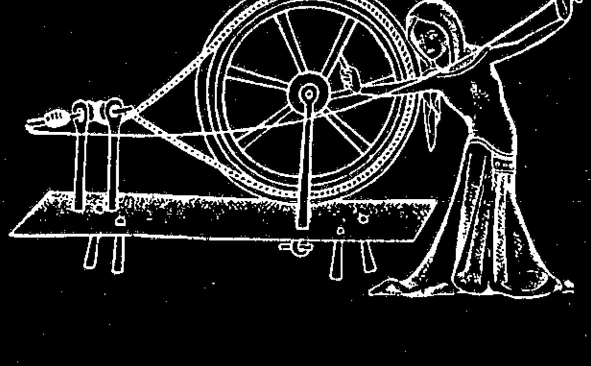
II EUROPEAN SYMPOSIUM on FLYWHEEL ENERGY STORAGE

Proceedings

Edited by G. GENTA



Torino, Italy, May 9-13, 1983

EXPERIMENTAL DETERMINATION OF ELASTIC MODULI OF FILAMENT WOUND COMPOSITES THROUGH RESONANCE DYNAMIC ANALYSIS

B. Picasso, F. Bertolino

Istituto di Meccanica, Università di Cagliari
Piazza D'Armi, Cagliari, Italia

ABSTRACT

This paper deals with the experimental determination of the elastic moduli of composite materials by means of dynamic analysis in resonance. Mechanical impedance measurements are carried out on a circular filament wound composite model in the frequency range 0-6000 cps. A detailed description of the experimental procedure and the measuring technique, based on a fast data acquisition system, is furnished. The main aim of the experimental work is the extension of the driving point impedance method for the determination of complex elastic moduli to circular filament wound composite specimens. The experimental resonant frequency values are compared with analytical results obtained by means of the finite element method for different values of the radial and circumferential E_θ elastic moduli. The comparison allowed to determine E_r and E_θ values of the material. Extensive experimental work is foreseen in order to check the applicability and reliability of the technique for filament wound composite materials.

1. INTRODUCTION

The determination of elastic properties of composite materials often poses particular problems due to the anisotropic structure of these materials and to the practical difficulties involved in extending the classical methods used in testing metallic materials to composites. The possibility of adopting traditional testing techniques for measuring elastic moduli is connected with the ability to manufacture specimens suitable for such tests. For materials with a substantial degree of anisotropy it is sometimes impossible to obtain tensile, compression or torsion specimens which represent in a statistically acceptable manner the strain behaviour for all directions and stress levels of interest. In this regard particular attention should be given to filament wound composite structures or machine members such as pipes, discs, shells of revolution, which are gaining ever-increasing importance in the mechanical and aerospace industries. Specimen manufacturing is in many cases a technological process distinct from machine member construction. One of the most significant features of composite materials is their behaviour under dynamic loading which can summarily be defined as "viscoelastic". Elastic moduli can no longer be defined as "constants" since they depend on dynamic loading frequency and are in general represented as complex quantities in order to account for the effect of internal damping. The complex Young's modulus is usually expressed in the following way [1]:

$$E^*_\omega = E_\omega(1 + i\delta_{E,\omega})$$

where E_ω, $\delta_{E,\omega}$ depend on the frequency ω of the dynamic load applied.

Several techniques which utilize specimens of various shapes excited to vibrate within a certain frequency range, have been proposed for the measurement of dynamic moduli. For a detailed description of such techniques the reader is referred for instance to [2]. The specimens are generally beam-shaped with rectangular cross section. The excitation point can be either at the midpoint or at one end of the beam. In the present and previous papers, the dynamic resonance technique was applied to circular filament wound thin specimens excited to vibrate in their centre.

In order to recall the main features of resonance techniques for elastic moduli determination, let us consider the simple system of Fig.1 where a mass M is connected to the base by means of an elastic element which plays the role of spring of the classical one d.o.f. model. The elastic forces can be expressed as KE^*_ω where the asterisk denotes a complex quantity. If the system is excited to vibrate by the sinusoidal oscillation of its base, the mechanical driving point impedance of the system can be written [1]:

$$Z^* = \frac{F^*}{V_1} = \frac{1}{\dfrac{i\omega}{KE^*_\omega} + \dfrac{1}{i\omega M}} = \frac{i\omega M \cdot KE^*_\omega}{KE^*_\omega - M\omega^2}$$

The dimensionless ratio $Z^*/i\omega M$ has the form:

$$\frac{Z^*}{i\omega M} = \frac{KE^*_\omega}{KE^*_\omega - M\omega^2} = \frac{KE_\omega(1 + i\delta_{E,\omega})}{KE_\omega(1 + i\delta_{E,\omega}) - M\omega^2} =$$

$$= \frac{1 + i\delta_{E,\omega}}{1 + i\delta_{E,\omega} - \frac{M\omega^2}{KE_\omega}}$$

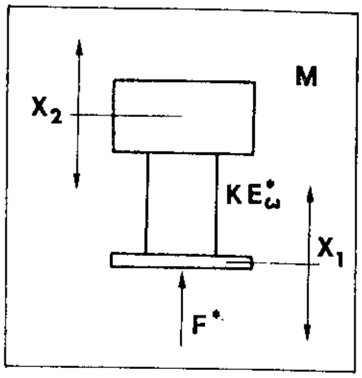


Fig. 1

Putting $\omega_o^2 = \frac{KE_o}{M}$ one obtains:

$$\frac{Z^*}{i\omega M} = \frac{1 + i\delta_{E,\omega}}{1 + i\delta_{E,\omega} - \frac{E_o}{E_\omega}\frac{\omega^2}{\omega_o^2}} = \frac{1 + i\delta_{E,\omega}}{\left(1 - \frac{E_o}{E_\omega}\frac{\omega^2}{\omega_o^2}\right) + i\delta_{E,\omega}}$$ and

$$\left|\frac{Z^*}{i\omega M}\right| = \frac{\left(1 + \delta^2_{E,\omega}\right)^{\frac{1}{2}}}{\left[\left(1 - \frac{E_o}{E_\omega}\frac{\omega^2}{\omega_o^2}\right)^2 + \left(\delta_{E,\omega}{}^2\right)\right]^{\frac{1}{2}}}$$

For materials commonly used in mechanical members the variation of E_ω with frequency is very small and for practical purposes can be neglected. The expression of the normalized driving point impedance shows that for $\omega = \omega_o$

$$\left|\frac{Z^*}{i\omega M}\right| = \frac{(1 + \delta^2_{E,\omega})^{\frac{1}{2}}}{\delta_{E,\omega}}$$

From the latter expression the loss factor $\delta_{E,\omega}$ can be derived through the measurement of the normalized driving point impedance in resonance. The measurement of the resonant frequency ω_o entails the evaluation of the elastic modulus E_o since $\omega_o^2 = KE_o/M$. Thus from the resonant frequency ω_o and amplitude of the driving point measurements one can evaluate the elastic modulus E_o and the loss factor δ_E. In practical applications to flexural vibrations of beam-shaped specimens the expression of the driving point impedance is of the type (valid for a free-free beam) [1]:

$$\left| \frac{z_o^*}{i\omega M} \right| = \frac{1}{n^* a} \frac{\sin h(n^* a)\cdot \cos(n^* a) + \cos h(n^* a)\cdot \sin(n^* a)}{1 + \cos h(n^* a)\cdot \cos(n^* a)}$$

where

$$n^* a = n\,a\,(1+i\delta_E)^{-1/4} = a \left[\frac{\omega^2 \rho}{r^2 \cdot E_\omega \cdot (1+\delta_{E,\omega})} \right]^{1/4} = a \left[\frac{\omega^2 \rho}{r^2 E_\omega} \right]^{1/4} \cdot (1+i\delta_{E,\omega})^{1/4}$$

Since the values of $n^* a$ at resonances and antiresonances of the beam can be easily calculated, one can, alternatively, determine the corresponding values of resonant frequencies ω_n, or if these quantities are measured experimentally arrive at the determination of the Young's modulus E_ω. Once again the loss coefficient $\delta_{E,\omega}$ can be derived from the measurement of the amplitude of the driving point impedance in resonance or through the well-known resonance bandwidth method [2].

For cylindrically orthotropic plates a closed form solution of the motion equations is very difficult to obtain and the problem of natural frequency determination is usually solved by approximate methods (Rayleigh-Ritz, or finite elements) [3]. The finite element method was used in the case at hand to obtain the natural axisymmetrical frequencies of the plate. A bending axisymmetrical element with three d.o.f. per node was employed and the effect of rotary inertia was taken into account [4]. Resonant frequencies were calculated for a wide range of values of the elastic moduli E_θ, E_r in order to demonstrate the dependence of natural frequencies on the elastic moduli.

Two approximations were introduced in this first stage of the analysis. The real part of the modulus was assumed to be independent of frequency. This assumption does not lead to any appreciable error in modulus determination, due to the fact that for most plastics and composite materials at room temperature frequency exerts only a very slight influence on modulus [1].

A secondary influence which was also neglected was the effect of damping on resonant frequencies. It is well known that compared with the corresponding no-damping values a shift in such frequencies does occur. Since in the analytical determination of frequencies damping was not considered, experimental damped frequencies were compared with analytical undamped ones. Such an approximation did induce an error in modulus determination but for materials such as epoxy based composites characterized by $\delta_E \leq 0{,}1$ the variation of natural frequency values due to the presence of internal damping is generally quite small. This does not mean that the determination of the loss factor is insignificant for engineering purposes; on the contrary this latter quantity is an indication of the capacity to absorb shocks and dynamic stress.

A further approximation consisted in neglecting the effect of impedance head mass on the values of resonant and antiresonant frequencies. A detailed experimental study has been conducted on this subject by Ziolkowsky [5] who pointed out that an additional mass on the centre of a free-free beam affects the values of resonant and antiresonant frequencies and the magnitude of the driving point impedance. However, the extent of this influence differs substantially for resonant and antiresonant frequencies. In fact resonant frequencies correspond to maximum amplitudes at the disc centre whilst in antiresonance this can be considered a modal point. For this reason the effect of the additional mass on frequencies and driving point impedances in antiresonance can be neglected.

2. IMPEDANCE MEASUREMENTS

The experimental set-up was based on a fast data acquisition system connected to a mini computer by means of an analog-to-digital converter. The analogic signals generated by the acceleration and force transducers were accessed to computer memory and then entered in FORTRAN program for signal analysis and impedance measurement. The maximum scanning speed on a single channel was about 17 KHz. For this reason, in order to avoid aliasing of input signals, a low pass filter was inserted into the measuring chain before the analog-to-digital converter (Fig.2).

The model used was a circular filament wound composite annular disc excited on its centre by a B & K electrodynamic exciter. Excitation force and vibration velocity were measured on the driving point by means of piezoelectric transducers. Force and vibration velocity data, converted into digital form were used by a FORTRAN subroutine to obtain impedance values for every step of excitation frequency. Figure 3 represents the normalized driving point $Z/i\omega M$ in logarithmic scale for the ordinates for frequencies ranging from 0 to 3.0 KHz. Figure 3 clearly shows resonant and antiresonant frequencies of the disc corresponding respectively to minimum and maximum impedance values. For elastic moduli determination the first three antiresonant frequencies were considered with 0,1,2 nodal circles. The elastic modulus determination is summarized in Fig.4 where the lines corresponding to the first three antiresonances are drawn with different graphics. The scales for these three frequencies are displaced so as to represent the three frequency values with a single horizontal line. In this way one can determine the moduli E_θ, E_r as those values which pertain to lines intersecting the horizontal line. This condition is only approximately verified because the first two frequencies indicate a modulus E_θ of 37000 N/mm^2 (point A in Fig.5), the first and third $E_\theta = 37200$ N/mm^2 (near point B in Fig. 5) and the second and third a value of E_θ near 39000 N/mm^2. The corresponding value of E_r is about 2000 N/mm^2, surprisingly low for filament wound composites. However, considering that previous research work brought to light the presence of defects and delaminations in the same material, this value could be

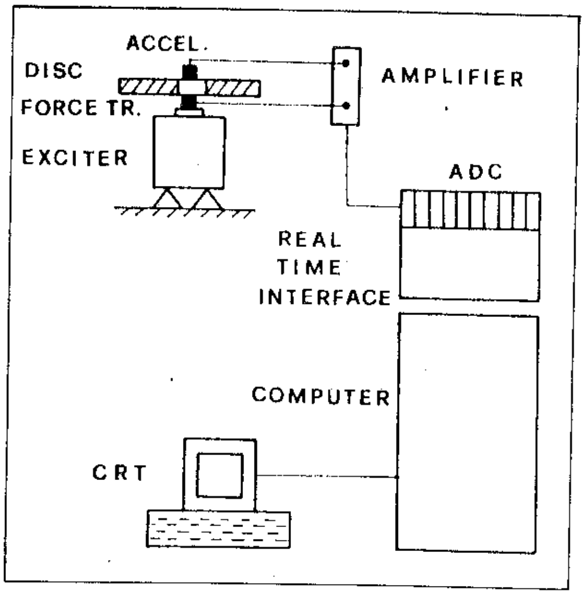


Fig.2

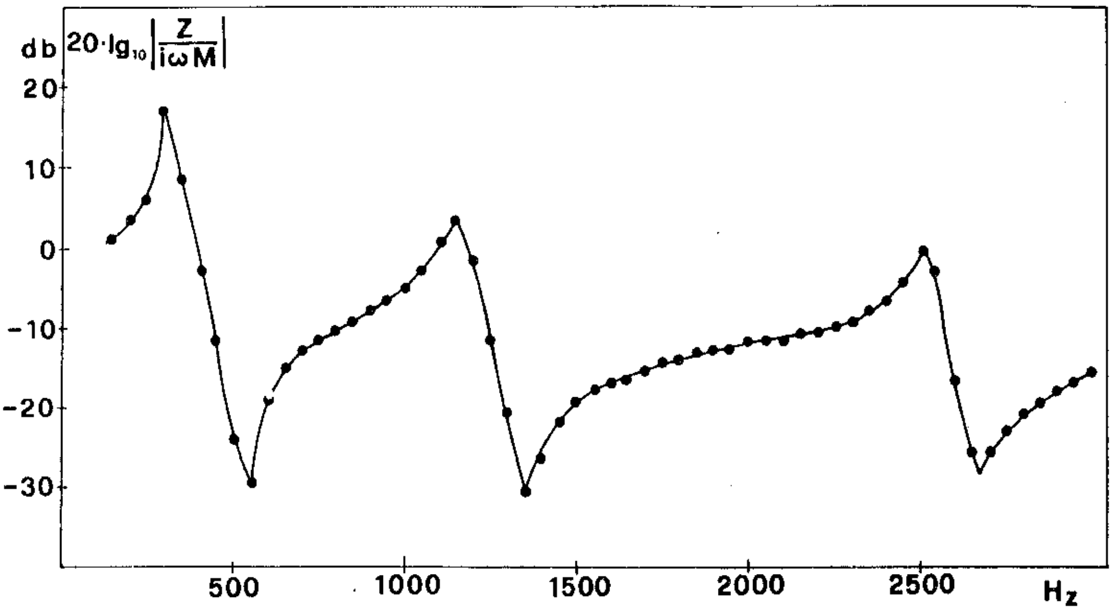


Fig.3

realistic. A comprehensive study of several models of the same materials will however provide some elucidation in this respect.

A few considerations are expedient in order to comment the results presented herein. Uncertainties in modulus determination may arise from two main sources, namely the inadequacy or approximation of the mathematical model used, which unfortunately cannot be evaluated "a priori", and the experimental test conditions which are affected by a number of disturbing factors in the analytical frequency determination. For this reason the finite element program was extensively tested on models of isotropic materials such as steel and plastic thin circular discs, thin conical shells, etc. In this instance analytical solutions were available for natural frequencies and normal modes and the verification of finite element results was facilitated. Furthermore, the elastic moduli of the materials were known with a good level of confidence and they were used as reference values for checking the reliability of the method. Results were generally good for all models tested but highlighted the fact that the higher the frequency order used for modulus determination, the poorer the accuracy of the results. This fact would seem to be attributable to the effect of shear, which increases with frequency.

In the experimental work on the anisotropic disc specimen, the thickness-to-radius ratio was maintained at a value of about 0.06 in order to keep shear effect to within negligible limits. For the same reason only the first three frequencies were utilized for modulus determination.

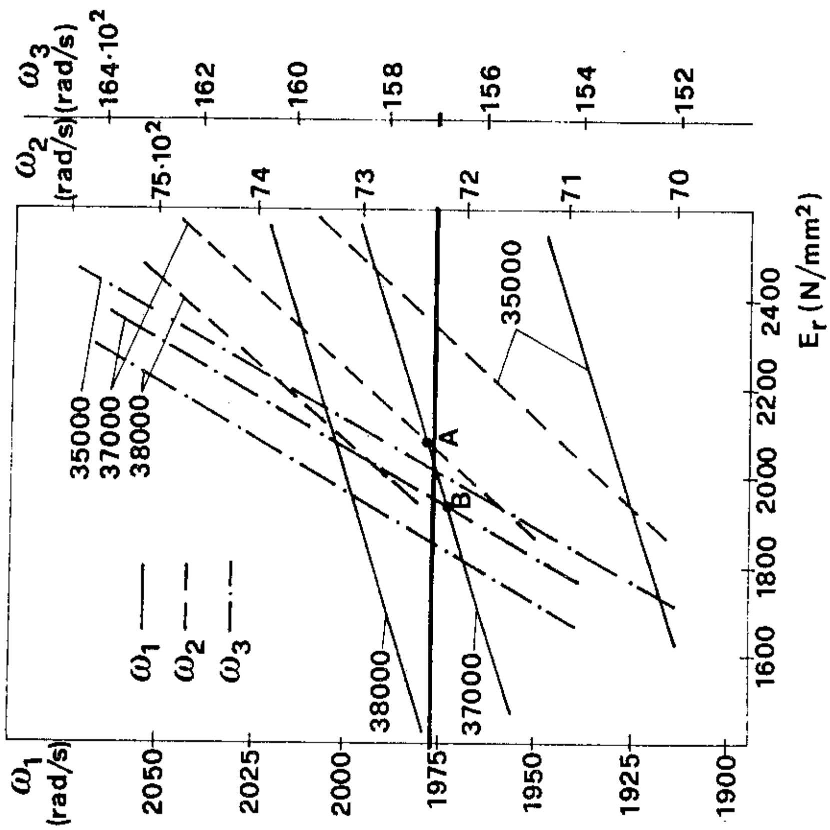

ω_2 (rad/s)
ω_3 (rad/s)
164·10²
162
160
158
156
154
152
75·10²
74
73
72
71
70
ω_1 (rad/s)
2050
2025
2000
1975
1950
1925
1900
1600
1800
2000
2200
2400
E_r (N/mm²)
35000
37000
38000
A
B
ω_1
ω_2
ω_3

Fig.5

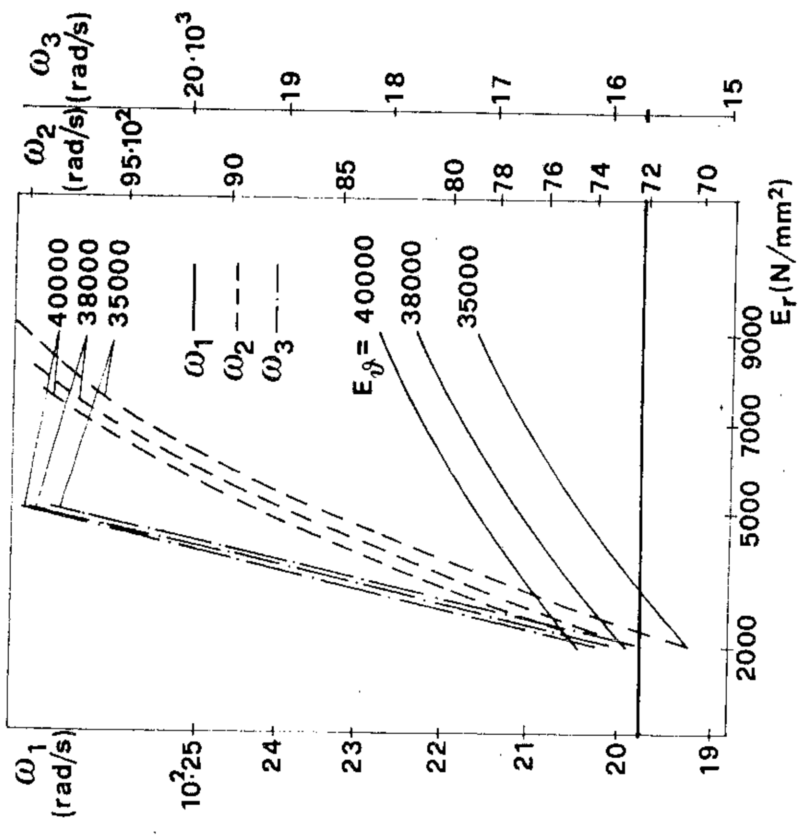

ω_2 (rad/s)
ω_3 (rad/s)
20·10³
19
18
17
16
15
95·10²
90
85
80
78
76
74
72
70
ω_1 (rad/s)
10²25
24
23
22
21
20
19
2000
5000
7000
9000
E_r (N/mm²)
40000
38000
35000
E_ϑ = 40000
ω_1
ω_2
ω_3

Fig.4

3. CONCLUSIONS

The results obtained with a filament wound composite disc would seem to justify, though with a certain measure of caution, a more concentrated research effort into the possibilities of using the dynamic method for determining elastic constants of composite materials. In future work an analytical solution to dynamic equilibrium equations will be sought in order to eliminate the uncertainties inherent in approximate numerical methods employed for finding the natural frequencies of the plate. A more extensive experimental study is also foreseen in order to check the reliability of the present results.

LIST OF SYMBOLS

E^{*}_{ω}	Complex Young's modulus
E_{ω}	Real part of complex modulus
$\delta_{E,\omega}$	Loss damping factor
ω	Circular frequency
Z^{*}	Complex impedance
$n = (\omega^2 \rho)/E_{\omega} \cdot r^2)$	Frequency parameter
$r = I/A$	Radius of giration of cross-section of beam
a	Half length of beam
M	Mass
ρ	Material mass density

REFERENCES

1 J.C. SNOWDON: "Vibration and Shock in Damped Mechanical System". John Wiley & Sons, Inc., New York, 1968.

2 B.E. READ, G.D. DEAN: "The Determination of Dynamic Properties of Polymers and Composites". Adam Hilger Ltd., Bristol, 1968.

3 S.G. LEKHNITSKII: "Anisotropic Plates". Gordon & Breach Science Publ. Inc., 1968.

4 F. BERTOLINO, B. PICASSO: "Analisi sperimentale di gusci conici a spessore sottile". IX Convegno Nazionale AIAS, Trieste-Grado, 1981.

5 W. ZIOLKOWSKI, A. SLIWINSKI: "The influence of the transmission function of the impedance head on the measurement of the complex elastic modulus of a viscoelastic beam by the driving point impedance method". Journal of Sound & Vibration 80 (2), 209-222, 1982.