Non-Univalent Approximation of Peano Curve for Global Optimization

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Abstract. In this article, multi-dimensional global optimization problems are considered, where the objective function is supposed to be Lipschitz continuous, multiextremal and without a known analytic expression (black-box). Non-Univalent approximation of Peano curve to reduce the problem to a univariate one satisfying the Hölder condition is employed. Geometric frameworks for construction of global optimization algorithms are discussed. Numerical experiments executed on 100 test functions taken from the literature show a promising performance of the algorithms.

INTRODUCTION

Global optimization is a rapidly growing area of numerical analysis (see [1, 2, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22] and references given therein). In this work, we consider the following global optimization problem

$$F^* = F(y^*) = \min F(y), \quad y \in D, \qquad D = \{y \in \mathbb{R}^N : a \le y_j \le b, \quad 1 \le j \le N\}, \tag{1}$$

where the objective black-box function $F(y)$ satisfies the Lipschitz condition with an unknown Lipschitz constant L, $0 < L < \infty$, i.e.,

$$|F(y') - F(y'')| \le L\|y' - y''\|, \quad y', y'' \in D, \tag{2}$$

and $\|\cdot\|$ denotes the Euclidean norm.

Space-filling curves which pass through every point of D are a powerful tool for solving this multivariate Lipschitz global optimization problem (see [2, 10, 11, 18, 19, 20]). In particular, as shown in [19], the problem (1), (2) is equivalent to the following one-dimensional problem

$$f(x^*) = F(y(x^*)) = \min F(y(x)), \quad x \in [0, 1], \tag{3}$$

where $y(x)$ is the Peano curve mapping the interval $[0, 1]$ in D. In addition, (see [19]) it can be proved that the function $f(x)$ satisfies the Hölder condition with the constant $H = 2L\sqrt{N+3}$, namely,

$$|f(x') - f(x'')| \le H|x' - x''|^{1/N}, \quad x', x'' \in [0, 1]. \tag{4}$$

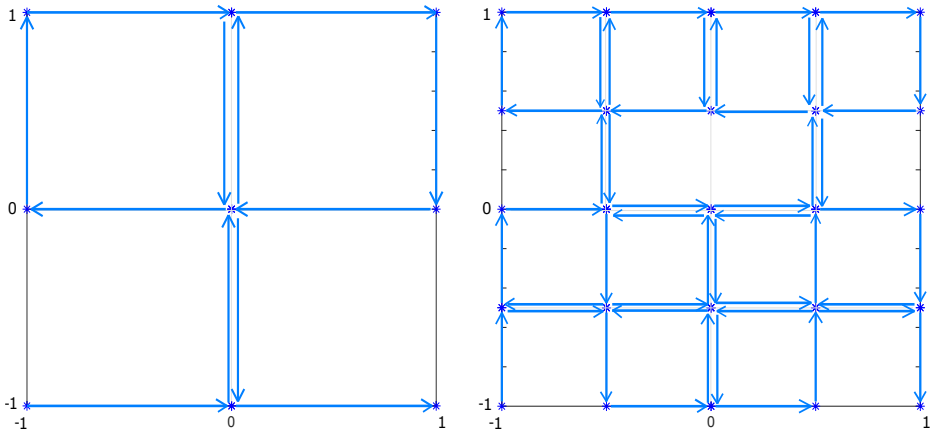

FIGURE 1. Image of Non-Univalent approximations of level 1 and 2.

Non-Univalent Approximation in Multidimensional Geometric Algorithm

Obviously, since Peano curve is a fractal generated by an iterative process, (see [19]), its computable approximations should be employed in the numerical algorithms. For this reason, following [19] let us now introduce the Non-Univalent Approximation of Peano Curve.

The Peano non-univalent approximation of level M is constructed subdividing the hyperinterval $D \subset \mathbb{R}^N$ into 2^{MN} subcubes with the edge length equal to $2^{-M}(b-a)$. Let us denote with $P(M,N)$ the grid composed by the vertices of these subcubes ($P(M,N) \subset P(M+1,N)$). Then the evolvent $n_M(x)$ maps the uniform grid $p(M,N)$ in $[0,1]$ composed by the points

$$w_j = \frac{j}{2^{(M+1)N} - 2^{MN}}, \quad 0 \le j \le 2^{(M+1)N} - 2^{MN}, \tag{5}$$

onto the grid $P(M,N)$. It reflects the property of the Peano curve $y(x)$: a point in $[0,1]$ could have several inverse images in $[0,1]$, at most 2^N. Figure 1 shows the image of the interval $[0,1]$ through the first and the second approximation of Peano curve using the Non-Univalent evolvent mapping $[0,1]$ to the square $[-1,1]^2$. The arrows indicate the order in which the vertices are produced if we consider the increasing order in the corresponding grid $p(M,N)$ of the interval $[0,1]$. In this way, since a point y in D can be characterized by its *multiplicity*, computing a single evaluation of the objective function $F(n_M(x))$ in a point $x' \in [0,1]$ (hereinafter this evaluation is called *trial*) we know that the values of the function will be the same for all the *inverse images* of $n_M(x')$ with respect the correspondence $n_M(x)$, i.e., some different inverse images $x', x'' \in [0,1]$ could have the same image $n_M(x') = n_M(x'')$ and therefore $F(n_M(x')) = F(n_M(x''))$ as shown in Fig. 2. In the present work, we employed the Multidimensional Geometric Algorithm (see [18]) making use of the just described Non-Univalent Approximation and the opportunity to evaluate only once the function $F(y)$ and then introduce this value to all multiple inverse images in the one-dimensional problem. Two different strategies to include inverse images in our trial points lead us to two different modifications GAP1 and GAP2. The first avoids to generate too small intervals in zones where we already have enough information, while GAP2 avoids excessive partition in many intervals if we are not in the vicinity of a global minimizer.

Numerical experiments

We compared methods GAP1,GAP2 with the original Multidimensional Geometric Algorithm MGA (see [18]) and the original method DIRECT algorithm from [8]. The number of trials was chosen as the comparison criterion.

We performed our experiments using Grishagins class of test functions which provides 100 two-dimensional multiextremal randomly generated test functions with known global minima y^* (see [4]). For the first series of experiments (see Tab. 1 and Fig. 3), where we compare DIRECT with our methods using approximations of level 10 of Peano curve, for each algorithm we used the following stopping criterion:

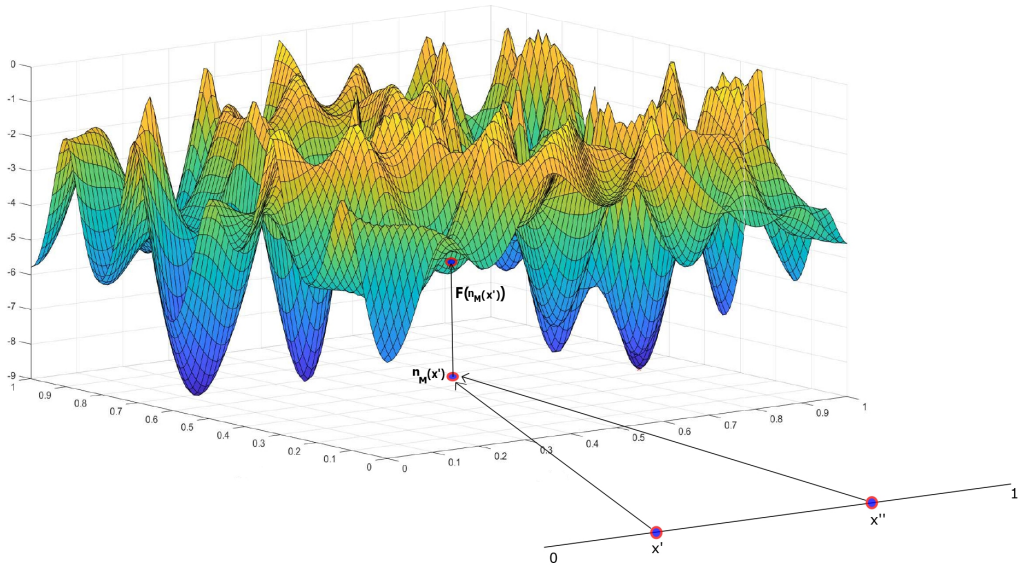

FIGURE 2. The value $F(n_M(x'))$ was assigned to both inverse images, x' and x''

TABLE 1. Average and maximum number of trials in numerical experiments on Grishagin's test functions using the stopping criterion a)

	DIRECT	**MGA**	**GAP1**	**GAP2**
average	184.48	206.33	188.87	190.74
max	1045	694	691	550

a) If an algorithm generated a point $y' \in D$ which satisfies the following condition

$$|y'(i) - y^*(i)| \leq \sqrt[N]{\Delta}(b-a), \quad 1 \leq i \leq N,$$

then the problem was considered to be solved and the algorithm stopped giving y' as an approximation of y^*. Figure 3 a) shows 1045 points of trials executed by DIRECT to find the global minimum of the problem and Fig. 3 b) presents 232 points of trials executed by the GAP2 to solve the same problem.

In the second series of experiments (see Tab. 2), where we compare methods using approximations of Peano curve among them, the following stopping criterion was applied:

b) The value $\varepsilon = 10^{-3}$ is fixed and the search terminates when the algorithm selects an interval $[x', x'']$ to perform the next trial which satisfies $|x' - x''|^{1/N} < \varepsilon$ or the maximum number of function evaluations (5000) is reached. When one executes tests with a class of 100 different functions it becomes inappropriate to use specific values of the reliability parameter of the method r for each function, hence in our experiments at most two values of this parameter have been fixed for the entire class (a detailed discussion of the choice of the parameter r can be found in [19]). In Tab. 2 the notation solved problems with $r_1(r_2)$" means that for GAP1 and GAP2 we used two different reliability parameters: r_1 for solving 99 problems and r_2 for the remaining one.

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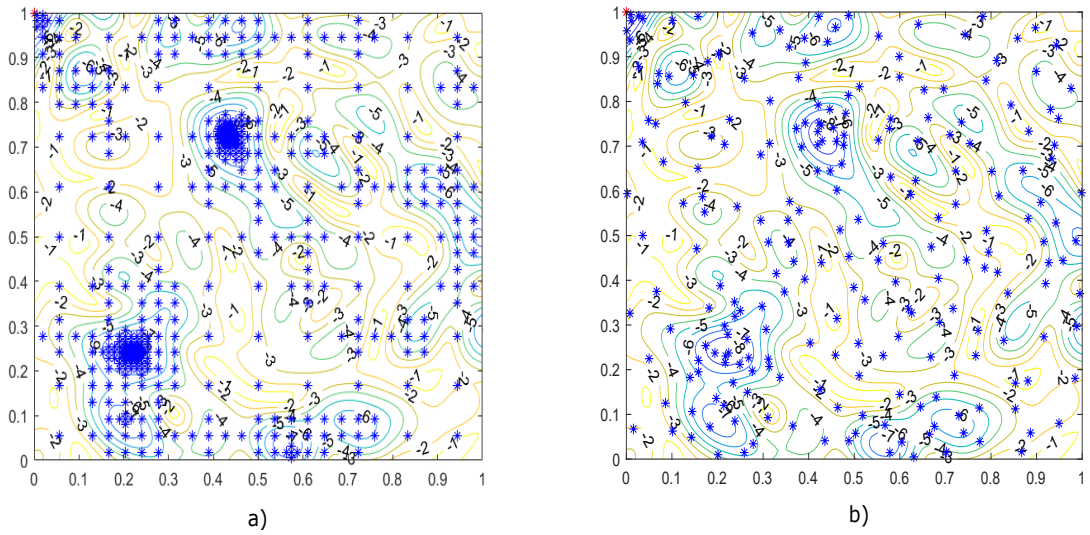

FIGURE 3. Contour lines of a test function and trial points generated by DIRECT a) and GAP2 b) during their work. Global minimizer is marked by the red symbol *"

TABLE 2. Average and maximum number of trials in Grishagin's test functions using the stopping criterion b)

	MGA	**GAP1**	**GAP2**
r_1	1.1	1.1	1.1
r_2	-	1.2	1.2
solved problems with $r_1(r_2)$	100(0)	99(1)	99(1)
average	1381.23	1422.28	1229.71
max	4706	4374	3227

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