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## A mathematical model of a solar collector augmented by a flat plate above reflector: Optimum inclination of collector and reflector

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### Abstract

In this study a theoretical analysis of a collector augmented by a bottom booster reflector is presented. An analytical model has been developed and used to estimate the solar irradiation passing through the transparent cover of a flat collector, both with and without a bottom reflector. The analytical model is based on the anisotropic sky model and takes into account a finite length system with different angular configurations and reciprocal shading and reflections between reflector and collector. Computer simulations have been carried out in order to find the optimum angles of the reflector with respect to the plane of the collector. Optimal inclinations of the collector and reflector for each month at 39° N latitude have been identified.

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### 1. Introduction

The simplest and most inexpensive means for increasing the solar energy flux incident on an absorbing surface is to attach to it one or more planar reflectors to the main harvester system the paper, Concentration devices can produce elevated operating temperature under clear sky conditions, but require good optical components, more precise construction techniques and generally a mechanism for tracking the sun. A reflector augmented flat-plate collector

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is, however, the best solution at utilizing both diffuse and beam (direct) radiation, while providing a moderate concentration with minimal tracking.

Many studies have been done to investigate the effect of booster reflectors on solar thermal collectors [1–13]. In particular Chiam [3], Garg and Hrishikesan [6] and Kostic et al. [12, 13] studied a solar thermal collector with top and/or bottom reflectors.

The top reflector extends from the upper edge of the collector and is inclined slightly from vertical, while the bottom reflector extends from the lower edge of the collector and is inclined slightly from horizontal.

#### Nomenclature

$A_c$  collector surface [m<sup>2</sup>];  $A_r$  reflector surface [m<sup>2</sup>];  $W_c$  collector width [m];  $W_r$  reflector width [m];  
 $l_c$  collector length [m];  $l_r$  reflector length [m];  $L_r$  reflector overhang [m];  
 $I_b$  beam hourly solar radiation component directly from the sun on a horizontal surface [kWh/m<sup>2</sup>]  
 $I_d$  total diffuse hourly radiation component from the sky on a horizontal surface [kWh/m<sup>2</sup>]  
 $I_{d,iso}$  hourly isotropic portion of the diffuse radiation on a horizontal surface [kWh/m<sup>2</sup>]  
 $I_{d,cs}$  hourly circumsolar portion of the diffuse radiation on a horizontal surface [kWh/m<sup>2</sup>]  
 $I$  total hourly radiation on a horizontal surface [kWh/m<sup>2</sup>];  $I_T$  total hourly radiation on a sloped surface [kWh/m<sup>2</sup>]  
 $I_0$  extraterrestrial hourly irradiation on a horizontal surface [kWh/m<sup>2</sup>]  
 $I_{b,\perp}$  hourly beam radiation perpendicular to both the collector and the reflector [kWh/m<sup>2</sup>];  
 $I_{b,\parallel}$  hourly beam radiation parallel to both the collector and the reflector plane [kWh/m<sup>2</sup>];  
 $G_{sc}$  solar constant 1367 [W/m<sup>2</sup>]  
 $A_i$  anisotropy index of the circumsolar diffuse radiation  
 $AC_{ns}$  ratio of the portion of the collector area not shadowed by the reflector, respect to the area of the collector surface;  
 $A_{r \rightarrow c}^{\alpha_r}$  reflected illuminated projection of the inclined reflector surface onto a tilted surface like the collector surface;  
 $F_{c \rightarrow s}$  view factor of the sky respect to the collector;  $F_{c \rightarrow g}$  view factor of the ground respect to the collector;  
 $F_{c \rightarrow r}$  view factor of the reflector respect to the collector;  $F_{r \rightarrow s}$  view factor of the sky respect to the reflector  
 $F_{r \rightarrow g}$  view factor of the ground respect to the reflector;  
 $n_1, n_2$  refractive indexes respectively of the glass and atmospheric air;  $n$  nth day of the year;  
 $K$  extinction factor of the transparent cover of the collector [m<sup>-1</sup>];  
 $l$  tick of the transparent cover of the collector [m];  
 $\alpha_{\perp}$  apparent solar altitude or profile angle;  $\alpha_s$  solar altitude;  $\beta_c$  collector tilt angle;  $\beta_r$  reflector tilt angle;  
 $\delta$  solar declination angle;  $\theta_z$  zenith angle;  $\theta_1$  angle of incidence;  $\theta_2$  angle of refraction;  $\rho_r$  reflector reflectance;  
 $\rho_g$  ground reflectance;  $\tau_b$  collector cover transmittance for beam radiation;  $\tau_d$  collector cover transmittance for diffuse radiation;  $\phi$  terrestrial latitude;  $\omega$  hour angle.

Garg and Hrishikesan [6] reported the optimum inclination of both the top and bottom reflectors on March, June and December when the collector inclination is horizontal or equal to the latitude where the collector is located. Kostic et al. [12] reported the optimum inclination of both the top and bottom reflectors throughout the year when the collector inclination is fixed at 45° N. Mc. Daniels and Lowndes [1], Rao et al. [7], Bollentin and Wilk [8] and Hussein et al. [9] studied the effect of the bottom reflector on the solar collector for Albany site. McDaniels and Lowndes [1] reported the effect of the inclination of both the reflector and the collector on solar radiation absorbed by the collector in winter.

For a conventional system, which is constituted by an absorbent surface with no reflector, the optimal angle can be easily determined and the angle varies depending on the season (or month) and on the period during which one wishes to optimize (maximize) the amount of energy on a collector surface.

With the reflector alone it is possible to obtain a maximum value of the uptaken energy on an year-round basis, either by changing the tilt angle of the collector at least 12 times through the year (in relation to the month), or by placing the collector in a certain fixed angular orientation as a function of the latitude alone.

Whilst results presented in the literature are in general agreement about beneficial effects of the reflector, it is not

trivial to find out which is the optimum configuration within the range analyzed in terms of latitude, angle positions and lengths of the absorber and reflector area. When the collector is built with a reflection device, the optimal angle for each month may differ for that of the conventional system. Also, the optimal angle of the reflector varies on a seasonal (or monthly) basis. So the optimum inclination of the collector as well as the optimum inclination of the booster reflector should be determined by considering the combination of these two inclinations to maximize the solar radiation incident onto the solar thermal collector. Numerical simulations have shown that, unlike the system consisting only of the collector, a stationary collector coupled with a stationary reflector cannot provide useful enhancements of solar beam energy on a year round basis, because of a wide separation of the optimum reflector angles.

In this paper we present a modeling study, preparatory to an experimental investigation of PV(photo-voltaic) and/or thermal collectors with the aim of predicting an enhancement of the radiation harvested by a solar collector on a year around basis operation, due to matching with the reflective surface, mounted at the foot of the collector device. In this regards we have performed a theoretical analysis on a tilted collector and reflector system in order to determine the pairs of optimal angles of collector and reflector for each month throughout a year at 39°N latitude (South Sardinia), and furthermore considering a stationary tilt collector coupled with an adjustable tilt reflector system to derive the optimal fixed angular position of the collector and the set of the optimal angular positions of the reflector for each month, which could maximize the solar energy received by the collector on a year round basis.

**2. The model**

The flat concentrator system to which we will refer, to quantify the amount of radiation incident on the collector absorber, is shown in Figure 1. The solar collector consisting of a single glass cover, an absorbing plate, and the flat plate reflector, with identical overhangs on both sides (equal to 0.5\**l<sub>c</sub>*), is assumed to be made of highly reflective materials. Beam and diffuse solar radiation coming directly from the sun, as well as the reflected solar radiation from the reflector and from the ground, are transmitted through the glass cover and therefore available onto the absorbing plate.

The model considers the anisotropy diffuse sky model formulated by Hay and Davis [14] and includes components of beam and diffuse irradiation, directly from the sun and from the sky dome respectively, and beam and diffuse irradiation reflected from the reflector and from the ground. The shading profiles of the three components of irradiation due to the reciprocal position of each surface, are taken into account too. A reflector below the collector south facing configuration is considered and both are considered to be of finite length and width, each with variable dimensions. Monthly average daily insolation data from the 10349 UNI Standard [15] are used as source of solar irradiation data. The total hourly incident solar irradiation on the tilted surface of the collector, *I<sub>T</sub>*, transmitted through the glass cover, it is the sum of a set of 6 irradiation fluxes involving the beam radiation and the two components of diffuse radiation, coming from the isotropic and circumsolar part of the sky, both those directly from the sun and those reflected from the surfaces seen by the tilted surface, assuming that the collector is able to see only the reflector, the sky and the ground surfaces. All optical proprieties, reflectance and transmittance, will be assumed to be independent of wavelength.

$$I_T = (I_b + I_{d,cs})\tau_b \left( R_{bc}AC_{ns} + R_{br} \frac{A_r}{A} \frac{\alpha_r}{\alpha_{rc}} \frac{A_r \alpha_r \cap A_c}{A_c} Q_r \right) + I_{d,iso}\tau_d (F_{c \rightarrow s} + F_{r \rightarrow s} Q_r F_{c \rightarrow r})\tau_d + I\tau_d Q_g F_{c \rightarrow g} + I\tau_d Q_g F_{r \rightarrow g} Q_r F_{c \rightarrow r} \tag{1}$$

$$I_T = (I_b + A_i I_d)\tau_b R_{bc}AC_{ns} + (I_b + A_i I_d)\tau_b R_{br} \frac{A_r}{A} \frac{\alpha_r}{\alpha_{rc}} \frac{A_r \alpha_r \cap A_c}{A_c} Q_r + (1 - A_i)I_d\tau_d F_{c \rightarrow s} + (1 - A_i)I_d\tau_d F_{r \rightarrow s} Q_r F_{c \rightarrow r} + I\tau_d Q_g F_{c \rightarrow g} + I\tau_d Q_g F_{r \rightarrow g} Q_r F_{c \rightarrow r} \tag{2}$$

Where *I<sub>b</sub>* is the beam hourly radiation from the sun on a horizontal surface, *I<sub>d,iso</sub>* and *I<sub>d,cs</sub>* are respectively the isotropic and the circumsolar hourly parts of the diffuse radiation component on a horizontal surface so that the total diffuse radiation component on a horizontal surface can, can be expressed as *I<sub>d</sub>* = *I<sub>d,iso</sub>* + *I<sub>d,cs</sub>* having neglected the horizon brightening diffuse radiation component, according to Hay and Davis anisotropic sky model [14].

The  $I_{d,cs}$  in turn can be expressed as:

$I_{d,cs} = A_i I_d$ , where  $A_i$  is the anisotropy index, which is a function of the transmittance of the atmosphere for beam radiation and then  $A_i = \frac{I_b}{I_0}$ , where  $I_0$  is the extraterrestrial hourly radiation on a horizontal surface and can be expressed as:

$$I_0 = \frac{12 \times 3600}{\pi} G_{sc} \left( 1 + 0.033 \frac{360-n}{365} \right) \times \left[ \cos \phi \cos \delta (\sin \omega - \sin(\omega - 15)) + \frac{\pi \cdot 15}{180} \sin \phi \sin \delta \right] \quad (3)$$

where  $G_{sc}$  is the solar constant (1367 W/m<sup>2</sup>) and  $\omega$  represents the hour angle.  $I$  is the total hourly radiation on a horizontal plane and is given by  $I = I_b + I_d$ . We assume that once "P" hits the ground surface, the reflected portion  $I_{Qg}$  is completely diffuse  $\tau_b$  and  $\tau_d$  are respectively the transmittances for beam and diffuse radiation of the transparent cover of the collector. The former is given by the product of the transmittance of an ideal transparent glass  $\tau_r$ , neglecting absorption in the cover material, and the absorption transmittance related to Bouguer's law, here named as  $\tau_a$ .

$$\tau_r = 0.5 \left( \frac{1-r_{\parallel}}{1+r_{\parallel}} + \frac{1-r_{\perp}}{1+r_{\perp}} \right) \text{ with } r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)}, \quad r_{\perp} = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)}, \quad \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_1}{n_2}, \quad \tau_a = e^{-\frac{K\ell}{\cos \theta_2}}$$

where  $n_1, n_2$  are refractive indexes respectively of the glass and atmospheric air,  $K$  is the extinction factor, and  $\ell$  is the thickness of the glass. The latter, for a wide range of conditions encountered in solar collector applications, can be approximated to the transmittance for beam radiation  $\tau_b$  calculated for an equivalent angle equal to 60°. Therefore  $\tau_d \cong \tau_b(60^\circ)$ . The geometric factors  $R_{bc} - R_{br}$  are:

$$R_{bc/br} = \frac{\cos \theta_{bc/br}}{\cos \theta_z} \quad (4)$$

The  $AC_{ns}$  area ratio factor is the ratio of the portion of the collector area as seen directly from the sun, therefore the portion of the collector area not shadowed by the reflector, with respect to the area of the collector surface; therefore the first term on the right hand side of Eq. 2 represents the sum of the beam with the circumsolar diffuse radiation incident on the tilted surface of the collector, starting from the radiation values on the horizontal surface, directly from the sun without any reflection.

$A_{r \rightarrow c}^{\alpha_r}$  is an area given by the reflected projection of the inclined reflector surface onto a tilted plane like the collector surface.

The result of the intersection indicated in Eq. (2),  $A_{r \rightarrow c}^{\alpha_r} \cap A_c$ , yields the portion of the collector area illuminated by the beam and circumsolar radiation reflected by the reflector. Since the reflected radiation from the sloped reflector to the sloped collector would be focused or spread, according to whether  $A_r \leq A_{r \rightarrow c}^{\alpha_r}$ , the intensity of the reflected radiation from the inclined reflector available on a collector plane is proportional to  $\frac{A_r}{A_{r \rightarrow c}^{\alpha_r}}$ .

" $Q_r$ " appearing in eq.(1) and (2) is the reflector reflectance. Therefore the second term represents the sum of the beam with the circumsolar diffuse radiation that hits the collector once it is reflected by the reflector surface.

$F_{c \rightarrow s}$  is the view factor of the sky with respect to the collector, therefore the third term represents the isotropic diffuse radiation component from the sky as seen by the collector.

$F_{r \rightarrow s}$  is the view factor of the sky with respect to the reflector and  $F_{c \rightarrow r}$  is the view factor of the reflector with respect to the collector, therefore the fourth term represents the isotropic diffuse radiation from the sky as seen by the reflector and then reflected on the collector

$F_{c \rightarrow g}$  is the view factor of the ground with respect to the collector and  $\rho_g$  is the ground reflectance, therefore the fifth term represents the total radiation from the sun and the sky as seen by the ground and then reflected on the collector surface

$F_{r \rightarrow g}$  is the view factor of the ground with respect to the reflector, therefore the sixth term represents the total radiation from the sun and the sky as seen by the ground and then reflected on the reflector and finally reflected on the collector surface.

As suggested by B. Perers and B. Karlssons [16] as regards the treatment of the beam radiation we introduce the quantity named “profile angle”. The radiation with directional propriety (beam and circumsolar radiation), coming directly from the sun, can be thought as subdivided into two mutually perpendicular components. One component is parallel to a vertical plane,  $I_{b\perp}$ , perpendicular to both the collector and the reflector planes, and the other component,  $I_{b\parallel}$ , is parallel to both the collector and the reflector planes. The parallel component does not provide solar flux incident on the collector and reflector surfaces and then can be neglected. The angle,  $\alpha_{\perp}$ , between the projection of the solar rays onto the vertical plane and the horizontal plane, is named apparent solar altitude or profile angle and it can be expressed as:

$$\tan \alpha_{\perp} = \frac{\tan \alpha_s}{\cos \gamma_s} \tag{5}$$

On the basis of the sketch depicted in figure 1a the component of the beam radiation in the perpendicular plane,  $I_{b\perp}$ , is given by

$$I_{b\perp} = I_{bn} \frac{\sin \alpha_s}{\sin \alpha_{\perp}} \tag{6}$$

The component of  $I_{bn}$  parallel to both planes of the collector and the reflector surfaces,  $I_{b\parallel}$ , can be left out as it gives no contribution to the radiation in the collector plane. The introduction of the profile angle offers an advantage to subdivide the calculations of the amount of radiation on the collector into five reference cases, according to the value assumed by the profile angle during the trajectory of the sun (Figure 1b).

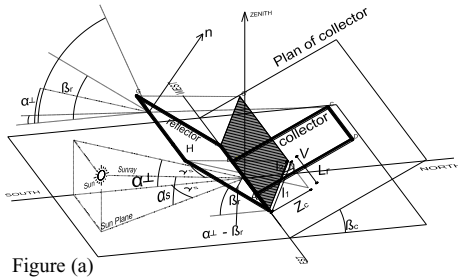


Figure (a)

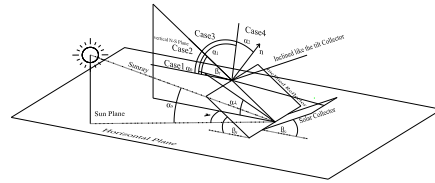


Figure (b)

Figure 1a. Representation of profile angle  $\alpha_{\perp}$  during the trajectory of the sun  $\alpha_s$  and representation of case 2. Figure 1b The cases, according to the value assumed by the profile angle

Before to introduce the possible five cases let  $\alpha_0$  be the angle of the direct beam radiation when, starting from the upper edge of the reflector, it reaches the top of the collector, with respect to the horizontal direction, as shown in figure 1b; let  $\alpha_1$  be the angle of the direct beam radiation for which specular reflected radiation gives rise to an angle equal to  $\alpha_0$  angle, and let  $\alpha_2$  be the angle of the direct beam radiation which specular reflected radiation give rise to an angle equal to the collector tilt  $\beta_c$

**Case 1**  $\alpha_{\perp} < \alpha_0$ . The profile angle is lower than the angle  $\alpha_0$ . The beam and circumsolar radiation do not reach neither the collector nor the reflector. The collector is only interested by the diffuse radiation.

$$AC_{ns} = \frac{A \alpha_r \cap A_c}{A \alpha_r \rightarrow c} = 0;$$

**Case 2**  $\alpha_{\perp} \geq \alpha_0$  but  $\alpha_{\perp} < \beta_r$ . The profile angle is greater than or equal to the angle  $\alpha_0$  but lower than the tilt reflector  $\beta_r$ . The direct beam and circumsolar radiation illuminate a portion of the collector surface, but not the

reflector surface; therefore the collector is interested by the direct radiation from the sun and the reflected beam radiation does not give any contribution.

$$AC_{ns} \in [0,1[; \frac{A_{r \rightarrow c}^{\alpha_r} \cap A_c}{A_{r \rightarrow c}^{\alpha_r}} = 0; Z_C = \frac{W_r \sin(\beta_r - \alpha_\perp)}{\sin(\alpha_\perp + \beta_c)}$$

$$\ell \sin \theta = v + L_r; \ell \cos \theta = Z_c \cos(\alpha_\perp + \beta_c) + W_r \cos(\beta_r - \alpha_\perp); \tan \theta = \frac{v + L_r}{Z_c \cos(\alpha_\perp + \beta_c) + W_r \cos(\beta_r - \alpha_\perp)};$$

$$\tan \theta = \cos \alpha_\perp \tan \gamma; v = \tan \gamma (W_r \cos \beta_r + Z_c \cos \beta_c) - L_r; l_1 \sin \eta = L_r + v; l_1 \cos \eta = Z_c \cos \beta_c$$

$$v = \tan \gamma (W_r \cos \beta_r + Z_c \cos \beta_c) - L_r; l_1 \eta = \tan \eta = \frac{Z_c \cos \beta_c}{L_r + v}; u = v \tan \eta$$

**Case 3**  $\alpha_\perp \geq \beta_m$  but  $\alpha_\perp < \alpha_1$ . The profile angle is greater than or equal to the angle  $\beta_m$  but lower than the  $\alpha_1$  angle. The reflector does not shadow the collector, not even partially, and therefore the direct beam and circumsolar radiation cover the whole collector. Moreover the reflected beam and circumsolar radiation, coming from the reflector, illuminate only a portion of the collector surface to the extent of  $A_{r \rightarrow c}^{\alpha_r} \cap A_c$ . As  $\alpha_\perp$  increases,  $A_{r \rightarrow c}^{\alpha_r} \cap A_c$  increases. In Case 3 the length of the illuminated portion of the collector due to the reflector is never equal or greater than the length of the collector  $l_c$ , and the length of the portion of the reflector surface, from which the reflected radiation comes and hits the collector, is always equal to the length of the reflector  $l_r$ , all the length of the reflector gives a contribution to increase the radiation onto collector. The reflected solar radiation on the collector coming from the reflector, related to case 3, can be treated as the direct solar radiation of case 2 with a virtual angle equal to  $\alpha_{virt} = 2\beta_r - \alpha_\perp$ , provided that the shadowed area of the virtual case 2 has to be considered as a reflected illuminated area.

$$AC_{ns} = 1; \frac{A_{r \rightarrow c}^{\alpha_r} \cap A_c}{A_{r \rightarrow c}^{\alpha_r}} \in [0,1[$$

**Case 4**  $\alpha_\perp \geq \alpha_1$  but  $\alpha_\perp < \alpha_2$ . The profile angle is greater than or equal to the angle  $\alpha_1$  but lower than the  $\alpha_2$  angle. The reflector does not shadow the collector, not even partially, and therefore the direct beam and circumsolar radiation cover the whole collector. Moreover the reflected beam and circumsolar radiation, coming from the reflector, illuminate only a portion of the collector surface to the extent of  $A_{r \rightarrow c}^{\alpha_r} \cap A_c$ . As  $\alpha_\perp$  increases,  $A_{r \rightarrow c}^{\alpha_r} \cap A_c$  decreases. The length of the illuminated portion of the collector due to the reflector is always equal to the length of the collector  $l_c$ , and the length of the portion of the reflector surface, from which the reflected radiation comes the reflected radiation and hits the collector surface, is always lower than the length of the reflector  $l_r$ , not all the length of the reflector gives a contribution to increase the radiation on the collector. The reflected solar radiation on the collector coming from the reflector, related to case 3, can be treated as the direct solar radiation of case 2 with a virtual angle equal to  $\alpha_{virt} = 2\beta_r - \alpha_\perp$  and a virtual width of reflector equal to

$$W_{r,virt} = \frac{W_r \sin(\alpha_{virt} + \beta_c)}{\sin(\alpha_\perp - \beta_r)}. \text{ The shadowed area of virtual case 2 has to be considered as a reflected illuminated area.}$$

$$AC_{ns} = 1; \frac{A_{r \rightarrow c}^{\alpha_r} \cap A_c}{A_{r \rightarrow c}^{\alpha_r}} \in [0,1[$$

**Case 5**  $\alpha_\perp \geq \alpha_2$ . The profile angle is greater than or equal to  $\alpha_2$  angle. The reflector does not shadow the collector, not even partially, and therefore the direct beam and circumsolar radiation cover the whole collector. Moreover the reflected beam and circumsolar radiation, coming from the reflector, don't reach any portion of the collector surface.

$$AC_{ns} = 1; \frac{A_{r \rightarrow c} \cap A_c}{A_{r \rightarrow c}} = 0$$

3. Results

In this paper the solar radiation, absorbed by the absorber surface of the collector, was calculated for representative average days for each month throughout the year according with what recommended by Klein [17]

The azimuth and altitude angle of the sun and the magnitude of direct and diffuse solar radiation on a horizontal surface were calculated with a refinement of 1 hour from sunrise to sunset as viewed by the collector.

The daily solar radiation received by the absorber element of a solar collector alone, i.e. without a reflector, is a direct function of the collector tilt  $\beta_c$  and, in Figure 2a, the trend of the total insolation received by the absorbing plate is depicted as a function of the  $\beta_c$  angle on a year round basis at a  $39^\circ$  N of latitude. In order to maximize the daily solar radiation in winter or in summer, the  $\beta_c$  angle respectively has to be respectively at least  $15^\circ$ - $20^\circ$  greater or lower than the latitude angle  $\phi$  since the solar altitude angle decreases in winter and increases in summer.

In five pairs of months (January and November, February and October, March and September, April and August, and May and July) would be almost the same, since the loci of the sun would be quite similar in each set of months. Stationary collector can provide useful enhancement of solar beam energy on a year round basis indeed the chart depicted in Figure 4b exhibits an optimal value of the angle value  $\beta_c$  (approximately equal to the latitude angle) which maximizes the yearly insolation.

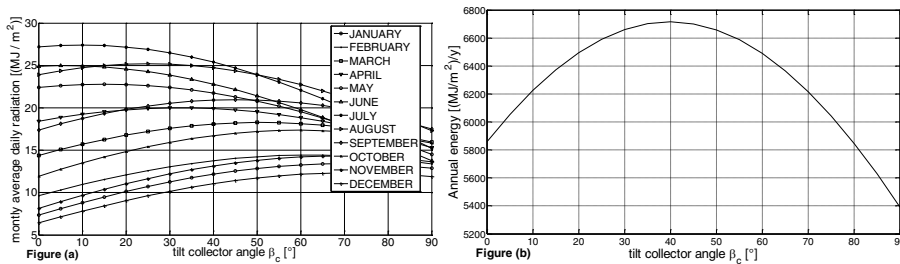


Figure 2 Monthly average daily (a) and annual (b) solar radiation received by a stationary collector to face the equator, as a function of the collector inclination  $\beta_c$  throughout the year at  $39^\circ$  N of latitude

In the synoptic Figure 3, we show the solar energy values that passes through the transparent cover of a collector as a function of both tilt angle of the collector  $\beta_c$  and the reflector  $\beta_r$ , for each month of the year.

In the Figure 3, the increment of solar radiation due to the reflector are depicted for each month of the year with respect to the value of radiation that could be picked up when the collector is without reflector, and is tilted in the optimum position of the corresponding month of the year under consideration. The y-axis represents an enhancement factor as the ratio of energy incident on a collector surface over a specified time period (a month) for an adjustable tilt collector/reflector system to that of a collector alone with its tilt optimized over the same time period. The figure 3 shows the optimum combinations of the inclinations  $\beta_c$  and  $\beta_r$ , that maximize the monthly solar radiation, and they vary considerably according to month. The optimum collector inclination  $\beta_c$  is lower in summer and higher in winter for the same reasons given in the results for a collector without the reflector as mentioned above. The absolute value of optimum reflector inclination  $|\beta_r|$  is lower in spring and autumn, and higher in summer and winter, since the solar altitude angle is larger in summer and lower in winter than in spring and autumn respectively. It is worth noting that an above reflector benefits from an off-optimum position during the winter period, and the most meaningful flux-augmentation during the 2 months on either sides of the winter solstice. Optimum collector inclinations  $\beta_c$  and optimum reflector inclinations  $\beta_r$  throughout the year are summarized in

Table 1. In figures 4a and 4b, a comparison between the annual amount of energy received by a stationary tilt collector coupled with an adjustable reflector is depicted. It concerns when the collector is kept at a constant angle position throughout the year and the reflector angle is changed through its optimal monthly positions, and when a collector, without reflector, is kept at its optimum yearly angular position. The picture depicted in Figure 4a exhibits a yearly optimal value of the collector angle  $\beta_c = 50^\circ$ . The Figure 4b shows the trend of the energy cumulated during the months as a function of a fixed collector yearly angle. The optimal reflector and collector angles for each month are shown over the surface. The Figure 5 is referred to a stationary tilt collector coupled with a stationary tilt reflector system and reveals an increase of solar radiation on the collector surface, when the collector and reflector are kept at a constant angular position throughout a year, with respect to the value of radiation that could be picked up when the collector is without the reflector and is kept tilted throughout the year in the position that optimizes the annual production.

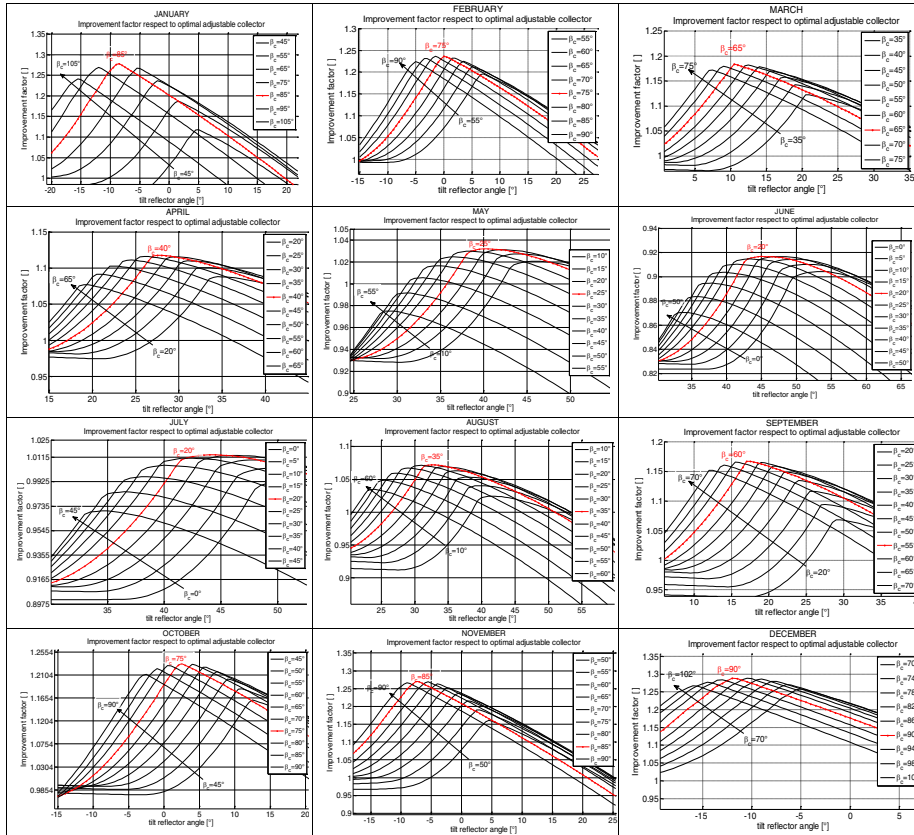


Figure 3 Effect of reflector and collector tilt angles on the calculated monthly irradiation received by the collector, for a reflector below collector configuration system throughout the year at 39° N, when the ratio of reflector and collector length is  $W_r/W_c = 1.0$  and the reflector have an overhang equal to  $0.5 l_c$ .



Table 1. Optimum inclinations of collector and reflector throughout a year

Month	conventional collector system	Collector augmented with reflector		Month	conventional collector system	Collector augmented with reflector	
	$\beta_c$	$\beta_c$	$\beta_r$		$\beta_c$	$\beta_c$	$\beta_r$
January	70°	85°	-8.5°	July	10°	20°	44°
February	62°	75°	0°	August	25°	35°	32.5°
March	51°	65°	10.5°	September	45°	55°	17.5°
April	32°	40°	28°	October	60°	75°	2.5°
May	15°	25°	40°	November	68°	85°	-7.5°
June	6°	20°	45°	December	72°	90°	-10°

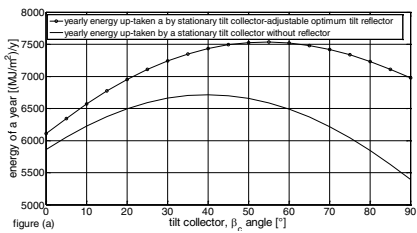


Figure 4a Comparison between the annual energy received by a stationary collector coupled with an adjustable reflector and that by a stationary collector without reflector. The tilt collector angle  $\beta_c$  is kept constant throughout a year.

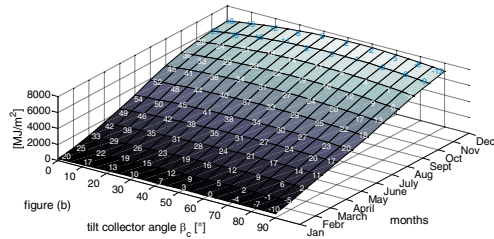


Figure 4b Stationary collector coupled with an optimum reflector: cumulative energy as a function of month and the tilt  $\beta_c$  of the collector. For each value  $\beta_c$  of the tilt collector, is shown, over the surface, the set of 12 optimal values of tilt reflector  $\beta_r$  [°]

Measures should be taken for adjusting of the reflector angle, if flux-augmentation is required throughout the year. However, a stationary augmented collector and reflector system may be well suited in order to maximize the energy production during a specific period of the year for a specific target: for example solar hot water systems application since augmentation can be restricted to the cooler months of the year without adversely affecting summer performance. Moreover it is also important to note that the annual amount of energy on the collector is mainly determined by the collector angular position and is slightly affected by the reflector tilt.

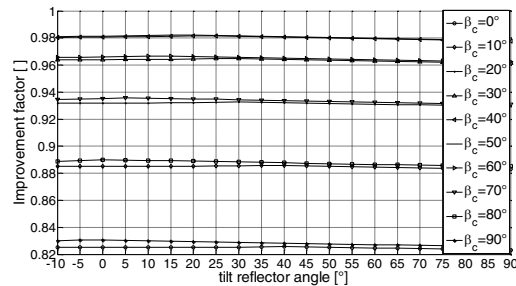


Figure 5. Enhancement factor of a stationary tilt collector coupled with a stationary tilt reflector system. The y-axis is the ratio of the amount of yearly energy on the surface of a collector coupled with a reflector to the yearly energy on the surface of a collector alone angled in its yearly optimal position.

#### 4. 4 Conclusion

We have performed a numerical analysis of a solar collector augmented by above reflector configuration. In this paper the reflector was assumed to have identical overhangs on both sides, and the width of the collector and the reflector are the same. We have determined the optimum inclination of both the collector and reflector throughout the year at 39° N latitude for an adjustable collector and reflector system, for a stationary collector coupled with an adjustable reflector and for a stationary collector coupled with a stationary above reflector configuration. We can summarize the results of this work as follows.

- 1) An adjustable above reflector can provide a meaningful solar flux augmentation on the absorbing plate of a solar collector, both with stationary and adjustable tilt, on a year round basis. The greatest benefit is obtained when the angles of both the collector and the reflector are oriented in the position according to which the reflector is inclined backwards in winter up to negative values, and forwards in summer: the optimum collector inclination is lower in summer and higher in winter. An above reflector configuration benefits from an off-optimum position during the winter period, when the enhancement factor reach the maximum value in the year.
- 2) A stationary collector and above reflector configuration can't provide useful enhancements on a year round basis. This stationary system may be well suited in order to maximize the energy production just during a specific period, without adversely affecting the yearly performance. The amount of energy on the collector surface is considerably affected by the angular position of the collector and is slightly affected by the tilt reflector.
- 3) A stationary collector coupled with an adjustable reflector exhibits a yearly optimal value of the collector angle,  $\beta_c = 50^\circ$  for which the solar energy received by the collector takes its absolute maximum value. Moreover every fixed yearly tilt collector value throughout a year may be relatively optimized by monthly optimal reflector angles.

#### References

- [1] McDaniel DK, Lowndes DH. Enhanced solar energy collection using reflector solar thermal collector combinations. *Solar Energy* 1975;17:277–83.
- [2] Taha IS, Eldighidly SM. Effect of off-south orientation on optimum conditions for maximum solar energy absorbed by flat plate collector augmented by plane reflector. *Solar Energy* 1980;25:373–9.
- [3] Chiam HF. Stationary reflector-augmented flat-plate collectors. *Solar Energy* 1982;29:65–9.
- [4] Arata AA, Geddes RW. Combined collector–reflector systems. *Energy* 1986;11:621–30.
- [5] Dang A. Collector, collector–reflector systems – an analytical and practical study. *Energy Convers Manage* 1986;26:33–9.
- [6] Garg HP, Hrishikesan DS. Enhancement of solar energy on flat-plate collector by plane booster mirrors. *Solar Energy* 1988;40:295–307.
- [7] Rao AVN, Chalam RV, Subramanyam S, Rao TLS. Energy contribution by booster mirrors. *Energy Convers Manage* 1993;34:309–26.
- [8] Bollentin JW, Wilk RD. Modeling the solar irradiation on flat plate collectors augmented with planar reflectors. *Solar Energy* 1995; 55:343.
- [9] Hussein HMS, Ahmad GE, Mohamad MA. Optimization of operational and design parameters of plane reflector-tilted flat plate solar collector systems. *Energy* 2000;25:529–42.
- [10] Pucar MDJ, Despica AR. The enhancement of energy gain of solar collectors and photovoltaic panels by the reflection of solar beams. *Energy* 2002;27:205–23.
- [11] Hellstrom B, Adsten M, Nostell P, Karlsson B, Wackelgard E. The impact of optical and thermal properties on the performance of flat plate solar collectors. *Renew Energy* 2003;28:331–44.
- [12] Kostic LT, Pavlovic TM, Pavlovic ZT. Optimal design of orientation of PV/T collector with reflectors. *Appl Energy* 2010;87:3023–9.
- [13] Kostic LT, Pavlovic TM, Pavlovic ZT. Influence of reflectance from flat aluminum concentrators on energy efficiency of PV/Thermal collector. *Appl Energy* 2010;87:410–6.
- [14] Hay, J. E. and J. A. Davies. Calculation of the Solar Radiation Incident on a inclined surface, in Proceedings of the first Canadian Solar Radiation Data Workshop, Ministry of Supply and Service, Toronto, Canada, p. 59 (1980)
- [15] 10349 uni standard riscaldamento e raffrescamento degli edifici, dati climatici
- [16] Perers B., Karlsson B., External reflectors for large solar collector arrays, simulation model and experimental result, *Solar Energy*, 51, pp 327–337 (1993)
- [17] Klein, S. A., Calculation of monthly average insolation on tilted surfaces, *Solar Energy*, 19, 325 (1977)