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Scenario Analysis for Optimization of Pumping Schedules in Complex Water Supply Systems Considering a Cost-Risk Balancing Problem

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Abstract

Optimization of pumping schedules in complex supply systems is considered when defining activation of emergency and costly water transfers under drought risk. An optimization procedure has been developed based on scenario analysis. The model allows identification of the optimal decision rules by balancing the risk of water shortages under different hydrological scenarios and the cost of pumping stations operating and maintenance. Scenario analysis optimization provide the resource management authority with information defining optimal activation thresholds for pumping stations assuring the water demand level fulfilment for users and activities (irrigational, civil, industrial, etc). The model application has been developed for optimization policy and energy conservation in shortage condition in the South-Sardinia (Italy) water supply system. The obtained results define a cost-risk trade-off considering water shortage probability and minimizing operative costs in shortage conditions.

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Keywords: Scenario Analysis optimization; Pumping schedules; Cost-risk balance.

1. Introduction

Treating the effectiveness of emergency and costly water transfers alleviating droughts, optimization of pumping schedules stress the system managers to the need of a robust approach defining activation rules. As recently recognized [1, 2] a powerful and intuitive way to incorporate the uncertainties in the management in complex supply systems is to use scenario analysis. Scenarios can be considered as alternative views of how the future will develop. The

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traditional planning framework is generally a single-future deterministic approach. To avoid the risk of system failure under conditions that can significantly change over time, a multiple scenario approach has proven to have advantages for making robust management decisions. Considering the inner structure of scenarios temporal evolution, it is possible to obtain a “robust” decision policy, minimizing the risk of wrong future decisions. Using a scenario-tree-based optimization model, a multistage decision making approach has been developed in previous papers [3, 4] to determine robust management of water resources systems, under uncertainty conditions.

In this paper, we use scenario analysis to identify the optimal water supply management using activation of emergency pumping stations in complex multi-reservoir systems. Multi-period scenario analysis has been developed in order to balance Operating and Management (O&M) costs and the risks that are originated from the possibility of water scarcity under possibility of critical hydrologic scenarios occurrences. The scenario analysis has been developed using mixed integer programming optimization and quadratic formulation of the objective function. Considering a multi-reservoir complex supply system, the main aim of this study is to provide the management authority with optimal reservoirs threshold levels for pumping stations activation, assuring an adequate water demand level fulfillment for multi-users’ activities (irrigational, civil, industrial etc.). The concept of “adequate level” of fulfillment must be defined considering both the risk of shortage occurrences, and cost of early warning and the consequently activation of emergency and costly water transfers. In the case of a “better scenario” occurrence (i.e.: in terms of higher hydrologic input to reservoirs) O&M regret costs should be made as a consequence that future scenarios should not require pumping the water.

2. Scenario analysis approach

Scenario analysis approach arises from considering that the future events can evolve with a set of different and statistically independent scenarios. The single scenario describes a possible realization of some sets of uncertain dates in the examined time-horizon. In a first step it is possible to work using a configuration with parallel scenarios, followed by a simulation aimed to confirm the reliability of each solution. Every solution is composed by a set of decision sequences that should be taken on each scenario and it is independent from those concerning the others scenarios [5, 6]. In a second step, a model that examines contextually a set of different scenarios can be elaborated. It responds to the possible temporal evolutions of some crucial dates, providing a “barycentric solution” to the multi-period decision problem. To consider the multi-period problem, the scenario-tree must be built in order to aggregate all scenarios in one model. This approach can be represented as a “tree-graph” according to appropriate *aggregation rules*. The formulation of these aggregation rules [1, 5] guarantee that the solution in any given period is independent of the information not yet available, in other words they can be considered as non-anticipative decision constraints. Two scenarios sharing a common initial portion of data must be considered together and partially aggregated with the same decision variables for the aggregated part in order to take into account the two possible evolutions in the subsequent diverse parts. In this way, the set of parallel scenarios is aggregated by producing the tree-structure. The root node of the tree corresponds to the beginning of time period $t = 1$. From this node, n_0 scenarios start and continue in parallel for T_1 periods. At $t = T_1$ each of n_0 scenarios is split into n_1 scenarios and all the obtained scenarios $n_0 n_1$ continue in parallel for a further T_1 periods until $t = 2T_1$, when each of them is split into n_2 additional scenarios.

An example of such a tree with $T_1 = 12$, $T = 36$, $K = 3$, $n_0 = 1$, $n_i = 3$, and $i = 1 : 2$ is shown in Fig. 1. In planning problems considering multi-reservoir supply system, one time period could corresponds to one month, and we take $T_1 = 12$, which corresponds to one year. This means that splitting occurs at the end of each year, which conforms to the common seasonal patterns of inflows and demands.

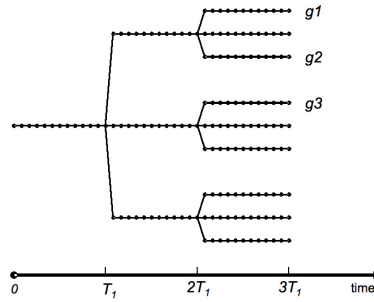


Fig.1. Scenario tree

A weight can be assigned to each scenario to characterize its relative importance. The weights could represent the probability of occurrence of each scenario, if this probability can be estimated by stochastic techniques or statistical tests based on historical data. The aggregated multistage stochastic programming model can be expressed as the collection of one deterministic model for each scenario $g \in G$ plus a set of congruity constraints representing the requirement that the subsets of decision variables corresponding to the indistinguishable part in each scenario must be equal among themselves [1, 5]:

$$\begin{aligned}
 \min \quad & \sum_{g \in G} p_g c_g^T x_g \\
 \text{s.t.} \quad & A_g x_g = b_g, \quad \forall g \in G \\
 & l_g \leq x_g \leq u_g, \quad \forall g \in G \\
 & x^* \in S
 \end{aligned} \tag{1}$$

where x_g represents the vector of decision variables in scenario g ; the vector c_g describes the unit cost of different activities like delivery cost, opportunity cost related to unsatisfied demand, opportunity cost of spilled water, and so on. The objective function is defined as the average of the cost objectives of all scenarios weighted with their probabilities p_g . The set of standardized equality constraints describes the relationships between storage, usage, spill, and exchange of water at different nodes and in subsequent time periods. The right hand sides b_g are formed from scenario data of inflows and demands. The lower and upper bounds l_g and u_g are defined by structural and policy constraints on the functioning of the system. All decision variables and data are scenario dependent, hence the index g . All constraints in equation (1) are collected from all scenarios and put in the aggregated model. The additional set of non-anticipative constraints $x^* \in S$ represents the congruity constraints derived by aggregation rules.

3. Cost – Risk balancing problem

The cost minimization point of view in the scenario optimization model developed in the previous section may not be sufficient and it should be enhanced by considering the associated risk of resource failure evaluations. For this reason, management optimization models should include the balance between costs and risks to which the end users are exposed [3, 4]. We assume that the resource in question is scarce and for this reason the demand could not be satisfied in many scenarios. In such scarcity situations, managers should develop an emergency policy to alleviate the effect of shortages. In order to do this a user should know in advance the reduced target level of demand satisfaction that the system manager is willing to deliver to him and that is independent of possible scenarios of uncertainty. Usually this new target level x_b will be less than the user’s demand due to inherent scarcity of the resource. The difference between original demands and reduced target x_b will represent the planned shortages which the user is asked to accept under drought conditions. Besides this planned shortage there can also be unplanned shortages when,

due to severe lack of resources under some scenarios; in this case supply is less than reduced target. In order to face this situation and develop an appropriate emergency policy, the proposed methodology tries to reach a “robust” decision policy, which can minimize the risk to take incorrect and harmful future decisions. The new target values x^b have to be barycentric in relation to future scenarios. With this kind of analysis we can reduce the deficit levels in the supply system, introducing some management preventive measures (such as decreasing the water distributed to the users or activating emergency transfers). In our models we should highlight the importance of flows between decision variables. Consequently, in the following model formulation the vector x_g will represent resource delivery in the multi-period graph under scenario g ; moreover, under scarcity scenarios we have to calculate the new demand target x^b , which has to be barycentric in relation to actual demand delivery x^g . The objective function tries to minimize the weighted distance of the flow values x^g related to the barycentric value x^b namely for each scenario g and period t .

The cost-risk balancing problem can be formulated modifying the objective function model (1) in a form containing both the risk and cost terms, as in [3, 4]:

$$\min (1-\lambda) \sum_{g \in G} p_g c_g(x_g) + \lambda \sum_{g \in G} p_g [w_g (\hat{x}_g - x^b)^2] \quad (2)$$

Here c_g is the cost associated with flow values x_g in pumping stations under scenario $g \in G$; $(\hat{x}_g - x^b)^2$ is the Euclidean measure of distance between the target delivery and actual delivery to demands and w_g is the cost of related unplanned deficits.

The first item of the objective function is a “cost function” and it tries to look for the system flows configuration that allows minimizing the costs supported by water system’s manager. The quadratic second item is the “risk function”, it could be considered like a non-linear “social function” in order to guarantee users’ major priority fulfillments, referring to future scenarios. In this way, giving a weight to both functions, we can find the solution of a cost-risk balancing problem. The relationship between cost function and risk function is regulated by the parameter λ called “weight factor” [3]. This parameter can vary between 0 and 1, where $\lambda = 0$ corresponds to the pure cost minimization problem, and for $\lambda = 1$ the problem becomes one of risk minimization. Intermediate values of λ provide different tradeoffs between costs and risks. We should build the efficient frontier in the space of risk-cost by solving our problem for different values of $0 < \lambda < 1$. As result of cost-risk-balancing process, it will be obtained the barycentric value x^b , which will be used like a new target value for a following deterministic modeling phase called re-optimization. The aim of the re-optimization phase is to evaluate the sensitivity of the system compared to the adopted solutions. Through this phase we can test the robustness of the system and plan a part of the decision problem considering the risk of wrong decisions caused by wrong assumptions due to adopted parameters

4. Optimization of Pumping Schedules using the multi-period network

Pump stations allow to supply some demand centers in critical scenarios with an increasing O&M economic burden that must be evaluated and accepted by system manager. The aim of this paper is to define the optimal pumping stations activation thresholds, mainly using reservoirs’ storage volumes like trigger values. Therefore, we have to define these values by a cost-risk-balancing process considering pumping energy and management costs (cost element) and water deficits for users (risk element) minimization. Moreover, in the case of a “better scenario” occurrence (i.e.: in terms of higher hydrologic input to reservoirs) a risk element could be given by the O&M pumping regret costs the manager should made in the past, as a consequence that future scenarios should not require pumping the water. The pump activation can be done on the single scenario and on different scenarios together, in this way we can define the barycentric activation value. In our multi-reservoirs systems, thresholds levels are mainly referred to reservoir’ storages that are used as supplies by demands centers. Thus, in order to optimize the activation rules, we must define a critical stored volume in reservoirs that may supply by gravity the downstream demand centers. To compose the multi-period model, preliminary we need to define the multi-period network starting from a basic graph given by a single-period, static draft of the system. The multi-period network construction needs to repeat it for every time-horizon period using multi-period rules, as defined in [8]. In this way it is possible to describe the complete behavior of the system in the whole extension of the time horizon. Schematically, in this multi-period representation, nodes could indicate water sources, demands, reservoirs situation in each period, while arcs represent the activities’

connections between them. Arcs are divided in two types: physical arcs describing spatial flow transfers and inter-period arcs describing in-time flow transfers. The multi-period graph is completed by a dummy node, which can represent an “external system” acting as a source or as a demand center in order to satisfy the general mass balance equation of the system in the multi-period horizon [8].

As previously defined, activation of a pumping stations are supposed to be dependent by the stored volume levels in reservoirs that supplies by gravity the downstream demand nodes. Therefore, to model pumps activation we insert a binary variable h_i to each i -th pump-station. This variable represents the on/off condition for the pump station as it can assume one or zero values. In the optimization model h_i is dependent by the stored level x_{vj} in the j -th reservoir. Moreover, it is necessary to introduce a new cost term in the objective function (2) multiplying h_i by an activation penalization coefficient CM_i that can be recognized as a fixed cost-management activation of the station. Nevertheless, the correctly balanced evaluation of these CM_i penalization coefficients is a complex task: it has been developed during a preliminary parametric analysis, considering only few sample scenarios. During this analysis, the penalization coefficient values (one for each pump station) are retrieved by model optimization as they are considered like variables to be optimized. Therefore, the objective function of the multi-period problem, considering activation of pumps, should be completed in the following form:

$$\min (1-\lambda) \sum_{g \in G} p_g c_g(x_g) + \lambda \sum_{g \in G} p_g [w_g (\hat{x}_g - x^b)^2] + \sum_{i \in P} h_i CM_i \quad (3)$$

Moreover, in the model it is necessary to introduce three new type of constrains in order to guarantee a correct operation of pumping stations:

$$\begin{aligned} a) \quad & xv_j - S_j < h_i BM \\ b) \quad & x_i = (1 - h_i) P_i \\ c) \quad & S_j < K_j \end{aligned} \quad (4)$$

The first type of constraint allows the i -th pump station activation if the stored volume in reservoir j is under the threshold value S_j . In this constrain the parameter BM (Big M) is a large scalar.

The second constraint guarantees that, in case of i -th pump station activation, the flow along the pumping arc starting from station i will be equal to its capacity P_i . The third constraint imposes an upper bound to the activation storage level S_j equal to the reservoir capacity K_j . To complete the multi-period model with these constraints it is necessary to guarantee the correct working of pumping stations. In general, the optimization process has been developed in the following main steps:

1. Parametric analysis on single scenario to define CM_i $i=1, NP$;
2. Single scenario optimization to define starting values of reservoir threshold levels S_j ;
3. Multi-scenario-tree optimization and barycentric values definition of threshold levels S_j ;
4. Re-optimization.

The optimization of pumping schedules using multi-period scenario analysis was implemented using the software GAMS [9], acronym of General Algebraic Modeling System. GAMS allows to combine the high efficacy in writing the optimization models with flexibility of data management, variables and constrains definition. Furthermore, GAMS can be interfaced with efficient solvers and others software in order to have an easier input/output data management.

5. Application to Southern-Sardinia water supply system

5.1. System description

To verify the potentiality of the proposed approach, we considered a schematization of the Southern-Sardinia (Italy)

water supply system, which has been schematized in the Fig. 2. Water resources are mainly given by five artificial reservoirs, reported in Tab. 1. Water demands have been grouped in six centers according to three different users: civil, irrigation and industrial. Annual volume of demands and deficit costs are reported in Tab. 2. As reported in the Sardinia-Region Water Plan, hydrological inflows in the reservoirs have been reconstructed from 1922 to 1992. Moreover, a study as been made in order to evaluate influences on historical data of climate changes occurred in last decades, particularly in the period following the end of the '80. On this basis, the application has been organized considering 4 hydrological scenarios, which are composed with a common root of 10 years and the following data diversified in 10 years scenarios.

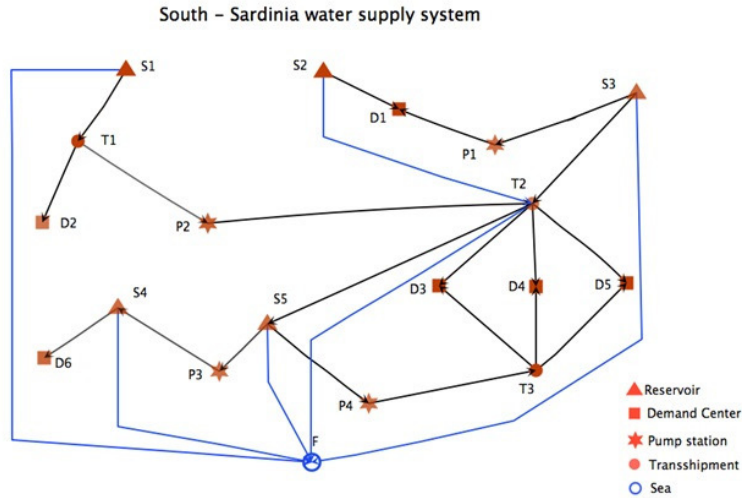


Fig. 2. Southern-Sardinia supply system

Table 1 Reservoirs features

Code	Reservoir Name	Capacity [106m3]
S1	Cantoniera	450
S2	Is Barrocos	12.24
S3	Flumendosa - Mulargia	623
S4	Bau Pressiu	8.25
S5	Cixerri	24

Pump stations allow supplying demand centers with an economic burden increased, namely incurring pumping costs in addition to the ordinary management costs. The sketch in Fig. 2 simplify the real configuration complexity of the system, but it can adequately correspond to reality analyzing flow configuration for values driven in pumping stations.

Thresholds levels for pumps activation are referred to stored volume in reservoirs that supply by gravity the downstream demand node. This could be related in some cases to a single reservoir or to 2 of them; in the second case the threshold value derives from the volumes in both reservoirs. Thus, in order to manage the lifting rule, Table 4 summarizes the functional dependences.

Table 2 Demand centers requirements and deficit costs

Code	Demand centres	Demand [106m3/year]	Deficit cost [€/103m3]
D1	Civil Sarcidano	11	250

D2	Irrigation Oristano	118	60
D3	Industrial Campidano	17	2560
D4	Irrigation Campidano	81	60
D5	Civil Campidano	90	250
D6	Civil Bau Pressiu	9	250

Table 3 Pump stations features

Code	Pump station name	Capacity [Mm ³ /month]	Pumping cost [€/m ³]
P ₁	Sarcidano	0.7	0.193
P ₂	Tirso - Campidano	5.2	0.2056
P ₃	Cixerri - Bau Pressiu	2.1	0.218
P ₄	Cixerri - Campidano	10.4	0.078

Table 4 Pump stations activation dependences

Pump station	Reservoir				
	S ₁	S ₂	S ₃	S ₄	S ₅
P1	0	1	0	0	0
P2	0	1	1	0	0
P3	0	0	0	1	0
P4	0	1	1	0	0

Table 5 Pump stations activation thresholds

Activation Threshold [Mm ³]	S1	S2	S3	S4
	1.624	35.451	5.07	37.558

6. Results and Conclusions

The barycentric values for activation thresholds has been obtained by solving the model (1) completed with (3) and (4) and using $\lambda = 0.5$, thus considering a equal balance between cost and risk elements. Obtained values of stored water activation thresholds in reservoirs are reported in Tab. 5. In the re-optimization phase the model considers these values like fixed-parameters inputs. As an example, the following Fig. 3 describes the operation of pump station P1 in 4 different hydrological scenarios. The blue line represents the storage volume, deduced the threshold value, while the red line represents the pumped volumes. The final result (after re-optimization phase) highlights only some small no-planned deficits, particularly in demand D4 ; in the second scenario a deficit of 0.0035 Mm³ in only one period of time-horizon was obtained. To calculate the real economic response of the system, an economic post-processor has been constructed considering only the real costs, namely the un-planned deficits and pumping costs. The annual pumping stations annual average costs [103 €/year], in the 4 considered scenarios, are the following: P1 = 245.00 ; P2 = 182.00 ; P3 = 1403.33 ; P4 = 83.20.

Scenario analysis optimization confirms its potentiality also considering this problem while reaching barycentric values among the 4 analyzed hydrological scenarios. The cost-risk balancing has contextually restricted deficit risks for the users and minimized the costs of the system in shortage conditions. This method guarantees almost the complete fulfillment of the water demand, through unplanned deficits minimization and rationalizing energy costs.

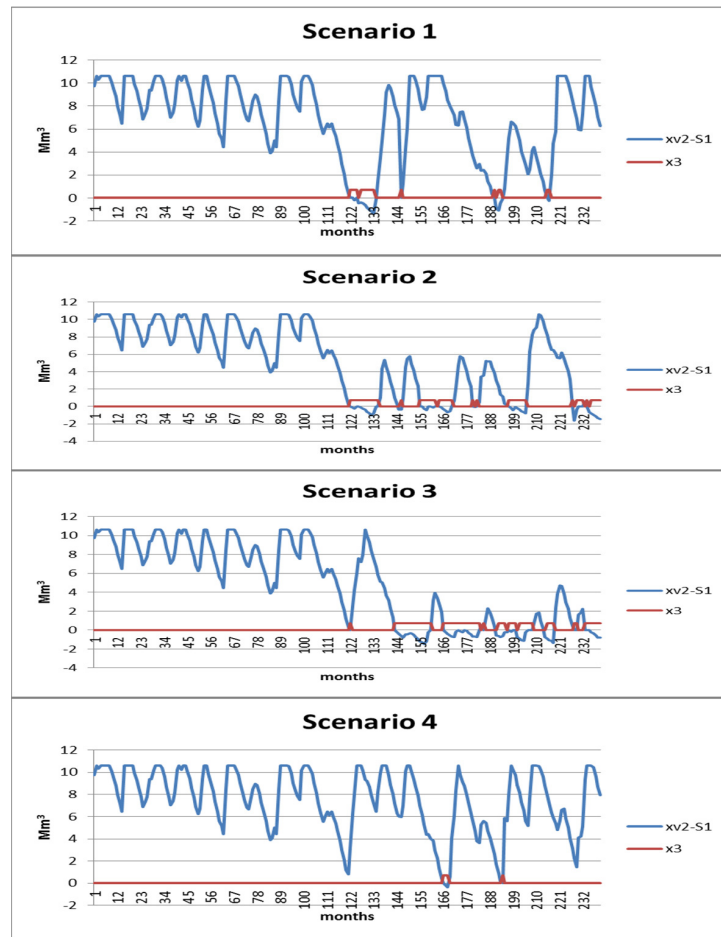


Fig. 3 Pumped volumes by P1 and storage volumes [Mm³] (minus threshold value S1)

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