\log(|R_{xt}|) + \epsilon_t' R_{xt}^{-1}\epsilon_t) \qquad (17)$$

The tests for the asymptotic consistency and normality of the estimates is based on the verification of the assumptions of completeness of the probability and measurability space of the functions (15) and (16). In particular, following along with the two-step Generalised Method of Moments (GMM) approach, moment conditions are imposed, with the vector-valued function $f(r_{xt}, \phi)$ such that $m(\phi_0) \equiv \mathrm{E}[f(r_{xt}, \phi_0)] = 0$ – with $m(\phi_0) \neq 0$ for $\phi \neq \phi_0$ – relative to the order moments $(1, 2, \dots, k)$ for each element of ϕ; replacing the theoretical values with the outcomes of empirical observations:

$$\hat{m}(\phi) \equiv \frac{1}{\mathrm{T}}\sum_{\mathrm{t}=1}^{\mathrm{T}} f(r_{xt}, \phi_0) \qquad (18)$$

Since the imposed conditions exceed the dimension of the vector ϕ and the resulting system would be indeterminate, the estimation proofs are achieved by minimising a generalised norm of conditions on the moments. The family of standards that the GMM model takes into consideration is defined by $\|\hat{m}(\phi)\|_W^2 = \hat{m}(\phi)' W \hat{m}(\phi)$, where the weighting matrix W, when calculated on the observed data set, is indicated by $\widehat{W}$. Therefore:

$$\hat{\phi} = \arg\min_{\phi\in\Psi}\left(\frac{1}{T}\sum_{t=1}^{T} f(r_{xt},\phi)\right)' \widehat{W}\left(\frac{1}{T}\sum_{t=1}^{T} f(r_{xt},\phi)\right) \quad (19)$$

Where Ψ is the parameter space and $\widehat{W}$ is precisely the weighting matrix, the following conditions, sufficient for the GMM estimator to be consistent, must be verified:

- $\widehat{W}_T \xrightarrow{p} W$, with W a positive semi-definite matrix;
- $W\mathrm{E}[f(r_{xt},\phi)] = 0$ only for $\phi = \phi_0$;
- the space of the possible parameters $\Theta \subset \mathbb{R}^k$ is compact;
- $f(r_x,\phi)$ is continuous in each ϕ with probability of 1;
- $\mathrm{E}\left[\sup_{\phi\in\Psi}\|f(r,\phi)\|\right] < \infty$.

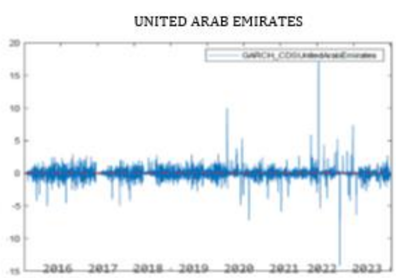
Wanting to obtain the most efficient GMM estimator in the asymptotic variance (Tasche, 2021) we should adopt $W \propto \Theta^{-1}$, which would then lead to the asymptotic distribution formula: $\sqrt{T}(\hat{\phi}-\phi_0) \xrightarrow{d} \mathcal{N}(0,(C'\Theta^{-1}C)'$, the matrices being $C = E(\nabla_\phi f(r_{xt},\phi_0))$ and $\Theta = \mathrm{E}(f(r_{xt},\phi_0)f(r_{xt},\phi_0)')$. The estimator reaches the Cramér–Rao limit.
Since r_{xt} i.i.d.:

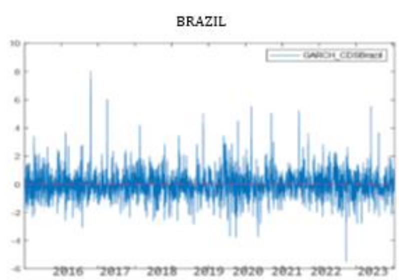
$$\widehat{W}_T(\hat{\phi}) = \left(\frac{1}{T}\sum_{t=1}^{T} f(r_{xt},\hat{\phi})f(r_{xt},\hat{\phi})'\right)^{-1} \quad (20)$$

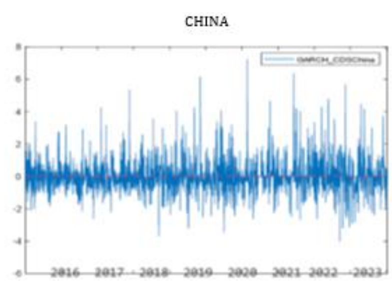
In the first step we will work on the equality between the weighting matrix and the identity matrix: $W = I$ and we can calculate the preliminary estimate $\hat{\phi}_{(1)}$. Next (step 2), $\widehat{W}_T(\hat{\phi}_{(1)})$ converges in probability to Θ^{-1}, and – by calculating $\hat{\phi}$ with this weighting matrix – the estimator will be asymptotically efficient.

Summary of the empirical results

Below are the results obtained by applying the mathematical-econometric models as developed and reported above; the log-likelihood findings are placed within the calculation of Akaike's information criterion (AIC). These statistical results indicate that there is a significant dynamic correlation in volatility trends.







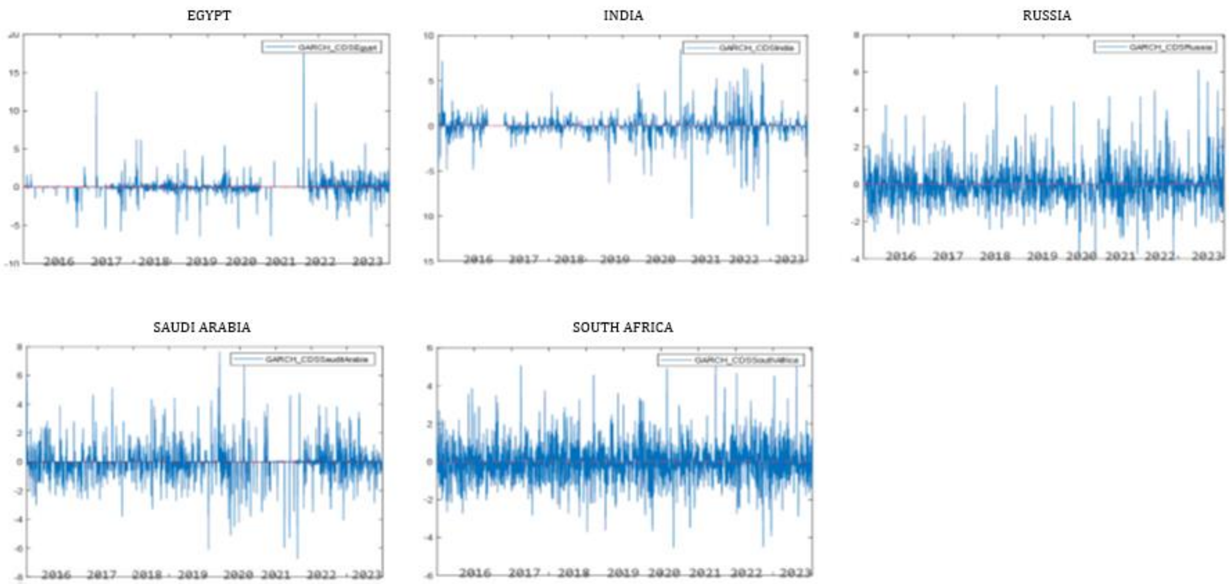


Figure 4: Residuals of model GARCH (CDS)

Table 2: ARIMA model estimation

	ARE	BRA	CHN	EGY	IND	RUS	SAU	ZAF
ARIMA MODEL ESTIMATION								
Constant	2.13740E-06	7.71700E-06	5.48980E-06	-2.91720E-07	-3.60150E-05	7.67390E-06	3.80200E-05	9.14080E-06
Standard error	2.00050E-06	1.06330E-05	6.98640E-06	1.28010E-06	5.70680E-05	7.67820E-06	5.71840E-05	1.26440E-05
t Statistic	1.06840E+00	7.25750E-01	7.85790E-01	-2.27890E-01	-6.31100E-01	9.99450E-01	6.64870E-01	7.22910E-01
P-Value	2.85340E-01	4.67990E-01	4.31990E-01	8.19730E-01	5.27980E-01	3.17580E-01	5.06130E-01	4.69740E-01
AR {1}	-7.47420E-01	-6.35510E-01	-5.91200E-01	-6.33520E-01	-6.23120E-01	-6.50140E-01	-6.40160E-01	-6.37120E-01
Standard error	1.03650E-02	8.93900E-03	1.10780E-02	9.65140E-03	8.98320E-03	1.30870E-02	8.89000E-03	1.53770E-02
t Statistic	-7.2109E+01	-7.10939E+01	-5.33649E+01	-6.56403E+01	-6.93648E+01	-4.96794E+01	-7.20094E+01	-4.14337E+01
P-Value	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500
AR {2}	-3.78350E-01	-3.08620E-01	-3.60430E-01	-2.60510E-01	-2.91710E-01	-2.58330E-01	-3.20070E-01	-3.49570E-01
Standard error	8.77490E-03	1.00780E-02	1.27910E-02	1.17290E-02	9.83890E-03	1.14060E-02	7.17120E-03	1.45940E-02
t Statistic	4.31170E+01	-3.06240E+01	-2.81790E+01	-2.22111E+01	-2.96490E+01	-2.26492E+01	-4.46323E+01	-2.39530E+01
P-Value	<1.000E-500	5.86340E-206	1.05890E-174	2.68040E-109	3.49520E-193	1.42200E-113	<1.000E-500	8.58960E-127
MA {1}	-9.99710E-01	-9.87800E-01	-9.92600E-01	-1.00000E+00	-9.33390E-01	-9.93040E-01	-9.33480E-01	-9.83440E-01
Standard error	1.84310E-03	3.31860E-03	3.93210E-03	7.71110E-03	4.19740E-03	3.67170E-03	4.51210E-03	4.02160E-03
t Statistic	-	2.97656E+	-	-	-	-	-	-

	5.4239E+02	02	2.52436E+02	1.29683E+02	2.22372E+02	2.70458E+02	2.06882E+02	2.44541E+02
Variance	1.18580E-03	1.38230E-03	1.63990E-03	1.15070E-03	1.23550E-03	1.77230E-03	1.31150E-03	1.08230E-03
Standard error	9.41860E-06	1.60480E-05	2.70880E-05	1.38320E-05	1.26330E-05	2.51220E-05	1.25210E-05	2.04680E-05
t Statistic	1.25900E+02	8.61361E+01	6.05388E+01	8.31899E+01	9.77937E+01	7.05460E+01	1.04746E+02	5.28779E+01
P-Value	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500

AIC	-7.81618E+03	-7.50847E+03	-7.15125E+03	-7.78227E+03	-7.70301E+03	-5.59666E+03	-7.57977E+03	-8.00349E+03
BIC	-7.78817E+03	-7.50847E+03	-7.12325E+03	-7.75431E+03	-7.67502E+03	-5.56977E+03	-7.55178E+03	-7.97548E+03

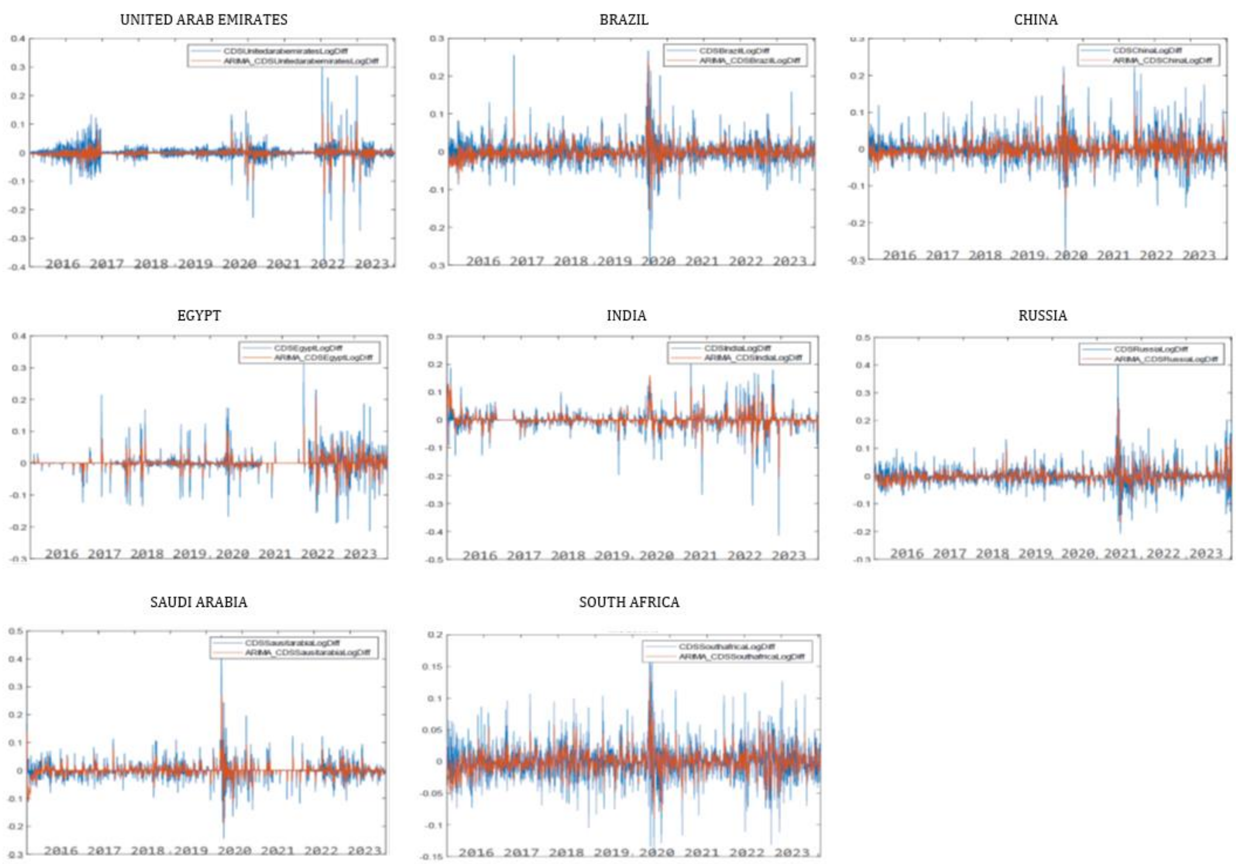


Figure 5: ARIMA processes (CDS)

Filtering the series with the ARIMA process returns an estimate of the parameters that appears appropriate from an econometric point of view, also by virtue of the AIC and BIC levels obtained.

Table 3: GARCH-DCC model estimation

	ARE	BRA	CHN	EGY	IND	RUS	SAU	ZAF
GARCH-DCC ESTIMATION	**MODEL**							

Constant	1.83390E-06	1.06610E-04	1.48480E-04	4.40300E-05	6.80800E-04	6.23380E-05	5.83530E-05	6.79380E-05
Standard error	2.11650E-07	7.01680E-06	1.10710E-05	1.13190E-06	9.94380E-06	6.57340E-06	5.83530E-05	7.86710E-06
t Statistic	8.66470E+00	1.51931E+01	1.34114E+01	3.89000E+01	6.84649E+01	9.48330E+00	1.70895E+01	8.63560E+00
P-Value	4.52870E-18	3.93170E-52	5.18340E-41	<1.000E-500	<1.000E-500	2.46470E-21	1.77580E-65	5.84220E-18
GARCH	9.19070E-01	6.63990E-01	6.92380E-01	8.49830E-01	4.85820E-01	7.89770E-01	8.03990E-01	8.13790E-01
Standard error	9.72310E-04	1.58230E-02	1.62880E-02	3.69410E-03	1.42190E-02	1.63990E-02	8.88990E-03	1.78890E-02
t Statistic	9.45245E+02	4.19635E+01	4.25094E+01	2.30052E+02	3.41670E-01	4.81582E+01	9.04388E+01	4.54911E+01
P-Value	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500	<1.000E-500
ARCH	8.09320E-02	2.16540E-01	1.91130E-01	1.14090E-01	2.45800E-01	1.60300E-01	1.17380E-01	9.63330E-02
Standard error	1.31350E-03	1.10320E-02	1.08780E-02	5.49900E-03	1.03390E-02	1.34990E-02	4.81580E-03	1.11290E-02
t Statistic	6.16155E+01	1.96288E+01	1.75694E+01	2.07466E+01	1.93440E-01	1.18757E+01	2.43738E+01	8.65570E+00
P-Value	<1.000E-500	8.78010E-86	4.22900E-69	1.31670E-95	7.47730E-256	1.58260E-32	3.24620E-131	4.90010E-18
AIC	-1.00088E+04	-8.91889E+03	-8.17202E+03	-9.05029E+03	-8.49101E+03	-6.80512E+03	-9.13190E+03	-8.88908E+03
BIC	-9.99197E+03	-8.90207E+03	-8.15521E+03	-9.03352E+03	-8.47421E+03	-6.78898E+03	-9.11511E+03	-8.87226E+03

Through the empirical analysis and the calculations carried out according to the procedures set out above, we have therefore verified the significance of the parameters and we have studied and confirmed the conditional dynamic correlation between the economies of the BRICS-plus, particularly in the presence of shocks, which also bring about mutual contagion (temporary increase in DCC) and even interdependence (increases in DCC leading to new, less unstable levels)

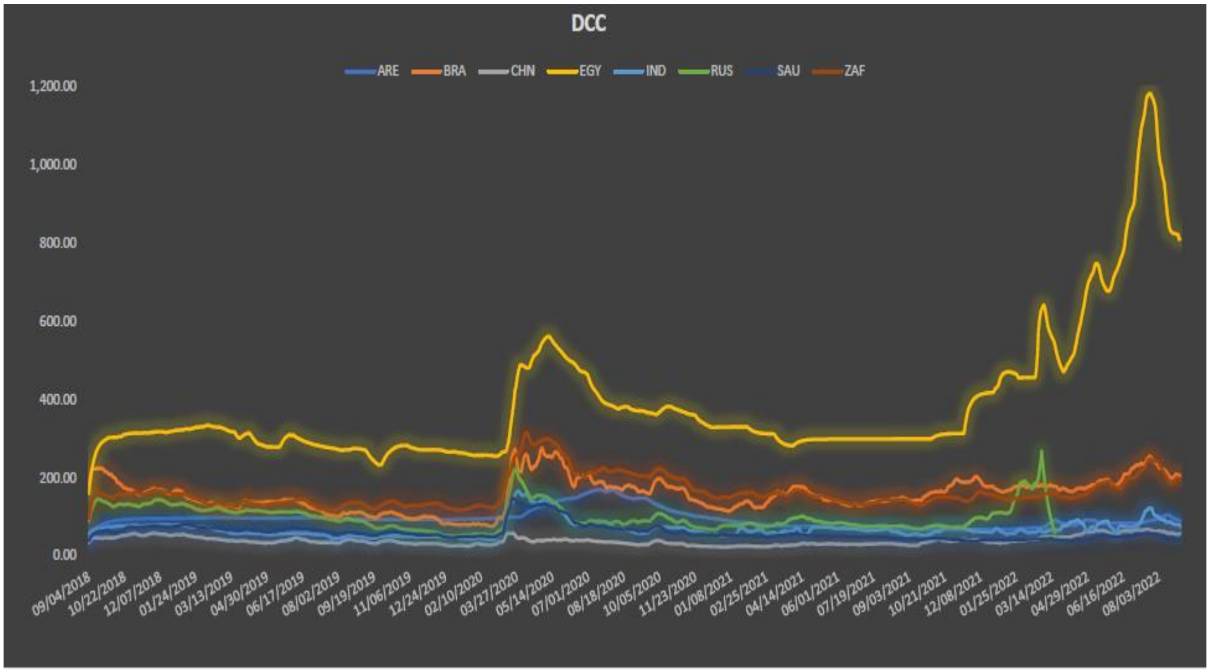


Figure 6: Estimation of volatilities in the GARCH-DCC process

A Further Consideration on the Probability of Sudden Peaks in Volatility

Let's look again at (9) and assuming that both distributions are normal Gaussian (with the mean = 0 and the variance = 1) and independent of each other, then: $a^2 + b^2 = 1$ and $Y_j = \alpha x_j + \sqrt{1-\alpha^2}\,\mathcal{j}_j$. With α known a priori, the above formula can indicate the process that preserves the system from significant perturbation of volatility: or rather, when Y remains below a certain threshold level L_ι – which should be verified contingently (this is a value subject to changes and updates, especially based on global context) – so that

$$\mathcal{P}\varsigma_\iota = Pr(Y < L_\iota) \qquad (21)$$

From which, $\mathcal{P}\varsigma_\iota = \Phi(L_\iota)$ and $L_\iota = \Phi^{-1}(\mathcal{P}\varsigma_\iota)$, with Φ as the cumulative probability function of the normal Gaussian (Glasserman and Pirjol, 2023; Salinas *et al*., 2019).

Then, from the foregoing, the paroxysmal state of volatility is verified when:

$$\otimes^n \left(\alpha x_j + \sqrt{1-\alpha^2}\,\mathcal{j}_j\right) < \Phi^{-1}(\mathcal{P}\varsigma_\iota) \qquad (22)$$

The factor $\mathcal{j}_j$ is considered unique and intrinsic to individual countries; therefore, let's solve the previous inequation on the basis of the macroeconomic context variable resulting from the dynamic correlation verified on the BRICS-plus partnership CDS series at 11:

$$x < \frac{\Phi^{-1}(\mathcal{P}\varsigma_\iota) - \sqrt{1-\alpha^2}\,\overline{\mathcal{J}_J}}{\alpha} \qquad (23)$$

We can write $\hat{\psi} = \frac{\Phi^{-1}(\mathcal{P}\varsigma_\iota) - \sqrt{1-\alpha^2}\,\overline{\mathcal{J}_J}}{\alpha}$, therefore $x < \hat{\psi}$. This formula measures the probability levels that the random vector should assume in order to prevent (probabilistically) paroxysmal perturbations in volatility, given the average of idiosyncratic factors $\overline{\mathcal{J}_J}$, based on the context of integrated macroeconomic conditions ($\mathcal{P}\varsigma_\iota | \overline{\mathcal{J}_J}$).

Conclusions: pay attention to the emergence of points of instability in the system

The relative dynamic economic system can be identified by:

$$\dot{x} = \mathfrak{F}(x, \mu) \qquad (24)$$

with

- $x \in \mathbb{R}^n$ system state vector, composed of inflation level, volatility of stock price, and inequality index);

- $\mu \in \mathbb{R}^m$ system operating parameters vector (monetary policies on nominal interest rates);
- $\mathfrak{F}: \Omega \subseteq \mathbb{R}^n \longrightarrow \Omega \subseteq \mathbb{R}^n$ system vector field.

The nonlinear system expressed in (24), given the assumptions, exhibits a certain degree of empirical randomness in the evolution of its dynamic variables. However, the randomness of the vector is only apparent: once the initial conditions have been established, the evolution of the system is deterministic (Ott, 2002); it is structurally stable when – at the set of parameter values μ – there exists a $\varepsilon > 0$ such that a homeomorphism subsists between the same system and

$$\dot{x} = \widetilde{\mathfrak{F}}(x, \mu) \qquad (25)$$

for all vector fields $\widetilde{\mathfrak{F}}: \Omega \subseteq \mathbb{R}^n \longrightarrow \Omega \subseteq \mathbb{R}^n$ with $\left\|\mathfrak{F}(x, \mu) - \widetilde{\mathfrak{F}}(x, \mu)\right\| < \varepsilon$. This means that there exists a transformation $o: \Omega \subseteq \mathbb{R}^n \longrightarrow \Omega \subseteq \mathbb{R}^n$, continuous with its inverse, which changes the trajectories of the system in (24) into those belonging to (25), even as preserving the direction.

In the case of the emerging economies observed – on the contrary – the related systems are structurally unstable, also in consideration of the deficiencies system operating parameters vector, and – especially whenever $x > \hat{\psi}$ – are instead characterized by a bifurcation: that is, these systems, which are in a condition of chaotic economic growth, are exposed to significant – and sudden – changes in stability, and in number and nature of solutions.

This happens when a steady state point $((x = x^*; \mu = \mu^*)$ where, e.g., $x^* = \hat{\psi})$ occurs (in the local state space), and the Jacobian matrix of the field $\mathfrak{F}$ at that point, $\mathcal{J}^* = \left.\frac{\partial \mathfrak{F}}{\partial x}\right|_{x=x^*, \mu=\mu^*}$, has at least one eigenvalue, $\lambda_i^{\mathcal{C}}$, with the real part zero (Tikjha and Gardini, 2020). The one or more eigenvalues $\lambda_i^{\mathcal{C}}$ with the real part zero are referred to as critical eigenvalues (Desogus and Casu, 2022). When one or more critical eigenvalues emerge, the system locally bifurcates.

If the critical eigenvalue presented by the matrix $\mathcal{J}^*$ is real, there is a real bifurcation, resulting in branches of stationary regime solutions without any periodic solutions.

Analysing these findings can be useful for implementing stability control measures, especially as *fluctuations* in macroeconomic variables become more pronounced and before the system becomes uncontrollable (Zungu *et al*., 2022).

Essentially, we have observed that one of the key factors for a country's economic development, once it has *de facto* assumed the necessary technological standing, is a lenient economic policy. Such a policy does not impose predetermined maximum output levels and may have short- and medium-term social costs in terms of income inequality and potential periods of reduced domestic consumption.

Nonetheless, such an easing of government economic control would require a rigorous scientific approach, supported by ongoing analyzes of the system's dynamics. Having thus demonstrated the existence of dynamic conditional correlation between the economies of the BRICS-plus countries – through the use of multivariate DCC-GARCH models in joint modelling obtained by filtering the series with a VAR model and specifically with an ARIMA analysis – and confirmed the significance of the parameters, contagion effects and even inter-group interdependence have been identified. In particular, the contagion effect was observed in temporary increases in dynamic conditional correlations.

The application of the presented model, capturing the relationships of gradual contagion and in light of the risks of losing control over the macroeconomic dynamics – first individual, then collective due to interdependencies – can offer an Early-Warning methodology with a warning signal threshold studied based on the mentioned steady state and pre-bifurcation points (x^*, μ^*): once the threshold is crossed, turbulent peaks of volatility form in the global commodity markets and derivative financial markets, as emerged from the reasoning carried out. The application of a control law should be based on findings of excessive growth in inflation or a macroeconomic spread that reaches a certain alert level; stabilization interventions can be monetary policy with intervention on the nominal interest rate, or through more structural economic policies.

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