



Optimal investment and fair sharing rules of incentives in virtual renewable energy communities

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Abstract

This paper examines two interconnected challenges in Renewable Energy Communities (REC) optimization: investment in renewable technologies and equitable sharing of incentives, usually offered by a central authority. Focusing on a REC under the so-called *virtual framework* and composed of a household and a biogas producer — common in rural and urban contexts — we analyze how investment decisions and incentive-sharing mechanisms impact community profitability and self-consumption. The household invests in photovoltaic panels to reduce energy purchases and monetize surplus generation, while the biogas producer either converts biogas into electricity or sells it on the gas market. We model this interaction as a leader-follower problem: an administrator (leader) defines the incentive-sharing rule, while a household and a biogas producer (followers) determine their optimal investments. Modeling the objective of the leader as a Nash bargaining problem and a Nash equilibrium for the followers' static game, we provide model-based insights into efficient REC design and policy implications for fostering sustainable community energy systems. The model is applied to this stylized case of a REC, under realistic data about the random variables and investment costs. We obtain structural insights into how specific incentive policies and members' investment decisions interact within this specialized configuration, highlighting trade-offs that can guide the strategic organization of similar community energy frameworks.

Keywords Stochastic modeling · Game theory · Environmental policy · Capacity expansion

1 Introduction

The global challenge of climate change and the ongoing energy transition represent an unprecedented issue for humanity. Addressing this challenge requires diverse solutions that engage all sectors of society. One promising approach to reducing CO₂ emissions is empow-

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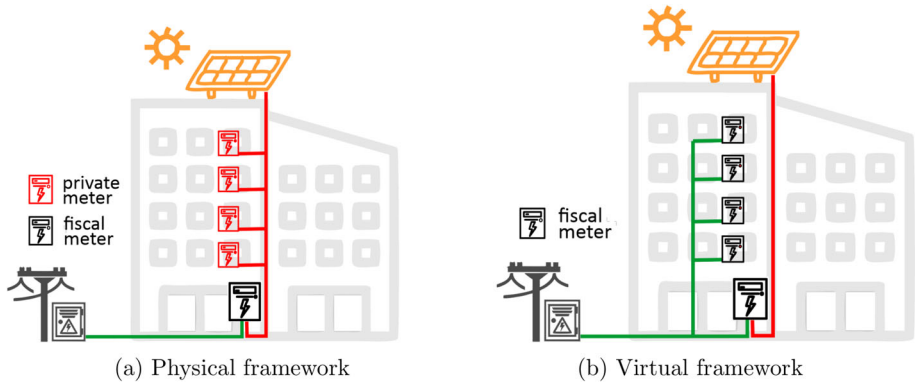


Fig. 1 Difference between physical and virtual REC frameworks

ering end-users to become both self-producers and self-consumers of Renewable Energy Sources (RES).

In this context, community-shared solar programs in the United States are now mandated by legislation in at least 15 States and the District of Columbia. Additionally, a growing number of electric utilities have voluntarily implemented these programs, as reported in Chan et al. (2017).

The European Union (EU) has introduced two key directives, namely Directive (EU) 2018/2001 [16] and Directive (EU) 2019/944 [17], which establish the regulatory framework enabling private households, small enterprises, and local public authorities to form Renewable Energy Communities (RECs). These communities facilitate the shared generation and consumption of renewable energy sources. The potential impact of RECs is substantial: in OECD European countries, approximately 450 million people collectively consumed over 3 trillion kWh of electricity in 2020 [14].

Heterogeneous *self-consumption* profiles within relatively small groups are expected to mitigate grid congestion and enhance the overall efficiency of national energy systems. However, integrating these entities into existing energy systems is inherently complex, as they can significantly alter both a country's energy mix and the economic equilibrium of institutional stakeholders, including generators, distributors, and retailers.

The implementation of EU directives across Member States is progressing at varying speeds and in accordance with different national regulations. A key decision faced by national regulators is whether to adopt a physical or virtual framework for RECs. For instance, Italy was among the first countries to implement the EU directives on RECs (Zhu et al., 2025), establishing a virtual framework for energy self-consumption (Italian Government, 2019; Italian Parliament, 2020; Italian Ministry of Ecological Transition, 2020). This pioneering approach has the potential to influence and inspire other EU countries seeking to leverage RECs for the energy transition. Figure 1 illustrates the distinction between physical and virtual frameworks, using the simplest case of a REC: a group of households within the same building equipped with a photovoltaic (PV) system. In the physical framework (Figure 1a), all households are directly connected to the PV system and equipped with private meters to track individual consumption. When PV generation at time t exceeds the building's aggregate energy demand, *self-consumption* occurs at no cost. Only the surplus energy is recorded by a fiscal meter and sold to the grid. Conversely, if PV generation is insufficient, the additional energy demand is supplied by the grid and allocated to each household based on private meter readings.

According to the virtual framework shown in Figure 1b, the REC members do not directly consume the energy generated by their plants. Under the virtual scheme, all the generation of a REC, as measured by the central fiscal meter, must be offered to the market. Since RECs are expected to be relatively small market participants, a central authority (e.g., GSE in Italy), which acts as a guarantor and promoter of the country's sustainable development, is established as a natural broker, guaranteeing a selling price calculated as an average of the gross price of electricity updated periodically. To satisfy their energy needs, the members entering a virtual REC must, therefore, continue to buy electricity from their retailers, with their consumption remaining under the control of individual fiscal meters. This is why, for them, self-consumption occurs only in a "virtual way": the central authority, by reading both the central and the individual meters, establishes, at every hour, how much energy has been produced by the PV system and collectively consumed by the members of the REC. The self-consumption of a REC is then defined on an hourly basis as the minimum of collective consumption and generation.

Since the members of a REC continue purchasing all the energy they consume while selling all the energy generated by their plants, the primary benefit of joining a REC lies in the incentive mechanism. More specifically, virtual RECs benefit from an incentive proportional to the total self-consumption accumulated over a given period, such as one year. While the virtual framework is undoubtedly more complex than the physical one, it offers a significant advantage in mitigating energy poverty within society. Given the definition of self-consumption, even members who join a REC solely as consumers without contributing to the initial investment in generation plants play a beneficial role. Their participation supports self-consumption and enhances REC revenues, particularly during peak generation periods when production exceeds demand.

However, a critical issue, which, to the best of our knowledge, has not yet been addressed in the literature, is the development of a fair and effective mechanism for distributing the incentive among the members of a virtual REC. As intuitively expected, the complexity of defining a fair incentive-sharing rule increases with the number of REC members, the diversity of their stakes, and their varying consumption profiles. A particularly critical factor is the stake of those members who invest in the generation infrastructure, as they must determine the optimal capacity for the REC. The viability of a REC depends crucially on the establishment of an equitable sharing rule for the self-consumption incentive, which directly influences members' economic decisions. This rule affects both the willingness of potential members to join and the retention of existing members, as it shapes their expected benefits.

This paper is organized as follows. Section 2 reviews the relevant bibliography and highlights the major contributions of this paper. Section 3 analyzes the optimal individual installation of potential REC members, i.e. in the absence of the incentives provided by joining the REC. In Section 4, we present a Stackelberg game combined with a Nash competitive game model for the members joining a REC, in the presence of an incentive based on self-consumption. In Section 5, we apply the model to real Italian data and develop a thorough sensitivity analysis of the equilibrium model. Section 6 concludes.

2 Literature review

A complete review of RECs can be found in the monograph (Löbbeck et al., 2022). The topic is addressed from multiple perspectives, in particular the definition of RECs across EU Member States and the U.S., as well as the regulatory frameworks governing these communities in

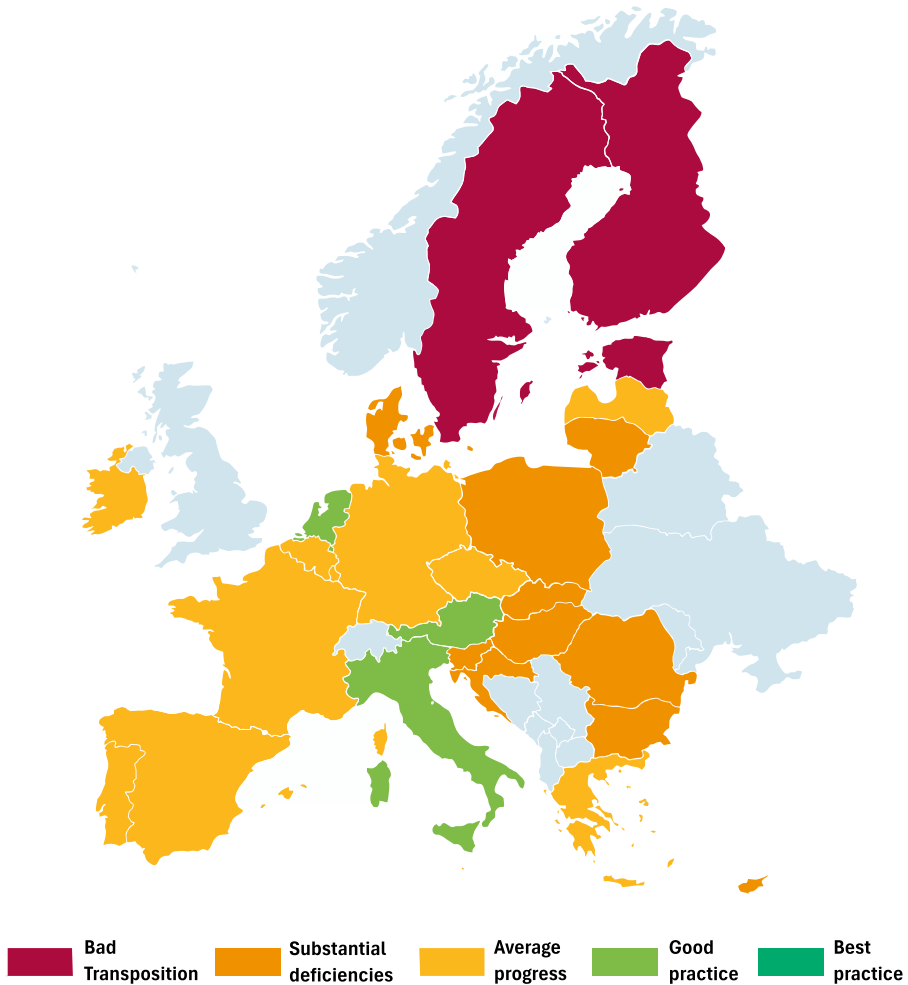


Fig. 2 Transposition of enabling frameworks and support schemes for RECs in EU Countries, Source: REScoop

each country. The authors highlight a general lack of adequate support mechanisms for RECs. The interested reader is referred to several contributions within the volume that offer policy recommendations aimed at better aligning regulatory directives with the development of RECs in Europe and the United States.

Drawing on the *Transposition Tracker for Enabling Frameworks and Support Schemes* (Ecoempower Consortium, 2024; REScoop.eu, 2025) and as we can see from Fig. 2, Italy, Austria and the Netherlands emerge as the most advanced Member States in terms of operational conditions for renewable energy communities, while other countries follow at different speeds.

While Italy fully implements the *Virtual Network Framework* that we discuss in detail below, Austria combines energy-sharing rights with reductions in use-of-system charges for intra-community exchanges and a market premium for part of the surplus sold to the grid. The

Netherlands offers a generally favorable framework, with investment support and special programs for energy cooperatives, but still lacks fully developed, REC-specific operational incentives for energy sharing. Portugal and Spain largely rely on exemptions or reductions of policy and grid charges for self-consumed or shared electricity, approximating a net-billing set-up, whereas France and Flanders (Belgium) provide only limited advantages through adapted network tariffs and tax treatment. Overall, existing schemes span explicit premiums on shared energy, targeted rebates of regulated charges, and low-incentive designs where the benefit of collective self-consumption is confined to avoided retail energy costs. A recent overview of REC development in Italy, Germany, and Spain is provided by Khorrami et al. (2025).

In this paper, we analyze the equilibrium of RECs operating under a virtual framework, with a particular focus on the Italian case, where some of the most advanced implementations have been developed. While several studies examine RECs within this regulatory context, they address only partial aspects of the problem. Existing contributions largely focus on either system design or regulatory features, or the joint analysis of strategic interaction and value allocation.

For instance, Cutore et al. (2023) study the optimal design and management of a REC under the Italian framework, focusing on maximizing total community value with photovoltaic and storage technologies. However, they abstract from the distribution of this value among REC members and rely exclusively on numerical methods, providing no analytical characterization of the underlying problem. Similarly, De Juan-Vela et al. (2023) examine the regulatory conditions for integrating storage systems and focus on maximizing shared energy, but do not analyze how incentive tariffs are allocated among participants.

Existing work studying strategic investment and fair community value distribution applies game-theoretic approaches, as they provide a structured framework for situations in which multiple participants interact to maximize collective benefits (Tushar et al., 2018). Several studies report cost reductions and enhanced benefits for REC members. The literature shows a trend toward using cooperative game theory for these purposes (Abada et al., 2017, 2025; Feng et al., 2020; Moncecchi et al., 2020; Norbu et al., 2021; Safdarian et al., 2021), with occasional applications of Stackelberg games (Li, 2020). In Safdarian et al. (2021), the authors propose a coalition game to distribute value among REC members. They propose a sharing mechanism called "minimizing the worst-case excess", which ensures the stability of their game, since Shapley value does not ensure stable coalitions. The model is applied to a REC in Finland composed of residential buildings with solar panels on their roofs and solutions are given numerically. The study in Abada et al. (2017) applies cooperative game theory to assess REC viability in terms of investment, considering a REC composed of n households willing to invest in PV panels. In particular, they analyze the distribution of the value of the community comparing Shapley value with pro-rata distribution. The same authors, in Abada et al. (2025), approach the viability of a REC with the same residential-PV production configuration, in terms of risk-sharing by applying coalitional game theory, distributing the value of the community through the MinVar allocation rule. Moncecchi et al. (2020) consider heterogeneous energy sources, and not only photovoltaic systems. They focus on optimizing the investment portfolio of a REC, accounting for variations in energy consumption and generation potential among members. Initially, the problem is solved under a centralized framework where a single entity determines the optimal REC size. In a second step, cooperative game theory is applied to fairly distribute the incentives received by the REC, using the Shapley value to allocate benefits and maximize its economic value. The results are obtained through numerical implementations. This optimization approach is extended in Zatti et al. (2021), where the design and operation of generation assets are optimized to maximize REC cash flows. In Norbu et al. (2021), a cooperative game-theoretic method

is introduced, based on the marginal contribution of players, for the fair redistribution of benefits from community-owned energy assets, considering distributed energy sources and storage systems. So far, even though the Shapley value is a sophisticated mechanism for value distribution in coalitions, for a large number of participants, computations become intractable, even in computational implementation. Moreover, it does not always ensure the stability of the coalition; therefore, alternative sharing mechanisms have been proposed. On the other hand, for a game with two participants, as in our case study, the Shapley value becomes trivial, allocating half of the value of the community to each player, missing the investment effort and demand contribution.

On the other hand, cooperative approaches may not fully capture the dynamics among REC members, as collaboration is not always guaranteed. This limitation is discussed in Todorean et al. (2023), where the authors address prosumer integration and decentralized decision-making using blockchain mechanisms alongside cooperative games.

The success of energy trading models within energy communities is affected by coordination challenges. An effective approach to encourage cooperation among members is the use of Nash bargaining theory, which enables bargaining-based cooperative energy trading that benefits both consumers and energy providers (Zhou & Lund, 2023). By ensuring Pareto optimality, Nash bargaining solutions incentivize energy trading, which yields greater consumer benefits and social welfare. In Wang and Huang (2018), an incentive mechanism algorithm is designed using Nash bargaining theory to encourage proactive energy trading and fair benefit sharing between interconnected micro-grids. In the spirit of incentivizing cooperation between participants, in Fan et al. (2018), the Nash bargaining framework is applied to coordinate energy distribution companies and demand response aggregators. Therefore, from the coordination perspective, such mechanisms ensure efficient energy sharing, increasing both individual member benefits and total community welfare. A notable study that considers the strategic behavior of members before joining a REC is Lilliu et al. (2021), which proposes an incentive mechanism based on a local energy market to promote renewable energy generation and self-consumption among individualistic players. They consider fixed energy production without distinguishing between the energy sources.

We should also mention contributions in the literature on dynamic investment decisions, such as Awerkin and Vargiolu (2021); Pagliarani et al. (2024), that develop models of progressive and irreversible capacity expansion. However, although this line of research is potentially relevant also for the decision process of REC members, its extension to the present setting leads beyond the class of analytically tractable models pursued in this paper.

Despite the richness of the aforementioned studies, a gap remains in the analytical characterization of RECs under both virtual and physical frameworks, where agents decide competitively with respect to their investment. Most existing works assume a cooperative environment where a central planner or a cooperative game dictates the distribution of benefits. While robust, these models often overlook the strategic 'veto power' inherent in virtual RECs, where dispatchable producers (like a biogas plant) and consumers have asymmetric bargaining positions. Furthermore, the results of those models are primarily numerical, focusing on specific case studies. Our work provides a stylized analytical characterization: in fact, by modeling the sharing rule as endogenously determined via a Stackelberg coordinator using Nash bargaining, we explicitly link the 'sharing rule' to the 'investment decision' in a way that illustrates how external coordination can support REC viability. Finally, whereas much of the literature focuses predominantly on photovoltaic technology adoption, our framework incorporates a multi-technology setup, pairing biogas generation with PV. While based on specific structural assumptions, this provides a conceptual framework for understanding the strategic incentives required for deep decarbonization in agricultural and urban settings.

2.1 Research questions and contributions

We consider a stylized yet economically meaningful REC composed of two agents: a biogas producer (BP) and a representative household. The latter can be interpreted as an aggregate of many homogeneous end-users of the same type, so that the model captures, in reduced form, the demand side of a two-technology community while preserving analytical tractability. The considered configuration is relevant in both agricultural areas, where biogas plants treat livestock manure, and urban settings, where they process organic waste.

A key feature of this setting—specific to the stylized virtual REC framework considered here—is the asymmetry in the strategic roles of the two agents. The BP supplies dispatchable renewable generation through the G2P technology, whereas the household contributes electricity demand and, potentially, additional PV capacity. Under the current assumption that the BP has no electricity demand, if it does not invest in the G2P unit, it contributes neither to generation nor to consumption; as a consequence, the REC cannot be formed. By contrast, the household may belong to the REC even with zero PV investment because, under a virtual framework, its demand is itself a source of shared self-consumption and of the associated incentive mechanism. In this sense, the BP's veto power is not contractual, but arises *de facto* from the structure of the model. We focus on this case because it yields a particularly sharp and analytically informative strategic environment, while still capturing a class of REC situations in which one member plays a pivotal role in the viability of the community.

Against this background, the paper addresses four tightly connected research questions, each corresponding to a specific analytical contribution of the model.

A first question concerns the interaction between the sharing rule and the investment decisions of heterogeneous members in a virtual REC. Since the profitability of each investment depends on how the self-consumption incentive is allocated, while the viability of each sharing rule depends, in turn, on the capacities installed by the members, the two dimensions cannot be studied separately. To address this issue, we develop a bi-level equilibrium model that allows us to characterize analytically their joint determination and to identify the corresponding equilibrium regimes.

Closely connected to this, the second research question focuses on whether the REC emerges as a Nash equilibrium of the decentralized investment game. This issue is especially relevant in a virtual framework, where REC viability depends jointly on market conditions, local demand characteristics, technological costs, and the incentive mechanism. To address it, we characterize analytically the equilibrium regimes of the game and distinguish the cases in which incentives activate investment from those in which they remain insufficient for REC formation.

This, in turn, raises a third question concerning how the equilibrium structure changes when the sharing rule is not left to decentralized interaction, but is selected by a coordinator. Since the members compete in investment decisions but cannot, in general, identify autonomously a viable common sharing rule, coordination becomes part of the economic problem rather than a merely institutional detail. To study this issue, we model the coordinator as a Stackelberg leader choosing the sharing rule according to a Nash bargaining criterion, and we use this framework to assess how coordination affects REC formation, equilibrium selection, and efficiency.

The fourth, distinct but related, question concerns the policy implications that can be drawn from the equilibrium analysis. To address it, the paper uses the equilibrium analysis to show when different technologies display different support needs, and therefore when efficient regulation requires not only the calibration of incentives, but also guidance on the internal allocation of the collective benefit and on the role of coordination.

3 Model

3.1 Notation

The following lists the variables and the parameters used in the model.

- Decision variables
 - y_b Power capacity investment in turbine G2P (MW), controlled by the biogas producer
 - y_h Power capacity investment in solar panels (MW), controlled by the household
 - β Percentage of the incentive allocated to the household, controlled by the coordinator
- State variables
 - p Initial gas price (€/MWh)
 - $P^p(t)$ Gas price at time t (€/MWh)
 - x_v Initial electricity spot price (€/MWh)
 - $X_v^{x_v}(t)$ Electricity spot price at time t (€/MWh)
 - x_c Initial electricity purchase price (€/MWh)
 - $X_c^{x_c}(t)$ Electricity purchase price at time t (€/MWh)
 - d Initial power demand (MW)
 - $D^d(t)$ Power demand at time t (MW)
- Parameters
 - η_v, m_v, Σ_v Jump intensity, mean and variance of the independent electricity price jumps
 - η_p, m_p, Σ_p Jump intensity, mean and variance of the independent gas price jumps
 - $\eta_{vp}, m_{vp}, \Sigma_{vp}$ Jump intensity, mean and variance of the common jumps
 - $\rho_c d$ Correlation between W_d and W_c
 - μ_p, σ_p Drift and volatility of the gas price (1/h)
 - μ_v, σ_v Drift and volatility of the spot electricity price (1/h)
 - μ_c, σ_c Drift and volatility of the purchase electricity price (1/h)
 - μ_d, σ_d Drift and volatility of power demand (1/h)
 - r Discount rate (1/h)

$r_v, r_p, r_c, r_d, r_{cd}$	Discount rates adjusted for the expected growth of the corresponding stochastic processes ($1/h$)
τ	Duration of the incentive scheme h
λ	Parameter of the exponentially distributed random variable τ
K_g	Total gas production capacity (m^3)
θ_h, θ_b	Maximum installation capacity for the household and biogas producer (MW)
c_b	Installation cost of the G2P turbine €/MW
c_h	Installation cost of solar panels €/MW
• Constants	
b	Gas-to-power conversion factor
α	Equivalent hours per year of peak level operation of photovoltaic panels
γ	Efficiency of the G2P turbine
Z	Incentive for self-consumption €/MWh

3.2 Optimal individual investment decision

In this section, we introduce the mathematical model describing the optimal investment problem that each member would solve individually. More precisely, we derive the expected profit that two potential REC members would obtain by investing in RES generation as independent entities. These results serve as a benchmark for comparison with the profits they would achieve by joining a REC. Both decision-makers determine their investment at time $t = 0$, under the assumption of an infinite time horizon, reflecting the indefinite lifespan of the installed plants.

RES generation investments are subject to multiple sources of risk. To represent this uncertainty, we define a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ satisfying the usual conditions, and rich enough to support the definition of a four-dimensional Lévy process

$$L = (L_p, L_v, L_c, L_d),$$

where L_p, L_v, L_c , and L_d denote respectively the log-returns of the geometric Lévy processes (GLP) used to model the gas price, the electricity selling price, the household electricity purchase price, and the household power demand. More in detail, we assume that this Lévy process can be decomposed as the sum of a 4-dimensional drift, a Brownian component, and a jump-diffusion component, i.e.,

$$L(s) = \mu s + AW(s) + \sum_{k=1}^{N_s} Y_k, \quad s > 0,$$

with $\tilde{\mu} = (\mu_p, \mu_v, \mu_c, \mu_d) \in \mathbb{R}^4$ vector of drifts, A a 4×4 matrix expressing the possible correlation of the Brownian components, W a 4-dimensional Brownian motion, N a Poisson process with intensity η and $(Y_k)_k$ i.i.d. random variables with values in $\mathbb{R}^4$ and joint distribution Π , representing the possible jumps. The correlation matrix of the Brownian components is given by $\Sigma := AA'$, so that the variance of the Brownian component of each process L_x is given by $\sigma_x^2 := \Sigma_{xx}$, for $x = p, v, c, d$. Moreover, we denote by v_x each marginal distribution of the x -component of the jumps, so that the i.i.d. jumps $(Y_k^x)_k$ in the component x have distribution given by v_x , $x = p, v, c, d$.

The choice of GLP to model the four processes is motivated by analytical tractability. At the same time, the Lévy framework is already rich enough to accommodate discontinuous shocks through its jump component, thus partially addressing abrupt market movements due, for instance, to regulatory or geopolitical events. While mean-reverting specifications may capture some empirical features more accurately, our analysis is centered on discounted expected profits and on the equilibrium effects of the sharing rule from given initial conditions. Under this perspective, and thanks to discounting and Assumption 3.2, the adopted GLP specification provides a parsimonious compromise between realism and analytical tractability.

Biogas Producer The BP has a total gas production capacity K_g (in m^3), with an hourly output of bK_g , where $b > 0$ is the MW/m^3 conversion factor. The producer sells this gas at the spot market price $(P^p(s))_{s \geq 0}$ (in $\text{€}/\text{MWh}$), which follows a geometric Lévy process with initial value p :

$$P^p(s) = pe^{L_p(s)}, \quad s \geq 0, \quad (1)$$

The BP can install a Gas-to-Power (G2P) turbine with capacity $y_b \leq \theta_b$, where θ_b represents the maximum feasible capacity. This turbine converts gas into electricity, which is sold at the spot electricity price $(X_v^{x_v}(s))_{s \geq 0}$. We model this price as a GLP with initial value x_v :

$$X_v^{x_v}(s) = x_v e^{L_v(s)}, \quad s \geq 0, \quad (2)$$

Installing the turbine reduces the amount of gas available for direct sale. Specifically, the gas output decreases to $(bK_g - y_b/\gamma)$, where γ denotes the G2P efficiency factor¹, while electricity production increases to y_b . The BP's profit functional is thus given by:

$$J_b^0(x_v, x_c, p, d, y_b, y_h) := \mathbb{E} \left[\int_0^\infty e^{-rs} X_v^{x_v}(s) y_b ds \right] + \mathbb{E} \left[\int_0^\infty e^{-rs} P^p(s) \left(bK_g - \frac{y_b}{\gamma} \right) ds \right] - c_b y_b, \quad (3)$$

where c_b represents the cost coefficient ($\text{€}/\text{MW}$) of installing a turbine with capacity y_b , and $r > 0$ is the discount rate reflecting the cost of capital.

Household To meet its electricity demand, the household cannot purchase energy directly from the market at the spot price X_v . Instead, they are typically compelled to buy electricity from a retailer at a different (often higher) price, which we assume to follow the GLP with initial value x_c :

$$X_c^{x_c}(s) = x_c e^{L_c(s)}, \quad s \geq 0. \quad (4)$$

This specification also encompasses the case in which the purchase price is fixed and identically equal to x_c over time, by simply considering the Lévy process L_c to coincide with 0.

The household's instantaneous power demand $(D^d(s))_{s \geq 0}$ is also modeled as a GLP:

$$D^d(s) = de^{L_d(s)}, \quad s \geq 0. \quad (5)$$

For consistency, we express demand D^d in MW.

The household can invest in PV panels with a capacity $y_h \leq \theta_h$ to reduce energy costs by selling surplus production at the electricity spot price X_v , where θ_h is typically constrained by the household's budget. The household's profit functional is given by:

$$J_h^0(x_v, x_c, p, d, y_b, y_h) := \mathbb{E} \left[\int_0^\infty e^{-rs} X_v^{x_v}(s) \alpha y_h ds \right]$$

¹ Typically, $\gamma \simeq 0.5\text{--}0.6$ for gas-fired power plants

$$- \mathbb{E} \left[\int_0^\infty e^{-rs} X_c^{x_c}(s) D^d(s) ds \right] - c_h y_h, \quad (6)$$

where α represents the PV panel's equivalent power production factor² and c_h is the installation cost per MW.

While constant PV generation is an evident simplification, we consider that, in expectation, the term αy_h can be safely seen as the constant long-run average rate of the PV generation process. Note that both plant installations are assumed to operate at full capacity at all times, with y_b representing the output of the G2P turbine and αy_h the output of the PV panels. Thus, in this framework, installed capacities coincide with generation rates.

Optimal stand alone investment To compute J_b^0 and J_h^0 , we make a quick survey of properties of GLP. The main property that we will use is the Lévy–Khintchine formula (see, e.g., (Cont and Tankov, 2004, Chapter 3)): for all $t > 0$ and $z \in \mathbb{R}^4$, the moment generating function of $L(t)$ is $\mathbb{E}[e^{\langle z, L(t) \rangle}] = e^{\psi(z)t}$, with $\psi(z)$ defined as

$$\psi(z) := \langle z, \mu \rangle + \frac{1}{2} z^T \Sigma z + \eta \int_{\mathbb{R}^4} (e^{\langle z, u \rangle} - 1) \Pi(du) \quad (7)$$

where $\Sigma = AA^T$ is the covariance matrix of the Brownian component of L . This allows us to reach conclusions about various expectations appearing in the profit functionals J_b^0 and J_h^0 .

Proposition 3.1 *For all $s \geq 0$ we have*

$$\mathbb{E}[e^{-rs} X_c^{x_c}(s)] = x_c e^{-r_c s}, \quad \text{with } r_c := r - \mu_c + \frac{1}{2} \sigma_c^2 + \eta \int_{\mathbb{R}} (e^y - 1) v_c(dy), \quad (8)$$

$$\mathbb{E}[e^{-rs} X_v^{x_v}(s)] = x_v e^{-r_v s}, \quad \text{with } r_v := r - \mu_v + \frac{1}{2} \sigma_v^2 + \eta \int_{\mathbb{R}} (e^y - 1) v_v(dy), \quad (9)$$

$$\mathbb{E}[e^{-rs} P^p(s)] = p e^{-r_p s}, \quad \text{with } r_p := r - \mu_p + \frac{1}{2} \sigma_p^2 + \eta \int_{\mathbb{R}} (e^y - 1) v_p(dy), \quad (10)$$

$$\mathbb{E}[e^{-rs} D^d(s)] = d e^{-r_d s}, \quad \text{with } r_d := r - \mu_d + \frac{1}{2} \sigma_d^2 + \eta \int_{\mathbb{R}} (e^y - 1) v_d(dy), \quad (11)$$

$$\mathbb{E}[e^{-rs} X_c^{x_c}(s) D^d(s)] = x_c d e^{-r_{cd} s}, \quad \text{with} \\ r_{cd} := r - \mu_c - \mu_d - \frac{1}{2} \sigma_c^2 - \frac{1}{2} \sigma_d^2 - \rho_{cd} \sigma_c \sigma_d - \eta \int_{\mathbb{R}^4} (e^{y_3 + y_4} - 1) \Pi(dy). \quad (12)$$

where ρ_{cd} denotes the correlation between the Brownian components of L_c and L_d .

Proof Choosing $zz = (0, 0, 1, 0)$ and using the Lévy–Khintchine formula we can compute

$$\mathbb{E}[e^{-rs} X_c^{x_c}(s)] = e^{-rs} \mathbb{E}[x_c e^{L_c(s)}] = e^{-rs} \mathbb{E}[x_c e^{\langle z, L(s) \rangle}] = x_c e^{-rs} e^{\left(\mu_c + \frac{1}{2} \Sigma_{cc} + \eta \int_{\mathbb{R}^4} (e^{y_3} - 1) v(dy)\right)s}$$

hence the result; similarly, we can obtain the corresponding results for $\mathbb{E}[e^{-rs} X_v^{x_v}(s)]$, $\mathbb{E}[e^{-rs} P^p(s)]$ and $\mathbb{E}[e^{-rs} D^d(s)]$. Finally, the last expectation follows from choosing $z =$

² Typically, α is related to the "equivalent hours per year": if a PV plant operates at peak-level equivalent for M hours annually, then $\alpha = \frac{M}{8760}$, with $M \in [1000, 1500]$ depending on location.

(0, 0, 1, 1) and computing

$$\begin{aligned}\mathbb{E}[e^{-rs} X_c^{x_c}(s) D^d(s)] &= e^{-rs} \mathbb{E}[x_c d e^{L_c(s) + L_d(s)}] = e^{-rs} \mathbb{E}[x_c d e^{\langle z, L(s) \rangle}] = \\ &= x_c d e^{-rs} e^{\left(\mu_c + \mu_d + \frac{1}{2} \Sigma_{cc} + \frac{1}{2} \Sigma_{dd} + \Sigma_{cd} + \eta \int_{\mathbb{R}^4} (e^{y_3 + y_4} - 1) v(dy)\right) s}\end{aligned}$$

□

The quantities r_c , r_v , r_p and r_d represent discount rates adjusted for the expected growth of the corresponding stochastic processes, while r_{cd} also represents the discount rate adjusted for the expected growth of the product between purchase price X_c and demand D . For the well-posedness of the profit functionals J_b^0 and J_h^0 , we need them to be strictly positive: this is the purpose of the next assumption.

Assumption 3.2 The parameters r_c , r_v , r_p , r_d and r_{cd} are strictly positive.

Under Assumption 3.2, it follows that:

$$J_b^0(x_v, x_c, p, d, y_b, y_h) = \frac{x_v y_b}{r_v} + \frac{p(bK_g - \frac{y_b}{\gamma})}{r_p} - c_b y_b = y_b g_b + \frac{pbK_g}{r_p}, \quad (13)$$

$$J_h^0(x_v, x_c, p, d, y_b, y_h) = \alpha \frac{x_v y_h}{r_v} - \frac{x_c d}{r_{cd}} - c_h y_h = y_h g_h - \frac{x_c d}{r_{cd}}. \quad (14)$$

The quantities

$$g_b := \frac{x_v}{r_v} - \frac{p}{\gamma r_p} - c_b, \quad g_h := \alpha \frac{x_v}{r_v} - c_h \quad (15)$$

represent the discounted net unit profits per MW for the BP and the household over an infinite time horizon. Since both profit functions are linear in y_b and y_h , the following result follows directly.

Lemma 3.3 *In the absence of incentives to join a REC, the optimal installation levels for the BP and household are given by:*

$$y_b^* = \begin{cases} \theta_b & \text{if } g_b \geq 0, \\ 0 & \text{if } g_b < 0, \end{cases} \quad \text{and} \quad y_h^* = \begin{cases} \theta_h & \text{if } g_h \geq 0, \\ 0 & \text{if } g_h < 0. \end{cases}$$

Remark 3.4 This result follows the classical "all-or-nothing" solution of linear investment optimization. Specifically, if the net unit profit g_b (for the BP) or g_h (for the household) is non-negative, the agent installs the maximum possible capacity; otherwise, no installation occurs. For the household, the expected revenue is given by $\frac{x_v}{r_v}$. If the marginal installation cost c_h exceeds this expected revenue, the optimal installation is zero. Similarly, for the BP, expected revenue is reduced by the gas price term $\frac{p}{r_p}$. If c_b exceeds expected revenues, no investment in G2P capacity occurs.

4 A model for virtual renewable energy communities

According to the Virtual Framework, a REC receives an incentive proportional to its self-consumption, which is then distributed among its members. In our case, both the BP and the household may revise their installation decisions from the previous section in light of these incentives. The specific incentive mechanism and its redistribution, together with their qualitative implications, are summarized in the following.

4.1 Key assumptions and implications

We assume that the REC is composed of two representative agents³, which are the same as before, i.e. a biogas producer and a household. In addition to these agents, since the REC operates as a single legal entity and receives the self-consumption incentive collectively, we assume that a designated coordinator is responsible for determining its distribution among the members. This coordinator can be interpreted as an entity (e.g., a municipality or an Energy Service Company (ESCO)) that selects the sharing rule to ensure the long-run viability and acceptability of the REC.

Key assumptions on the incentive and on the agents' behavior are the following:

1. Both members can contribute to the total power generated by the REC: the household installs a capacity y_h of PV panels providing αy_h units of power, while the BP installs a capacity y_b of G2P turbines generating y_b units of electricity from gas conversion: this latter operation consumes y_b/γ units of gas, while the PV panels are assumed to operate without costs. Conversely, only the household contributes to the REC's consumption, which is structural, equal to its demand $D(t)$ for all $t > 0$, and independent of its installation y_h .
2. For this virtual REC, its total self-consumption at any time t is defined as:

$$q(t) = \min(D^d(t), \alpha y_h + y_b). \quad (16)$$

3. A central authority rewards a REC with a flat tariff Z for every unit of self-consumed energy (as defined in Equation (16)) over a period $[0, \tau]$; here τ is a random variable representing the duration of the incentive scheme, which accounts for the possibility that the policy may be discontinued at an uncertain future time.
4. In order to share this self-consumption incentive among the REC's members, we assume that the coordinator applies a Nash bargaining (NB) criterion. More specifically, the coordinator will attribute a proportion β of the total incentive to the household, and the remaining proportion $1 - \beta$ to the biogas producer: the choice of β is known to both the biogas producer and the household at the REC's creation. The exact mathematical mechanism of the NB will be described in detail later.
5. Given the choice of β made by the coordinator, the biogas producer and the household can decide their installation (y_b and y_h , respectively) in order to maximize their respective total profits, which now also incorporate their respective share of cumulative self-consumption incentive.
6. All the choices of the agents, i.e. β for the coordinator, y_b for the biogas producer and y_h for the household, are made at time 0 and assumed to be static, i.e. they will be constant during all the REC's life.

This framework has the following implications. Firstly, investment decisions and the incentive-sharing rule now interact in a nontrivial manner, as opposed to the case without incentives, where (as seen in Lemma 3.3) each agent maximizes their own payoff independently of the others. More specifically, if either g_b or g_h is negative, the incentives may shift a zero-investment decision into a positive investment. In this case, as we will demonstrate, the optimal investment can possibly be less than the maximum capacity limit.

Secondly, self-consumption plays a central role in the incentive system adopted for virtual RECs. Increasing self-consumption depends not only on the generation capacity $\alpha y_h + y_b$

³ as discussed in Section 2.1, these representative agents can possibly aggregate the common behavior of many agents similar to them

but also on the aggregate demand of the REC, $D^d(t)$. This structure supports the mitigation of *energy poverty*, as individuals who join the REC without making an initial investment still contribute by increasing aggregate demand, thereby enhancing the REC's overall self-consumption.

Last but not least, the agents' payoffs can still be expressed in closed form. More in detail, given their installations y_b and y_h , the profits now include the respective shares of the incentive, which adds to J_b^0 and J_h^0 as defined in Equations (13) and (14). These incentive shares are both proportional to the expected self-consumption cumulated by the REC over its lifetime, defined as

$$w(y_h, y_b, d) := \mathbb{E} \left[\int_0^\tau e^{-rs} q(s) ds \right]. \quad (17)$$

We will find that, in the case in which τ is exponentially distributed and independent of W_d , w can be found in closed form as a function of y_b , y_h and d , and depends on τ only through its parameter.

4.2 Optimal sharing rule and capacity decisions within a virtual REC

We model this situation as a Stackelberg game, where the coordinator acts as the strategic leader and the two REC members act as followers.

If $\beta \in (0, 1)$ represents the share of the incentive allocated to the household, with $1 - \beta$ assigned to the BP, the coordinator solves the following optimization problem:

$$\max_{\beta \in (0,1)} (J_h(x_v, x_c, p, d, y_b, y_h, \beta) - d_h) (J_b(x_v, x_c, p, d, y_b, y_h, \beta) - d_b), \quad (18)$$

where $J_h(x_v, x_c, p, d, y_b, y_h, \beta)$ and $J_b(x_v, x_c, p, d, y_b, y_h, \beta)$ denote the profits of the household and the BP under REC participation, while d_h and d_b denote their respective *disagreement points*, that is, their outside-option profits in the absence of REC formation. They are given by:

$$d_h := J_h^0(x_v, x_c, p, d, y_h^*, y_b^*), \quad d_b := J_b^0(x_v, x_c, p, d, y_h^*, y_b^*). \quad (19)$$

where y_b^* and y_h^* are defined in Lemma 3.3.

The Nash bargaining criterion is adopted here not merely as a fairness principle, but as a coordination device consistent with the institutional problem faced by a virtual REC. Since participation is voluntary and the two members are heterogeneous in both technological role and outside option, the sharing rule must guarantee each of them a strictly positive and sufficiently balanced gain relative to stand-alone investment. At the same time, the members compete in investment decisions and cannot, in general, determine autonomously a viable common allocation rule. For this reason, the bargaining problem is assigned to the coordinator, who selects β by taking the disagreement points (d_h, d_b) as the relevant benchmark. In this way, the sharing rule is not imposed ad hoc, but is grounded in the members' outside options and in the requirement that REC participation be individually acceptable to both agents. Since the objective function in (18) is the product of two terms, it reaches zero whenever either term is zero. This structure prevents highly asymmetric solutions in which one member benefits disproportionately at the expense of the other.

Alternative sharing rules could clearly be considered. Equal sharing (i.e. letting $\beta = 1/n$, where n is the number of members in the REC) would provide the simplest benchmark, but would ignore differences in investment costs, outside options, and technological roles. Proportional rules (as considered in Abada et al. (2017) and Norbu et al. (2021)) would require

Stackelberg game: single leader, multiple followers

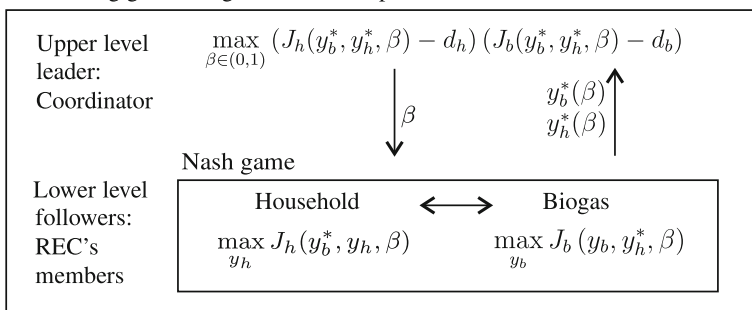


Fig. 3 Representation of the bilevel problem, with a Nash game nested within a Stackelberg game

specifying the proportionality basis, such as installed capacity, monetary investment, or contribution to shared self-consumption; since capacities are themselves equilibrium outcomes, such rules would lead to a different fixed-point problem. Welfare-based allocations (e.g. Abada et al. (2025)) with participation constraints would constitute a natural optimization benchmark, but would emphasize aggregate surplus rather than the balanced distribution of gains relative to outside options. Finally, a Shapley-inspired rule (applied in e.g. Moncecchi et al. (2020), Abada et al. (2017)) would require defining a cooperative value function and, in a two-member setting such as ours, would either reduce to equal sharing or to an equal split of surplus above disagreement payoffs. For this reason, the present paper focuses on Nash bargaining as a criterion consistent with voluntary participation, outside-option payoffs, and the coordinating role assigned to the REC administrator, while leaving a systematic theoretical and numerical comparison of alternative allocation principles to future research.

To solve the problem (18), the coordinator must compute J_h and J_b as the best responses of the followers (y_h^* , y_b^*), given its choice of β . We now define the profit functions J_h and J_b for the BP and the household, respectively. The BP's profit is expressed as:

$$J_b(x_v, x_c, p, d, y_b, y_h, \beta) := J_b^0(x_v, x_c, p, d, y_b, y_h) + (1 - \beta)Zw(y_h, y_b, d),$$

where J_b^0 represents the BP's profit in the absence of incentives, as computed in Equation (13), and the term $(1 - \beta)Zw(y_h, y_b, d)$ accounts for the incentive received by the BP, where w is the function defined in Equation (17). Similarly, the household's profit is given by:

$$J_h(x_v, x_c, p, d, y_b, y_h, \beta) := J_h^0(x_v, x_c, p, d, y_b, y_h) + \beta Zw(y_h, y_b, d),$$

where J_h^0 is the household's profit without incentives, as computed in Equation (14).

For each fixed β , the members of the REC seek Nash equilibrium (NE) points (y_h^*, y_b^*) satisfying:

$$\begin{cases} J_h(x_v, x_c, p, d, y_h, y_b^*, \beta) \leq J_h(x_v, x_c, p, d, y_h^*, y_b^*, \beta) & \forall y_h \in [0, \theta_h], \\ J_b(x_v, x_c, p, d, y_h^*, y_b, \beta) \leq J_b(x_v, x_c, p, d, y_h^*, y_b^*, \beta) & \forall y_b \in [0, \theta_b]. \end{cases} \quad (20)$$

We assume a designated coordinator determines a "fair" sharing rule, allowing the two members to compete in optimizing their respective installation capacities within a structured framework.

We solve our leader-follower framework using a two-step procedure. First, we determine the NE of the competitive installation game between the REC members for a given β . Then,

using the resulting optimal capacities $y_h^*(\beta)$ and $y_b^*(\beta)$, we solve the leader's NB problem, as illustrated in Fig. 3.

4.3 Lower-level problem: Nash equilibrium between the REC members

4.3.1 Preliminary results

In this section we compute the expected value of the household and BP profits. Let us begin with the following lemma.

Lemma 4.1 *Assume that $r_d > 0$. Assume further that τ is independent of D and exponentially distributed with parameter $\lambda > 0$. Then the function w , defined in Equation (17), is of class $C^2((\mathbb{R}^+)^3)$, and is equal to*

$$w(y_h, y_b, d) := \begin{cases} B_1(\alpha y_h + y_b)^{1-m_2} d^{m_2} + \frac{d}{r_d + \lambda} & d < \alpha y_h + y_b \\ C_1(\alpha y_h + y_b)^{1-m_1} d^{m_1} + \frac{\alpha y_h + y_b}{r + \lambda} & d \geq \alpha y_h + y_b, \end{cases} \quad (21)$$

with B_1 and C_1 negative constants given by

$$B_1 = \frac{m_1 \mu_d - r - \lambda}{(r + \lambda)(r_d + \lambda)(m_2 - m_1)}, \quad C_1 = \frac{m_2 \mu_d - r - \lambda}{(r + \lambda)(r_d + \lambda)(m_2 - m_1)} \quad (22)$$

and $m_1 < 0$, and $m_2 > 1$ solutions of the characteristic equation $f_v(m) = 0$, with f_v defined as

$$f_v(m) := \mu_d m + \frac{1}{2} \sigma_d^2 m^2 + \eta \int_{\mathbb{R}^4} (e^{m y_4} - 1) v(dy) - (r + \lambda), \quad (23)$$

Proof See Appendix. □

Remark 4.2 In the case when D has no jumps, i.e., when $v_d \equiv 0$, the characteristic equation simplifies to an algebraic second-degree equation, with solutions

$$\bar{m}_{1,2} := \frac{-\mu_d \mp \sqrt{\mu_d^2 + 2(r + \lambda)\sigma_d^2}}{\sigma_d^2}$$

We now examine the competitive game of power installation between the community members, seeking NE. Since each player's optimal decision depends on the best response of the other, we derive explicit analytical expressions for their strategies, ensuring a NE characterization.

4.3.2 Household's best response

Let us begin with the household's best response for a fixed strategy $y_b \in [0, \theta_b]$ of the BP. We want to find y_h^* such that

$$J_h(x_v, x_c, p, d, y_h^*, y_b, \beta) = \max_{y_h \in [0, \theta_h]} J_h(x_v, x_c, p, d, y_h, y_b, \beta). \quad (24)$$

Proposition 4.3 For $\beta \in (0, 1)$ and $y_b \in [0, \theta_b]$ fixed, the best response y_h^* for the household to the strategy y_b of the BP is

$$y_h^* = \begin{cases} \theta_h, & g_h \geq 0, \\ 0, & g_h + \frac{\alpha Z \beta}{r+\lambda} \leq 0, \\ \max \left(\min \left(\frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - y_b \right), \theta_h \right), 0 \right) & g_h < 0 < g_h + \frac{\alpha Z \beta}{r+\lambda} \wedge \beta > \beta_h, \\ \max \left(\min \left(\frac{1}{\alpha} \left(d \left(\frac{g_h}{\beta_h} + \frac{\alpha Z}{r+\lambda} \right)^{1/m_1} - y_b \right), \theta_h \right), 0 \right) & g_h < 0 < g_h + \frac{\alpha Z \beta}{r+\lambda} \wedge \beta \leq \beta_h, \end{cases} \quad (25)$$

where β_h is defined as

$$\beta_h := \frac{-g_h}{\alpha Z (C_1(1 - m_1) + \frac{1}{r+\lambda})} = \frac{-g_h}{\alpha Z B_1(1 - m_2)}. \quad (26)$$

and it is strictly positive if and only if $g_h < 0$. In addition, if $\beta_h < \beta$, then the aggregate installation exceeds the power demand, i.e., $\alpha y_h^* + y_b > d$. On the other hand, if $\beta_h \geq \beta$, then the aggregate installation does not exceed the community demand, i.e., $\alpha y_h^* + y_b \leq d$. The equality is verified when $\beta = \beta_h$. In conclusion, β_h is a threshold that determines whether the optimal joint installation exceeds the power demand.

Remark 4.4 The reward functional is strictly concave due to the presence of the incentive, which adds a concave nonlinearity to the linear functional without incentives. This results in the existence of a unique maximizer y_h^* : since this maximizer must be found in the compact set $[0, \theta_h]$, several cases arise. If the net unitary gain g_h is non-negative, then the optimum is reached by installing the maximum possible power: in fact, this was already optimal without the incentive and remains so in this case. Conversely, a zero optimal installation is now optimal only if g_h , summed with the expected value over time of the household's incentive share $\alpha \frac{Z\beta}{r+\lambda}$, is still non-positive.

Finally, we have internal solutions $y_h^* \in (0, \theta_h]$ when the net gain g_h is negative but becomes positive when summed with the household's incentive share $\alpha \frac{Z\beta}{r+\lambda}$. Since the functional form of the incentive in Equation (21) assumes two analytic expressions according to the values of d and $\alpha y_h + y_b$, here we have two functional forms for the inner maximum point y_h^* : it turns out that the choice between these two forms is uniquely determined by whether β is greater than or less than a given constant β_h .

Remark 4.5 As Equations (6), (15), and (25) show, although the cost of electricity $X_c(s)$ affects the level and volatility of household profit, it does not affect the investment decision. Indeed, notice that y_h only affects the household's expected revenues and investment expenditure (first integral and last term in Equation (6)). Correspondingly, no parameter of that process enters into the household's best response. This also partially extends to the other state variables. In fact, we notice that, exactly as in the stand alone case treated in Section 3.2, the parameters of the electricity selling price X_v influence the household's optimal installation only through their presence in g_h through Equations (9) and (15); in turn, the parameters of the demand influence the household's optimal installation only through the quantities $m_{1,2}$ as defined in Equation (23). We will also see later that this effect also influences both the Nash equilibria and the biogas producer's optimal installation, which are impacted by the parameters of P only through g_b .

4.3.3 Biogas producer's best response

Let us move to the BP's best response problem. For a fixed $y_h \in [0, \theta_h]$, we want to find y_b^* such that

$$J_b(x_v, x_c, p, d, y_h, y_b^*, \beta) = \max_{y_b \in [0, \theta_b]} J_b(x_v, x_c, p, d, y_h, y_b, \beta). \quad (27)$$

Proposition 4.6 For $\beta \in (0, 1)$ and $y_h \in [0, \theta_h]$ fixed, the best response y_b^* for the BP to the strategy of the household is

$$y_b^* = \begin{cases} \theta_b, & g_b \geq 0 \\ 0, & g_b + \frac{Z(1-\beta)}{r+\lambda} \leq 0 \\ \max \left(\min \left(d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2} - \alpha y_h, \theta_b \right), 0 \right), & g_b < 0 < g_b + \frac{Z(1-\beta)}{r+\lambda} \wedge \beta < \beta_b \\ \max \left(\min \left(d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1} - \alpha y_h, \theta_b \right), 0 \right), & g_b < 0 < g_b + \frac{Z(1-\beta)}{r+\lambda} \wedge \beta \geq \beta_b \end{cases} \quad (28)$$

where β_b is defined as

$$\beta_b := 1 + \frac{-g_b}{ZB_1(m_2 - 1)} = \frac{g_b}{Z(C_1(1 - m_1) + \frac{1}{r+\lambda})} + 1. \quad (29)$$

We have that $1 - \beta_b > 0$, if and only if, $g_b < 0$. In addition, if $\beta_b > \beta$, then the aggregate installation exceeds the power demand, i.e., $\alpha y_h + y_b^* > d$. On the other hand, if $\beta_b \leq \beta$, then the aggregate installation does not exceed the community demand, i.e., $\alpha y_h + y_b^* \leq d$. The equality holds when $\beta = \beta_b$. In conclusion, β_b is a threshold which determines whether the optimal joint installation exceeds the power demand or not.

Remark 4.7 As in the case of the household, discussed in Remark 4.4, the interpretation of this result is also straightforward and is analogous to the household's case. We highlight that again, we have two functional forms for the inner maximum point y_b^* , with the choice between these two forms being uniquely determined by β being greater or less than a given constant β_b , which is such that $1 - \beta_b$ has the opposite sign of g_b .

Remark 4.8 One can easily notice from the definitions of β_b and β_h that

$$ZB_1(m_2 - 1) = \frac{g_h}{\alpha\beta_h} = \frac{g_b}{1 - \beta_b}$$

As previously discussed, the solution $y_h^* = 0$ is consistent with the creation of a REC, since it represents the case of a household contributing to the REC's self-consumption through its energy demand. However, we exclude the case $y_b^* = 0$, since it is not consistent with the creation of a REC, since we assumed that the BP has zero power demand.

4.3.4 Nash equilibria

Now, we study the case where both players simultaneously apply the best response to the other player's strategy. As we saw in the previous sections, we will have different equilibria depending on the parameter values. For simplicity, we suppose that $d \leq \alpha\theta_h + \theta_b$.

Proposition 4.9 For $\beta \in (0, 1)$ fixed, the energy community is created if and only if $g_b + \frac{Z}{r+\lambda}(1-\beta) \geq 0$, $y_h^* \geq 0$ and $y_b^* > 0$, with y_h^* and y_b^* defined respectively by Propositions 4.3 and 4.6. In the case where $y_h^* \in [0, \theta_h]$ and $y_b^* \in (0, \theta_b]$, the following strategies (y_h^*, y_b^*) are NE.

1. If $g_h \geq 0$, $g_b \geq 0$, then

$$y_h^* = \theta_h, \quad y_b^* = \theta_b. \quad (30)$$

2. If $g_h + \frac{\alpha Z}{r+\lambda}\beta \leq 0$, $g_b \geq 0$, then

$$y_h^* = 0, \quad y_b^* = \theta_b. \quad (31)$$

3. If $g_h < 0 < g_h + \frac{\alpha Z}{r+\lambda}\beta$, $g_b \geq 0$ and

3.1. $\beta > \beta_h$, then

$$y_h^* = \max \left\{ \min \left\{ \frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - \theta_b \right), \theta_h \right\}, 0 \right\}, \quad y_b^* = \theta_b. \quad (32)$$

3.2. $\beta \leq \beta_h$, then

$$y_h^* = \max \left\{ \min \left\{ \frac{1}{\alpha} \left(d \left(\frac{\frac{g_h}{\beta_h} + \frac{\alpha Z}{r+\lambda}}{\frac{g_b}{\beta} + \frac{\alpha Z}{r+\lambda}} \right)^{1/m_1} - \theta_b \right), \theta_h \right\}, 0 \right\}, \quad y_b^* = \theta_b. \quad (33)$$

4. If $g_h \geq 0$, $g_b < 0 < g_b + \frac{Z(1-\beta)}{r+\lambda}$ and

4.1. $\beta < \beta_b$, then

$$y_h^* = \theta_h, \quad y_b^* = \max \left\{ \min \left\{ d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2} - \alpha\theta_h, \theta_b \right\}, 0 \right\}. \quad (34)$$

4.2. $\beta \geq \beta_b$, then

$$y_h^* = \theta_h, \quad y_b^* = \max \left\{ \min \left\{ d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1} - \alpha\theta_h, \theta_b \right\}, 0 \right\}. \quad (35)$$

5. If $g_h + \frac{\alpha Z\beta}{r+\lambda} \leq 0$, $g_b < 0 < g_b + \frac{Z(1-\beta)}{r+\lambda}$ and

5.1. $\beta < \beta_b$, then

$$y_h^* = 0, \quad y_b^* = \max \left\{ \min \left\{ d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2}, \theta_b \right\}, 0 \right\}. \quad (36)$$

5.2. $\beta \geq \beta_b$, then

$$y_h^* = 0, \quad y_b^* = \max \left\{ \min \left\{ d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1}, \theta_b \right\}, 0 \right\}. \quad (37)$$

6. In the case $g_h \in \left(-\frac{\alpha Z\beta}{r+\lambda}, 0\right)$, $g_b \in \left(-\frac{Z(1-\beta)}{r+\lambda}, 0\right)$, there exists a unique value $\beta = \beta_n$, defined as

$$\beta_n := \frac{\beta_h}{1 - \beta_b + \beta_h} = \frac{g_h}{g_h + \alpha g_b}, \quad (38)$$

such that the two members find NE (y_h^*, y_b^*) . β_n is always between β_b and β_h , i.e. either $\beta_n \in (\beta_h, \beta_b)$ or $\beta_n \in (\beta_b, \beta_h)$ and, depending on which of these cases is true, we have:

6.1. If $\beta_n \in (\beta_h, \beta_b)$, any pair (y_h^*, y_b^*) , which satisfies

$$\alpha y_h^* + y_b^* = d \left(\frac{ZB_1(m_2 - 1)}{g_h + g_b} \right)^{1/m_2} = d \left(\frac{1 - \beta_n}{1 - \beta_b} \right)^{1/m_2} > d \quad (39)$$

6.2. If $\beta_n \in (\beta_b, \beta_h)$, any pair (y_h^*, y_b^*) , which satisfies

$$\alpha y_h^* + y_b^* = d \left(\frac{ZC_1(m_1 - 1)}{g_h + g_b + \frac{Z}{r+\lambda}} \right)^{1/m_1} = d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta_n} + \frac{Z}{r+\lambda}} \right)^{1/m_1} < d \quad (40)$$

If $g_b + \frac{Z}{r+\lambda}(1 - \beta) \leq 0$, the REC is not created, and the following decisions are NE.

7. If $g_h \geq 0$, $g_b + \frac{Z}{r+\lambda}(1 - \beta) \leq 0$, then

$$y_h^* = \theta_h, \quad y_b^* = 0. \quad (41)$$

8. If $g_h + \alpha \frac{Z}{r+\lambda} \beta \leq 0$, $g_b + \frac{Z}{r+\lambda}(1 - \beta) \leq 0$, then

$$y_h^* = 0, \quad y_b^* = 0. \quad (42)$$

9. If $g_h < 0 < g_h + \alpha \frac{Z}{r+\lambda} \beta$, $g_b + \frac{Z}{r+\lambda}(1 - \beta) \leq 0$. As the BP does not invest, the REC cannot be created. Consequently, the household misses the necessary profitability and its installation is also zero:

$$y_h^* = 0, \quad y_b^* = 0. \quad (43)$$

Proof See Appendix. $\square$

Economic discussion.

Proposition 4.9 is relevant both for environmental policymakers and for the players involved. From an economic perspective, the different equilibria therein convey distinct policy implications. NE 6.1 and 6.2 represent the socially desirable outcome. In these cases, incentives ensure REC viability and turn out to be economically efficient, since they are decisive in convincing both players to install a positive renewable capacity and, at the same time, the joint investment of the two players closely matches the expected consumption d within the community.

By contrast, NE 1., 2., 3.1, 3.2, 4.1, 4.2, and 7, in which one member would invest even without incentives and independently of the level of the expected energy demand d inside the REC, suggest potential over-subsidization. In particular, under NE 1, 2, 3.1, and 3.2, the BP installs up to its boundary value (θ_b) ; symmetrically, under NE 1., 4.1, 4.2 and 7 the household invests up to $\text{tp}(\theta_h)$.

Finally, in other equilibria (2., 5.1, 5.2, 7., 8., and 9.) incentives are insufficient for one or both players to trigger the investments. In particular, under equilibria 7., 8., and 9, the REC fails to emerge at all because of the (implied) veto power held by the BP. Symmetrically, under equilibria 2, 5.1, and 5.2 the household does not invest in any PV capacity, yet the REC is created anyway, since the household joins the community as a pure consumer.

These results suggest that designing an incentive scheme for a REC under a virtual framework is far from straightforward from a policy perspective. In particular, which of the nine Nash equilibria characterized in Proposition 4.9 may arise in practice is not entirely under the control of the regulator. Indeed, key quantities such as g_h and g_b are entirely determined by

market conditions, while other relevant parameters, such as the expected electricity demand within the REC, d , and its volatility, σ_d , are shaped by local conditions. Naturally, the incentive level Z remains a central policy variable. In particular, it may support the emergence of the socially desirable equilibria NE 6.1 and NE 6.2. However, outside these comparatively rare cases, a uniform incentive level Z , namely one that is not differentiated across technologies, often happens to induce investment by one member while being redundant for the other. This asymmetry suggests that a technology-specific incentive design may be preferable, notwithstanding the evident difficulties involved in calibrating differentiated support levels.

A final remark concerns the sharing rule β . Even more than the incentive level Z , this parameter plays a pivotal role in determining which equilibrium emerges and, in some cases, whether the REC is formed at all. Indeed, an inappropriate choice of β may undermine the viability of the REC even under generous incentives Z , whereas a suitable sharing rule may sustain REC formation even when the incentive level is less favorable. With the exception of NE 6.1 and NE 6.2, where β is determined endogenously by the two REC members, all the other Nash equilibria require the intervention of a coordinator to fix a viable sharing rule, since two competing agents would generally be unable to determine it autonomously.

4.4 Upper-level problem: Stackelberg solution for the coordinator

The coordinator pursues the viability of the energy community: to this end, it sets a NB criterion to balance the economic advantage of both players in entering it, as introduced in the problem statement (18). Therefore, it seeks for the value β^* , which optimizes the following problem:

$$\max_{\beta \in (0,1)} F(d, y_h^*, y_b^*, \beta), \quad (44)$$

where F is defined as

$$F(d, y_h^*, y_b^*, \beta) = (J_h(x_v, x_c, p, d, y_h^*, y_b^*, \beta) - d_h)(J_b(x_v, x_c, p, d, y_h^*, y_b^*, \beta) - d_b).$$

d_h and d_b are the disagreement points defined in (19) for the household and BP, respectively.

Notice again that in solving this problem the coordinator acts strategically, considering y_h^* and y_b^* , as the optimal responses of the two players to its decision, β . Depending on the sign of g_h and g_b , F becomes

$$F(d, y_h^*, y_b^*, \beta) = \begin{cases} \beta(1-\beta)Z^2w^2(\theta_h, \theta_b, d) & g_h \geq 0, g_b \geq 0, \\ \beta(1-\beta)Z^2w^2(y_h^*, \theta_b, d) + (1-\beta)Zw(y_h^*, \theta_b, d)g_hy_h^* & g_h < 0, g_b \geq 0, \\ \beta(1-\beta)Z^2w^2(\theta_h, y_b^*, d) + \beta Zw(\theta_h, y_b^*, d)g_by_b^* & g_h \geq 0, g_b < 0, \\ \beta(1-\beta)Z^2w^2(y_h^*, y_b^*, d) + \beta Zw(y_h^*, y_b^*, d)g_by_b^* & \\ + (1-\beta)Zw(y_h^*, y_b^*, d)g_hy_h^* + y_b^*y_h^*g_bg_h & g_h \leq 0, g_b \leq 0. \end{cases} \quad (45)$$

We apply first order conditions to determine the maximizing β : $\frac{\partial F(d, y_h^*, y_b^*, \beta)}{\partial \beta} = 0$. The first case ($g_h \geq 0$ and $g_b \geq 0$) is the simplest, and the first order condition is solved trivially for $\beta = \frac{1}{2}$.

In the second and third cases of Eq. (45) the solution to the first order condition must be found numerically and it determines a unique solution (y_h^*, y_b^*) .

The fourth case ($g_h \leq 0, g_b \leq 0$) includes the relevant case where $g_h \in (-\alpha Z\beta/(r+\lambda), 0)$ and $g_b \in (-Z(1-\beta)/(r+\lambda), 0)$, which corresponds to NE 6. of Proposition 4.9. Under this equilibrium, the coordinator can only fix $\beta = \beta_n$, otherwise no NE is possible. From the

same proposition we know that, given β_n , there are infinitely many combinations (y_h^*, y_b^*) , which are all NE and satisfy the conditions of Eqs. (39) and (40). Among them, the particular NE

$$(y_h^*, y_b^*) : \alpha y_h^* = y_b^* = \frac{\alpha y_h^* + y_b^*}{2}, \quad (46)$$

where the sum $\alpha y_h^* + y_b^*$ has the property of maximizing F for $\beta = \beta_n$.

5 Application

The analytical framework developed in Sections 3 and 4 yields, within the proposed setting, a closed-form characterization of the equilibrium structure. Existence conditions, optimal investment levels, the coordinator's sharing rule, and the thresholds governing transitions across regimes are derived explicitly as functions of the model parameters.

This feature distinguishes the present approach from optimization and equilibrium models relying primarily on numerical methods, where the properties of the solution are typically observed ex post from simulation outputs. The calibrated solution and the sensitivity analysis developed below are therefore not intended as a general empirical validation of virtual RECs. Rather, they illustrate how the structural properties established in Propositions 4.3, 4.6, and 4.9, together with the characterization of the coordinator's problem, materialize in the case of a two-technology REC, namely PV and G2P, over empirically relevant parameter ranges reflecting investment costs, technological features, and key regulatory elements of the virtual framework adopted in Italy. Consequently, the numerical values obtained in the calibrated case should be read as case-specific evidence, whereas the threshold structure and the equilibrium regimes are structural implications of the model.

5.1 Data description and parameter estimation

For this empirical application, we focus on the case of Italy, which is the first example worldwide of a virtual framework to incentivize RECs. To estimate the GLP parameters, we use publicly accessible real data. The electricity spot price data correspond to hourly measurements in €/MWh of the Italian national price (PUN) taken from January 2, 2014 to February 29, 2024, while the gas price data correspond to daily measurements in €/MWh of the Italian PSV natural gas, taken from January 2, 2014 to February 29, 2024⁴. The power demand data correspond to monthly measurements in MWh of residential power demand in Italy, taken from January 2021 to December 2024⁵. Finally, for the electricity purchase price X_c , we assume a flat tariff, as is usual for household consumers. Analyzing the period 2017-2024 in Italy⁶, we let the purchase cost of electricity be equal to $x_c = 170$ €/MWh. Notice that the purchase price has no effect on the household's decision to enter the REC;

⁴ Both time series are available on the website of the Gestore del Mercato Elettrico (GME), at <https://www.mercatoelettrico.org/it/download/DatiStorici.aspx> for power prices and at <https://www.mercatoelettrico.org/It/download/DatiStoriciGas.aspx> for gas prices

⁵ The series is available on the website of Autorità di Regolazione per Energia, Reti e Ambiente (ARERA) at <https://www.arera.it/dati-e-statistiche/dettaglio/analisi-dei-consumi-dei-clienti-domestici>.

⁶ See Eurostat at https://doi.org/10.2908/NRG_PC_204_C

according to the virtual framework, an individual, after entering a REC, continues to buy energy from their retailer at the usual price.

Before estimating the parameters of the stochastic processes, we preliminarily remove seasonality from the data. This methodological step is essential because our problem concerns *initial capacity investment* rather than short-term operational management. Hence, the relevant stochastic dynamics are those describing long-run uncertainty in prices and demand, while cyclical fluctuations linked to weather or consumption patterns are transient and largely cancel out over the lifetime of the plants. Moreover, retaining strong periodic components would artificially inflate the estimated volatilities, confounding structural uncertainty with predictable cycles. By filtering them out, we obtain deseasonalized series that better represent the persistent stochastic behavior governing long-run profitability. Finally, this approach aligns naturally with the use of GLP models for electricity and gas prices, as well as for demand. GLPs are suited for capturing multiplicative random growth over long horizons, whereas seasonal variations are deterministic and therefore removed before calibration. In summary, deseasonalization ensures that the estimated parameters reflect genuine long-term risk, consistent with the economic horizon of investment decisions.

The procedure that we choose in order to deseasonalize data is

harmonic regression (Bloomfield, 2007). The harmonic regression consists of finding the significant frequencies of a given time series at discrete times $(X(t_i))_{i=1,\dots,I}$ and representing it as a sum of trigonometric functions plus some noise. Suppose $f_k, k = 1, \dots, n$, are the significant frequencies; thus, for our data set, we approximate the seasonality as

$$X(t_i) = \sum_{k=1}^n A_k \cos(2\pi f_k t_i) + B_k \sin(2\pi f_k t_i) + \epsilon(t_i), \quad (47)$$

where $(\epsilon(t_i))_{i=1,\dots,I}$ is some noise. For the power demand and the electricity and gas spot prices, we suppose that the logarithm of the observations can be described by Equation (47). In these cases, the noise ϵ is described by BMs with drift. The significant frequencies for the three data series and the corresponding amplitudes are presented in Table 6 in Appendix B.

After removing seasonalities, the next step is to remove jumps from the various time series. To do this, for a given time series $(Y_n)_{n=0,\dots,N}$ we consider a new time series consisting of log-yields, defined as

$$Z_n := \log \frac{Y_n}{Y_{n-1}}, \quad n = 1, \dots, N$$

On this modified time series, we use the by-now classical threshold method introduced in Mancini (2009). It consists of estimating the variance $\hat{\sigma}^2$ of this time series (for example, with the least squares estimator as in Brigo et al. (2008), see below), and then collecting every observation Z_n whose absolute value exceeds a threshold⁷, putting them out of the time series as a "jump". We then iterate this method, recalculating $\hat{\sigma}^2$ on the time series without jumps and isolating jumps, until this procedure stabilizes. We find that the time series for D presents no jumps, while the time series for P and X_v present jumps. We thus estimate the jump intensities η_p and η_v for the two time series, respectively; and η_{vp} for the common jumps. For both time series, we assume that the independent jumps have a one-dimensional Gaussian distribution, while the common jumps have a bivariate Gaussian distribution: these parameters are estimated by using standard statistical techniques. All the results on the jump estimators are collected in Table 1.

⁷ we used $3\hat{\sigma}$ as the threshold for all of these time series.

Table 1 Jump parameters for the processes P and X_v . The first two lines report the parameters for the isolated jumps of P and X_v , respectively; the third line reports the parameters for their common jumps.

Jump intensity	Value (1/h)	Jump Mean	Value	Jump Variance	Value
η_p	0.00283079	m_p	0.0062521	Σ_p	0.0489095
η_v	0.00306604	m_v	-0.00691772	Σ_v	0.567686
η_{vp}	0.00033	m_{vp}	$\begin{pmatrix} 0.061807 \\ -0.032154 \end{pmatrix}$	Σ_{vp}	$\begin{pmatrix} 0.319565 & -0.007881 \\ -0.007881 & 0.122683 \end{pmatrix}$

Table 2 Initial condition and volatility parameters for the processes of electricity (selling) price, electricity (purchase) cost, gas spot price and power demand

Initial condition	Value	Volatility parameter	Value (1/ $\sqrt{h}$)	Drift parameter	Value (1/h)
x_v	80 €/MWh	σ_v	0.062037	μ_v	0.000677
x_c	170 €/MWh	σ_c	0	μ_c	0
p	35 €/MWh	σ_p	0.006536	μ_p	$1.3809 \cdot 10^{-5}$
d	0.30 MW	σ_d	0.0027	μ_d	$-3.0361 \cdot 10^{-6}$

After removing seasonalities and jumps, we estimate the volatility parameters and the drift of the three GLPs associated with power demand, electricity price, and gas price by using the least squares estimator as in Brigo et al. (2008), see Table 2.

In our empirical application, we adopt the standard assumption that the stochastic processes for demand D , the electricity selling price X_v , and the gas price P evolve as martingales under the physical measure. This choice is well established in the energy literature, as it avoids taking a stand on long-run growth trends – which are notoriously hard to estimate and even harder for REC members to agree upon – while preserving the relevant risk structure through the volatilities. From a practical viewpoint, it also matches the natural benchmark used by agents making long-term investment decisions: lacking reliable information about persistent trends, REC participants typically assume that “the world will proceed as it is”. Martingale dynamics thus provide a neutral real-world expectation.

This calibration choice should therefore be read as a benchmark for long-run investment analysis, rather than as an empirical description of trend formation, extreme-event perception, or attitude toward risk.

To implement this assumption, we adjust the drift parameters of the three GLPs by setting μ_d , μ_v and μ_p so that the formulas in Proposition 3.1 give $r_x = r$ for $x \in \{d, v, p\}$, while leaving the volatilities and jumps unchanged.

This assumption has an important modeling implication. Since all equilibrium quantities depend on the GLP parameters only through the r_x , any simultaneous change in these parameters that keeps $r_x = r$ leaves the equilibrium unchanged. Therefore, varying e.g. a volatility parameter while compensating for the drift to maintain the martingale property does not generate meaningful comparative statics – except in the case of σ_d (recall that we found no jumps in the time series of demand). In fact, demand volatility σ_d enters not only through r_d , but also through the terms m_1 and m_2 , and hence through the coefficients B_1 and C_1 that determine the analytical expression of the expected self-consumption unit incentive w . For this reason, changes in σ_d cannot be neutralized by adjusting μ_d : they genuinely affect

Table 3 Reference values of the parameters used in the application

Parameter	Value	Parameter	Value	Parameter	Value
c_h	1 600 000 €/MW	K_g	300 m ³	r	$3.4247 \cdot 10^{-6}$ 1/h
c_b	1 500 000 €/MW	α	1 500/8 760	Z	110 €/MWh
θ_h	0.3125 MW	γ	0.4	Budget	500 000 €
θ_b	0.20 MW	b	0.01056 MWh/m ³	λ	0.00001

Table 4 Parameters of the realistic example

Parameter	β_h	β_b	g_h	g_b
Value	-4.5744	-0.2043	$2.4 \cdot 10^6$	$-3.69 \cdot 10^6$

the structure of the REC. Consequently, the only volatility parameter for which sensitivity analysis is economically informative is the demand volatility σ_d .

5.2 Optimal solution

Table 3 presents the parameter values used in the application of our model.

The value for r corresponds to an annual discount rate of 3%. The values for c_h and c_b are average (unit) investment costs commonly observed for PV and small-sized gas-turbine generation plants. In general, the upper bounds θ_h and θ_b can be determined either by technical (e.g., available roof surface for the PV plants) or economic (e.g., budget) reasons, whichever applies first. A maximum installation capacity of $\theta_b = 0.2$ MW is being considered, taking into account both the existing support framework and the available capacity K_g . We set a capacity $K_g = 300$ m³, which is a typical size for medium biogas producers. We also assume that the household's investment capacity is bounded only by a budget constraint of 500 000 €; thus, considering the costs of PV installation at $c_h = 1\,600\,000$ €/MW, we have a maximum PV capacity of $\theta_h = 0.3125$ MW. The value $Z = 110$ €/MWh is the example of the Italian incentive tariff for RECs. We set a constant solar radiation of 1500 operational hours per year and obtain a value of $\alpha = 0.1712$, which corresponds to the usual ratio of the operating hours of a PV panel in a year. For the G2P efficiency factor γ , we consider the technical value reported for some biogas-based turbine manufacturers, which is approximately 0.4⁸.

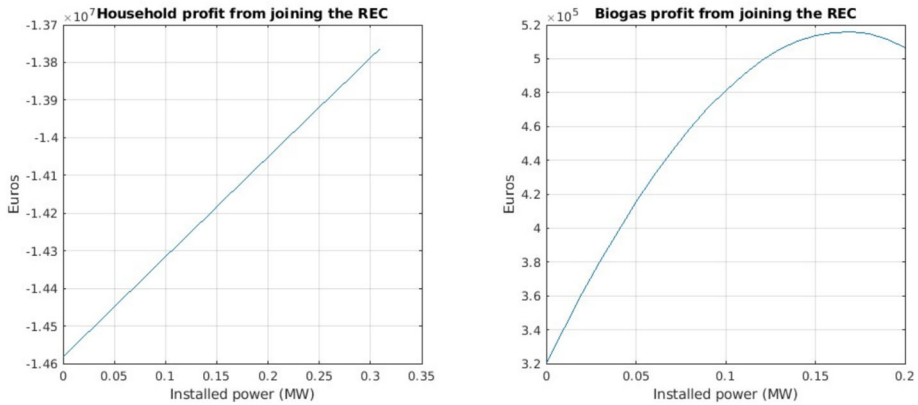
The values in Table 4, which result from those in Table 3, are key to assessing the kind of equilibrium. We see that our case corresponds to NE 4.2 ($g_h > 0$, $g_b < 0$ and $\beta > \beta_b$) in Equation (35), Proposition 4.9. Under this type of NE, the household does not need any incentive to invest in PV panels. However, the BP invests (and enters the REC) only if $g_b + Z \frac{(1-\beta)}{(r+\lambda)} > 0$. This is a case where the coordinator, by fixing the value of β^* according to problem (44), determines both the birth of the REC and a fair sharing rule for the incentives among the two members. Table 5 shows the optimal installations for the members (y_h^* , y_b^*) and the fair sharing rule β^* .

Fig. 4 shows the profits for the household (left) and the BP (right), considering the solution value of $\beta^* = 0.2543$. Both profit plots are calculated strategically; that is, they include the optimal investment choice of the other member.

⁸ See, for example, https://www.cat.com/en_US/products/new/power-systems/electric-power/gas-generator-sets.html

Table 5 Optimal solution for the example

Parameter	y_h^*	y_b^*	β^*
Value	0.3125 MW	0.1668 MW	0.2543

**Fig. 4** Household and BP best response profits

We see that the profits for the household increase monotonically, so the optimal capacity investment for it coincides with the corresponding maximum capacity boundaries. In contrast, for the biogas producer, we observe a concave function, which however is not globally increasing, so its optimal capacity investment does not exceed its maximum boundaries. This effect is due to the biogas producer's need and incentives to invest in G2P turbines. In fact, as we see in Table 4, g_b is negative; therefore, the biogas producer needs these incentives in order to invest and enter into the REC. The resulting total generation capacity of this REC ($\alpha y_h^* + y_b^* = 0.2203$ MW) is lower than the average internal energy demand, i.e., $d = 0.3$ MW. This equilibrium corresponds to the expression (35) of Proposition 4.9.

We observe that, in this case, the self-consumption incentive favors the birth of the REC, as the BP would not have installed any capacity without its presence. Finally, this incentive is unnecessary for the household, as the corresponding installation would have reached the maximum even without the presence of incentives, since its net unit profit g_h is positive.

5.3 Sensitivity analysis

We conduct a sensitivity analysis of the optimal solution with respect to the parameters x_v , p , d , r , c_h , c_b , and σ_d . From the analysis developed in Section 4 and the discussion in Section 5.1, we have already obtained that changes in the parameters σ_v and σ_p do not influence the equilibrium established in Section 5. Using the parameter values established in Section 5 as a reference, we vary each parameter individually while keeping the others constant. The purpose of this exercise is twofold: on the one hand, to assess the quantitative sensitivity of the calibrated example; on the other hand, to verify whether the qualitative conclusions of the model—namely the emergence of distinct equilibrium regimes, the role of the sharing rule in sustaining REC formation, and the different responses of the two technologies to the incentive mechanism—persist over non-trivial parameter ranges.

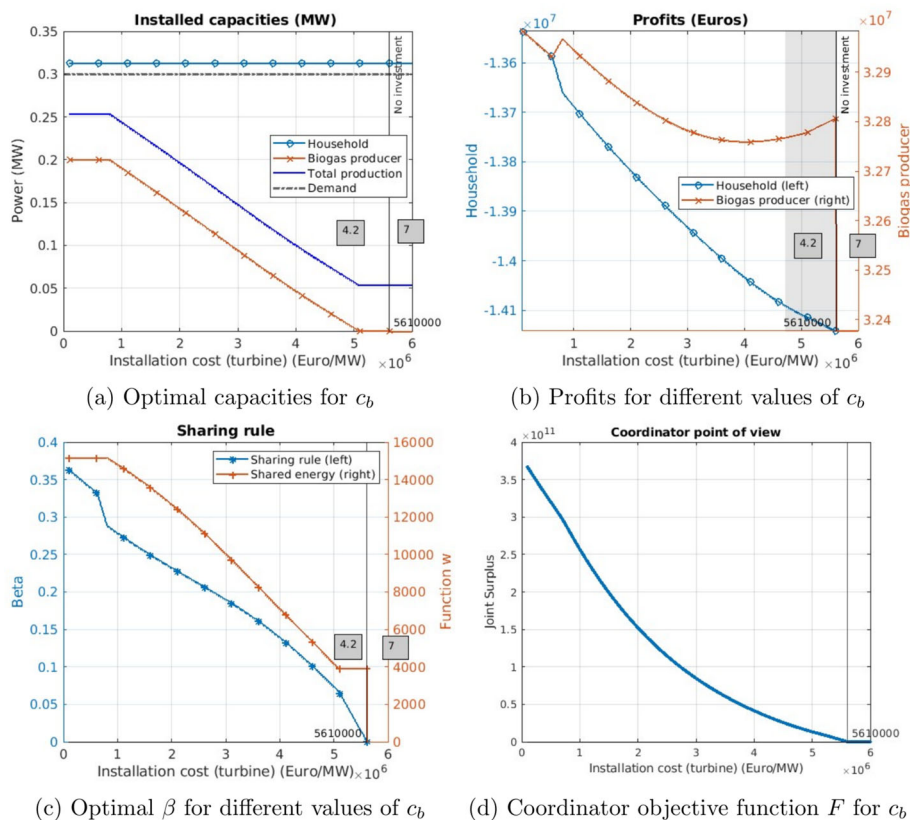


Fig. 5 Sensitivity w.r.t. c_b

Through graphical plots, we illustrate the effects of these parameters on investment decisions (y_h^* , y_b^*), profits (J_h , J_b), the fair sharing rule (β), and the joint surplus (F).

Our focus is on the parameter intervals in which investment decisions, sharing rules, or equilibrium types effectively change. The discussion provides a graphical characterization of the regions in which the model switches across qualitatively different equilibria.

It is based on four figures, showing how the optimal installed capacities (y_h , y_b), the expected profits (J_h , J_b), the optimal sharing rule, the discounted incentive (β , w), and finally the coordinator objective F , change depending on the given parameter.

In scenarios where the community is not formed, we set $\beta^* = 0$ as a convention for graphical representation. Our focus is on the parameter intervals where significant impacts on investment decisions are observed.

5.3.1 Sensitivity with respect to the PV cost c_h

Figure 5 illustrates how the turbine installation cost c_b affects the viability of G2P investment and, consequently, the structure of the REC.

This parameter is of particular interest because it directly governs the competitiveness of the biogas producer's conversion option relative to gas sale, and therefore plays a pivotal role in determining whether the BP remains an active investing member of the community.

As c_b increases, the profitability of the BP declines, and the model transitions from a joint-investment equilibrium (NE 4.2) to a stand-alone PV configuration (NE 7). For low and intermediate turbine costs, from $c_b = 100\,000\text{ €/MW}$ up to $c_b = 5\,610\,000\text{ €/MW}$, the equilibrium remains in NE 4.2. In this region, the coordinator is able to sustain G2P investment by allocating a sufficiently large share of the incentive to the BP. At the entry point, the optimal sharing rule is $\beta^* \approx 0.36$, which allows the BP to install its maximum feasible capacity (Fig. 5a). As c_b rises, the coordinator progressively shifts the incentive towards the BP, reducing β (Fig. 5c) to compensate for the rising capital cost. This transfer temporarily stabilizes the BP's profit (Fig. 5b), but cannot prevent a steady decline in its optimal capacity and in the shared energy w .

The decline in G2P investment directly reduces the social value generated inside the REC: the joint surplus F decreases monotonically with c_b (Fig. 5d), reflecting the diminishing contribution of dispatchable renewable power as turbines become more expensive.

When the turbine cost reaches $c_b = 5\,610\,000\text{ €/MW}$, the incentive is no longer sufficient to make G2P investment profitable. At this point the system switches to NE 7: the BP installs zero capacity (Fig. 5a), the sharing rule collapses to $\beta = 0$ (Fig. 5c), no energy is shared, and the REC effectively reduces to a stand-alone PV investment by the household. Beyond this threshold, the coordinator cannot create a viable REC involving the BP, as the cost of the turbine exceeds any realistic incentive support.

5.3.2 Sensitivity with respect to the PV cost c_h

This is one of the most informative sensitivity analyzes, in particular because it highlights equilibrium type 6.2

and makes the competitive structure of the game especially transparent. The parameter c_h is particularly relevant because a reduction in the PV installation cost not only strengthens the household's incentive to invest, but may also displace the BP's investment in the G2P turbine by altering the equilibrium sharing rule and the composition of aggregate renewable capacity within the REC.

As we can see in Figure 6a, increasing PV panel costs monotonically reduce the total installation of the household. This is explained by considering the budget constraint, as $\theta_h = \text{budget}/c_h$. Higher values of cost c_h reduce θ_h , as the household maintains a fixed budget. At the beginning, for installation costs c_h below $320\,000\text{ €/MW}$, the system operates under NE type 7. In this range, the household's installed capacity decreases rapidly as costs increase, while the BP installation capacity remains equal to 0. In the following range, the system transitions to NE type 4.2, where the sharing rule β initially rises steeply from zero to approximately 0.31. Subsequently, within the cost interval from $320\,000\text{ €/MW}$ to around $4\,100\,000\text{ €/MW}$, β decreases slowly to approximately 0.25, ensuring that the BP receives an adequate incentive to achieve full investment capacity. As highlighted in Figure 6a, the household also continues to invest, using all the available budget. Under this sharing rule, both members install a total capacity that is not much lower than the average demand d .

Between $4\,100\,000\text{ €/MW}$ and $4\,800\,000\text{ €/MW}$, the community enters NE type 6.2. In this region, the only sharing rule that is a NE is the endogenous value β_n . Unlike the previous interval, where β^* is selected by the coordinator to maximize the joint surplus, here the sharing rule results from the strategic interaction between the two members. Consequently, competition extends from the installation decisions (y_h, y_b) to the incentive-sharing rule itself, allowing the member with greater bargaining power to leverage this additional degree of freedom. This mechanism explains the sharp oscillations observed in the profit curves: upon entering NE 6.2, the biogas producer initially benefits, but as the household becomes

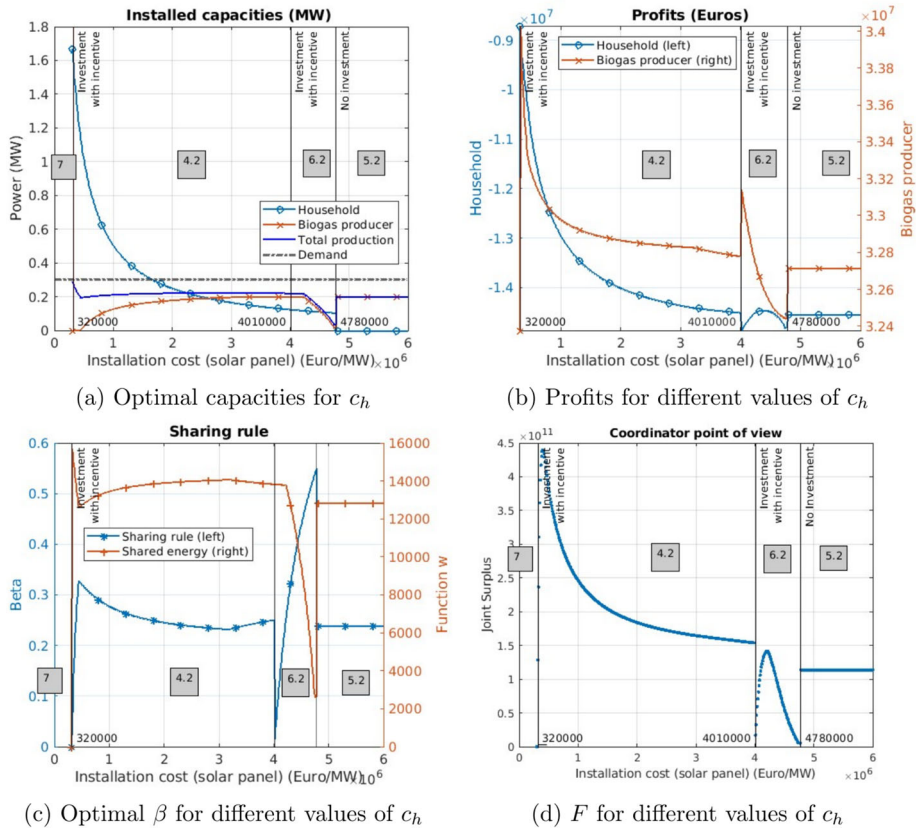


Fig. 6 Sensitivity w.r.t. c_h

increasingly dependent on a larger incentive share to sustain its investment, the advantage shifts in its favor. Overall, this intensified strategic interaction reduces both the total shared energy and the joint surplus.

When the PV panel cost exceeds 4800000 €/MW, the household can no longer invest due to the budget constraint, and the system transitions to NE type 5.2. In this equilibrium, the sharing rule β is again determined by the coordinator, and competition between the two members is (again) restricted to the installation decisions only. As a result, both individual profits increase, and the total surplus improves compared to the values obtained in NE 6.2.

5.3.3 Sensitivity with respect to the risk free rate r

Figures 7a–7d show the sensitivity of the equilibrium with respect to the risk-free rate r .

This parameter is of particular interest because it simultaneously affects the discounted value of market revenues, the stand-alone profitability of both technologies, and the expected value of the incentive mechanism. Hence, varying r does not merely rescale payoffs, but may alter the strategic balance between the two members and, with it, the equilibrium configuration of the REC.

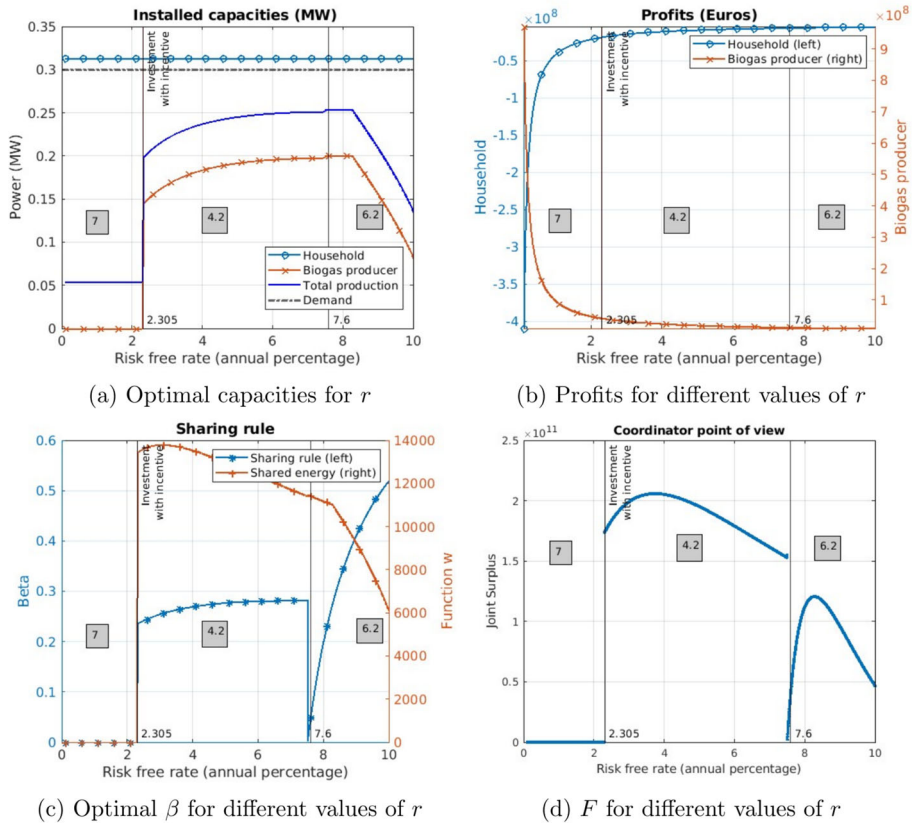


Fig. 7 Sensitivity w.r.t. r

The vertical lines at $r = 2.305\%$ and $r = 7.6\%$ mark the thresholds at which the type of Nash equilibrium changes, and the gray boxes indicate the prevailing equilibrium in each interval: NE 7 for $r \in [0, 2.305\%]$, NE 4.2 for $r \in (2.305\%, 7.6\%]$, and NE 6.2 for $r > 7.6\%$.

The change in equilibrium type does not significantly impact the profits of the two players. In panel 7b, the household's profit increases with r solely due to discounting, while for the same reason, the BP's profit also increases. In short, as the present value of future cash flows vanishes, their net profits converge to the negative of their initial investment costs. Thus, discounting has a dominant effect on profits over other factors.

For low discount rates, $r \in [0, 2.305\%]$, the system is in NE type 7. As shown in Fig. 7a, the household invests at its maximum PV capacity, whereas the BP does not invest in the G2P turbine. Since the BP remains outside the community, the REC is not created: the sharing rule β is set to zero and the shared energy w is identically zero (Fig. 7c), so the joint surplus F in Fig. 7d is also zero. When the discount rate exceeds 2.305%, the REC becomes viable and the system switches to NE 4.2, which persists up to $r = 7.6\%$. In this regime, both members invest: the household maintains its PV installation at the upper bound, while the BP installs a positive G2P capacity that first increases and then slightly decreases with r (Fig. 7a). The coordinator chooses a sharing rule β around 0.3, with only mild variation across this interval (Fig. 7c). This allocation devotes a substantial portion of the incentive to

the BP, compensating for its higher exposure to the discount rate. As r rises, a downward trend in shared energy appears, w (right axis of Fig. 7c), and the hump-shaped behavior of the joint surplus F in Fig. 7d. In this interval, competition between the two agents concerns only the installation decisions, while the sharing rule is optimally set by the coordinator to maximize F .

A qualitatively different behavior appears when r crosses the second threshold at 7.6%, where the equilibrium changes to NE 6.2. Here, both members require a share of the incentive to make participation in the REC preferable to their outside options. In line with the theoretical analysis of NE 6.2, the sharing rule can no longer be freely chosen by the coordinator: it is uniquely pinned down at β_n , which depends on r .

This behavior is clearly visible in Fig. 7c: when r crosses 7.6% and the system enters NE 6.2, the equilibrium value of β experiences a sharp drop, reflecting a sudden shift of bargaining power in favor of the BP and a reallocation of the incentive towards its technology. As r increases further, the equilibrium sharing rule progressively moves back in favor of the household, and the BP reacts by reducing its G2P investment (Fig. 7a). These adjustments are consistent with the strong decrease in shared energy w and with the marked reduction in the joint surplus F (Figures 7c and 7d), while the individual profit curves in Fig. 7b remain almost flat within this high- r regime, close to the agents' initial ($t = 0$) investment cost.

5.3.4 Sensitivity with respect to the power demand volatility σ_d

The four plots in Figure 8 illustrate the sensitivity of the optimal solution with respect to the volatility of the energy demand σ_d . This parameter deserves special attention because, unlike the other volatility coefficients, it affects the equilibrium not only through discounting but also through the analytical structure of the incentive term w ; for this reason, it is the only volatility parameter among those used in the model showing economic relevance.

A fact to keep in mind when explaining the results in this section is that the function defining self-consumption, $q = \min(D^d(t), \alpha y_h + y_b)$ in Eq. (16), is concave in the demand. As a consequence, increasing σ_d while keeping the expected demand d constant reduces the expected value of q and the incentive. Panels 8a–8d confirm this effect. For low levels of volatility, $\sigma_d \in [0, 0.0095]$, the equilibrium is of type 4.2, as indicated by the gray label in each subplot. In this regime, both members invest: the household installs PV capacity at its upper bound, while the BP chooses a positive level of G2P capacity (Fig. 8a). As σ_d increases within this interval, the BP's optimal capacity decreases monotonically, reflecting the deterioration of the expected incentive associated with a more volatile demand process, whereas the household's capacity remains constant (up to the budget).

Figure 8c shows how the coordinator adjusts the sharing rule and the resulting shared energy. For $\sigma_d \in [0, 0.0095]$, the optimal value of β gradually declines from about 0.32 to approximately 0.2, which allocates a larger fraction of the incentive to the BP in order to keep its investment profitable despite the increasing volatility. Over the same range, the shared energy w follows a decreasing pattern, consistently with the concavity argument above: higher volatility reduces expected self-consumption even though the average demand remains fixed.

The impact on profits and on the coordinator's objective can be seen in Figures 8b and 8d. In NE 4.2, the household's profit decreases monotonically with σ_d , consistently with the reduction in expected self-consumption. The joint surplus F also displays a steep downward trend and approaches zero when σ_d is close to 0.0095 (Figure 8d). An interesting feature of Figure 8b is the slight increase in the BP's profit within the gray shaded region, just before the transition from NE 4.2 to NE 7. This behavior reflects the flexibility option in the hands of

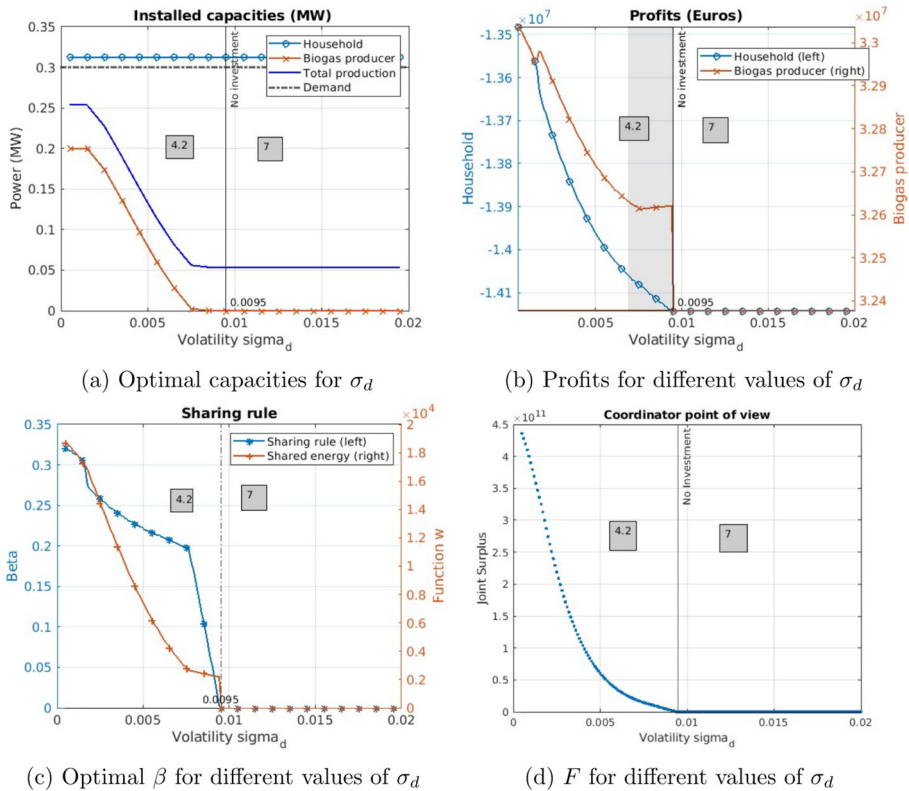


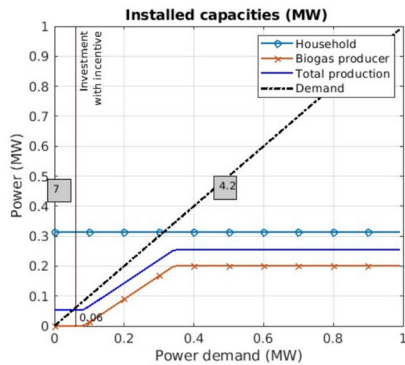
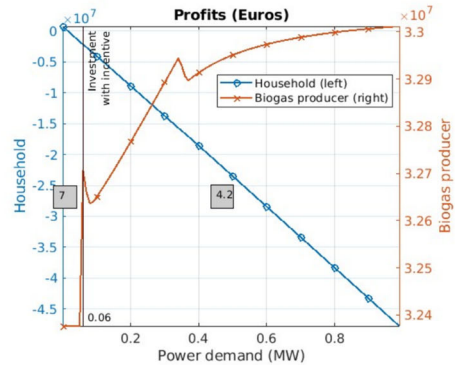
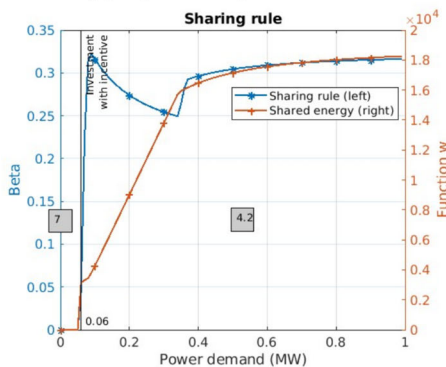
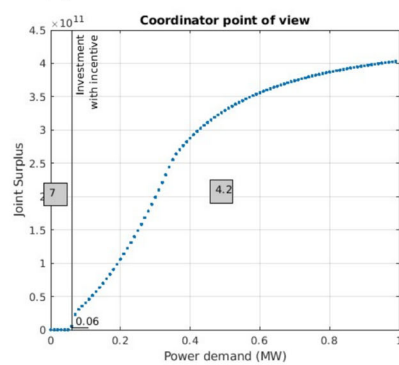
Fig. 8 Sensitivity w.r.t. σ_d

the BP: as demand volatility rises, the BP optimally reduces its G2P investment (Figure 8a), freeing a larger share of gas to be sold on the external market, whose price process maintains the same volatility. The additional gas revenues more than compensate, in that narrow range, for the loss due to lower shared energy, and the BP's profit experiences a small uptick even though the overall surplus continues to fall.

Beyond the threshold $\sigma_d = 0.0095$, the REC is no longer viable for the BP and the equilibrium switches to type 7. In this high-volatility regime, the BP optimally chooses not to invest, so the community is not formed: β and w collapse to zero (Figure 8c), the household invests on its own at the same PV capacity as before (Figure 8a), and the joint surplus F remains at zero (Figure 8d).

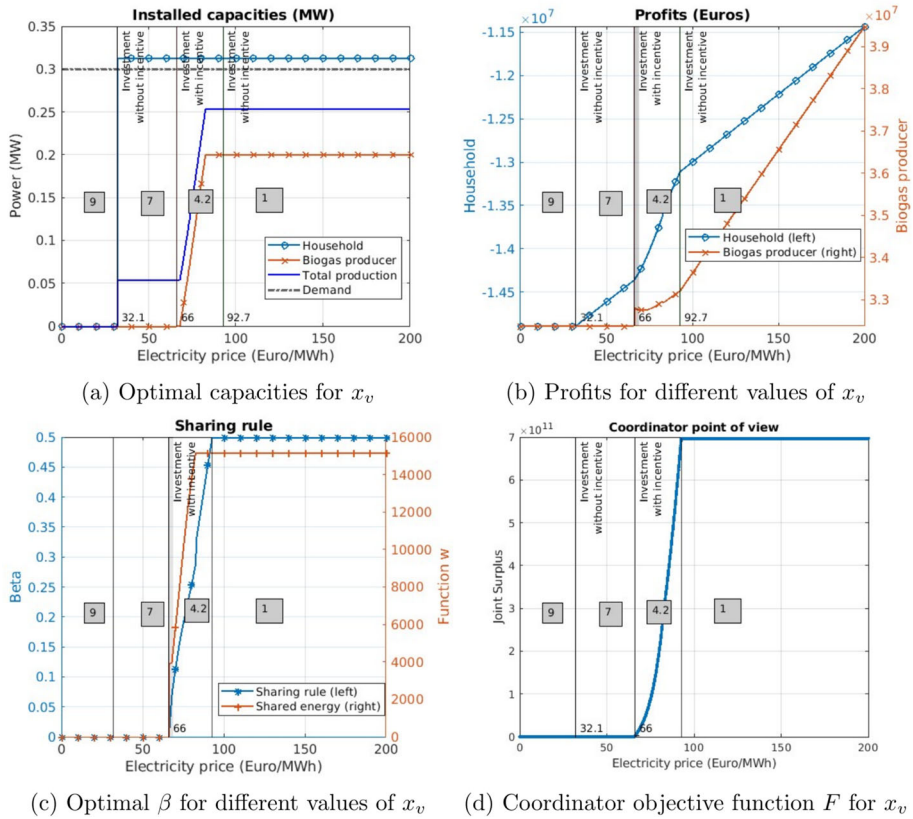
5.3.5 Sensitivity with respect to the power demand d

The four panels in Figure 9 display the impact of the average demand level d on the equilibrium outcome. This sensitivity is especially relevant in the virtual REC framework because demand is not only a determinant of individual energy expenditure but also the key enabling factor for shared self-consumption and, therefore, for the very formation of the REC. For this reason, varying d reveals the threshold at which the household's demand becomes sufficiently large to sustain the BP's entry and activate the REC creation.

(a) Optimal capacities for d (b) Profits for different values of d (c) Optimal β for different values of d (d) F for different values of d **Fig. 9** Sensitivity w.r.t. d

The vertical line at $d = 0.06$ MW separates NE 7, for $d \in [0, 0.06]$, from NE 4.2, for $d > 0.06$. For low demand (NE 7), the household installs PV capacity at its upper bound, while the BP does not invest (panel 9a). Since the BP stays out of the community, the REC is not formed: β and the shared energy w are zero (panel 9c), and the joint surplus F is also zero (panel 9d).

Once d exceeds 0.06 MW, the REC becomes active (NE 4.2). The BP starts investing, and its optimal G2P capacity increases with demand, whereas the household keeps a PV capacity close to its upper bound (panel 9a). At the same time, β jumps from zero to a positive value and then stabilizes around 0.3, while w rises quickly and then flattens as capacity constraints become binding (panel 9c). In panel 9b, the BP's profit increases with d , benefiting from higher demand and the incentive, whereas the household's profit decreases approximately linearly, reflecting higher net expenditure. From the coordinator's perspective, F is increasing in d (panel 9d), indicating that additional demand enlarges the economic scope of the REC.

Fig. 10 Sensitivity w.r.t. x_v

5.3.6 Sensitivity with respect to the electricity price x_v

Figure 10 illustrates how the expected selling price of electricity x_v selects different equilibrium regimes and, through them, the shape of the REC. This sensitivity is especially informative because x_v shows the most extreme sequence of Nash equilibria: both players do not invest, stand-alone PV investment, biogas producer's entry sustained by the incentive mechanism, and finally a regime in which both technologies are profitable even without the incentive.

For very low prices, up to about $x_v \simeq 32$ €/MWh, both technologies are structurally unprofitable even in the presence of incentives. The model settles in NE 9: neither the household nor the BP invests (panel 10a), the REC is not formed, and the coordinator's surplus F is zero (panel 10d).

Once x_v exceeds this first threshold, PV becomes marginally profitable for the household, while the G2P unit remains unattractive. In the interval $32.1 \leq x_v \leq 66.0$, the equilibrium is NE 7: the household behaves as a stand alone investor, choosing a positive PV capacity independently of β (panel 10a), while the BP does not install any turbines and there is still no REC.

The key qualitative change occurs when x_v crosses 66.0 €/MWh. At this point, electricity revenues are high enough, in combination with the self-consumption incentive, to make

G2P investment profitable, so the system moves to NE 4.2. Both members now invest, and self-consumption inside the REC becomes substantial. Economically, the BP is the marginal player: it joins only if it receives a sufficiently large fraction of the incentive. The coordinator therefore chooses a low value of β at the entry point and then gradually shifts part of the tariff back to the household as x_v rises and the BP becomes less dependent on support (panel 10c). In this regime, both individual profits and the joint surplus F increase steeply with x_v (panels 10b and 10d), reflecting the rapid growth of shared energy.

A further threshold, $x_v = 93.0 \text{ €/MWh}$, marks the transition to NE 1. At such high prices, both PV and G2P are profitable on their own: the two players install at (or very close to) their technical upper bounds irrespective of the incentive (panel 10a). The REC still generates a large surplus, but the tariff becomes a pure transfer within an already profitable configuration. Consistent with the Nash-bargaining objective, the optimal sharing rule converges towards an approximately symmetric split (panel 10c), while capacities, shared energy, and joint surplus plateau because technical and budget constraints are binding (panel 10d). This regime highlights the risk of over-subsidization at high x_v : additional public support affects only how rents are divided, without changing investment or REC viability.

5.3.7 Sensitivity with respect to the gas price p

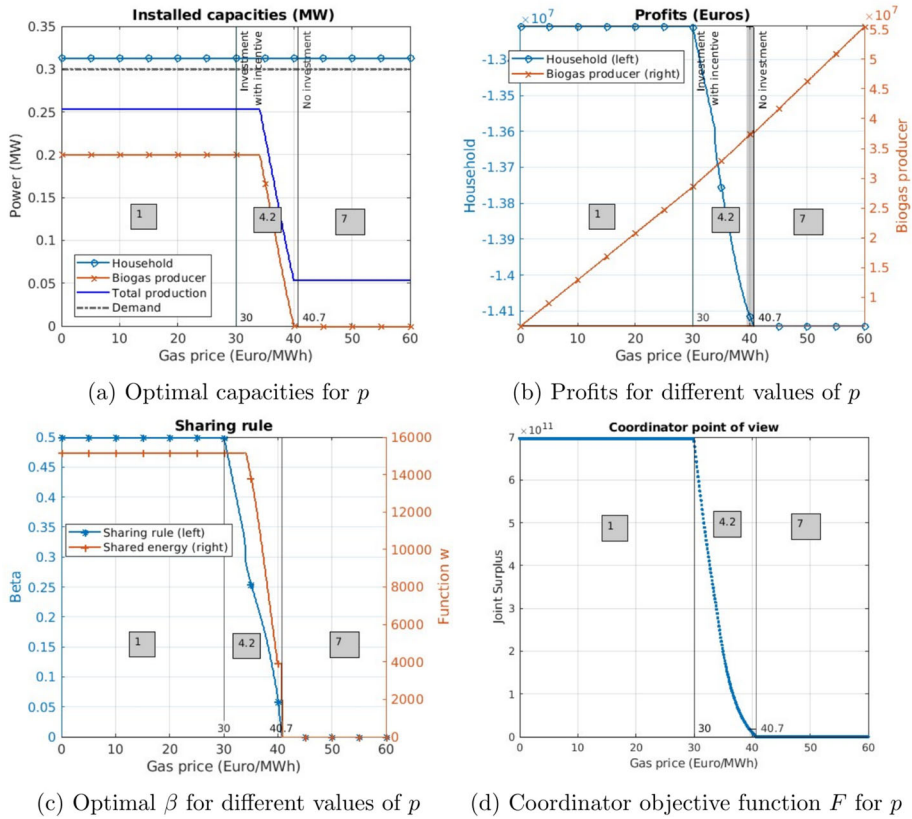
Figure 11 shows how the gas price p affects the structure and viability of the REC by selecting different equilibrium regimes.

This parameter is especially relevant because it governs the outside option of the BP: as p increases, selling gas on the external market becomes more attractive than converting it into electricity within the REC. The corresponding sensitivity therefore makes transparent one of the central economic tensions of the model.

For low gas prices, up to $p = 30 \text{ €/MWh}$, the system is in NE 1. Gas is cheap, so the G2P technology is highly profitable, and the BP has a strong incentive to supply electricity to the REC. Both agents invest up to their technical/budget limits (panel 11a), shared energy w is high (panel 11c), and the joint surplus F reaches its maximum plateau (panel 11d). Given this favorable environment, the coordinator has a perfectly balanced allocation of the incentive, with β equal to 0.5.

When p exceeds 30 €/MWh , the opportunity cost of using gas in the REC instead of selling it on the market rises. The equilibrium moves to NE 4.2 and joint investment becomes progressively more fragile. The BP reduces its optimal G2P capacity, and total community production falls back towards demand (panel 11a). To keep the BP inside the REC, the coordinator must tilt the incentive in its favor: β decreases markedly, transferring a growing share of the tariff to the BP (panel 11c). In parallel with this rebalancing, both the shared energy w and the joint surplus F drop sharply as p increases (panels 11c and 11d).

At $p = 40.7 \text{ €/MWh}$ the BP no longer invests, so the REC collapses to a stand-alone PV project: the household installs only a small PV capacity, just enough to satisfy part of its demand (panel 11a). The sharing rule and shared energy both drop to zero (panel 11c), and the joint surplus F coincides with the disagreement outcome (panel 11d). High gas prices, therefore, undermine the economic rationale for including dispatchable biogas generation in the REC and forbid the creation of the REC.

Fig. 11 Sensitivity w.r.t. p

6 Conclusions

From a broader economic and policy perspective, the findings of the paper suggest that the contribution of virtual RECs to the objectives underlying the EU framework is not ensured by the mere introduction of support schemes. If RECs are to promote active participation in the energy transition, local renewable deployment, and socially valuable forms of self-consumption, our framework highlights that their institutional design should ideally also satisfy a criterion of economic efficiency. In the present context, this means aligning decentralized investment incentives with viable and socially desirable REC configurations while avoiding both underinvestment and technology-insensitive over-subsidization. Under this perspective, the sharing rule and the role of coordination emerge as not secondary design details but as integral components of an efficient REC architecture.

With respect to the first research question (which concerned the interaction between the sharing rule and the investment decisions), our analytical results illustrate that in a virtual REC with heterogeneous members, the sharing rule is not a mere redistribution device but a determinant of the equilibrium investment pattern within our framework. The two investments are intrinsically complementary; yet their viability is strategically governed by β , which may select joint investment, one-sided investment, or no REC formation. The sensitivity analysis confirms that this mechanism is driven primarily by the relative profitability

of the two technologies, which shifts the members' best responses, and by the level and volatility of demand, which govern the value of shared self-consumption and, hence, the effectiveness of the incentive.

As for the second research question (conditions under which Nash equilibria emerge), our framework indicates that the emergence of the REC as a Nash equilibrium is regulated by precise threshold conditions. The analytical characterization of the equilibrium regimes sharply distinguishes between joint-investment configurations, one-sided investment, and REC failure. The sensitivity analysis confirms that these regime changes are driven primarily by the relative profitability of PV and G2P investment and by the level and volatility of demand, which govern the extent and stability of shared self-consumption.

Turning to the third research question (role of the coordinator), our results suggest that the coordinator is not an accessory institutional layer, but in many cases, a critical element for REC viability under the assumed market structures. The equilibrium analysis reveals that, in several regions of the parameter space, the members cannot autonomously identify a common sharing rule capable of sustaining joint investment, even when public support is available. In this sense, the coordinator acts as an equilibrium-selection device, making REC formation implementable and, in some cases, improving both individual payoffs and aggregate efficiency. A broader implication suggested by our framework is that REC regulation may be incomplete with respect to its own objectives: by supporting community formation without explicitly addressing the internal coordination problem, it risks leaving unresolved a mechanism that is essential for REC viability in a non-negligible set of situations.

A distinct, though closely related, issue is raised by the fourth research question (which concerned policy implications): our analysis suggests that the effective regulation of virtual RECs requires particular attention to the design of the incentive mechanism. As the model and its application to historical data indicate, heterogeneous technologies may display markedly different support needs. Therefore, within our stylized setting a uniform incentive scheme cannot, in general, ensure economically efficient REC formation. This makes the allocation of the collective benefit a regulatory issue in its own right, rather than a purely internal matter of the community. In this respect, the current Italian framework, despite providing an advanced virtual REC scheme, may not yet appear complete: by remaining silent on the principles governing the distribution of the incentives inside a REC, it potentially leaves unresolved a coordination problem that our model suggests, in many cases, to be crucial for the REC to achieve the very objectives the regulation is meant to pursue.

While the specific structural assumptions adopted here—such as the two-agent setup and stylized stochastic processes—were necessary to maintain analytical tractability, they point directly to several promising paths for future research. One involves the separation of the profit model for PV investment to distinguish between daylight and nighttime periods, which would create additional opportunities for the two technologies to integrate, with G2P generation sustaining self-consumption when PV halts generation. Another interesting extension, connected to the previous one, is the introduction of battery energy storage, motivated by its obvious support for the self-consumption of RECs and by the expected reduction in its cost. The choice of alternative criteria for the sharing rule would also be extremely interesting. However, alternative criteria in many cases require significant changes in the economic assumptions and in the mathematical settings, so this line of research would benefit from a dedicated future study.

Finally, a challenging extension could address the fact that a REC is a dynamic entity that can, for example, decide to increase the installation of biogas and/or photovoltaic power over time, even though such a line of development would probably require abandoning the analytical tractability maintained here.

A Appendix

A.1 Proofs

Proof of Lemma 4.1 First of all we notice that w depends on $y_b, y_h \geq 0$ only as parameters, as the nontrivial dependence is on d . For this reason, in this proof we will explicitly indicate only the dependence on d . Thus, if we assume that $\tau \sim \text{Exp}(\lambda)$ is independent of D , then its randomness can be eliminated from the computation of w in this way:

$$\begin{aligned}
 w(d) &= \mathbb{E} \left[\int_0^\tau e^{-rs} \min \{ D^d(s), \alpha y_h + y_b \} ds \right] = \\
 &= \mathbb{E} \left[\mathbb{E} \left[\int_0^\tau e^{-rs} \min \{ D^d(s), \alpha y_h + y_b \} ds \middle| \mathcal{F}_\tau \right] \right] = \\
 &= \mathbb{E} \left[\int_0^\tau e^{-rs} \mathbb{E} \left[\min \{ D^d(s), \alpha y_h + y_b \} \middle| \mathcal{F}_\tau \right] ds \right] = \\
 &= \mathbb{E} \left[\int_0^\tau e^{-rs} \mathbb{E} \left[\min \{ D^d(s), \alpha y_h + y_b \} \right] ds \right] = \\
 &= \int_0^\infty \lambda e^{-\lambda u} \int_0^u e^{-rs} \mathbb{E} \left[\min \{ D^d(s), \alpha y_h + y_b \} \right] ds du = \\
 &= \int_0^\infty e^{-rs} \mathbb{E} \left[\min \{ D^d(s), \alpha y_h + y_b \} \right] \int_s^\infty \lambda e^{-\lambda u} du ds = \\
 &= \int_0^\infty e^{-(r+\lambda)s} \mathbb{E} \left[\min \{ D^d(s), \alpha y_h + y_b \} \right] ds = \\
 &= \mathbb{E} \left[\int_0^\infty e^{-(r+\lambda)s} \min \{ D^d(s), \alpha y_h + y_b \} ds \right]
 \end{aligned}$$

where we use the independence between τ and D in the fourth equality and the Tonelli's theorem in the sixth and eighth.

We can easily check that, with this new formulation, w satisfies the ordinary differential equation (ODE)

$$(r + \lambda)w(d) - \mathcal{L}w(d) = \min(d, \alpha y_h + y_b) \quad (48)$$

where

$$\mathcal{L}u(d) = \left(\mu_d + \frac{1}{2} \sigma_d^2 \right) du'(d) + \frac{1}{2} \sigma_d^2 d^2 u''(d) + \eta \int_{\mathbb{R}^4} [u(de^y) - u(d)] v(dy)$$

see e.g. (Pham, 2009, Remark 3.5.6). Now, the general solution of Equation (48) is a linear combination of one particular solution and the solutions of the corresponding homogeneous equation $(r + \lambda)w(d) - \mathcal{L}w(d) = 0$. It is easy to verify that the functions $w(d) = d^m$ are solutions of this homogeneous equation if and only if m satisfies $f_v(m) = 0$, with f_v defined in Equation (23).

We now prove that two such values of m exist, together with the properties announced in Lemma 4.1. First of all, we notice that f_v is convex and $\lim_{m \rightarrow -\infty} f_v(m) = \lim_{m \rightarrow +\infty} f_v(m) = +\infty$. This, along with the facts that $f_v(0) = -(r + \lambda) < 0$ and $f_v(1) = \mu_d - r - \lambda = -r_d - \lambda < 0$, gives us the existence of two points $m_1 < 0$ and $m_2 > 1$ such that $f_v(m_i) = 0$, $i = 1, 2$.

Once this is established, the general solution of Equation (48) is

$$w(d) = \begin{cases} Ad^{m_1} + Bd^{m_2} + \frac{d}{r_d + \lambda} & d < \alpha y_h + y_b \\ Cd^{m_1} + Dd^{m_2} + \frac{\alpha y_h + y_b}{r + \lambda} & d \geq \alpha y_h + y_b \end{cases} \quad (49)$$

where A , B , C and D are constants to be determined. As we want Equation (17) to hold, we want $w(0) = 0$; therefore, we must have $A = 0$. On the other hand, it is also clear from Equation (17) that $w(y_h, y_b, d) \leq \frac{\alpha y_h + y_b}{r}$, thus w must stay bounded when $d \rightarrow \infty$, which gives $D = 0$. Since Equation (48) is a linear second-order ODE with continuous coefficients, we expect its solution to be of class $C^2(\mathbb{R}^+)$. For this reason, we impose the value matching and smooth pasting conditions at $d = \alpha y_h + y_b$, which imply

$$\begin{cases} B(\alpha y_h + y_b)^{m_2} + \frac{\alpha y_h + y_b}{r_d + \lambda} = C(\alpha y_h + y_b)^{m_1} + \frac{\alpha y_h + y_b}{r + \lambda} \\ Bm_2(\alpha y_h + y_b)^{m_2-1} + \frac{1}{r_d + \lambda} = Cm_1(\alpha y_h + y_b)^{m_1-1}, \end{cases} \quad (50)$$

which gives

$$\begin{cases} B = (\alpha y_h + y_b)^{1-m_2} B_1 \\ C = (\alpha y_h + y_b)^{1-m_1} C_1, \end{cases} \quad (51)$$

with B_1 and C_1 as in Equation (22). With this determination, one can easily check that the function w is increasing and concave on $\mathcal{R}^+$, and C^2 in d . We can easily check that w is C^2 also in y_h and y_b . In fact, it is C^2 outside the region $\{yybb + \alpha y y_h = dd\}$. Moreover,

$$\begin{aligned} w_{y_b} \big|_{\alpha y_h + y_b = d^+} &= w_{y_h} \big|_{\alpha y_h + y_b = d^+} = B_1(1 - m_2) = w_{y_b} \big|_{\alpha y_h + y_b = d^-} \\ &= w_{y_h} \big|_{\alpha y_h + y_b = d^-} = C_1(1 - m_1) + \frac{1}{r + \lambda} \end{aligned} \quad (52)$$

which is easily obtained from the system (50) by simplifying for $(\alpha y_h + y_b)$ in the first equation and subtracting the second. The fact that the second derivatives are also continuous at $\{\alpha y_h + y_b = d\}$ can be verified with some algebra.

By applying the Itô formula to the process $(e^{-(r+\lambda)t} w(D(t)))_t$ from 0 to T , we have

$$\begin{aligned} e^{-(r+\lambda)T} w(D^d(T)) &= w(d) + \int_0^T e^{-(r+\lambda)s} \left(\mathcal{L}w(D^d(s)) - (r + \lambda)w(D^d(s)) \right) ds \\ &\quad + \int_0^T e^{-(r+\lambda)s} D^d(s) w'(D^d(s)) (AdW(s))|_d \\ &\quad + \sum_{k=1}^{N_\tau} \left(w(D^d(\tau_k-) e^{Y_k}) - w(D^d(\tau_k-)) - \eta \int_{\mathbb{R}^d} [w(D^d(\tau_k-) e^y) - w(D^d(\tau_k-))] v(dy) \right) \\ &= w(d) - \int_0^T e^{-(r+\lambda)s} \min\{D^d(s), \alpha y_h + y_b\} ds + \\ &\quad + \int_0^T e^{-(r+\lambda)s} D^d(s) w'(D^d(s)) (AdW(s))|_d \\ &\quad + \sum_{k=1}^{N_\tau} \left(w(D^d(\tau_k-) e^{Y_k}) - w(D^d(\tau_k-)) - \eta \int_{\mathbb{R}^d} [w(D^d(\tau_k-) e^y) - w(D^d(\tau_k-))] v(dy) \right). \end{aligned}$$

One can easily check that w is concave; thus, $|w'(d)| \leq w'(0) < +\infty$ for all $d \geq 0$: by using the Levy-Khinchine formula (7), this implies

$$\mathbb{E} \left[\int_0^T e^{-2(r+\lambda)s} D^2(t) (w'(D(t)))^2 \sigma_d^2 dt \right]$$

$$\begin{aligned}
&\leq \mathbb{E} \left[\int_0^T e^{-2(r+\lambda)s} D^2(t) (w'(0))^2 \sigma_d^2 dt \right] \\
&= (w'(0)d)^2 \sigma_d^2 \int_0^T e^{-2s(r+\lambda)} \mathbb{E} \left[e^{2L_d(s)} \right] ds \\
&= (w'(0)d)^2 \sigma_d^2 \int_0^T e^{-2s(r+\lambda-\mu_d-\sigma_d^2-\frac{1}{2} \int_{\mathbb{R}} (e^{2y}-1) v_d(dy))} ds < \infty
\end{aligned}$$

as, by assumption, all exponential moments of L_d are finite. Thus, the expectation of the stochastic integral above is zero, as well as the expectation of the jump part, and we have that, for all $T > 0$,

$$w(d) = \mathbb{E} \left[e^{-(r+\lambda)T} w(D^d(T)) \right] + \mathbb{E} \left[\int_0^T e^{-(r+\lambda)s} \min\{D^d(s), \alpha y_h + y_b\} ds \right] \quad (53)$$

Finally, since for all $d \geq 0$ we have

$$0 \leq w(d) \leq \lim_{d \rightarrow +\infty} w(d) = \frac{\alpha y_h + y_b}{r + \lambda},$$

then

$$\lim_{T \rightarrow \infty} \left| \mathbb{E} \left[e^{-(r+\lambda)T} w(D^d(T)) \right] \right| \leq \lim_{T \rightarrow \infty} e^{-(r+\lambda)T} \frac{\alpha y_h + y_b}{r + \lambda} = 0,$$

and by letting $T \rightarrow +\infty$ in Equation (53), we get the result in Equation (17). $\square$

Proof of Proposition 4.3 We can find the point of maximum of J_h using a first order condition. The partial derivative of J_h w.r.t. y_h is given by

$$\frac{\partial J_h}{\partial y_h} = \begin{cases} g_h + \alpha Z \beta B_1 d^{m_2} (1 - m_2) (\alpha y_h + y_b)^{-m_2} & d < \alpha y_h + y_b \\ g_h + \alpha Z \beta (C_1 d^{m_1} (1 - m_1) (\alpha y_h + y_b)^{-m_1} + \frac{1}{r+\lambda}) & d \geq \alpha y_h + y_b. \end{cases} \quad (54)$$

Since $\frac{\partial J_h}{\partial y_h}$ is continuous in y_h , we search for the critical point $\frac{\partial J_h}{\partial y_h} = 0$. As $y_h \in [0, \theta_h]$ we also have to consider the extreme points 0 and θ_h .

Since J_h is C^1 and concave in y_h , we can determine whether the maximum is attained when the aggregate installation is greater than the demand or not, by observing the sign of $\frac{\partial J_h}{\partial y_h} \big|_{y_h = \frac{d-y_b}{\alpha}}$. In fact $\frac{\partial J_h}{\partial y_h} \big|_{y_h = \frac{d-y_b}{\alpha}} \leq 0$ if and only if the maximum $y_h^* \leq \frac{d-y_b}{\alpha}$.

We have that

$$\frac{\partial J_h}{\partial y_h} \bigg|_{y_h = \frac{d-y_b}{\alpha}} = g_h + \alpha Z \beta B_1 (1 - m_2)$$

thus, since $B_1 < 0$ and $m_2 > 1$, we have that $\frac{\partial J_h}{\partial y_h} \big|_{y_h = \frac{d-y_b}{\alpha}} > 0$ if and only if $\beta > \beta_h$, this latter being defined in Equation (26).

Let us move into the solution of problem (24). Let us suppose $\beta > \beta_h$, so that $\frac{\partial J_h}{\partial y_h} \big|_{y_h = \frac{d-y_b}{\alpha}} > 0$: then we know that the maximum is attained when the installation exceeds the demand, i.e. when $d < \alpha y_h + y_b$. the first-order condition obtained from (54) is solved, in the case $d < \alpha y_h + y_b$, for

$$\alpha Z \beta B_1 d^{m_2} (m_2 - 1) (\alpha y_h + y_b)^{-m_2} = g_h$$

As the left-hand side is always negative, the first-order condition has a solution only if $g_h < 0$. In this case, $\beta > \beta_h > 0$ and the solution is

$$y_h^* = \frac{1}{\alpha} \left(d \left(\frac{\alpha Z \beta B_1 (m_2 - 1)}{g_h} \right)^{1/m_2} - y_b \right) = \frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - y_b \right).$$

If $y_h^* \in [0, \theta_h]$, then y_h^* is the maximum. If $y_h^* < 0$ then the maximum is 0 and if $y_h^* > \theta_h$ then the maximum is θ_h . Moreover, if $\alpha Z B_1 (m_2 - 1) < g_h$, then $\beta_h < 1$. Summarizing,

$$y_h^* = \max \left\{ \min \left\{ \frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - y_b \right), \theta_h \right\}, 0 \right\}. \quad (55)$$

Instead, if $g_h \geq 0$, then the first-order condition is not satisfied, $\frac{\partial J_h}{\partial y_h}$ is always positive and the maximum of J_h is reached for $y_h^* = \theta_h$. In this case $\beta_h < 0$.

Let us now suppose that $\beta \leq \beta_h$, i.e. $\frac{\partial J_h}{\partial y_h} \big|_{y_h = \frac{d - y_b}{\alpha}} \leq 0$. Then for $d \geq \alpha y_h + y_b$, the first order condition is solved for

$$\alpha Z \beta C_1 d^{m_1} (m_1 - 1) (\alpha y_h + y_b)^{-m_1} = g_h + \frac{\alpha Z \beta}{r + \lambda},$$

as the left-hand side is always positive, the first order condition has a solution only if $g_h + \frac{\alpha Z \beta}{r + \lambda} > 0$, and the solution is

$$y_h^* = \frac{1}{\alpha} \left(d \left(\frac{\alpha Z \beta C_1 (m_1 - 1)}{g_h + \frac{\alpha Z \beta}{r + \lambda}} \right)^{1/m_1} - y_b \right) = \frac{1}{\alpha} \left(d \left(\frac{\frac{g_h}{\beta_h} + \frac{\alpha Z}{r + \lambda}}{\frac{g_h}{\beta} + \frac{\alpha Z}{r + \lambda}} \right)^{1/m_1} - y_b \right).$$

Moreover if $g_h < 0$, then $\beta_h > 0$. If $y_h^* \in [0, \theta_h]$, then y_h^* is the maximum. If $y_h^* < 0$ then the maximum will be attained at zero, and if $y_h^* > \theta_h$ then the maximum will be attained at θ_h . Summarizing,

$$y_h^* = \max \left\{ \min \left\{ \frac{1}{\alpha} \left(d \left(\frac{\frac{g_h}{\beta_h} + \frac{\alpha Z}{r + \lambda}}{\frac{g_h}{\beta} + \frac{\alpha Z}{r + \lambda}} \right)^{1/m_1} - y_b \right), \theta_h \right\}, 0 \right\}. \quad (56)$$

On the other hand, if $g_h + \frac{\alpha Z \beta}{r + \lambda} \leq 0$, the derivative is always negative and the maximum is always attained at $y_h^* = 0$. Again in this case $\beta_h > 0$. $\square$

Proof of Proposition 4.6 We can find the maximum point of J_b using a first order condition. The partial derivative of J_b w.r.t. y_b is given by

$$\frac{\partial J_b}{\partial y_b} = \begin{cases} g_b + Z(1 - \beta) (B_1(1 - m_2)(\alpha y_h + y_b)^{-m_2} d^{m_2}) & d < \alpha y_h + y_b \\ g_b + Z(1 - \beta) \left(C_1(1 - m_1)(\alpha y_h + y_b)^{-m_1} d^{m_1} + \frac{1}{r + \lambda} \right) & d \geq \alpha y_h + y_b. \end{cases} \quad (57)$$

Since $\frac{\partial J_b}{\partial y_b}$ is continuous in y_b , we search for the critical point $\frac{\partial J_b}{\partial y_b} = 0$. As $y_b \in [0, \theta_b]$ we also have to consider the extreme points 0 and θ_b .

Since J_b is C^1 in y_b and it is concave, we can determine whether the maximum is attained when the aggregate installation exceeds the demand or not, by observing the sign of $\frac{\partial J_b}{\partial y_b} \big|_{y_b = d - \alpha y_h}$. In fact $\frac{\partial J_b}{\partial y_b} \big|_{y_b = d - \alpha y_h} \leq 0$ if and only if the maximum $y_b^* \leq d - \alpha y_h$.

We have that

$$\frac{\partial J_b}{\partial y_b} \bigg|_{y_b = d - \alpha y_h} = g_b + Z(1 - \beta) B_1(1 - m_2) = g_b + Z(1 - \beta) \left(C_1(1 - m_1) + \frac{1}{r + \lambda} \right)$$

thus, since $B_1 < 0$ and $m_2 > 1$, we have that $\frac{\partial J_b}{\partial y_b}|_{y_b=d-\alpha y_h} > 0$ if and only if $1 - \beta > 1 - \beta_b$ with β_b defined in Equation (29).

Let us move into the solution of problem (27). Let us suppose $\beta < \beta_b$, i.e. $\frac{\partial J_b}{\partial y_b}|_{y_b=d-\alpha y_h} > 0$, then we know that the maximum is attained when the installation exceeds the demand, i.e. $d < \alpha y_h + y_b$: the first-order condition is then solved for

$$g_b = Z(1 - \beta) (B_1(m_2 - 1)(\alpha y_h + y_b)^{-m_2} d^{m_2}).$$

Since the right-hand side is always negative, the first order condition has a solution only if $g_b < 0$ and the solution is

$$y_b^* = d \left(\frac{Z(1 - \beta)B_1(m_2 - 1)}{g_b} \right)^{1/m_2} - \alpha y_h = d \left(\frac{1 - \beta}{1 - \beta_b} \right)^{1/m_2} - \alpha y_h.$$

If $y_b^* \in [0, \theta_b]$, then it is the maximum. If $y_b^* > \theta_b$, then we select θ_b as the optimal point; if instead $y_b^* < 0$, we select $y_b^* = 0$. Moreover, if $g_b > ZB_1(m_2 - 1)$, then $\beta^b > 0$. Summarizing,

$$y_b^* = \max \left\{ \min \left\{ d \left(\frac{1 - \beta}{1 - \beta_b} \right)^{1/m_2} - \alpha y_h, \theta_b \right\}, 0 \right\}. \quad (58)$$

In the case $g_b \geq 0$, the derivative is always positive and the maximum is attained at $y_b^* = \theta_b$.

In analogy with the previous case, let us now suppose $\beta \geq \beta_b$, i.e. $\frac{\partial J_b}{\partial y_b}|_{y_b=d-\alpha y_h} \leq 0$, in this case $d \geq \alpha y_h + y_b$. The first order condition is now solved for

$$g_b + \frac{Z(1 - \beta)}{r + \lambda} = Z(1 - \beta) (C_1(m_1 - 1)(\alpha y_h + y_b)^{-m_1} d^{m_1}).$$

Since the right-hand side is always positive, the first order condition is solved only if $g_b + \frac{Z(1-\beta)}{r+\lambda} > 0$ and the solution is

$$y_b^* = d \left(\frac{Z(1 - \beta)C_1(m_1 - 1)}{g_b + \frac{Z(1-\beta)}{r+\lambda}} \right)^{1/m_1} - \alpha y_h = d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1} - \alpha y_h. \quad (59)$$

If $y_b^* \in [0, \theta_b]$, then it is the maximum. If $y_b^* > \theta_b$, then we select θ_b as the optimal point; if instead $y_b^* < 0$, we select $y_b^* = 0$. If, moreover, $g_b < 0$, then $\beta^b < 1$. Furthermore, if it is verified that $ZB_1(m_2 - 1) < g_b$, then $\beta^b > 0$. Summarizing,

$$y_b^* = \max \left\{ \min \left\{ d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1} - \alpha y_h, \theta_b \right\}, 0 \right\}. \quad (60)$$

In the case $g_b + \frac{Z(1-\beta)}{r+\lambda} \leq 0$, the derivative is always negative and the maximum is attained at $y_b^* = 0$. $\square$

Proof of Proposition 4.9 We start by proving that the equilibria presented in (30),(31),(32), (33), (34), (35), (36), (37), (39) and (40) are NE. In all these cases the community is formed if the BP investment is positive. Moreover, notice that, in all the cases in equations (30),(31),(32), (33), (34), (35), (36) and (37), one of the two players (b or h) has an optimal installation level which does not depend on the other player's decision, as either its net unit profit $g_{b,h}$ is strictly positive, thus the corresponding optimal strategy $y_{b,h}^* = \theta_{b,h}$, or

its marginal cost is too high to justify installing anything even with incentives, thus $y_{b,h}^* = 0$. In all these cases, the presented NE follow quite easily.

We now pass to the only nontrivial case, i.e. when $g_h + \frac{\alpha Z \beta}{r+\lambda} > 0 > g_h$ and $g_b + \frac{Z(1-\beta)}{r+\lambda} > 0 > g_b$. First of all, in this case we have $\beta_h > 0$ and $1 - \beta_b > 0$, thus one can easily verify from Equation (38) that $\beta_n > \beta_h$ if and only if $\beta_h < \beta_b$, and $\beta_n < \beta_b$ if and only if $\beta_h < \beta_b$, hence β_n must lie in the interval with extrema β_b and β_h . Moreover, we see that the value of β_n comes from the computation of the best responses of the household and BP. From Propositions 4.3 and 4.6 we know that, if $\beta_h < \beta_b$, if we take any $\beta \in (\beta_h, \beta_b)$, the best responses are

$$y_h^* = \max \left\{ \min \left\{ \frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - y_b \right), \theta_h \right\}, 0 \right\}, \quad (61)$$

$$y_b^* = \max \left\{ \min \left\{ d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2} - \alpha y_h, \theta_b \right\}, 0 \right\}. \quad (62)$$

Assume now that we have an internal NE, i.e.

$$y_h^* = \frac{1}{\alpha} \left(d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} - y_b^* \right), \quad (63)$$

$$y_b^* = d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2} - \alpha y_h^*. \quad (64)$$

The system given by Equations (63) and (64) has solutions if and only if

$$\alpha y_h^* + y_b^* = d \left(\frac{\beta}{\beta_h} \right)^{1/m_2} = d \left(\frac{1-\beta}{1-\beta_b} \right)^{1/m_2},$$

which, after some algebra, is true if and only if $\beta = \beta_n$ as defined in Equation (38). Conversely, in the case $\beta_b < \beta_h$, the same argument brings us to

$$\alpha y_h^* + y_b^* = d \left(\frac{\frac{g_h}{\beta_h} + \frac{\alpha Z}{r+\lambda}}{\frac{g_b}{\beta} + \frac{\alpha Z}{r+\lambda}} \right)^{1/m_1} = d \left(\frac{\frac{g_b}{1-\beta_b} + \frac{Z}{r+\lambda}}{\frac{g_b}{1-\beta} + \frac{Z}{r+\lambda}} \right)^{1/m_1},$$

However, by Remark 4.8, we have $\frac{g_h}{\alpha \beta_h} = \frac{g_b}{1-\beta_b}$, which after some algebra brings, as in the previous case, a solution if and only if $\beta = \beta_n$ as defined in Equation (38).

To fix the ideas, suppose now $\beta_n \in (\beta_h, \beta_b)$ (the same arguments can be applied for $\beta_n \in (\beta_b, \beta_h)$). Assume also for simplicity that $d \left(\frac{1-\beta_n}{1-\beta_b} \right)^{1/m_2}$ is not greater than both θ_b and θ_h . In this case we check if any strategy of the form

$$\begin{aligned} \alpha y_h^* &= \delta d \left(\frac{1-\beta_n}{1-\beta_b} \right)^{1/m_2}, \\ y_b^* &= (1-\delta) d \left(\frac{1-\beta_n}{1-\beta_b} \right)^{1/m_2}, \end{aligned}$$

with $\delta \in (0, 1)$, is a NE by checking Equation (20).

For the household profit we have

$$J_h(x_v, x_c, p, d, y_h, y_b^*, \beta) = y_h g_h - \frac{x_c d}{r_{cd}} + \beta_n Z w(y_h, y_b^*, d) \quad (65)$$

which we can maximize in y_h , finding that

$$\begin{aligned} \arg \max_{y_h \in [0, \theta_h]} J_h(x_v, x_c, p, d, y_h, y_b^*) \\ = \frac{\delta d}{\alpha} \left(\frac{1 - \beta_n}{1 - \beta_b} \right)^{1/m_2}. \end{aligned} \quad (66)$$

On the other hand, for the BP profit we have

$$J_b(x_v, x_c, p, d, y_h^*, y_b) = y_b g_b + \frac{pbK_g}{r_p} + (1 - \beta_n)Zw(y_h^*, y_b, d)$$

which we can maximize in y_b by using Proposition 4.6, finding that

$$\arg \max_{y_b \in [0, \theta_b]} J_b(x_v, x_c, p, d, y_h^*, y_b) = (1 - \delta)d \left(\frac{1 - \beta_n}{1 - \beta_b} \right)^{1/m_2}. \quad (67)$$

By (66) and (67) the strategies (65) satisfy Equation (20) and therefore any linear combination $\delta \in (0, 1)$ is a NE.

In all the other possible cases in (61), the system is solvable for any $\beta \in (0, 1)$ and the value for the optimum β is computed by solving the coordinator's problem (44). $\square$

B Parameter estimation for seasonalities

(Table 6).

Table 6 Significant frequencies and parameter estimation in Equation (47) for the three observed time series (all estimations are statistically significant at the 99.9% level)

Demand D			Electricity price X_v			Gas price P		
Freq. (1/h)	Amplitude		Freq. (1/h)	Amplitude		Freq. (1/d)	Amplitude	
f_k	A_k	B_k	f_k	A_k	B_k	f_k	A_k	B_k
0.416667	0.02667	0	0.00004	-0.010513	0.033065	0.0003817	-0.36363	0.46082
0.0833333	0	0.03726	0.0001167	-0.115432	0.148645	0.0007634	0	-0.56797
0.1666667	0.12217	0.05014	0.0059508	0.011536	-0.093213	0.001145	0.16201	-0.40630
0.25	-0.04156	0.05358	0.0833333	-0.14971	0.025015			
0.33333	0.02983	0	0.0119016	0.017194	-0.045613			
			0.0416667	-0.096227	0.022029			

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