

Quantum information, cognition and music.

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Submitted to Journal:
Frontiers in Psychology

Specialty Section:
Cognition

Article type:
Original Research Article

Manuscript ID:
163769

Received on:
08 Aug 2015

Revised on:
15 Sep 2015

Frontiers website link:
www.frontiersin.org

Conflict of interest statement

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest

Keywords

Turing machines, quantum computation, Quantum information, semantics, music cognition

Abstract

Word count: 70

Parallelism represents an essential aspect of human mind/brain activities. One can recognize some common features between psychological parallelism and the characteristic parallel structures that arise in quantum theory and in quantum computation. The article is devoted to a discussion of the following questions:

- 1) a comparison between classical probabilistic Turing machines and quantum Turing machines;
- 2) possible applications of the quantum computational semantics to cognitive problems;
- 3) parallelism in music.

Quantum information, cognition and music.

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2 ABSTRACT

3 Parallelism represents an essential aspect of human mind/brain activities. One can recognize
4 some common features between psychological parallelism and the characteristic parallel structures
5 that arise in quantum theory and in quantum computation. The article is devoted to a
6 discussion of the following questions:

- 7 1. a comparison between classical probabilistic Turing machines and quantum Turing machines.
- 8
- 9 2. Possible applications of the quantum computational semantics to cognitive problems.
- 10 3. Parallelism in music.

1 INTRODUCTION

11 Parallelism represents an essential aspect of the activities of human brain and mind. One can recognize
12 some common features between psychological parallelism and the characteristic parallel structures that
13 arise in quantum theory and in quantum computation, being responsible for the extraordinary efficiency
14 and speed of quantum computers.

15 Quantum parallelism and classical parallelism are deeply different, although it is sometimes claimed that
16 quantum Turing machines are nothing but special examples of classical probabilistic Turing machines.¹
17 But what exactly are quantum Turing machines? So far, the literature has not provided a rigorous “institutional”
18 concept of *quantum Turing machine*. Some definitions seem to be based on a kind of “imitation”
19 of the classical definition of *Turing machine*, by referring to a *tape* (where the symbols are written) and
20 to a *moving head* (which changes its position on the tape).² These concepts, however, seem to be hardly
21 applicable to physical quantum computers. Both in the classical and in the quantum case, it is expedient
22 to consider a more abstract concept: the notion of *state machine*, which neglects both tapes and moving

¹ See, for instance, Penrose, 1994.

² See, for instance, Fouché et al., 2007.

heads. Every finite computational task realized in different computational models proposed in the literature can be simulated by a state machine.³ In order to compare classical and quantum parallelism, we will analyze the concepts of (classical) *deterministic state machine*, (classical) *probabilistic state machine* and *quantum state machine*. On this basis we will discuss the question: to what extent can quantum state machines be simulated by probabilistic state machines? (Sections 2-3).

In the investigation about possible links between *quantum structures* and *psychological structures* a useful tool is represented by a special form of quantum logical semantics (called *quantum computational semantics*) that has been inspired by the theory of quantum computation. We will see how this semantics can be naturally applied to a formal analysis of musical compositions, where *parallel structures*, *ambiguity*, *holism* and *contextuality* play an essential role (Sections 4-5).⁴

Our analysis seems to confirm a general conjecture that has been defended and discussed in different research-fields: the basic concepts of the quantum-theoretic formalism (which had for a long time been regarded as mysterious and potentially paradoxical) seem to have a *universal* interest that goes beyond the domain of microphysical phenomena.

2 CLASSICAL DETERMINISTIC AND PROBABILISTIC MACHINES

We will first introduce a formal definition for the notion of *deterministic state machine*. On this basis, *probabilistic state machines* will be represented as stochastic variants of deterministic machines, which are able to calculate different outputs with different probability-values.

Definition 1. *Deterministic state machine.*

A *deterministic state machine* is an abstract system M based on the following elements:

1. A finite set S of *internal states*, which contains an *initial state* s_{in} and includes a set of *halting states* $S_{halt} = \{s_{halt_j} | j \in J\}$.
2. A finite alphabet, which can be identified with the set $\{0, 1\}$ of the two classical bits. Any *register* represented by a bit-sequence $w = (x_1, \dots, x_n)$ is a *word* (of length n). Any pair (s, w) consisting of an internal state s and of a word w represents a possible *configuration* of M , which is interpreted as follows: M is in the internal state s and w is the word written on an ideal tape.
3. A set of words that represent possible *word-inputs* for M .
4. A *program*, which is identified with a finite sequence of *rules*:

$$(R_0, \dots, R_t).$$

Each R_i is a partial function that transforms configurations into configurations. We may have: $R_i = R_j$ with $i \neq j$. The number i , corresponding to the rule R_i , represents the i -th step of the program. The following conditions are required:

- 4.1 The rule R_0 is defined for any configuration (s_0, w_0) , where s_0 is the initial state s_{in} and w_0 is a possible word-input. We have:

$$R_0 : (s_0, w_0) \mapsto (s_1, w_1),$$

where s_1 is different from the initial state and from all halting states (if $t \neq 0$).

³ See, for instance, Savage, 1998 and Gudder, 1999.

⁴ Some basic intuitive ideas of the quantum computational semantics are close to the “quantum cognition approach” that has been extensively developed in recent times (see, for instance, Aerts et al., 2005a, 2005b, Aerts et al., 2014). In both theories *concepts* and *thoughts* are represented as special abstract entities that can be described in the framework of the quantum-theoretic formalism. The technical developments of the two approaches are, however, different.

4.2 For any i ($0 < i < t$),

$$R_i : (s_i, w_i) \mapsto (s_{i+1}, w_{i+1}),$$

where s_{i+1} is different from all $s_i, \dots, s_0$ and from all halting states.

4.3 $R_t : (s_t, w_t) \mapsto (s_{t+1}, w_{t+1})$,

where s_{t+1} is a halting state.

Each configuration (s_{i+1}, w_{i+1}) represents the *output* for the step i and the *input* for the step $i + 1$.

The concept of *computation* of a deterministic state machine can be now defined as follows.

Definition 2. *Computation of a deterministic state machine.*

A *computation* of a deterministic state machine M is a finite sequence of configurations

$$((s_0, w_0), \dots, (s_{t+1}, w_{t+1})),$$

where:

1. w_0 is a possible word-input of M .
2. $s_0, \dots, s_{t+1}$ are different internal states of M such that: $s_0 = s_{in}$ and s_{t+1} is a halting state.
3. For any i ($0 \leq i \leq t$),

$$(s_{i+1}, w_{i+1}) = R_i((s_i, w_i)),$$

where R_i is the i -th rule of the program.

The configurations (s_0, w_0) and (s_{t+1}, w_{t+1}) represent, respectively, the *input* and the *output* of the computation; while the words w_0 and w_{t+1} represent, respectively, the *word-input* and the *word-output* of the computation.

Apparently, each deterministic state machine is devoted to a single task that is determined by its program.

Let us now turn to the concept of *probabilistic state machine*. The only difference between deterministic and probabilistic state machines concerns the program, which may be stochastic in the case of a probabilistic state machine (PM). In such a case, instead of a sequence of rules, we will have a sequence $(Seq_0, \dots, Seq_t)$ of sequences of rules such that:

$$Seq_0 = (R_{0_1}, \dots, R_{0_r})$$

.....

$$Seq_t = (R_{t_1}, \dots, R_{t_l}).$$

Each rule R_{i_j} (occurring in the sequence Seq_i) is associated to a probability-value p_{i_j} such that:

$$\sum_j p_{i_j} = 1.$$

From an intuitive point of view, p_{i_j} represents the probability that the rule R_{i_j} be applied at the i -th step. A deterministic state machine is, of course, a special case of a probabilistic state machine characterized by the following property: each sequence Seq_i consists of a single rule R_i .

Any probabilistic state machine naturally gives rise to a graph-structure for any choice of an input-configuration $conf_0 = (s_0, w_0)$. As an example, consider the following simple case: a probabilistic state machine PM whose program consists of two sequences, each consisting of two rules:

$$Seq_0 = (R_{0_1}, R_{0_2})$$

$$Seq_1 = (R_{1_1}, R_{1_2}).$$

73 The graph associated to PM for the configuration $conf_0$ is illustrated by Figure 1.

74 How do probabilistic machines compute? In order to define the concept of *computation* of a proba-
75 bilistic machine, let us first introduce the notions of *program-path* and of *computation-path* of a given
76 probabilistic machine.

77 **Definition 3.** *Program-path and computation-path.*

78 Let PM be a probabilistic state machine with program $(Seq_0, \dots, Seq_t)$.

- A *program-path* of PM is a sequence

$$\mathcal{P} = (R_{0_h}, \dots, R_{i_j}, \dots, R_{t_k}),$$

79 consisting of t rules, where each R_{i_j} is a rule from Seq_i (probabilistically independent of all other
80 rules of $\mathcal{P}$).

- For any choice of an input (s_0, w_0) , any program-path $\mathcal{P}$ determines a sequence of configurations

$$\mathcal{CP} = ((s_0, w_0), \dots, (s_i, w_i), \dots, (s_{t+1}, w_{t+1})),$$

81 where $(s_{i+1}, w_{i+1}) = R_{i_j}(s_i, w_i)$ and R_{i_j} is the i -th element of $\mathcal{P}$. This sequence is called the
82 *computation-path* of PM determined by the program-path $\mathcal{P}$ and by the input (s_0, w_0) .

83 The configuration (s_{t+1}, w_{t+1}) represents the output of $\mathcal{CP}$.

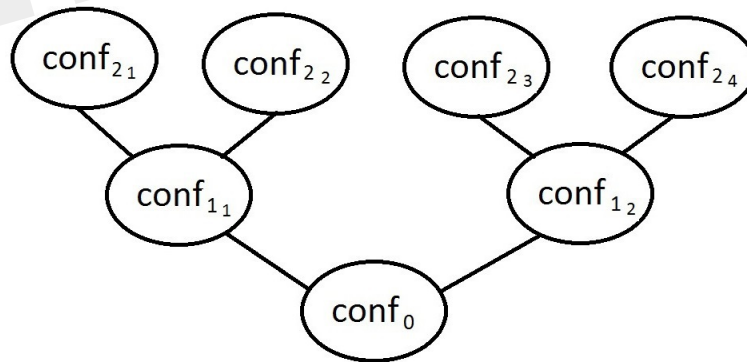


Figure 1. The graph of PM.

Any program-path $\mathcal{P} = (R_{0_h}, \dots, R_{i_j}, \dots, R_{t_k})$ has a well determined probability-value $p(\mathcal{P})$, which is defined as follows (in terms of the probability-values of its rules):

$$p(\mathcal{P}) := p_{0_h} \cdot \dots \cdot p_{i_j} \cdot \dots \cdot p_{t_k}.$$

As expected, the probability-value of a program-path $\mathcal{P}$ naturally determines the probability-values of all corresponding computation-paths. It is sufficient to put:

$$p(\mathcal{CP}) := p(\mathcal{P}).$$

Consider now the set $\mathbf{P}_{\mathbf{PM}}$ of all program-paths and the set $\mathbf{CP}_{\mathbf{PM}}$ of all computation-paths of a probabilistic machine $\mathbf{PM}$. One can easily show that:

$$\sum_i \{p(\mathcal{P}_i) | \mathcal{P}_i \in \mathbf{P}_{\mathbf{PM}}\} = \sum_i \{p(\mathcal{CP}_i) | \mathcal{CP}_i \in \mathbf{CP}_{\mathbf{PM}}\} = 1.$$

84 On this basis the concept of *computation* of a probabilistic state machine can be defined as follows.

85 **Definition 4.** *Computation of a probabilistic state machine.*

86 A *computation* of a probabilistic state machine $\mathbf{PM}$ with input (s_0, w_0) is the system of all computation-
87 paths of $\mathbf{PM}$ with input (s_0, w_0) .

Unlike the case of deterministic state machines, a computation of a probabilistic state machine does not yield a unique output. For any choice of a configuration-input (s_0, w_0) , the computation-output is a system of possible configuration-outputs (s_{t+1}^i, w_{t+1}^i) , where each (s_{t+1}^i, w_{t+1}^i) corresponds to a computation-path $\mathcal{CP}_i$. As expected, each (s_{t+1}^i, w_{t+1}^i) has a well determined probability-value that is defined as follows:

$$p((s_{t+1}^i, w_{t+1}^i)) := \sum_i \{p(\mathcal{CP}_i) | \text{the configuration-output of } \mathcal{CP}_i \text{ is } (s_{t+1}^i, w_{t+1}^i)\}.$$

88 One can easily show that the sum of the probability-values of all configuration-outputs of any machine
89 $\mathbf{PM}$ is 1.

3 QUANTUM STATE MACHINES

The strong parallelism that characterizes quantum computers is based on two quantum-theoretic notions that have been often described as mysterious and potentially paradoxical: *superposition* and *entanglement*. For the readers who are not expert of quantum theory it is expedient to recall some concepts of the quantum formalism that are used in quantum computation.⁵ The basic idea is that any *piece of quantum information* is mathematically represented as a possible *state* of a quantum system that can store and transmit the information in question. In the simplest situations one is dealing with a single particle S (say, an electron or a photon), whose “mathematical environment” is a special example of a vector space: the two-dimensional Hilbert space $\mathbb{C}^2$, based on the set of all ordered pairs of complex numbers. The canonical (orthonormal) basis of $\mathbb{C}^2$ consists of the two following unit-vectors:

$$|0\rangle = (1, 0); \quad |1\rangle = (0, 1),$$

which represent, in this framework, the two classical bits (0 and 1), or (equivalently) the two classical truth-values (*Falsity* and *Truth*). A *pure state* corresponds to a *maximal piece of information* that cannot be consistently extended to a richer knowledge. Such state is represented as a unit-vector $|\psi\rangle$ that can be expressed as a *superposition* of the two elements of the canonical basis of $\mathbb{C}^2$:

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle,$$

90 where c_0 and c_1 (also called *amplitudes*) are complex numbers such that $|c_0|^2 + |c_1|^2 = 1$.

91 The physical interpretation of $|\psi\rangle$ (also called *qubit-state* or, briefly, *qubit*) is the following: the physical
92 system S in state $|\psi\rangle$ might satisfy the physical properties that are *certain* for the bit $|0\rangle$ with probability

⁵ A survey of quantum computation theory can be found, for instance, in Nielsen et al., 2000.

93 $|c_0|^2$ and might satisfy the physical properties that are *certain* for the bit $|1\rangle$ with probability $|c_1|^2$. Due to
 94 the characteristic indeterminism of quantum theory, the pure state $|\psi\rangle$ is at the same time a *maximal and*
 95 *logically incomplete piece of information* that cannot *decide* some important physical properties of the
 96 system S . Accordingly, from an intuitive point of view, one can say that $|\psi\rangle$ describes a kind of *cloud of*
 97 *potential properties* that might become *actual* when a measurement is performed. Measuring a physical
 98 quantity (by means of an apparatus associated to the canonical basis) determines a sudden transformation
 99 of the qubit $|\psi\rangle$ either into the bit $|0\rangle$ or into the bit $|1\rangle$. Such transformation is usually called *collapse of*
 100 *the wave-function*.

Not all states associated to a physical system S are pure. Non-maximal pieces of information can be represented as *mixtures of pure states* (special examples of operators called *density operators*). In the space $\mathbb{C}^2$ a density operator ρ can be represented as a convenient finite sum of projection-operators:

$$\rho = \sum_i w_i P_{|\psi_i\rangle},$$

101 where w_i are real numbers such that $\sum_i w_i = 1$, while each $P_{|\psi_i\rangle}$ is a projection-operator that projects
 102 along the direction of $|\psi_i\rangle$. Notice that such representation is not generally unique. A density operator
 103 that cannot be represented as a projection $P_{|\psi\rangle}$ is called a *proper mixture*. While pure states codify an
 104 essential indetermination of some relevant properties of the quantum system under investigation, mixtures
 105 may correspond to an *epistemic uncertainty* of the observer. Unlike pure states (which always satisfy
 106 some well-determined properties), there are mixtures that cannot decide any (non-trivial) property of the
 107 associated system. An example of this kind is the state $\rho = \frac{1}{2}I$, where I is the identity operator of the
 108 space $\mathbb{C}^2$.

As happens in classical information theory, quantum computation also needs complex pieces of information, which are supposed to be stored by composite quantum systems (generally consisting of n subsystems). Accordingly, one can naturally adopt the quantum-theoretic formalism for the mathematical representation of composite physical systems, based on the use of *tensor products* (special examples of products).⁶ While a single qubit is a unit-vector of the space $\mathbb{C}^2$, a pure state representing a complex piece of information can be identified with a unit-vector of the n -fold tensor product of $\mathbb{C}^2$:

$$\otimes^n \mathbb{C}^2 = \underbrace{\mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2}_{n\text{-times}} \text{ (with } n \geq 1\text{)}.$$

Such vectors are called *quregisters*. The canonical basis of the space $\otimes^n \mathbb{C}^2$ consists of all *registers*, products of bits that have the following form:

$$|x_1\rangle \otimes \dots \otimes |x_n\rangle \text{ (where any } x_i \text{ is either 0 or 1)}.$$

Instead of $|x_1\rangle \otimes \dots \otimes |x_n\rangle$, it is customary to write $|x_1, \dots, x_n\rangle$. Any quregister can be represented as a superposition of registers:

$$|\psi\rangle = \sum_i c_i |x_{i_1}, \dots, x_{i_n}\rangle,$$

109 where c_i are complex numbers such that $\sum_i |c_i|^2 = 1$.

110 A tensor product $|\psi_1\rangle \otimes \dots \otimes |\psi_n\rangle$ (of n quregisters) is often briefly indicated by: $|\psi_1\rangle \dots |\psi_n\rangle$.

⁶ The basic property of the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ of two (finite-dimensional) Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ is the following: $\mathcal{H}_1 \otimes \mathcal{H}_2$ is a Hilbert space that properly includes an isomorphic image of the Cartesian product $\mathcal{H}_1 \times \mathcal{H}_2$ (consisting of all ordered pairs of vectors that belong to the spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively). Furthermore, $\mathcal{H}_1 \otimes \mathcal{H}_2$ contains all possible superpositions of its elements. A vector $|\psi\rangle$ of $\mathcal{H}_1 \otimes \mathcal{H}_2$ is called *factorized* iff $|\psi\rangle$ corresponds to a pair $(|\psi_1\rangle, |\psi_2\rangle) \in \mathcal{H}_1 \times \mathcal{H}_2$. In such a case, it is customary to write: $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$. Of course, not all vectors of $\mathcal{H}_1 \otimes \mathcal{H}_2$ are factorized.

Quantum computation makes essential use of some characteristic quantum states that are called *entangled*. In order to illustrate the concept of entanglement from an intuitive point of view, let us refer to a simple paradigmatic case. We are concerned with a composite physical system S consisting of two subsystems S_1 and S_2 (say, a two-electron system). By the quantum-theoretic rules that concern the mathematical description of composite systems, all states of S shall live in the tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Hilbert spaces associated to the systems S_1 and S_2 , respectively. The observer has a *maximal information* about S : a *pure state* $|\psi\rangle$ of $\mathcal{H}$. What can be said about the states of the two subsystems? Due to the form of $|\psi\rangle$, such states cannot be pure: they are represented by two identical *mixtures*, which codify a “maximal degree of uncertainty”. A typical possible form of $|\psi\rangle$ is the following *Bell-state*:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0, 0\rangle + |1, 1\rangle),$$

111 which lives in the space $\mathbb{C}^2 \otimes \mathbb{C}^2$, whose canonical basis consists of the four vectors
 112 $|0, 0\rangle, |0, 1\rangle, |1, 0\rangle, |1, 1\rangle$.

113 This gives rise to the following physical interpretation: the global system S might satisfy the properties
 114 that are *certain* either for the state $|0, 0\rangle$ or for the state $|1, 1\rangle$ with probability-value $\frac{1}{2}$. At the same time,
 115 $|\psi\rangle$ determines that the *reduced state* of both subsystems (S_1 and S_2) is the mixture $\frac{1}{2}\mathbf{I}$. Although it is
 116 not determined whether the state of the global system S is $|0, 0\rangle$ or $|1, 1\rangle$, the two subsystems S_1 and S_2
 117 can be described as “entangled”, because in both possible cases they would satisfy the same properties,
 118 turning out to be *indistinguishable*. As a consequence, any measurement performed by an observer either
 119 on system S_1 or on system S_2 would instantaneously transform the *potential* properties of both subsystems
 120 into *actual* properties (by *collapse of the wave-function*).

121 The celebrated “Einstein-Podolsky-Rosen paradox” (*EPR*) is based on a similar physical situation. As
 122 is well known, what mainly worried Einstein was the possibility of “non-local effects”: the subjective
 123 decision of an observer (who may choose among different *incompatible observables* to be measured on the
 124 system S_1) seems to determine the instantaneous emergence of an actual property for the system S_2 , which
 125 might be very “far” from S_1 (possibly inaccessible by means of a light-signal). Interestingly enough, in
 126 the framework of quantum computation, entangled states have been often used as a powerful resource,
 127 even from a technological point of view (for instance, in the applications to teleportation-phenomena and
 128 to quantum cryptography).

129 As expected, quantum computation cannot be identified with a “static” representation of pieces of infor-
 130 mation. What is important is the dynamic *process* of information that gives rise to quantum computations.
 131 Such process is mathematically performed by *quantum logical gates* (briefly, *gates*): special examples of
 132 *unitary operators* that transform quregisters into quregisters in a reversible way. Since in quantum theory
 133 the time-evolution of all physical systems is mathematically described by unitary operators, one can say
 134 that quantum computations can be regarded as the time-evolution of some special quantum objects.

135 We will now introduce the definition of *quantum state machine*, which represents a quantum counter-
 136 part of the classical notion of *deterministic state machine*. From an intuitive point of view, any quantum
 137 state machine can be regarded as a kind of quantum superposition of many classical deterministic state
 138 machines. Some definitions of *quantum Turing machine* discussed in the literature are based on a strong
 139 idealization: no limit is assumed for the length of the registers occurring in a computation. This corre-
 140 sponds to the classical assumption according to which a Turing machine is equipped with an infinite tape.
 141 We will consider a more realistic concept, closer to physical quantum computers, which are of course
 142 always bound to a limited memory.

143 **Definition 5.** *Quantum state machine.*

A *quantum state machine* is an abstract system $\mathbf{QM}$ associated to a (finite-dimensional) Hilbert space $\mathcal{H}^{\mathbf{QM}}$ whose unit-vectors $|\psi\rangle$ represent possible pure states of a quantum system that could physically

implement the computations of the state machine. The space $\mathcal{H}^{\text{QM}}$ has the following form:

$$\mathcal{H}^{\text{QM}} = \mathcal{H}^H \otimes \mathcal{H}^S \otimes \mathcal{H}^W.$$

144 The following conditions are required:

- 145 1. $\mathcal{H}^H$ (which represents the halting-space) is the space $\mathbb{C}^2$, where the two elements of the canonical
146 basis ($\{|0\rangle_H, |1\rangle_H\}$) correspond to the states “the machine does not halt” and “the machine halts”,
147 respectively.
- 148 2. $\mathcal{H}^S$ (which represents the internal-state space) is associated to a finite set $\mathcal{S}$ of classical internal states.
149 We require that $\mathcal{H}^S = \otimes^m \mathbb{C}^2$, where 2^m is the cardinal number of $\mathcal{S}$. Accordingly, the set $\mathcal{S}$ can be
150 one-to-one associated to a basis of $\mathcal{H}^S$.
- 151 3. $\mathcal{H}^W$ (which represents the word-space) is identified with a Hilbert space $\otimes^n \mathbb{C}^2$ (for a given $n \geq 1$).
152 The number n determines the length of the registers $|x_1, \dots, x_n\rangle$ that may occur in a computation.
153 Shorter registers $|x_1, \dots, x_h\rangle$ (with $h < n$) can be represented in the space $\otimes^n \mathbb{C}^2$ by means of
154 convenient *ancillary bits*.

Let B^{QM} be a basis of $\mathcal{H}^{\text{QM}}$, whose elements are unit-vectors having the following form:

$$|\varphi_i\rangle = |h_i\rangle |s_i\rangle |x_{i_1}, \dots, x_{i_n}\rangle,$$

155 where $|h_i\rangle$ belongs to the basis of $\mathcal{H}^H$, while $|s_i\rangle$ belongs to the basis of $\mathcal{H}^S$.

Any unit-vector $|\psi\rangle$ of $\mathcal{H}^{\text{QM}}$ that is a superposition of basis-elements $|\varphi_i\rangle$ represents a possible *computational state* of QM. The expected interpretation of a computational state

$$|\psi\rangle = \sum_i c_i |h_i\rangle |s_i\rangle |x_{i_1}, \dots, x_{i_n}\rangle$$

156 is the following:

- 157 • the machine in state $|\psi\rangle$ might halt with probability $|c_i|^2$ (if $|h_i\rangle = |1_H\rangle$) or with probability
158 $1 - |c_i|^2$ (if $|h_i\rangle = |0_H\rangle$).
- 159 • the machine in state $|\psi\rangle$ might correspond to the classical configuration $(s_i, (x_{i_1}, \dots, x_{i_n}))$ with
160 probability $|c_i|^2$.
- 161 Hence, the state $|\psi\rangle$ describes a kind of *quantum co-existence* of different classical deterministic
162 configurations.

4. The set of *possible inputs* of QM is identified with the set of all computational states that have the following form:

$$|\psi\rangle = \sum_i |0_H\rangle |s_{in}\rangle |x_{i_1}, \dots, x_{i_n}\rangle.$$

5. Like a deterministic state machine, a quantum state machine QM is characterized by a *program*. In the quantum case, a program is identified with a sequence of unitary operators of $\mathcal{H}^{\text{QM}}$:

$$(U_0, \dots, U_t),$$

163 where we may have: $U_i = U_j$ with $i \neq j$.

164 The following conditions are required:

- a) for any possible input $|\psi_0\rangle$, $U_0(|\psi_0\rangle) = |\psi_1\rangle$ is a superposition of basis-elements having the following form:

$$|h_i^1\rangle |s_i^1\rangle |x_{i_1}^1, \dots, x_{i_n}^1\rangle,$$

165 where all s_i^1 are different from s_{in} and $|h_i^1\rangle = |0_H\rangle$, if $t \neq 0$.

- b) For any j ($0 < j < t$), $U_j(|\psi_j\rangle) = |\psi_{j+1}\rangle$ is a superposition of basis-elements having the following form:

$$|0_H\rangle |s_i^{j+1}\rangle |x_{i_1}^{j+1}, \dots, x_{i_n}^{j+1}\rangle.$$

- c) $U_t(|\psi_t\rangle) = |\psi_{t+1}\rangle$ is a finite superposition of basis-elements having the following form:

$$|1_H\rangle |s_{halt_j}\rangle |x_{i_1}^{t+1}, \dots, x_{i_n}^{t+1}\rangle.$$

166 The concept of *computation* of a quantum state machine can be now defined in a natural way.

167 **Definition 6.** *Computation of a quantum state machine.*

Let **QM** be a quantum state machine, whose program is the operator-sequence $(U_0, \dots, U_t)$ and let $|\psi_0\rangle$ be a possible input of **QM**. A *computation* of **QM** with input $|\psi_0\rangle$ is a sequence of computational states of **QM**

$$\mathcal{QC} = (|\psi_0\rangle, \dots, |\psi_{t+1}\rangle),$$

168 such that: $|\psi_{i+1}\rangle = U_i(|\psi_i\rangle)$, for any i ($0 \leq i \leq t$).

169 The vector $|\psi_{t+1}\rangle$ represents the output of the computation, while the density operator $Red^3(|\psi_{t+1}\rangle)$ (the
170 reduced state of $|\psi_{t+1}\rangle$ with respect to the third subsystem) represents the word-output of the computation.

171 Like all abstract notions of *quantum computer*, the concept of *quantum state machine* gives rise to some
172 critical questions that have been often discussed in the literature. Two important problems (which cannot
173 have any counterpart in the case of classical computation) are the following:

- 174 • How shall we interpret the operation of “reading the output” of a computation of a given machine?
- 175 What is the role of the *collapse of the wave-function* during a reading-action?
- 176 • Is it possible to measure the halting state without disturbing the configuration-state?

Consider now a quantum state machine whose program is

$$(U_0, \dots, U_t).$$

Each U_i naturally determines a corresponding word-operator U_i^W , defined on the word-space $\mathcal{H}^W$. Generally, it is not guaranteed that all word-operators are unitary. But it is convenient to refer to quantum state machines that satisfy this condition. In this way, any quantum state machine (whose word-space is $\otimes^n \mathbb{C}^2$) determines a *quantum circuit*, consisting of a sequence of unitary operators (*gates*):

$$(U_0^W, \dots, U_t^W),$$

177 where n represents the *width*, while $t + 1$ represents the *depth* of the circuit.

178 To what extent can quantum state machines be simulated by classical probabilistic state machines? In
179 order to discuss this important question, let us refer to a celebrated quantum experiment, based on the
180 *Mach-Zehnder interferometer* (represented by Figure 2).

181 The physical situation can be sketched as follows. Consider a photon-beam (possibly consisting of a
182 single photon) and assume that $|0\rangle$ describes the state of photons moving along the x direction, while
183 $|1\rangle$ describes the state of photons moving along the y direction. All photons go through a first *beam*
184 *splitter* that “splits” them giving rise to the following effect: within the box each photon follows a path
185 corresponding either to the x -direction or to the y -direction with probability $\frac{1}{2}$. Soon after, on both paths,
186 all photons are reflected by a *mirror* that inverts their direction. Finally, the photons pass through a second
187 beam splitter that determines the output-state. Suppose that all photons entering into the interferometer-
188 box are moving in the x -direction. According to a “classical way of thinking” we would expect that the

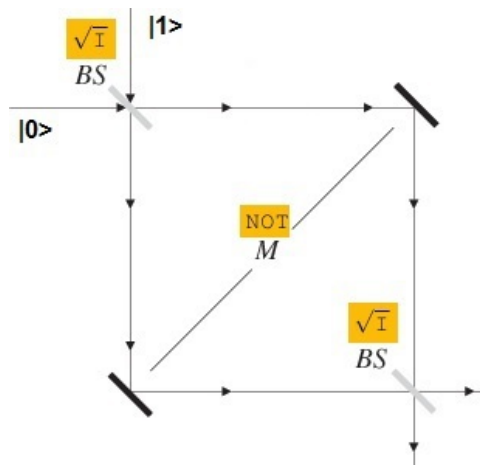


Figure 2. The Mach-Zehnder interferometer.

189 photons detected at the end of the process will move either along the x -direction or along the y -direction
 190 with probability $\frac{1}{2}$. The result of the experiment is, instead, completely different: the Mach-Zehnder
 191 interferometer *always* transforms the input-state $|0\rangle$ into the output-state $|0\rangle$; while the input-state $|1\rangle$ is
 192 transformed into $|1\rangle$.

From a mathematical point of view, such a “surprising” result can be explained by using, in an essential way, the concept of superposition. The apparatuses (used in the Mach-Zehnder experiment) can be mathematically represented by two important gates. A beam splitter can be regarded as a physical implementation of the *Hadamard-gate* $\sqrt{I}$ (also called *square root of identity*), which is defined as follows (on the canonical basis of $\mathbb{C}^2$):

$$\sqrt{I}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle); \quad \sqrt{I}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

193 Apparently, the Hadamard-gate transforms the two classical bits $|0\rangle$ and $|1\rangle$ into two (different) genuine
 194 superpositions. As a consequence, within the Mach-Zehnder box a photon in state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ turns out
 195 to satisfy at the same time two alternative properties: the property of moving along the x -direction and
 196 the property of moving along the y -direction. We have here a characteristic quantum parallelism: a single
 197 photon “goes along” two different paths at the same time! Metaphorically, situations of this kind have
 198 been sometimes compared to the puzzling behavior of a “quantum skier” who runs at the same time on
 199 the left and on the right side of a given tree (see Figure 3).

200 The second apparatus of the Mach-Zehnder interferometer (the mirror), can be regarded as a physical
 201 implementation of another important gate, the *negation* NOT (a quantum generalization of the classical
 202 negation), which is defined as follows:

$$\text{NOT}|0\rangle = |1\rangle; \quad \text{NOT}|1\rangle = |0\rangle.$$

Accordingly, the *Mach-Zehnder circuit* can be identified with the following sequence of three gates (all defined on the space $\mathbb{C}^2$):

$$(\sqrt{I}, \text{NOT}, \sqrt{I}).$$

203 Let us now apply the Mach-Zehnder circuit to the input $|0\rangle$. We obtain:

$$204 \quad \sqrt{I} : |0\rangle \mapsto \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle); \quad \text{NOT} : \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \mapsto \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle); \quad \sqrt{I} : \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \mapsto |0\rangle.$$



Figure 3. The quantum skier.

205 We can see, in this way, how the Mach-Zehnder circuit transforms the input-state $|0\rangle$ into the output-state
 206 $|0\rangle$. In a similar way, the input-state $|1\rangle$ is transformed into the output-state $|1\rangle$.

207 Is there any natural “classical counterpart” for the Hadamard-gate? A natural candidate might be a par-
 208 ticular example of a probabilistic state machine that we can conventionally call *the classical probabilistic*
 209 *NOT-state machine* (PM^{NOT}). Such machine can be defined as follows:

- 210 • The set of possible word-inputs of PM^{NOT} is the set of words $\{(0), (1)\}$.
 • The program of PM^{NOT} consists of the following sequence of rules:

$$\text{Seq}_0 = (R_{0_1}, R_{0_2}),$$

211 where:

212 $R_{0_1} : (s_{in}, (x)) \mapsto (s_{halt_j}, (x))$ and $p(R_{0_1}) = \frac{1}{2}$;

213 $R_{0_2} : (s_{in}, (x)) \mapsto (s_{halt_j}, (1 - x))$ and $p(R_{0_2}) = \frac{1}{2}$.

Consider, for instance, the input $(s_{in}, (0))$. The output will be the following set:

$$\{(s_{halt_j}, (0)), (s_{halt_j}, (1))\}.$$

214 On this basis, a “classical probabilistic Mach-Zehnder state machine” would determine (for the word-input
 215 (0)) the word-graph illustrated by Figure 4.

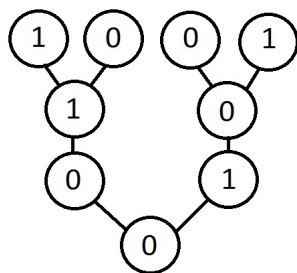


Figure 4. A word-graph for a “classical probabilistic Mach-Zehnder state machine”.

Such a machine turns out to compute both the words (0) and (1) with probability $\frac{1}{2}$. Interestingly enough, this is the same probabilistic result that is obtained in the quantum case, when one performs a measurement inside the interferometer-box. In such a case, photons behave like “normal skiers”, who pass *either* at the right *or* at the left side of a tree (where *or* represents here, of course, the exclusive disjunction).

The arguments we have developed seem to confirm the following conjecture: the characteristic superposition-patterns, that may occur during a quantum computation (when no measurement is performed during the computation-process), cannot be generally represented by probabilistic state machines. Quantum parallelism (based on superpositions) and classical parallelism are deeply different.

4 QUANTUM PARALLELISM, PSYCHOLOGICAL PARALLELISM AND QUANTUM COMPUTATIONAL SEMANTICS

What kind of similarity can be recognized between quantum parallel structures and different forms of psychological parallelism? Trying to represent the human mind as a kind of system of quantum state machines would be, of course, naive and misleading. In spite of many important results in the framework of neurosciences, the complex network that connects human conscious and unconscious thoughts is still quite mysterious.⁷ Quantum-like superpositions can be reasonably applied to represent some aspects of such complex networks. Even quantum *interference phenomena* (with the characteristic constructive and destructive effects) can find some natural psychological interpretations.

According to an interesting hypothesis (discussed by the neuroscientist Edoardo Boncinelli⁸), the mysterious emergence of an *act of consciousness* can be represented as a sudden transition from a parallel structure to a linear one. Is it reasonable to conjecture that such transition could be described as a kind of “psychological collapse of the wave-function”?

In the investigations about possible links between *quantum structures* and *psychological structures* a useful tool is represented by a special form of quantum logical semantics (called *quantum computational semantics*) that has been naturally inspired by the theory of quantum computation.⁹

Let us briefly recall the basic ideas of this semantics. We can refer a first-order language $\mathcal{L}$, whose non-logical alphabet contains individual terms (variables and names), predicates and sentential constants. Interpreting the language $\mathcal{L}$ means associating to any formula α a *meaning*, identified with a piece of quantum information that can be stored by a quantum system. Accordingly, any possible meaning of α is represented by a possible (pure or mixed) state of a quantum system: generally, a density operator ρ_α that lives in a Hilbert space $\mathcal{H}^\alpha$, whose dimension depends on the linguistic complexity of α .

The logical operators of $\mathcal{L}$ are associated to special examples of Hilbert-space operations that have a characteristic dynamic behavior, representing possible computation-actions. The logical connectives are interpreted as particular (reversible) gates, like the negation NOT, the Hadamard-gate $\sqrt{I}$, the Toffoli-gate T (which allows us to define a reversible conjunction AND). At the same time, the logical quantifiers ($\forall$, $\exists$) are interpreted as possibly irreversible quantum operations. Since the universe of discourse (which the language refers to) may be indeterminate, the use of quantum quantifiers may give rise to a reversibility-breaking, which is quite similar to what happens in the case of measurement-phenomena.

Due to the characteristic features of quantum holism, meanings turn out to behave in a *holistic* and *contextual* way: the density operator ρ_α (which represents the *global* meaning of a formula α) determines the *contextual meanings* of all parts of α (which can be obtained by applying the *reduced-state function* to ρ_α). As a consequence, it may happen that the meaning of a formula is an entangled pure state, while the

⁷ As is well known, the literature devoted to the study of parallel structures in the mind/brain-behavior is very rich. As an example, one can refer to some important contributions of A. Damasio (see, for instance, Damasio, 1999).

⁸ See Boncinelli, 2012.

⁹ See Dalla Chiara et al., 2005, Dalla Chiara et al., 2010, Dalla Chiara et al. (in press) .

meanings of its parts are proper mixtures. In such cases, the meaning of a global expression turns out to be more precise than the meanings of its parts. It is also admitted that one and the same formula receives different contextual meanings in different contexts.

As an example, consider the atomic sentence “Alice is pretty” (formalized as $\mathbf{Pa}$). In order to store the information expressed by this sentence, we need three quantum objects whose states represent the pieces of information corresponding, respectively, to the predicate $\mathbf{P}$, to the name $\mathbf{a}$ and to the truth-degree according to which the individual denoted by the name $\mathbf{a}$ satisfies the property denoted by the predicate $\mathbf{P}$. Accordingly, the meaning of the sentence $\mathbf{Pa}$ can be identified with a (pure or mixed) state $\rho_{\mathbf{Pa}}$ living in the tensor-product space $\mathcal{H}^{\mathbf{Pa}} = \otimes^3 \mathbb{C}^2$. In order to obtain the contextual meanings of the linguistic parts of $\mathbf{Pa}$ it is sufficient to consider the two reduced states $\text{Red}^1(\rho_{\mathbf{Pa}})$ and $\text{Red}^2(\rho_{\mathbf{Pa}})$, which describe (respectively) the states of the first and of the second subsystem of the quantum object that stores the information expressed by the sentence $\mathbf{Pa}$. From a logical point of view, $\text{Red}^1(\rho_{\mathbf{Pa}})$ and $\text{Red}^2(\rho_{\mathbf{Pa}})$ can be regarded as two *intensional meanings*: a property-concept and an individual concept, respectively; while $\rho_{\mathbf{Pa}}$ represents a *propositional concept* (or *event*).

Like formulas, sequences of formulas also can be interpreted according to the quantum computational rules. As expected, a possible meaning of the sequence $(\alpha_1, \dots, \alpha_n)$ will be a density operator $\rho_{(\alpha_1, \dots, \alpha_n)}$ living in a Hilbert space $\mathcal{H}^{(\alpha_1, \dots, \alpha_n)}$, whose dimension depends on the linguistic complexity of the formulas $\alpha_1, \dots, \alpha_n$.

In this framework one can develop an abstract theory of *vague possible worlds*. Consider a pair

$$W = ((\alpha_1, \dots, \alpha_n), \rho_{(\alpha_1, \dots, \alpha_n)}),$$

consisting of a sequence of formulas and of a density operator that represents a possible meaning for our sequence. It seems reasonable to assume that W describes a *vague possible world*, a kind of *abstract scene* where most events are characterized by a “cloud of ambiguities”, due to quantum uncertainties. In some cases W might be exemplified as a “real” scene of a theatrical play or as a vague situation that is described either in a novel or in a poem. And it is needless to recall how ambiguities play an essential role in literary works.

As an example, consider the following vague possible world:

$$W = ((\mathbf{Pab}), \rho_{\mathbf{Pab}}),$$

where $\mathbf{Pab}$ is supposed to formalize the sentence “Alice is kissing Bob”, while $\rho_{\mathbf{Pab}}$ corresponds to the pure state

$$|\Psi\rangle_{\mathbf{Pab}} = |\varphi\rangle \otimes \frac{1}{\sqrt{2}}(|0, 1\rangle + |1, 0\rangle) \otimes |1\rangle,$$

where $|\varphi\rangle$ lives in the space $\mathbb{C}^2$, while $|\Psi\rangle_{\mathbf{Pab}}$ lives in the space $\otimes^4 \mathbb{C}^2$. Here the reduced state of $|\Psi\rangle_{\mathbf{Pab}}$ that describes the pair (Alice, Bob) has the typical form of an entangled state; consequently, the states describing the two individuals Alice and Bob are two identical mixed states. In the context $|\Psi\rangle_{\mathbf{Pab}}$ Alice and Bob turn out to be indistinguishable: it is not determined “who is who” and “who is kissing whom”. It is not difficult to imagine some “real” theatrical scenes representing ambiguous situations of this kind.

5 A QUANTUM SEMANTICS FOR MUSIC

An abstract version of the quantum computational semantics can be applied to a formal analysis of musical compositions, where both *musical ideas* and *extra-musical meanings* are generally characterized by some essentially vague and ambiguous features.¹⁰

¹⁰ See Dalla Chiara et al., 2012.

287 Any musical composition (say, a sonata, a symphony, an opera,...) is, generally, determined by three
 288 elements:

- 289 • a *score*;
- 290 • a set of *performances*;
- 291 • a set of musical *thoughts* (or *ideas*), which represent possible *meanings* for the *musical phrases*
 292 written in the score.

293 While scores represent the syntactical component of musical compositions, performances are physical
 294 events that occur in space and time. From a logical point of view, we could say that performances are,
 295 in a sense, similar to *extensional meanings*, i.e. well determined systems of objects which the linguistic
 296 expressions refer to.

297 Musical thoughts (or ideas) represent, instead, a more mysterious element. Is it reasonable to assume
 298 the existence of such ideal objects that are, in a sense, similar to the *intensional meanings* investigated
 299 by logic? Is there any danger to adhere, in this way, to a form of *Platonism*? When discussing semantic
 300 questions, one should not be “afraid” of Platonism. In the particular case of music, a composition cannot
 301 be simply reduced to a score and to a system of sound-events. Between a score (which is a system of
 302 signs) and the sound-events created by a performance there is something intermediate, represented by the
 303 musical ideas that underlie the different performances. This is the abstract environment where normally
 304 live both composers and conductors, who are accustomed to study scores without any help of a material
 305 instrument.

Following the rules of the quantum semantics, *musical ideas* can be naturally represented as superposi-
 tions that ambiguously describe a variety of co-existent thoughts. Accordingly, we can write:

$$|\mu\rangle = \sum_i c_i |\mu_i\rangle,$$

306 where:

- 307 • $|\mu\rangle$ is an abstract object representing a musical idea that *alludes* to other ideas $|\mu_i\rangle$ (possible *variants*
 308 of $|\mu\rangle$ that are, in a sense, all co-existent);
- 309 • the number c_i measures the “weigh” of the component $|\mu_i\rangle$ in the context $|\mu\rangle$.

310 As happens in the case of composite quantum systems, musical ideas (which represent possible mea-
 311 nings of *musical phrases* written in a score) have an essential *holistic* behavior: the meaning of a global
 312 musical phrase determines the *contextual meanings* of all its parts (and not the other way around).

313 An important feature of music is the capacity of *evoking* extra-musical meanings: subjective feelings,
 314 situations that are vaguely imagined by the composer or by the interpreter or by the listener, real or virtual
 315 theatrical scenes (which play an essential role in the case of lyric operas and of *Lieder*). The interplay
 316 between musical ideas and extra-musical meanings can be naturally represented in the framework of
 317 our quantum semantics, where extra-musical meanings can be dealt with as special examples of vague
 318 possible worlds.

We can refer to the abstract tensor product of two spaces

$$MSpace \otimes WSpace,$$

319 where:

- 320 • $MSpace$ represents the space of musical ideas $|\mu\rangle$.

- 321 • $WSpace$ represents the space of vague possible worlds, dealt with as special examples of abstract
 322 objects $|w\rangle$ that can be evoked by musical ideas.

Following the quantum-theoretic formalism, we can distinguish between *factorized* and *non-factorized* global musical ideas. A factorized global musical idea will have the form:

$$|M\rangle = |\mu\rangle \otimes |w\rangle.$$

But we might also meet entangled global musical ideas, having the form:

$$|M\rangle = c_1(|\mu_1\rangle \otimes |w_1\rangle) + c_2(|\mu_2\rangle \otimes |w_2\rangle).$$

- 323 As is well known, music gives rise to a special kind of psychological experience, where some complex
 324 parallel structures are consciously grasped, in a way that may appear miraculous. Paradigmatic examples
 325 arise, for instance, in the case of trios or quartets of lyric operas. In such cases, the listener perceives a
 326 *global polyphonic structure*; at the same time, he/she is able to follow (at least to a certain extent) the
 327 different melodic lines and even the different thoughts and feelings of the characters who are singing. As
 328 an example, it may be interesting to consider three great masterpieces of the history of lyric operas: the
 329 quartet of Act 1 in Beethoven's *Fidelio*, the quartet of Act 3 in Verdi's *Rigoletto* and the trio of Act 3
 330 of *Der Rosenkavalier* by Richard Strauss. The parallel structures that arise in these three examples have
 331 some significant differences both from the musical and from the semantic point of view.

- 332 In *Fidelio*'s quartet the psychological contraposition between the four characters (Marzelline, Leonore,
 333 Rocco, Jaquino) is realized by means of a single musical theme that is successively sung by the four
 singers (Figure 5).

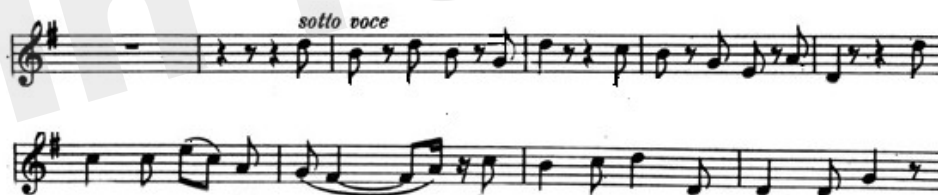


Figure 5. The quartet-theme.

- 334 It is amazing how Beethoven succeeds in expressing, by one and the same theme, different attitudes and
 335 emotions: the joyful hope of Marzelline, the doubts and the anguish of Leonore, the paternal satisfaction
 336 of Rocco, the jealous rage of Jaquino. The whole context is dominated by strong ambiguities and anta-
 337 gonistic elements: the contrast between an improbable family-portrait and the cruel jail-environment, the
 338 contradictions of Rocco (who is at the same time a fond father and an accomplice of the prison-system),
 339 the sexual ambiguity of Leonore, the loving heroin who has disguised herself as a man (*Fidelio*), in the
 340 attempt to save her husband, the prisoner Florestan. The musical result is an extraordinary and highly
 341 emotional polyphonic construction based on very simple musical components.

- 343 The structure of *Rigoletto*'s quartet is completely different. All characters are associated to specific
 344 musical themes that are repeated with some variations. The leading musical idea is represented by the
 345 wonderful theme sung by the Duke of Mantova at the very beginning (Figure 6).¹¹

¹¹ *Fairest daughter of love, I am a slave of your charms; with but a single word you could relieve my every pain. Come touch my breast and feel how my heart is racing. With but a single word you could relieve my every pain.*



Figure 6. The Duke.

346 Like Mozart's *Don Giovanni*, Verdi's Duke is a cynic seducer, who may appear sweet and sincere with
 347 his victims. And music often exalts a paradoxical co-existence of contradictory psychological attitudes.
 348 All contrasts are emphasized in the quartet by the sordid environment, where a crime is going to be
 349 committed. Maddalena's answer to the Duke is based on a fully different theme, a *staccato*-sequence of
 sixteenth-notes (Figure 7).¹²



Figure 7. Maddalena.

350

351 Both the music and the text reflect Maddalena's ambiguity: she is a prostitute who is playing a traditional
 352 seductive role; at the same time she is also instrumental to a murder-project. Gilda's entrance (soon after

¹² Ah! Ah! That really makes me laugh, talk like that is cheap enough.

353 Maddalena's first phrase) determines a sudden dramatic change. What Gilda sings is a cry of sorrow,
 354 interrupted by some short pauses and *appoggiaturas* that seem to describe desperate sobs (Figure 8).¹³



Figure 8. Gilda.

354

355 One has often discussed the reasons that may have led Gilda to her unreasonable sacrifice for an unwor-
 356 thy man who had deceived her. Representing Gilda as a naive and modest girl is, however, misleading and
 357 in contrast with the greatness expressed by the music. Gilda's death-choice can be perhaps better under-
 358 stood as a suicide, caused by an unendurable disillusion. Rigoletto's role in the quartet is musically less
 359 "visible". His mind is completely absorbed in the vengeance-project ("la vendetta") that shall be shortly
 360 accomplished. From a musical point of view, the quartet is constructed as a polyphonic structure, where
 361 the four voices are interlaced, each preserving its own musical, semantic and psychological autonomy.

¹³ Ah, these are the loving words the scoundrel spoke once to me! O wretched heart betrayed, do not break of sorrow.

362 *Der Rosenkavalier* by Strauss belongs to a musical and literary world that is somewhat far both from
 363 *Fidelio* and from *Rigoletto*. Different forms of ambiguity are exalted in this opera, which is characterized
 364 by an extraordinary unity of music and text, written by the great poet Hugo von Hofmannsthal. The theme
 365 of sexual ambiguity is here developed by the character of Octavian, the *Rosenkavalier* whose role is sung
 366 by a mezzo-soprano. Although Octavian may recall Mozart's Cherubino, ambiguities are in Strauss' opera
 367 more sophisticated: in two different situations Octavian disguises himself as a woman in order to make
 368 fun of the rude fiancé of the fascinating girl Sophie. Interestingly enough, some interpreters of the role
 369 of Octavian have told how often they have been puzzled by their "oscillating identity" during the opera's
 370 performance.

371 A different and deeper "identity-question" is evoked in a splendid aria sung by the *Marschallin* in Act
 372 1. After a passionate night spent with her lover Octavian, the lady is troubled by some sad thoughts about
 373 the flowing of time and the mysterious co-existence of different identities of one and the same person in
 374 different stages of life. She sings:

375 *Aber wie kann das wirklich sein,*
 376 *dass ich die kleine Resi war,*
 377 *und dass ich einmal die alte Frau sein werd'*
 378
 379 *Wie kann denn das geschehen?*
 380 *Wie macht denn das der liebe Gott?*
 381 *Wo ich doch immer die gleiche bin.*
 382 *Und wenn er's schon so machen muss,*
 383 *warum lasst er mich denn zuschaun dabei*
 384 *mit gar so klarem Sinn?*
 385 *Warum versteckt er's nicht vor mir?*
 386 *Das alles ist geheim, so viel geheim.*¹⁴

387 One is dealing with an extraordinary poetic and musical representation of a "hard" scientific and
 388 philosophical problem, that modern philosophers of science usually call "the genidentity-question".¹⁵

389 The trio performed at the end of the opera by three female voices (the *Marschallin*, Sophie, Octavian) is
 390 a wonderful polyphonic construction, where the three characters express different thoughts and feelings,
 391 which are not generally associated to some specific musical themes (unlike the case of *Rigoletto*'s quartet).
 392 The main theme is sung at the very beginning by the *Marschallin* (Figure 9).¹⁶

393 By this deeply moving musical phrase the *Marschallin* expresses her extreme act of love, which is to
 394 renounce love. Her choice might recall what Violetta Valery sings in Verdi's *La Traviata*:

395 *Dite alla giovine sì bella e pura*¹⁷

¹⁴ But how can it be that I was the little Resi and that I shall be the old lady. How can it come to pass? How can God decree it so? While, in fact, I am always the same. And if indeed it must be so, why does he let me look at it so clearly? Why does he not hide it to me? All this is a mystery, a great mystery.

¹⁵ The term "genidentity", which refers to the problematic *identity of individuals through time* has been introduced by Kurt Lewin (in his doctoral thesis in 1922) and has been further investigated by Hans Reichenbach and many other scholars. See, for instance, Reichenbach, 1928.

¹⁶ I promised to love him in the right way, even to love his love for another woman.

¹⁷ Tell the beautiful and pure girl



Figure 9. The Marschallin.

396 although Violetta and the Marschallin are, of course, completely different characters.

397 Sophie's entrance in the trio is, in a sense, surprising. She joins in, in the final part of the Marschallin's
 398 first phrase, just upon the critical word "andern" ("other"). Her intervention creates a sudden brief disso-
 399 nance (a minor-second chord), which immediately disappears when the two women (who are both in love
 400 with Octavian) harmonically conclude the phrase at a distance of a minor-third. What Sophie perceives
 401 is a strange religious atmosphere that she cannot really understand, since she is not aware of the *liason*
 402 between Octavian and the Marschallin. The *incipit* of the main theme (the characteristic imprinting of the
 403 whole trio) is then immediately transposed to a different key (from *D* flat major to *A* major) by Octavian,
 404 whose initial attitude seems to be mainly dominated by embarrassing doubts and questions. But finally the
 405 reasons of love prevail over all doubts. At the end of the trio, while the two young lovers sing an expected
 406 "dich habe ich lieb" ("I love you"), the Marschallin concludes with an enigmatic phrase:

407 *als wie halt Männer das glücklich sein verstehen.*¹⁸

408 singing the last note alone over a perfect tonic chord.

409 The three examples of polyphonic constructions, created by Beethoven, Verdi and Strauss, are all chara-
 410 cterized by strong unitary conceptions, based on complex parallel networks of harmonic, melodic, timbric
 411 and semantic relationships (which have been extensively analyzed in musicological literature¹⁹). At the
 412 same time, one can easily recognize some significant differences that distinguish the three cases, both
 413 from the musical and from the semantic point of view. The structure of *Fidelio's* quartet is very close to
 414 a *canon-form*, where the entrance of each voice is associated to a specific semantic connotation. Rigo-
 415 letto's quartet is, instead, dominated by strong musical contrasts that reflect the conflicting feelings of
 416 four human beings, living in a highly dramatic situation. Finally, Strauss' trio seems to propose a kind
 417 of musical and semantic "peaceful resolution". The trio is perceived by the listener as a strongly unitary
 418 musical idea that evolves in time. The three female voices are in a sense "entangled", sometimes creating
 419 the illusion that a single voice is singing (as happens in the case of some entangled quantum objects,
 420 whose parts are indistinguishable). Such musical situations can be naturally represented in the framework

¹⁸ as far as men can understand happiness.

¹⁹ See, for instance, Solomon, 1998, Budden, 1983, Principe, 2004.

of the quantum musical semantics, where *musical thoughts* are dealt with as holistic ideal objects that vaguely allude to a (possibly infinite) variety of co-existing ideas.

The analysis proposed in this article has concerned questions that belong to worlds apparently “far apart”: the theory of quantum computers, psychology, logical semantics and music. A common pattern that arises in all these fields is a frequent and sometimes essential emergence of some characteristic parallel structures. We have seen how the quantum-theoretic concepts of *superposition* and *entanglement* have inspired the development of a “bridge-theory” (based on the quantum computational semantics) that can be usefully applied to a formal representation of different kinds of phenomena where parallelism plays a relevant role.

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Figure 1.JPEG

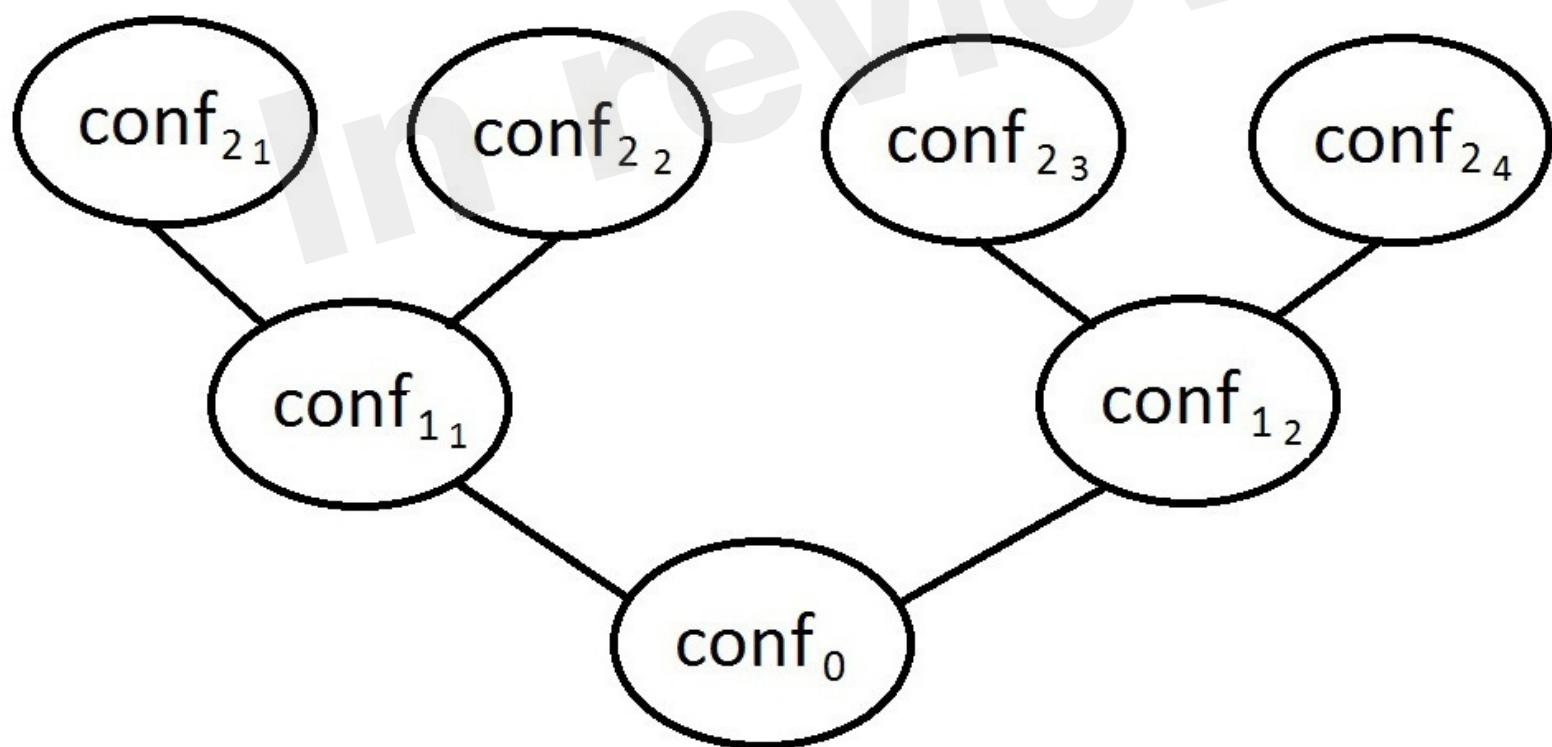


Figure 2.JPEG

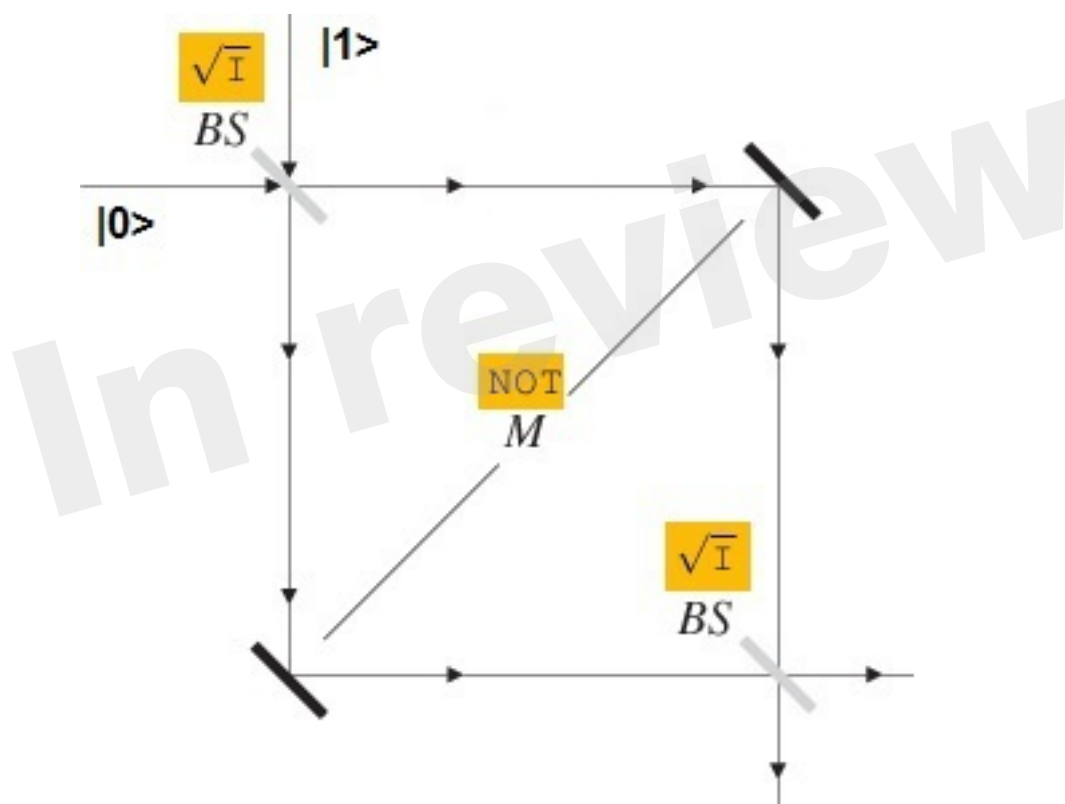


Figure 3.JPEG



Figure 5.JPEG



Figure 6.JPEG

Andante.

Bel-la fi-glia del-la - mo - re, schia - vo
son de' vez - zi tuo - i; con un detto, un det - to sol tu
puo - i le mie pe - ne, le mie pe - ne con - so - lar. Vieni, e
sen-ti del mio co - re il fre - quente pal - pi - tar, con un
detto, un det - to sol tu puo - i le mie pe - ne, le mie pe - ne con - so -
lar.

Figure 7.JPEG



Figure 8.JPEG

A musical score for a vocal piece, likely an aria or duet, in E-flat major (three flats). The score consists of six staves of music. The lyrics are in Italian and are written below the notes. The music features various melodic lines, including some with trills and ornaments. The lyrics are: "Ah! co-sì par-lar d'a -", "mo - re", "a - me pur l'in-fa-me ho u - di - to!", "In - fe -", "li - - - ce cor tra - di - to, per an -", "go - - - scia non scop - piar, no, no, non scop -", and "piar.".

Ah! co-sì par-lar d'a -

mo - re

a - me pur l'in-fa-me ho u - di - to! In - fe -

li - - - ce cor tra - di - to, per an -

go - - - scia non scop - piar, no, no, non scop -

piar.

Figure 9.JPEG

Hab'

p

— mir's ge - lobt, ihn lieb — zu ha - ben in der rich - ti - gen Weis', — daß ich

selbst sein Lieb' zu ei - ner an - - - dern — noch — lieb

hab'!