

On the universality of thermodynamics and η/s ratio for the charged Lovelock black branes

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ABSTRACT: We investigate general features of charged Lovelock black branes by giving a detailed description of geometrical, thermodynamic and holographic properties of charged Gauss-Bonnet (GB) black branes in five dimensions. We show that when expressed in terms of effective physical parameters, the thermodynamic behaviour of charged GB black branes is completely indistinguishable from that of charged Einstein black branes. Moreover, the extremal, near-horizon limit of the two classes of branes is exactly the same as they allow for the *same* $\text{AdS}_2 \times R_3$, near-horizon, *exact* solution. This implies that, although in the UV the associated dual QFTs are different, they flow in the IR to the same fixed point. The calculation of the shear viscosity to entropy ratio η/s confirms these results. Despite the GB dual plasma has in general a non-universal temperature-dependent η/s , it flows monotonically to the universal value $1/4\pi$ in the IR. For negative (positive) GB coupling constant, η/s is an increasing (decreasing) function of the temperature and the flow respects (violates) the KSS bound.

KEYWORDS: Holography and condensed matter physics (AdS/CMT), Black Holes

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Contents

| | | |
|----------|--|-----------|
| 1 | Introduction | 1 |
| 2 | Black brane solutions of Lovelock gravity | 4 |
| 2.1 | Universality of black brane thermodynamics in Lovelock gravity | 5 |
| 3 | 5d Reissner-Nordström black brane solution | 6 |
| 4 | Gauss-Bonnet solution | 7 |
| 4.1 | 5d GB black brane | 7 |
| 4.2 | Singularities | 8 |
| 4.3 | f_- branch | 9 |
| 4.3.1 | Near horizon extremal solution | 12 |
| 4.4 | Near horizon metric as exact solution of equations of motion | 12 |
| 4.5 | f_+ branch | 13 |
| 5 | Charged GB black brane thermodynamics | 14 |
| 5.1 | Large temperature | 15 |
| 5.2 | Small temperature | 15 |
| 5.3 | Excitations near extremality and near-horizon limit | 16 |
| 6 | Shear viscosity to entropy ratio | 17 |
| 6.1 | η/s for the charged GB black brane | 18 |
| 6.2 | η/s in the large and small T_H regime | 19 |
| 6.2.1 | Large T_H | 20 |
| 6.2.2 | Small T_H | 20 |
| 6.3 | η/s in the extremal case | 21 |
| 6.4 | Dependence on the time rescaling factor N | 22 |
| 7 | Summary and outlook | 22 |
| A | The black hole case | 24 |

1 Introduction

Since its first formulation, Lovelock gravity [1] has been a fruitful and widely explored subject [2–5]. The peculiarity of the theory is to be a higher curvature gravity theory with second-order field equations for the metric. This characteristic not only allows to avoid some of the shortcomings of generic higher-derivative theories (such as ghosts in the linearized excitation spectrum and ill-posed Cauchy problem) but also enables us to

derive exact black hole (and black brane) solutions of the theory. As a consequence, the thermodynamics of Lovelock black holes is well known and has several interesting, nontrivial features. One of these features is that the thermal entropy [6, 7] and the holographic entanglement entropy [8] of a black hole depend on the higher-curvature gravitational couplings. It is also well understood that there are in these theories new types of phase transitions that also depend on the value of the gravitational couplings [9–12].

Lovelock gravity is interesting also from the holographic point of view. The higher curvature terms in the action correspond, on the gauge theory side of the AdS/CFT correspondence, to corrections due to finite \mathcal{N} (rank of the gauge group) and finite t’Hooft coupling λ_{tH} . Thus, Lovelock gravity allows to investigate finite \mathcal{N} and λ_{tH} effects without having some of the undesirable features of higher curvature gravity theories.

Among the Lovelock gravity theories, one of the most investigated cases, that will also be the subject of this paper, is the five-dimensional ($5d$) Gauss-Bonnet (GB) theory. Specifically, GB gravity is the 2nd-order Lovelock gravity, i.e., it includes only quadratic curvature corrections in the Einstein-Hilbert action. The main reason to study $5d$ GB in the AdS/CFT framework is that the dual QFT lives in four spacetime dimensions. Hence, $5d$ GB gravity can be used to describe $1/\mathcal{N}$ corrections to relativistic QFTs with a gravitational dual. Particular attention has been devoted to the low-frequency hydrodynamic limit, $\omega, k \ll T$, where ω is the frequency, k is the wavelength number and T is the temperature of the dual thermal QFT. In this limit, the theory describes a sort of “GB plasma” for which transport coefficients can be calculated using the rules of the AdS/CFT correspondence.

A quantity, which plays a distinguished role in the hydrodynamic regime of thermal QFTs with gravitational duals is the shear viscosity to entropy density ratio η/s . It has been shown that η/s attains an *universal* value $1/4\pi$ for all gauge theories with Einstein gravity duals [13–20]. This fact motivated the formulation of a fundamental bound $\eta/s \geq 1/4\pi$, known as Kovton, Son and Starinets (KSS) bound [21, 22], which also found support from energy-time uncertainty principle arguments in the weakly coupled regime [22] and known experimental data for quark-gluon plasma [22, 23]. However, it was soon realized that higher curvature gravity theories may generically violate the bound [24]. This is, in particular, true for GB gravity theories with a positive coupling constant.

Violation of the KSS bound of higher curvature gravity theories can be understood as generated by finite- \mathcal{N} , finite- λ_{tH} effects and traced back to the inequality of the two central charges of the dual QFT [25, 26]. Nevertheless, this does not answer the question about the possible existence of general bounds on η/s lower than the KSS one. The GB gravity, owing to its features, is the most promising playground for trying to answer this question. Progress in this direction has been achieved by imposing causality and positivity of energy to the QFT dual to GB gravity [27–29]. These requirements imply some constraints on the GB coupling parameter, which in turn translate into a bound on η/s lower than the KSS bound [27–29]. However, the hydrodynamic transport coefficients of a theory are expected to be determined by IR physics whereas causality requirements are in the domain of the UV behavior of the dual QFT. The existence of a fundamental bound of the previous kind for the GB plasma would, therefore, imply an interplay between IR and UV physics, whose meaning is presently not clear.

In a parallel, very recent, development it has been shown that generically the KSS bound is violated if translation invariance is broken [30–33, 55, 56]. If translation symmetry is preserved in the IR, η/s tends to a constant as $T \rightarrow 0$, whereas it scales as a positive power of T when translation symmetry is broken. Although the breaking of translation symmetry prevents a purely hydrodynamic interpretation of η , this result strongly indicates that bounds on η/s are completely determined by IR physics and insensitive to the UV regime of the theory.

A promising way to tackling this kind of problems is to consider gravitational backgrounds in which η/s flows as a function of the temperature and for which an IR fixed point exists at $T = 0$. Following this indication, in this paper, we will focus on the charged $5d$ GB black brane solutions (BB) for which it is known that the ratio η/s flows as a function of the temperature [34].

We will start by investigating the general Lovelock BB solution as a thermodynamic system. We will show that, when expressed in terms of effective physical parameters, the thermodynamic behavior of charged Lovelock BB is completely indistinguishable from that of charged Einstein BB. We then proceed by focusing on the $5d$ GB case and investigating in detail the geometrical properties of the charged GB black brane. We show that the theory allows for two branches of solutions continuously connected through a branch-point singularity. Holographically they represent flows between two different CFTs through a singularity. Moreover, we show that at extremality, in the near horizon regime, the charged GB black brane has *exactly the same* $AdS_2 \times R_3$ geometry of the Einstein charged black brane. In fact, in the near horizon regime the contributions of the higher-curvature terms to the field equations vanish and the $AdS_2 \times R_3$ solution of Einstein-Maxwell gravity in $5d$ is also the exact solution of GB gravity in $5d$.

In terms of the dual QFT description this means that, although in the UV the associated dual QFTs for Einstein and GB gravity are different, in the IR they flow to the same fixed point. We then calculate the shear viscosity to entropy ratio η/s for the extremal and non-extremal case, using the simple method recently proposed in refs. [30, 31, 35]. Whereas in the non-extremal case we find a non-universal, monotonically increasing (for negative GB coupling parameter) or decreasing (for positive GB coupling parameter) temperature-dependent expression for η/s , in the extremal case we find the universal value $1/4\pi$. Thus, charged Gauss-Bonnet gives an example of a higher curvature gravity theory in which the IR behaviour of the dual theory respects the universal bound for η/s and is completely independent from the UV regime.

The structure of the paper is as follows. In section 2 we briefly review some relevant features of black brane solutions of Lovelock-Maxwell gravity and show the universality of their thermodynamic behaviour. In section 3 we review the Reissner-Nordström (RN) BB solutions of $5d$ Einstein-Maxwell, including its extremal limit and its $AdS_2 \times R_3$, extremal, near horizon geometry. In section 4 we discuss the charged black brane solution of GB gravity, paying particular attention to the geometry of the solution and the extremal, near horizon regime. In section 5 we discuss the charged GB black brane thermodynamics, and we consider in detail the thermodynamic behaviour at small and large temperature. In section 6 we discuss the shear viscosity to entropy ratio for the GB plasma and compute the

value both for $T \neq 0$ and $T = 0$. We also present a discussion about the large T and small T behaviour. Finally, in section 7 we draw our conclusions. In the appendix A we briefly discuss the black hole solutions of the GB theory, i.e., the solution with spherical horizons.

2 Black brane solutions of Lovelock gravity

Let us consider black branes that are solutions of Lovelock higher curvature gravity in d -dimensional spacetime. To describe the static, electrically charged, radially symmetric AdS Lovelock BB, we use the following line element and electromagnetic (EM) field

$$ds^2 = -f(r) N^2 dt^2 + f(r)^{-1} dr^2 + \frac{r^2}{L^2} d\Sigma_{d-2}^2, \quad F = \frac{Q}{r^{d-2}} dt \wedge dr, \quad (2.1)$$

where $d\Sigma_{d-2}^2$ denotes the $(d-2)$ -dimensional space with zero curvature and planar topology, whereas L is related to the cosmological constant $\hat{\alpha}_{(0)}$ by $L^{-2} = \hat{\alpha}_{(0)}/(d-1)(d-2)$.

Notice that the metric in eq. (2.1) differs from that in the usual Schwarzschild gauge by a (constant) rescaling $t \rightarrow Nt$ of the time coordinate t . As we will see later in this paper this rescaling is necessary in order to have a unit speed of light in the dual CFT. Using the rescaled Lovelock coupling constants

$$L^{-2} = \alpha_0 = \frac{\hat{\alpha}_{(0)}}{(d-1)(d-2)}, \quad \alpha_1 = \hat{\alpha}_{(1)}, \quad \alpha_k = \hat{\alpha}_{(k)} \prod_{n=3}^{2k} (d-n) \quad \text{for } k \geq 2, \quad (2.2)$$

the field equations read

$$\sum_{k=0}^{k_{\max}} \hat{\alpha}_{(k)} \mathcal{G}_{ab}^{(k)} = 8\pi G_N \left(F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right), \quad (2.3)$$

where G_N is the d -dimensional Newton's constant and each of the Einstein-like tensors $\mathcal{G}^{(k)a}{}_b$ defined by

$$\mathcal{G}^{(k)a}{}_b = -\frac{1}{2^{(k+1)}} \delta_{b e_1 f_1 \dots e_k f_k}^{a c_1 d_1 \dots c_k d_k} R_{c_1 d_1}{}^{e_1 f_1} \dots R_{c_k d_k}{}^{e_k f_k}, \quad (2.4)$$

independently satisfies a conservation law $\nabla_a \mathcal{G}^{(k)a}{}_b = 0$. The higher-curvature terms contribute to the equations of motion only for $d > 2k$. For $d = 2k$ the higher-curvature corrections are topological, and they vanish identically in lower dimensions. Setting $\hat{\alpha}_{(k)} = 0$ for $k \geq 2$, one can recover the standard form of general relativity. In the notation (2.2), the field equations (2.3) reduce to the requirement that $f(r)$ solves the following polynomial equation of degree $k_{\max} = \lfloor \frac{d-1}{2} \rfloor$ (see e.g., [2–5, 36–38])

$$\mathcal{P}(f) = \sum_{k=0}^{k_{\max}} \alpha_k \left(\frac{\kappa - f}{r^2} \right)^k = \frac{\omega_d M_{\text{ADM}}}{N r^{d-1}} - \frac{8\pi G_N Q^2}{(d-2)(d-3) r^{2d-4}}. \quad (2.5)$$

Here M_{ADM} is the ADM mass of the black brane and ω_d is

$$\omega_d = \frac{16\pi G_N}{(d-2)} \frac{L^{d-2}}{V^{d-2}} \quad (2.6)$$

where V^{d-2} is the volume of the $(d - 2)$ -dimensional space with curvature $\kappa = 0$. The electric charge Q of the brane is

$$Q = \frac{L^{d-2}}{2V_{d-2}} \int *F. \tag{2.7}$$

2.1 Universality of black brane thermodynamics in Lovelock gravity

Interestingly, even without knowing $f = f(r)$ in eq. (2.5) explicitly, it is possible to find the thermodynamic quantities characterizing the Lovelock black brane solution [2, 39, 40]. Let r_+ denotes the radius of the event horizon, determined as the largest root of $f(r) = 0$. Introducing the effective mass M and temperature T related to the usual ADM mass M_{ADM} and Hawking temperature T_H by the relations

$$M = \frac{M_{\text{ADM}}}{N}, \quad T = \frac{T_H}{N}, \tag{2.8}$$

the black brane mass M , the temperature T , the entropy S , and the gauge potential Φ are given by [2, 41]

$$M = \frac{1}{\omega_d L^2} r_+^{d-1} + \frac{V_{d-2}}{2(d-3)L^{d-2}} \frac{Q^2}{r_+^{d-3}}, \tag{2.9}$$

$$\begin{aligned} T &= \frac{1}{2\pi N} \frac{1}{\sqrt{g_{rr}}} \left. \frac{d\sqrt{-g_{tt}}}{dr} \right|_{r=r_+} \\ &= \frac{1}{4\pi r_+} \left[(d-1) \left(\frac{r_+}{L} \right)^2 - \frac{8\pi G_N Q^2}{(d-2)r_+^{2(d-3)}} \right], \end{aligned} \tag{2.10}$$

$$S = \frac{V^{d-2}}{4G_N} \left(\frac{r_+}{L} \right)^{d-2}, \quad \Phi = \frac{V_{d-2}}{(d-3)L^{d-2}} \frac{Q}{r_+^{d-3}}. \tag{2.11}$$

The rescaling of the physical parameters (2.8) of the Lovelock BB having the dimensions of energy is essentially due to the presence of the constant N^2 in the metric. The two time coordinates t and Nt correspond to using two different units to measure the energy. However, when we deal with Einstein-Hilbert branes the rescaling of the time coordinate is not necessary and we will simply set $M = M_{\text{ADM}}$ and $T = T_H$. Notice that the area-law for the entropy S always hold for the generic Lovelock black brane.

A striking feature of these thermodynamic expressions is that *they do not depend on the Lovelock coupling constants α_k for $k \geq 2$ but only on α_0 and α_1* , i.e., they depend only on the cosmological constant and on Newton constant. This means that the thermodynamic behaviour of the BB in Lovelock theory is universal, in the sense that *it does not depend on the higher order curvature terms* but only on the Einstein-Hilbert term, the cosmological constant and the matter fields content (in our case the EM field). This implies, in turn, that as thermodynamic system the charged BBs of Lovelock gravity are indistinguishable from the Reissner-Nordström BBs of Einstein-Hilbert gravity. Notice that this feature is not shared by the black hole solutions of the theory, i.e., solutions with spherical or hyperbolic horizons. In fact, in the Lovelock thermodynamic expressions (see refs. [2, 41]) the dependence on the Lovelock coupling constants $\alpha_{k \geq 2}$ is introduced by the dependence on the curvature κ of the $(d - 2)$ -dimensional spatial sections. This dependence drops out when $\kappa = 0$.

We remark, however, that the universal thermodynamic behaviour of charged Lovelock black branes is strictly true only when we choose $N = 1$ in the metric (2.1). As we will see later in this paper, the parameter N has to be fixed in terms of the Lovelock coupling constants $\alpha_{k \geq 2}$. Hence, the ADM mass and the Hawking temperature of the Lovelock BB will depend on $\alpha_{k \geq 2}$. The universality of the Lovelock BB thermodynamics is recovered simply by rescaling the units we use to measure the energy, i.e., by using in eqs. (2.9) and (2.10) the effective parameters M and T instead of M_{ADM} and T_H .

In the following, we provide a detailed calculation for the case $k_{\text{max}} = 2$, i.e., GB gravity in five spacetime dimensions, which is the most interesting case from the AdS/CFT point of view. However, we expect that most of our considerations can be easily generalized to every charged BB solution of Lovelock gravity in generic dimensions.

3 5d Reissner-Nordström black brane solution

Let us preliminary review some known facts about the RN BB solutions of Einstein-Maxwell gravity. Setting $\alpha_k = 0$ for $k \geq 2$ and $d = 5$ in Eq (2.4), we have standard GR equations sourced by an electromagnetic field. For this choice of the parameters, eq. (2.5) is a linear equation in f that gives the following solution:

$$f = \alpha_0 r^2 - \frac{\omega_5 M}{r^2} + \frac{4\pi G_N Q^2}{3 r^4}, \tag{3.1}$$

where ω_5 is given by eq. (2.6) and G_N is the five dimensional Newton's constant. Performing the asymptotic limit $r \rightarrow \infty$, the function (3.1) reduces to $f = r^2/L^2$, i.e., AdS₅ with AdS length $L^2 = \alpha_0^{-1}$. The ratio L^3/G_N is proportional to the central charge c of the dual CFT. The central charge c can be defined as the coefficient of the large temperature expansion of the free energy (see section 5.1). The condition for the validity of classical AdS gravity in the bulk is $c \gg 1$. In most of the established examples of the AdS/CFT correspondence $c \propto \mathcal{N}$, where the limit $c \gg 1$ is referred to as the large \mathcal{N} limit.

Setting $r^2 = Y$ in eq. (3.1), the RN BB horizons are determined by the cubic equation

$$Y^3 - \omega_5 M L^2 Y + \frac{4\pi}{3} G_N L^2 Q^2 = 0. \tag{3.2}$$

This equation has two positive roots for

$$M^3 \geq 12\pi^2 \frac{G_N^2 Q^4}{\omega_5^3 L^2}, \tag{3.3}$$

which gives the extremal (BPS [42, 43]) bound for the RN black brane in 5d. In general, we will have an inner and outer horizon, when the bound is saturated the two horizons merge at r_0 and the RN BB becomes extremal. In the extremal case, eq. (3.2) has a double root at $Y_0 = \sqrt{\omega_5 M L^2/3}$ and $f(r)$ can be factorized in the following way

$$f(r) = \frac{1}{L^2 r^4} (r^2 + r_0^2) (r - r_0)^2 (r + r_0)^2, \quad r_0 = \left(\frac{\omega_5 M L^2}{3} \right)^{1/4}. \tag{3.4}$$

The extremal near-horizon geometry can be determined expanding the metric near r_0 and keeping only the leading term in the metric

$$f(r) = \frac{12}{L^2}(r - r_0)^2, \tag{3.5}$$

a simple translation of the radial coordinate $r \rightarrow r + r_0$ gives the $\text{AdS}_2 \times R_3$ extremal near-horizon geometry with AdS_2 length l

$$ds^2 = -\left(\frac{r}{l}\right)^2 dt^2 + \left(\frac{l}{r}\right)^2 dr^2 + \left(\frac{r_0}{L}\right)^2 d\Sigma_3^2, \quad l^2 = \frac{L^2}{12}. \tag{3.6}$$

The extremal solution given in eq. (3.4) is a soliton interpolating between the asymptotic AdS_5 geometry in the UV and the $\text{AdS}_2 \times R_3$ geometry (3.6) in the IR.

4 Gauss-Bonnet solution

We use the form (2.1) with coupling constant (2.2). Moreover, from now on we set $\alpha_1 = 1$ in order to recover the usual Newtonian limit. For $k = 2$ and generic curvature κ , eq. (2.5) reduces to a quadratic equation

$$\alpha_2 \frac{(\kappa - f)^2}{r^4} + \frac{(\kappa - f)}{r^2} + \alpha_0 - \frac{\omega_d M}{r^{d-1}} + \frac{8\pi G_N Q^2}{(d-2)(d-3)r^{2d-4}} = 0, \tag{4.1}$$

from which one obtains two possible solutions, f_{\pm} . In the following, we will refer to the solution f_- as the ‘*Einstein branch*’ because it approaches the Einstein case when the Gauss-Bonnet coupling α_2 goes to zero and to f_+ as the ‘*Gauss-Bonnet branch*’ [10]. The quadratic eq. (4.1) gives the following necessary condition requirement for the existence of f_{\pm} for large r :

$$1 - 4\alpha_0\alpha_2 \geq 0. \tag{4.2}$$

When this inequality is violated, the space becomes compact because of the strong nonlinear curvature [10]. Therefore, there is no asymptotic ‘AdS region’ and consequently no proper black hole with standard asymptotics.

4.1 5d GB black brane

In this subsection, we discuss the special case of 5d GB BB ($\kappa = 0$). It is easy to check that for $d = 5$ and $\kappa = 0$, then eq. (4.1) reduces to the following equation

$$\alpha_2 \frac{f^2}{r^4} - \frac{f}{r^2} + \alpha_0 - \frac{\omega_5 M}{r^4} + \frac{4\pi Q^2}{3r^6} = 0 \tag{4.3}$$

and the two branches are respectively

$$f_{\pm} = \frac{r^2}{2\alpha_2} \left[1 \pm \sqrt{1 - 4\alpha_0\alpha_2} \sqrt{1 + \frac{4M\alpha_2\omega_5}{(1 - 4\alpha_0\alpha_2)r^4} - \frac{16\pi G_N}{3} \frac{Q^2\alpha_2}{1 - 4\alpha_0\alpha_2} \frac{1}{r^6}} \right]. \tag{4.4}$$

In case of positive GB coupling $\alpha_2 > 0$ that satisfy the condition (4.2), the two branches describe two asymptotically AdS_5 spacetimes, however, from eq. (4.4) one can see that f_+ has no zeroes, hence the f_+ branch does not describe a BB but a solution with no event horizon. Thus, only the f_- branch describes a BB solution.

Let us now study the asymptotic geometry of the GB BB. At leading order for $r \rightarrow \infty$ the metric coefficient $g_{tt} = N^2 f(r)$ in eq. (2.1) becomes

$$g_{tt} \rightarrow N^2 \frac{r^2}{2\alpha_2} (1 \pm \sqrt{1 - 4\alpha_0\alpha_2}) . \quad (4.5)$$

In order to have the boundary of the asymptotic AdS_d conformal to $(d-1)$ -Minkowski space with speed of light equal to 1, $ds^2 \approx \alpha_0 r^2 (-dt^2 + d\Sigma_3^2)$, the constant N^2 has to be chosen as

$$N^2 = \frac{1}{2} (1 \mp \sqrt{1 - 4\alpha_0\alpha_2}) , \quad (4.6)$$

where we have the + sign for the f_- branch, the BB solution, while the - sign has to be used when we consider the f_+ branch.

In the AdS/CFT correspondence, the central charge c of the dual CFT is determined by the AdS length. Thus, the CFTs dual to GB gravity in both branches have central charge different from the RN case. Only in the $\alpha_2 \rightarrow 0$ limit the central charge of the f_- branch coincides with that of the CFT dual to the RN theory. However, naive computation of the central charge in terms of the AdS length does not work in this case because of the rescaling of the time coordinate. We will compute c in section 5 using the scaling law of the mass and entropy as a function of the temperature.

For $\alpha_2 < 0$, only the f_- branch is asymptotically AdS. Conversely, the f_+ branch describes a spacetime which is asymptotically de Sitter (dS) and can be therefore relevant as a cosmological solution.

4.2 Singularities

To determine the position of the singularities of the spacetime we calculate the scalar curvature for both the f_{\pm} branches:

$$R^{(\pm)} = \mp \frac{1}{2} \frac{\beta r^2 (20r^{10} + 30\sigma r^6 - 31\rho r^4 + 6\sigma^2 r^2 - 9\rho\sigma) \pm 20r^3 (r^6 + r^2\sigma - \rho)^{3/2} + 2\beta\rho^2}{\alpha_2 r^3 (r^6 + \sigma r^2 - \rho)^{3/2}} , \quad (4.7)$$

where the \pm sign refers respectively to the f_{\pm} branches. To simplify expressions we used (here and after) the following notation

$$\beta = \sqrt{1 - 4\alpha_0\alpha_2}, \quad \sigma = \frac{4\alpha_2\omega_5 M}{\beta^2}, \quad \rho = \frac{16\pi G_N \alpha_2 Q^2}{3\beta^2}, \quad e = \frac{1}{\beta^2} - 1 = \frac{4\alpha_0\alpha_2}{\beta^2}, \quad Y = r^2. \quad (4.8)$$

There are curvature singularities at $r = 0$ and at the zeroes of the argument of the square root in eq. (4.7) (branch-point singularities). The position of the physical singularities of the spacetime is therefore determined by the pattern of zeroes of the function $g(Y)$, with

$$g(Y) = Y^3 + \sigma Y - \rho. \quad (4.9)$$

The singularity will be located at the biggest positive zero Y_1 of $g(Y)$ or at $r = 0$ when $g(Y)$ has no zeroes for positive Y . The singularity at $Y = Y_1$ is a branch point singularity. The pattern of zeroes of $g(Y)$ is determined by the signs of the coefficients ρ, σ and the discriminant $\Delta = \left(\frac{\rho}{2}\right)^2 + \left(\frac{\sigma}{3}\right)^3$.

- For $\sigma > 0$, the function $g(Y)$ is a monotonic increasing function of Y with a single zero which, depending on the sign of ρ , will be positive $Y = Y_1$ ($\rho > 0$) or negative ($\rho < 0$). The physical spacetime singularity will be therefore located at $r = \sqrt{Y_1}$ for $\rho, \sigma > 0$ and at $r = 0$ for $\rho < 0, \sigma > 0$.
- For $\sigma < 0$, the function $g(Y)$ is an oscillating function with a maximum at negative Y and a minimum at positive Y , it may therefore have one, two or three zeros. For $\sigma < 0, \rho > 0$, $g(Y)$ has at least a positive zero. For $\sigma < 0, \rho < 0$ we have a positive zero for $\Delta \leq 0$ and no positive zeros for $\Delta > 0$. For $\Delta = 0$ we have a double zero of $g(Y)$ so that Y_1 is not anymore a branch point singularity. In this latter case the singularity is at $r = 0$.

Summarising, the physical singularity is always located at $r = \sqrt{Y_1}$ unless $\sigma > 0, \rho < 0$ or $\sigma < 0, \rho < 0, \Delta \geq 0$ in which case the singularity is at $r = 0$.

4.3 f_- branch

In this subsection, we study in detail the horizons of the f_- branch, solution of eq. (4.4), describing the GB black brane. In general the BB will have an inner ($r = r_-$) and outer ($r = r_+$) event horizon. The BB becomes extremal when $r_+ = r_-$. Using the notation (4.8), (4.9), one finds that the necessary condition for the existence of the BB is the positivity of the argument in the square root in eq. (4.4), i.e., $g(Y) \geq 0$. The position of the event horizon(s) is determined by the *positive* roots of the cubic equation

$$h(Y) = eY^3 - \sigma Y + \rho = 0. \tag{4.10}$$

We will first consider the case $\alpha_2 > 0$, which corresponds to $\sigma, \rho, e > 0$ (since also $\alpha_0 > 0$). The condition for the existence of real roots of the function $h(Y)$ can be easily found: the function $h(Y)$ has a maximum (minimum) for, respectively

$$Y = Y_{M,m} = \pm \sqrt{\frac{\sigma}{3e}} = \pm \sqrt{\frac{\omega_5 M L^2}{3}} \tag{4.11}$$

also, $h(Y = 0) = \rho > 0$, hence the cubic equation (4.10) always has a negative root. The existence of other roots is determined by the sign of $h(Y_m)$. We will have two (one) positive roots hence a BB with two (one) event horizons for $h(Y_m) \leq 0$, i.e., for

$$\rho \leq \frac{2}{3} \sigma \sqrt{\frac{\sigma}{3e}}. \tag{4.12}$$

Using eq. (4.8), the previous inequality can be written in terms of the charge Q and the effective mass M and gives *the same* Bogomol'nyi-Prasad-Sommerfield (BPS) bound (3.3) found in the RN case. However, the BPS bound is modified when we instead express it in terms of the ADM mass:

$$M_{\text{ADM}}^3 \geq 12N^3 \pi^2 \frac{G_N^2 Q^4}{\omega_5^3 L^2}. \tag{4.13}$$

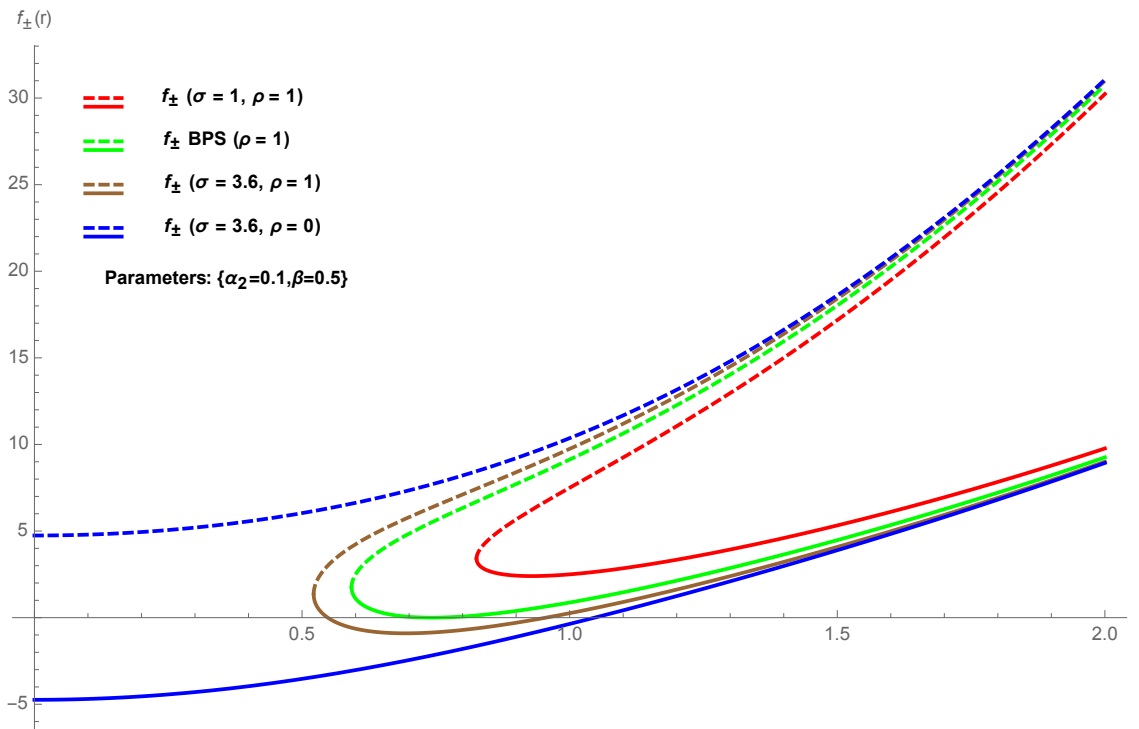


Figure 1. Behaviour of the metric functions f_{\pm} for $\alpha_2, M > 0$ and selected values of the other parameters. The dashed (solid) lines describe the f_+ branch (f_- branch). The red, green, brown and blue solid lines describe respectively a naked singularity, an extremal, two-horizon and vanishing charge BB geometry. The corresponding dashed lines describe spacetimes with a naked singularity.

When the bound is saturated, the inner and outer horizon merge at $r_- = r_+$, the BB becomes extremal, and the solution describes a soliton. The striking feature of the BPS bound (4.12) is that the *BPS bound of 5d Gauss-Bonnet BB does not depend on the Lovelock coupling constant, and it is exactly the same one gets for GR ($\alpha_2 = 0$), i.e., for the 5d Reissner-Nordström BB.* When M does not satisfy the inequality (4.12), the spacetime describes a naked singularity. For $\alpha_2 > 0$, the condition $M > 0$ implies $\sigma, \rho > 0$ and the function $g(Y)$ is a monotonic increasing function which cuts the Y -axis at the point Y_1 , and, in view of the previous discussion, it also gives the position of the singularity. Since, the position of the event horizon Y_h is determined by the equation

$$\beta\sqrt{g(Y_h)} = Y\sqrt{Y_h}, \tag{4.14}$$

from which follow that $g(Y_h) > 0$ hence $Y_h > Y_1$, this checks that in the region where the bound (4.12) holds the condition $g(Y) > 0$ is always satisfied and that the physical singularity is always shielded by two (one in the extremal case) event horizons.

The behaviour of the metric function f_- for $\alpha_2, M > 0$ and selected values of the other parameters is shown in figure 1. The solid red, green and brown lines describe respectively a naked singularity, extremal and two-horizon BB geometry. The solid blue line represents a zero-charge, BB solution with single horizon.

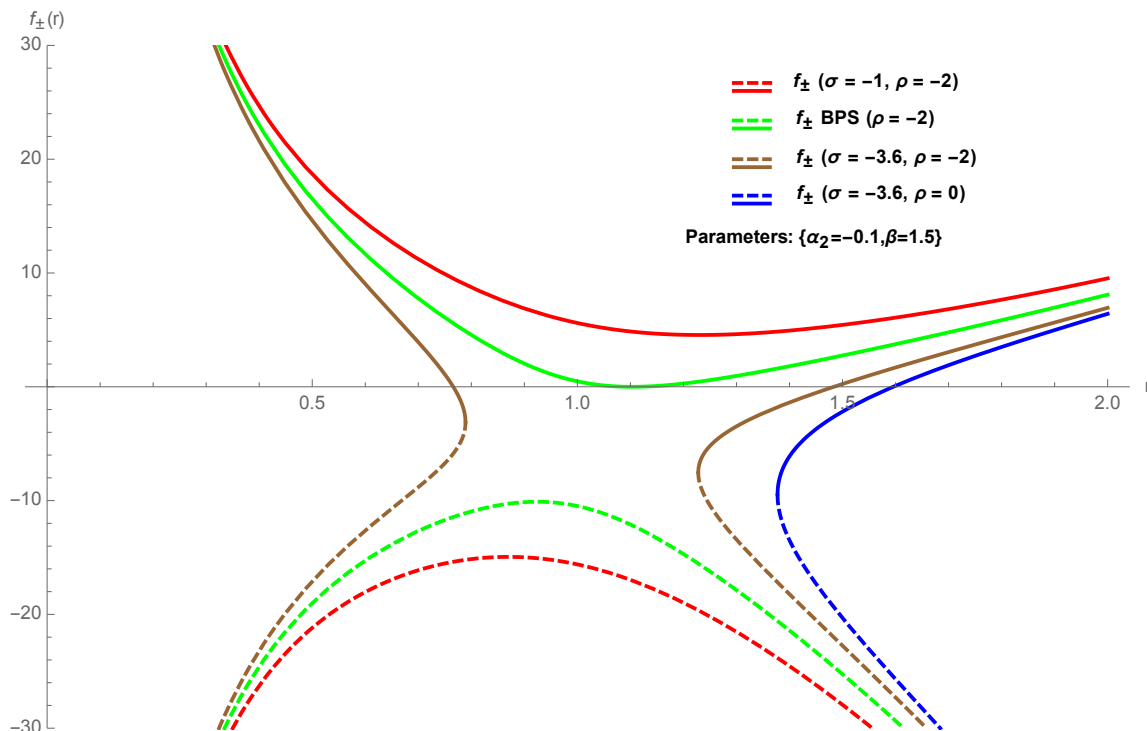


Figure 2. Behaviour of the metric functions f_{\pm} for $\alpha_2 < 0, M > 0$ and selected values of the parameters. The dashed (solid) lines describe the f_+ branch (f_- branch). The red, green, brown, blue solid lines describe respectively a naked singularity, an extremal, single-horizon, vanishing charge BB geometry. The corresponding dashed lines describe cosmological solutions with a singularity which approach asymptotically to the dS spacetime.

The case $\alpha_2 < 0, M > 0$ gives exactly the same BPS bound. Now, we have $\sigma, \rho, e < 0$. The function $h(Y)$ in eq. (4.10) always has a negative root and a minimum (maximum) for

$$Y = Y_{m,M} = \pm \sqrt{\frac{\sigma}{3e}} = \pm \sqrt{\frac{\omega_5 M L^2}{3}}. \tag{4.15}$$

The conditions for the existence of two positive roots become $|\rho| \leq \frac{2}{3}|\sigma|\sqrt{\frac{\sigma}{3e}}$ leading to the same BPS bound (4.12). However, there is a crucial difference from the $\alpha_2 > 0$ case. When $\alpha_2 < 0$, the condition $M > 0$ implies $\sigma, \rho < 0$. Taking into account that $|e| < 1$ owing to (4.2), we see that the condition $\Delta < 0$ implies the BPS bound (4.12). This means that the two horizons are separated by a region in which the solution does not exist. The spacetime breaks into two disconnected parts. The physical part, having an asymptotic AdS region, describes a BB with singularity shielded by a *single* event horizon. The behaviour of the metric function f_- for $\alpha_2 < 0$ and selected values of the other parameters is shown in figure 2. The solid red, green and brown lines describe respectively a naked singularity, extremal and single-horizon BB geometry. The solid blue line represents a zero-charge, BB solution with horizon.

4.3.1 Near horizon extremal solution

When the bound (4.12) is saturated, the BB becomes extremal and the metric function (4.4) has a double zero at

$$Y_h = Y_m = \sqrt{\frac{\sigma}{3e}} = \sqrt{\frac{\omega_5 M L^2}{3}}, \quad (4.16)$$

thus, the solution f_- can be factorized as

$$f_-^{(\text{ex})}(Y) = \frac{e\beta^2}{2\alpha_2} \frac{(Y + 2Y_m)(Y - Y_m)^2}{Y^2 + \beta\sqrt{Y^4 + \sigma Y^2 - \rho Y}}. \quad (4.17)$$

This solution represents the extremal GB soliton.

Let us now consider the near-horizon geometry. In this regime, the solution (4.17) can be expanded around $r = r_0 = \left(\frac{\sigma}{3e}\right)^{1/4}$. At the leading order the Einstein branch reads

$$f_-^{(\text{ex})}(r) = 12\alpha_0(r - r_0)^2. \quad (4.18)$$

Translating the radial coordinate $r \rightarrow r + r_0$ and rescaling the time coordinate as $t \rightarrow t/N$ we get the extremal, near-horizon geometry:

$$ds^2 = -\left(\frac{r}{l}\right)^2 dt^2 + \left(\frac{l}{r}\right)^2 dr^2 + \left(\frac{r_0}{L}\right)^2 d\Sigma_3^2, \quad l^2 = \frac{1}{12\alpha_0}, \quad (4.19)$$

i.e., $\text{AdS}_2 \times R_3$ with the AdS_2 length l being determined uniquely by α_0 . Thus, the extremal near-horizon geometry does not depend on α_2 and fully coincides with the extremal near-horizon geometry (3.6) one gets in the RN case.

4.4 Near horizon metric as exact solution of equations of motion

In this section, we will show that the near-horizon solution given in eq. (4.19) is an exact solution of the equations of motion (EOM). For the GB case, eqs. (2.3) read

$$\begin{aligned} R_{ab} - \frac{1}{2}Rg_{ab} &= \frac{6}{L^2}g_{ab} + 8\pi G_N \left(F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{cd}F^{cd} \right) \\ &+ \frac{\alpha_2}{2}g_{ab} \left(R_{cdef}R^{cdef} - 4R_{cd}R^{cd} + R^2 \right) \\ &+ \alpha_2 \left(-2RR_{ab} + 4R_{ac}R_b{}^c + 4R_{cd}R_a{}^c{}_b - 2R_{acde}R_b{}^{cde} \right). \end{aligned} \quad (4.20)$$

We note that, since the eq. (4.19) describes a spacetime with $\text{AdS}_2 \times R_3$ geometry, the contribution to the curvature tensors coming from the planar geometry R_3 vanishes. Thus, the EOM includes only the contribution of the AdS_2 part of the metric which is a two dimensional maximally symmetric space.

For a generic n -dimensional maximally symmetric space with $R = \Lambda$ the two terms in eqs. (4.20), that are quadratic in the curvature tensors, are given respectively by

$$\alpha_2 \Lambda^2 \frac{(n-2)(n-3)}{2n(n-1)}, \quad -2\alpha_2 \Lambda^2 \frac{(n-2)(n-3)}{n^2(n-1)}. \quad (4.21)$$

These relations are consequence of the fact that the GB term in the action is topological for $d = 4$ and identically vanishes for $d = 2$ and $d = 3$. The previous equations imply that in

the case of the $\text{AdS}_2 \times R_3$ geometry, the contributions given by the GB terms to the EOM vanish; therefore, the near horizon metric (4.19) is an *exact* solution of both Einstein and GB EOM. In particular, the latter reduces to the usual Einstein-Maxwell equations in 5d.

Summarising, we have seen that the $\text{AdS}_2 \times R_3$ geometry is not only a near horizon approximation but it is an exact solution of the field equations of GB-Maxwell gravity. The presence of two exact extremal solutions (the extremal soliton interpolating through a throat region the $\text{AdS}_2 \times R_3$ geometry with the asymptotic AdS geometry and the $\text{AdS}_2 \times R_3$ geometry itself) is a typical feature of extreme black branes describing BPS states (see e.g. refs. [44, 45]). The two solutions correspond to two different extremal limits. As we will see in section 5, the presence of two different extremal, exact, solutions gives rise to a non-trivial extremal thermodynamic behaviour.

4.5 f_+ branch

This branch does not describe a BB but a spacetime with a singularity for every value of the parameters $Q \neq 0, M \neq 0$. Depending on the value of the parameter α_2 we have either a spacetime with a naked singularity (for $\alpha_2 > 0$) or a cosmological, asymptotically de Sitter (dS) solution with a singularity (for $\alpha_2 < 0$.) This follows from the above discussion of the singularities of the scalar curvature (4.7). In the f_+ branch the spacetime always has a singularity, which can be located at $r = 0$ or $r = \sqrt{Y_1}$ depending on the values of the parameters. This is consistent with the results of ref. [36], according to which the f_+ branch is unstable and contains ghosts.¹

For $M, \alpha_2 > 0$, the metric functions for the f_+ branch are the dashed lines shown in figure 1. An interesting, peculiar feature is that in this case, all the solutions of the f_- branch are continuously connected with the solution of the f_+ branch passing through the singularity. This feature has a simple analytic explanation. In the cases under consideration the singularities are the zeros of the function $g(Y)$ and when $g(Y) = 0$ then $f_+ = f_-$. This fact can have interesting holographic implications: we have two CFTs with different central charges connected through the same singularity.

For $M > 0$ and $\alpha_2 < 0$, the f_+ branch describes a cosmological solution with a singularity. The corresponding metric functions are shown (dashed lines) in figure 2. Also in this case the solutions of the f_- branch with an horizon are continuously connected with the solution of the f_+ branch passing through the singularity. We have now an asymptotically AdS solution continuously connected through a cosmological singularity to a late de Sitter geometry. On the other hand, the solutions of the f_- branch describing a naked singularity are disconnected from the cosmological solutions.

For $\alpha_2, M < 0$, the f_+ branch describes a cosmological solution with a singularity with late de Sitter behaviour, whereas the f_- branch describes an asymptotically AdS spacetime

¹In principle, one could have hoped to have a regular spacetime when the function $g(Y)$ has a double zero at positive Y . In fact in this case the branch point singularity is removed and if the spacetime in the region $Y_1 \leq Y < \infty$ is geodesically complete we have regular, solitonic geometry. The function $g(Y)$ has a double zero at positive Y for $\sigma, \rho < 0, \Delta = 0$, but unfortunately the spacetime cut at $Y = Y_1$ thus it is not geodesically complete.

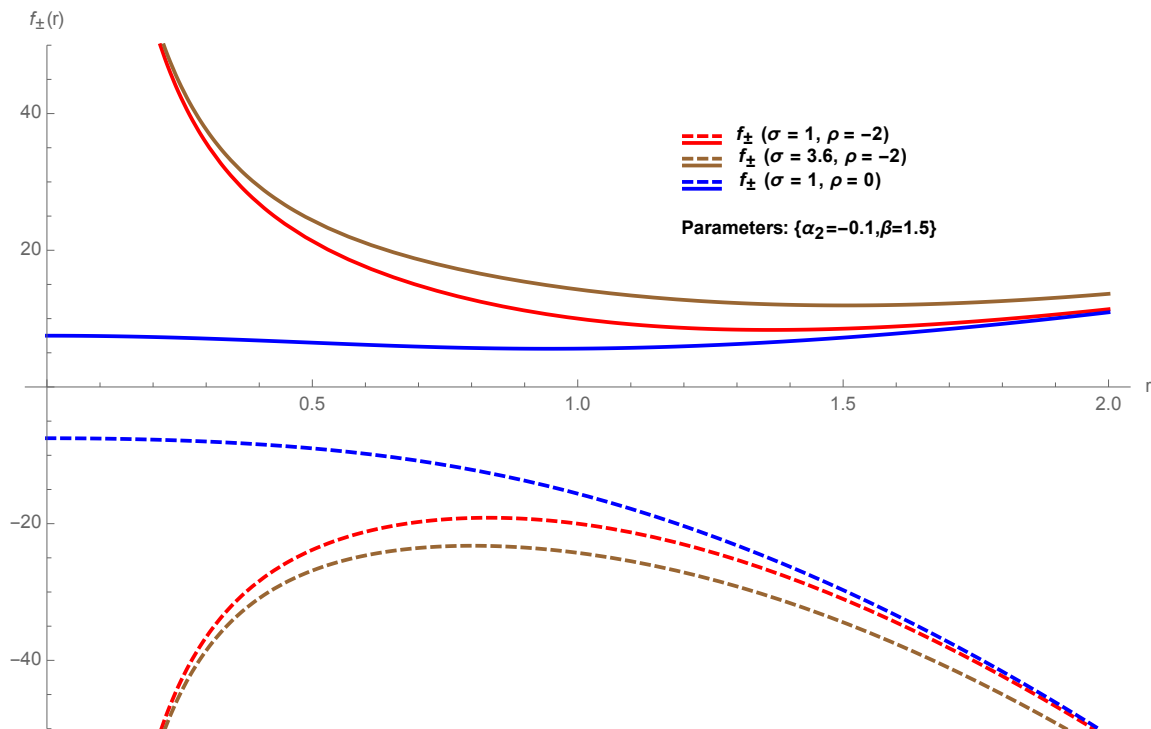


Figure 3. Behaviour of the metric functions f_{\pm} for $\alpha_2, M < 0$ and selected values of the other parameters. The dashed (solid) lines describe the f_+ branch (f_- branch). The solid lines describe spacetimes with naked singularities, whereas the dashed lines describe cosmological, asymptotically dS solutions with a singularity.

with a naked singularity. However, here the two branches are disconnected. The metric functions for this case are shown in figure 3.

It should be stressed that in the $Q = 0$ case, the f_+ branch has ghosts in the spectrum [36]. We naturally expect this to extend to the charged case and is consistent with the intrinsic instability of these branch of solutions connected with the presence of naked singularities.

5 Charged GB black brane thermodynamics

In this section, we will study the thermodynamics of the GB BB solutions, *i.e.*, solutions in the f_- branch and make a comparison with the Reissner-Nordström black branes.

The effective thermodynamic potentials $M = M_{\text{ADM}}/N, S, \Phi$ and the temperature $T = T_H/N$ can be written as functions of the horizon radius r_+ and the charge Q by specializing eqs. (2.9), (2.10), (2.11) to $d = 5$. We obtain the following equations

$$\begin{aligned}
 M &= \frac{r_+^4}{\omega_5 L^2} \left(1 + \frac{4\pi G_N Q^2 L^2}{3 r_+^6} \right), & T &= \frac{1}{\pi L^2} \left(r_+ - \frac{2\pi G_N Q^2 L^2}{3 r_+^5} \right), \\
 S &= \frac{V_3}{4G_N} \left(\frac{r_+}{L} \right)^3, & \Phi &= \frac{V_3}{2L^3} \frac{Q}{r_+^2},
 \end{aligned}
 \tag{5.1}$$

that satisfy the first principle $dM = TdS + \Phi dQ$. As pointed out in section 2.1, because of the universality of the thermodynamic behaviour, the thermodynamic relations (5.1) hold for both for the charged GB and the RN BB. The only difference is that for the GB brane, with metric function (4.4), M and T are the effective parameters whereas in the RN case $M = M_{\text{ADM}}$ and $T = T_H$.

In order to have a clear and complete description of the GB BB thermodynamics, one should eliminate r_+ from the eqs. (5.1) and write $M(T, Q), S(T, Q)$. This parametrization cannot be done in analytic form because we have to solve a 6th grade equation in r_+ . Thus, we will derive the explicit scaling behaviour of M and S as a function of the temperature in the large and small T limit. These relations will shed light on the holographic interpretation of the solutions. The functions $M(T, Q)$ and $S(T, Q)$ can be obtained in implicit form by using the second equation in (5.1) as an implicit definition of the function $r_+(T, Q)$, and they read

$$M(T, Q) = \frac{r_+^3}{\omega_5 L^2} (3r_+ - 2\pi L^2 T), \quad S(T, Q) = \frac{V_3}{4G_N} \left(\frac{r_+}{L}\right)^3. \quad (5.2)$$

Let us now consider separately the two limits of interest: $T \rightarrow \infty$ and $T \rightarrow 0$.

5.1 Large temperature

The limit $T \rightarrow \infty$ corresponds to large radius BB, i.e., $r_+ \rightarrow \infty$. In this regime, the temperature scales linearly with r_+

$$T \simeq \frac{r_+}{\pi L^2} \quad (5.3)$$

and, at the leading order, we get for M and S

$$M = \frac{3V_3 L^3}{16\pi G_N} (\pi T)^4, \quad S = \frac{V_3 L^3}{4G_N} (\pi T)^3. \quad (5.4)$$

This is exactly the scaling behaviour one expects for a UV fixed point described by a CFT₄.

Because of the universality of the thermodynamic behaviour, the relations (5.4) hold for both the RN and the GB BB. In the former case, eqs. (5.4) hold when $M = M_{\text{ADM}}, T = T_H$, in the latter when M, T are given by the effective values in eq. (2.8). Thus, for the GB BB, mass and entropy acquire a $1/N^3$ factor.

The central charge c of the associated CFT is determined by the proportionality factor and can be easily calculated. In the case of the RN BB, when $M = M_{\text{ADM}}$ and $T = T_H$ in eq. (5.1), we have $c \propto L^3/G_N$. On the other hand, in the GB BB case, we have seen that the same thermodynamic relations (5.1) hold for M, T given by the effective values in eq. (2.8) and we will get from eqs. (5.4)

$$c \propto \frac{L^3}{N^3 G_N}. \quad (5.5)$$

5.2 Small temperature

The $T \rightarrow 0$ thermodynamic behaviour corresponds to extremal BBs in which the BPS bound (4.13) is saturated. This is achieved at non vanishing, constant value of the BB

radius

$$r_+ = \left(\frac{2\pi G_N L^2 Q^2}{3} \right)^{1/6} \equiv r_0 \tag{5.6}$$

that corresponds, as expected for BPS states, to the extremal brane $T = 0$ state with non vanishing mass and entropy given by

$$M_{\text{ext}} = \frac{3r_0^4}{2\omega_5 L^2}, \quad S_{\text{ext}} = \frac{V_3}{4G_N} \left(\frac{r_0}{L} \right)^3. \tag{5.7}$$

We can now expand in Taylor series the temperature near r_0 to obtain

$$T \simeq \frac{3}{\pi L^2} \left[2(r_+ - r_0) - \frac{5}{r_0} (r_+ - r_0)^2 \right], \tag{5.8}$$

and the behaviour of M and S near $T = 0$ is of the form

$$M - M_{\text{ext}} = \frac{2r_0^2}{3\omega_5} (\pi L T)^2 + \mathcal{O}(T^4), \quad S - S_{\text{ext}} = \frac{\pi r_0^2 V_3}{8G_N L} T + \mathcal{O}(T^2). \tag{5.9}$$

Again, universality of the thermodynamic behaviour imply that the relations in eq. (5.9) hold both for the RN and for the GB BB. For the RN case, the relations take the same form with $M = M_{\text{ADM}}$ and $T = T_H$. For the GB case, when we express the relations (5.9) in terms of ADM mass and Hawking temperature we get

$$\begin{aligned} M_{\text{ADM}} &= N M_{\text{ext}} + \frac{2r_0^2}{3N\omega_5} (\pi L T_H)^2 + \mathcal{O}(T^4) \\ S &= S_{\text{ext}} + \frac{\pi r_0^2 V_3}{8NG_N L} T_H + \mathcal{O}(T^2). \end{aligned} \tag{5.10}$$

5.3 Excitations near extremality and near-horizon limit

An important feature of the RN BB, which in view of the previous results extends to the charged GB BB, is that the semiclassical analysis of its thermodynamic behavior breaks down near extremality [44, 45]. In fact, the energy of an Hawking radiation mode is of order T_H and the semiclassical description breaks down when this energy is comparable with the energy above extremality $M - M_{\text{ext}}$ given by eq. (5.9). This results in an energy gap for excitations above extremality [44], which in the case under consideration is $E_{\text{gap}} \sim (N\omega_5)/L^2 r_0^2$. The fact that the extremal limit is singular, can be also understood in geometrical terms. It has been observed that at extremality the geometry splits into two spacetimes: an extremal black hole and a disconnected AdS space [46].

The presence of this energy gap has important consequences for what concerns the spectrum of BB excitations near extremality. In particular, whereas in the extremal case the near-horizon geometry is given, as shown in section 4.3.1, by $\text{AdS}_2 \times R_3$, finite energy excitations of $\text{AdS}_2 \times R_3$ are suppressed. Analogously to the RN case in $4d$ [44], one can consider near-horizon limits not restricted to zero temperature and excitation energy. These limits are obtained by letting the $5d$ Planck length L_P go to zero, holding fixed some of the other physical parameters of the BB (energy, charge and temperature).

6 Shear viscosity to entropy ratio

The *universality* of the shear viscosity η to entropy density s ratio for Einstein-Hilbert gravity represents a very important result of the gauge/gravity correspondence. First found for the hydrodynamic regime of the QFT dual to black branes and black holes of the Einstein-Hilbert theory [13, 21, 22], the KSS bound $\eta/s \geq 1/4\pi$ has been extended to a variety of cases. These include Einstein-Hilbert gravity with all possible matter terms in the action, hence, among others the QFT dual to Reissner-Nordström 5d gravity [21, 22], the important case of the quark-gluon plasma (see e.g. [47]) and charged solutions in gravity theories with curvature squared corrections [48–51]. It has been also conjectured that the KSS bound holds for any fluid in nature. For a detailed discussion on the shear viscosity to entropy ratio see refs. [13, 21, 22, 24, 34, 41, 47, 52–54].

The KSS bound seems to lose its universality when one introduces, in the Einstein-Hilbert action, higher powers of the curvature tensors. This is, for instance, the case of Lovelock (and Gauss-Bonnet) gravity we are discussing in this paper. In particular, the KSS bound depends on the coupling constant for the higher curvature terms [24].

Following the notation of [24], we rewrite the GB BB solution (4.4) as follows

$$f_- = \frac{r^2}{2\lambda L^2} \left[1 - \sqrt{1 - 4\lambda \left(1 - \frac{\omega_5 M L^2}{r^4} + \frac{4\pi G_N Q^2 L^2}{3 r^6} \right)} \right], \quad (6.1)$$

where $\alpha_0 \alpha_2 = \alpha_2 / L^2 = \lambda$. In 5d Gauss-Bonnet gravity, the shear viscosity to entropy ratio is [24]

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 - 4\lambda). \quad (6.2)$$

The KSS bound still holds if $\lambda \leq 0$ but is violated for $0 < \lambda \leq 1/4$ (the upper bound follows from eq. (4.2)). The dependence of the bound from the coupling constant λ makes the bound not anymore universal as in the Einstein-Hilbert theory. In terms of the dual gauge theory, the curvature corrections to the Einstein-Hilbert action correspond to finite \mathcal{N} and λ_{tH} effects. It has been argued that the universality of the KSS bound strictly holds in the limit $\mathcal{N} \rightarrow \infty$ whereas, in general, finite \mathcal{N} effects will give lower bounds for η/s [17].

A crucial issue is that the relation (6.2) seems to allow for arbitrary violations of the KSS bound. However, consistency of the QFT dual to bulk GB gravity as a relativistic field theory constrains the allowed values of λ . For instance, in [27–29] it was found that causality and positivity of the energy for the dual QFT describing the Gauss-Bonnet plasma require $-7/36 < \lambda < 9/100$ implying $4\pi\eta/s > 16/25$, a bound lower than the KSS bound. On the other hand, the hydrodynamic description of the dual GB plasma is valid in the IR regime, i.e., for $\omega, k \ll T$, whereas causality is determined by the propagation of modes in the $\omega, k > T$, UV regime. Thus, the existence of lower bounds for η/s implies a higher non-trivial relationship between the transport properties in the IR and causality requirements in the UV regime of the QFT dual to GB gravity.

Recent investigations have shown that if translation symmetry is broken in the IR then one may have strong violation of the KSS bound even in the context of Einstein gravity, in the form of $\eta/s \sim T^{2\nu}$, $\nu \leq 1$ [30, 55, 56]. Although, for these backgrounds,

the breaking of translational invariance prevents an hydrodynamical interpretation of the viscosity, this behaviour of η/s is clearly related to the emergence of extremely interesting physics in the far IR.

A way to shed light on these questions is to investigate the behaviour of η/s in the case of a gravitational bulk background for which there is a temperature flow of η/s . The charged GB BB represents a nice example of this behaviour, particularly in view of the universality of the IR $\text{AdS}_2 \times R_3$ fixed point. This will be the subject of the next three subsections.

6.1 η/s for the charged GB black brane

A standard way to calculate the shear viscosity for a QFT is by using the Kubo formula for the transverse momentum conductivity

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{xy}^R, \tag{6.3}$$

where G_{xy}^R is the retarded Green function for the T_{xy} component of the stress-energy tensor.

The application of the usual AdS/CFT procedure for the computation of correlators gives for the U(1)-charged Gauss-Bonnet black brane in five dimensions [34, 52]

$$\eta = \frac{s}{4\pi} \left[1 - 4\lambda \left(1 - \frac{a}{2} \right) \right], \tag{6.4}$$

where $a = \frac{4\pi}{3} \frac{G_N Q^2 L^2}{r_+^6}$, and s is the entropy density S/V following from (5.1).

A drawback of the usual computation of the shear viscosity is that it does not work in the extremal $T = 0$ case because the metric function has a double zero at the horizon. For this reason, η in the case of extremal BB cannot be simply computed by taking the $T_H = 0$ limit in eq. (6.4). Building on [57], a method of dealing with this problem has been developed in [14]. Recently, a very simple and elegant formula for computing correlators of the form (6.3) in QFTs dual to a gravitational bulk theory has been proposed in [35] (see also [30, 31]). This method also works in the extremal case; thus, in the following, we will use it to compute η for the charged GB BB.

Considering perturbations $g_{ab} = g_{ab}^{(0)} + h_{ab}$ of the background (6.1), at the linear level the field equations (4.20) give for the $h_x^y(t, r) = \phi(r)e^{-i\omega t}$ component of the perturbation

$$\partial_r \left[\sqrt{\gamma(r)} f_-(r) F(r) \partial_r \phi \right] + \omega^2 \frac{\sqrt{\gamma(r)} F(r)}{N^2 f_-(r)} \phi = 0, \tag{6.5}$$

where $\gamma(r) = (r/L)^3$ is the determinant of the spatial metric, $f_-(r)$ is given by eq. (6.1) and $F = N \left(1 - \frac{\lambda L^2}{r} \partial_r f_-(r) \right)$. Notice that in the background (6.1), the component h_x^y decouples from the other perturbation modes.

Let us first consider the non extremal black brane. The extremal case will be discussed in section 6.3. Following ref. [35] we now denote with $\phi_0(r)$ the time independent solution of (6.5) which is regular on the horizon $r = r_+$ and such that $\phi_0 \rightarrow 1$ as $r \rightarrow \infty$. The

other linearly independent solution $\phi_1(r)$ of eq. (6.5) behaves as $1/r^4$ for $r = \infty$ and can be computed using the Wronskian method,

$$\phi_1 = \phi_0 \int_r^\infty \frac{dr}{\phi_0^2 \sqrt{\gamma} F f_-}. \quad (6.6)$$

Expanding near the horizon $r = r_+$ we get at leading order

$$\phi(r) = -\frac{1}{\phi_0(r_+)} \frac{\ln(r - r_+)}{4\pi T_H \sqrt{\gamma(r_+)} \left[1 - 4\lambda\left(1 - \frac{a}{2}\right)\right]}, \quad (6.7)$$

where T_H is the Hawking temperature of the BB and a is defined as in eq. (6.4). Solving now eq. (6.5) near the horizon with infalling boundary conditions and for small ω , one gets at leading order in ω

$$\phi(r) = \phi_0(r_+) \left(1 - \frac{i\omega}{4\pi T_H} \ln(r - r_+)\right). \quad (6.8)$$

Comparing eq. (6.7) with eq. (6.8) and expanding near the $r \rightarrow \infty$ boundary of AdS, one gets

$$\phi(r) = 1 + i\omega \phi_0^2(r_+) \sqrt{\gamma(r_+)} \left[1 - 4\lambda\left(1 - \frac{a}{2}\right)\right] \frac{1}{r^4}. \quad (6.9)$$

The usual AdS/CFT rules for computing boundary correlators tell us that the retarded Green function is $1/(16\pi G_N)$ the ratio between normalizable and non-normalizable modes, so that we have

$$\eta = \frac{s}{4\pi} \phi_0(r_+)^2 \left[1 - 4\lambda\left(1 - \frac{a}{2}\right)\right]. \quad (6.10)$$

Because $\phi_0(r)$ goes to 1 as $r \rightarrow \infty$ and must be regular on the horizon, we have $\phi_0(r_+) = 1$ and eq. (6.10) reproduces correctly the previous result (6.4).

Now, the second eq. (5.1) can be used to define, implicitly, the horizon radius as a function of the BB Hawking temperature and the electric charge, thus allowing us to write also the shear viscosity (6.4) as a function of T_H and Q

$$\eta(T_H, Q) = \frac{1}{16\pi G_N} \left(\frac{r_+(T_H, Q)}{L}\right)^3 \left[1 - 4\lambda \frac{\pi L^2 T_H}{N r_+(T_H, Q)}\right]. \quad (6.11)$$

In the same way, the entropy density in eq. (5.1) can be written as a mere function of T_H and Q , so that we can write the shear viscosity to entropy ratio in the form

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 - 4\lambda \frac{\pi L^2}{N r_+(T_H, Q)} T_H\right]. \quad (6.12)$$

It is interesting to investigate the behaviour of η/s at large and small T_H . In fact, as we have seen in the previous sections, in these limits the BB allows for a simple thermodynamic description. We, therefore, expect this to be true also for the shear viscosity to entropy ratio. This will be the subject of the next sections.

6.2 η/s in the large and small T_H regime

The behavior of the shear viscosity (6.11) for large and small temperatures can be investigated in a way similar to that used for the BB thermodynamics.

6.2.1 Large T_H

For large T_H , the Hawking temperature is given by eq. (5.3), thus leading to the following expression for the shear viscosity in eq. (6.11),

$$\eta = \frac{1}{16\pi G_N} \left(\frac{\pi L T_H}{N} \right)^3 (1 - 4\lambda). \quad (6.13)$$

The shear viscosity at large T_H scales as T_H^3 . In this limit, the entropy density also depends on the temperature as T_H^3 (see eq. (5.4)), the shear viscosity to entropy density ratio approaches eq. (6.2) and reduces to the universal value $1/4\pi$ when $\lambda \rightarrow 0$. This is rather expected, because at large T_H the contribution of the electric charge can be neglected.

6.2.2 Small T_H

To investigate the small T_H behaviour we invert eq. (5.8) and we write the horizon radius as

$$r_+ - r_0 \simeq \frac{\pi L^2}{6N} T_H, \quad (6.14)$$

where r_0 is defined by eq. (5.6). At small temperature the subleading term in the shear viscosity scales linearly in T_H

$$\eta \simeq \frac{1}{16\pi G_N} \left(\frac{r_0}{L} \right)^3 \left[1 + \left(\frac{1}{2} - 4\lambda \right) \frac{\pi L^2 T_H}{N r_0} \right]. \quad (6.15)$$

The behavior of the entropy density in the small T_H regime is given by the second equation in (5.9). Hence, in this limit, also the subleading term of the shear viscosity to entropy density ratio scales linearly

$$\frac{\eta}{s} \simeq \frac{1}{4\pi} \left[1 - 4\lambda \frac{\pi L^2 T_H}{N r_0} \right]. \quad (6.16)$$

The result $\eta/s = 1/4\pi$ for $T_H = 0$ has been already found and discussed in the literature in the case of the RN solution [14, 57]. It has been argued that at small temperatures, the dual QFT behaves as a “strange RN metal”. The optical conductivity exhibits the generic perfect-metal behaviour, but although we have a non-vanishing ground-state entropy, for the strange metal hydrodynamics continues to apply and energy and momentum can diffuse.

In the limit $T_H = 0$, the ratio becomes $\eta/s = 1/4\pi$ attaining the universal value one expects from the KSS bound. This result is what one naturally expects in view of the fact that at $T_H = 0$ the near-horizon solution of the GB brane gives exactly the same $\text{AdS}_2 \times R_3$ geometry of the RN solution. However, extra care is needed when one takes the $T_H \rightarrow 0$ limit in eq. (6.12). Taking $T_H \rightarrow 0$ directly in eq. (6.12) is not save for several reasons. First, as discussed in section (5.3) the semiclassical description for the BB breaks down at small temperature when the energy gap above extremality prevents excitations with finite energy. Second, as noted by Cai [34], although the $T_H \rightarrow 0$ limit is well defined, the usual computation of the shear viscosity to entropy ratio fails in the extremal case because the metric function as a double zero at the horizon. Third, also the computations of section 6.1 do not hold for $T_H = 0$ because the expressions (6.7) and (6.8) are ill defined for $T_H = 0$. However, the general method based on [35] and used in section 6.1 for calculating η , works also for extremal BB.

6.3 η/s in the extremal case

Let us now extend the calculations of η described in section 6.1 to the case of the extremal brane. In the extremal case the function f_- given by eq. (6.1) and its first derivative vanish when evaluated on the horizon. We have therefore at leading order near the horizon

$$f_-(r_+) = f'_-(r_+) = 0, \quad F(r_+) = N, \quad f_-(r) \simeq k(r - r_+)^2, \quad (6.17)$$

where k is some non zero constant. Using the previous expression in (6.6) one gets

$$\phi_1(r) = \frac{1}{kN\phi_0(r_+)\sqrt{\gamma(r_+)}} \frac{1}{(r - r_+)}. \quad (6.18)$$

On the other hand the near-horizon, small ω expansion gives now

$$\phi(r) = \phi_0(r_+) \left[1 + \frac{i\omega}{kN(r - r_+)} \right]. \quad (6.19)$$

Comparing eqs. (6.18) and (6.19), near the $r \rightarrow \infty$ boundary of AdS₅ we find the expansion

$$\phi(r) = 1 + i\omega\phi_0^2(r_+)\sqrt{\gamma(r_+)} \left(\frac{1}{r^4} \right), \quad (6.20)$$

from which follows the shear viscosity

$$\eta = \frac{s}{4\pi} \phi_0(r_+)^2. \quad (6.21)$$

Using the same argument used in section (6.1) to infer that $\phi_0(r_+) = 1$, we get for the shear viscosity to entropy ratio of the extremal GB black brane the universal value

$$\frac{\eta}{s} = \frac{1}{4\pi}. \quad (6.22)$$

It is interesting to notice that the universality of η/s for the extremal GB BB is a direct consequence of the universality of the AdS₂ \times R_3 , extremal, near-horizon geometry. In fact the extremal, near-horizon metric background (4.19) does not depend on λ . The other source for a λ - or Q -dependence of η is the function F in eq. (6.5). However, this contribution, hence the dependence of η from λ and Q , is removed by the condition $f'(r_+) = 0$, which implies that near the horizon the two-dimensional sections of the metric behave as AdS₂.

To conclude, let us now discuss the global behaviour of η/s as a function of the temperature in order to gain some insight about the η/s bounds. Taking into account that $r_+(T_H)$ is a monotonically increasing function, one easily finds that also the function $P(T_H) = \pi L^2 T_H / (N r_+) = 1 - 2\pi G_N Q^2 L^2 / (3r_+^6)$ in eq. (6.12) is a monotonically increasing function of T_H , with $P(0) = 0$ and $P(\infty) = 1$. The global behaviour of η/s in eq. (6.12) therefore is ruled by the sign of λ . For $\lambda < 0$, η/s is a monotonically *increasing* function of T_H , which raises from its minimum value $1/4\pi$ at $T_H = 0$ to its maximum value $(1 + 4|\lambda|)/4\pi$ for $T_H = \infty$, in full agreement with the KSS bound. On the other hand, for $0 < \lambda < 1/4$, η/s is a monotonically *decreasing* function of T_H , which drops from its maximum value $1/4\pi$ at $T_H = 0$ to its minimum value $(1 - 4\lambda)/4\pi$ for $T_H = \infty$, violating the KSS bound. Similar arguments on the existence of a global minimum for the η/s ratio in the case of scalar field coupled with a higher derivative gravity theory have been put forward in ref. [58].

6.4 Dependence on the time rescaling factor N

To conclude, let us briefly summarise the main facts concerning the role played by the factor N , which controls the rescaling of the time coordinate. The rescaling of the time coordinate in the metric (2.1) is compelling if one requires a unit speed of light in the dual CFT. This rescaling of the time coordinate implies that all the thermodynamical parameters, having the dimensions of energy (like e.g., mass and temperature), scale as $1/N$. As a consequence, the thermodynamical behaviour of GB (more in general Lovelock) black brane is universal (*i.e.*, independent from higher curvature terms in the action) only if one expresses thermodynamical relations in terms of the effective parameters (2.8). In particular, this implies that the central charge of the dual CFT at the UV fixed point scales as $1/N^3$ (see eq. (5.5)). On the other hand, the IR fixed point $\text{AdS}_2 \times S^3$ does not depend on N . This has a counterpart in the behaviour of the shear viscosity to entropy ratio. In the IR, the ratio η/s attains the universal, N -independent, value $1/4\pi$. Conversely, away from the IR fixed point, η/s shows a non trivial dependence on N . From eq. (6.12) we get

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 - 4N\pi L^2(1 - N^2) \frac{T_H}{r_+} \right]. \quad (6.23)$$

When the electric charge is set to zero, the ratio T_H/r_+ in eq. (6.12) is $N/(\pi L^2)$ and η/s reaches the value in eq. (6.2), as one expects. On the other hand, the dependence of η/s on T_H and N in the generic charged case is rather puzzling.

Given the universality of the thermodynamic behaviour of GB BB described in the previous sections, one would naively expect also the shear viscosity to entropy ratio to be universal, *i.e.*, that eq. (6.23) becomes the same as in the RN case just by using the effective temperature $T = T_H/N$ instead of T_H . However, this is not the case. Only for $N = 1$, which corresponds to $\alpha_2 = 0$, *i.e.*, exactly the RN case, η/s assumes the universal value $1/4\pi$, while for a generic N we have a quite complicated dependence on N and T_H . This behaviour indicates strongly that the transport features of the dual QFT in the hydrodynamic regime contain more information about the underlying microscopic theory than that contained in the thermodynamic description.

7 Summary and outlook

In this paper, we have discussed in detail geometrical, thermodynamic and holographic properties of charged $5d$ GB black branes. Although our discussion has been mainly confined to the GB case, we expect that most of our results can be generalised to Lovelock gravity theories in any spacetime dimensions.

We have shown that the particular combination of GB higher curvature terms added to the Einstein gravity action have three main effects:

- (1) They introduce a new branch of brane solutions, which are however not black branes but describe naked singularities. The global structure of the RN geometry of Einstein gravity is preserved only for $\alpha_2 > 0$. For $\alpha_2 < 0$ the spacetime splits into two disconnected regions (an inner and outer region), with the external region having

a single event horizon also in the non-extremal case. An interesting feature is that the solutions of the two branches may be, in some cases, continuously connected one with the other through the singularity. When this is the case, they describe transitions of the kind: $\text{AdS}_5 \rightarrow \text{singularity} \rightarrow \text{AdS}_5$, $\text{AdS}_5\text{-black brane} \rightarrow \text{singularity} \rightarrow \text{AdS}_5$ or $\text{AdS}_5\text{-black brane} \rightarrow \text{singularity} \rightarrow \text{dS}_5$. Although, it is known that one of the two branches of the solution (f_+) is unstable [36] one expects that the first two of these transitions have a holographic interpretation as the flow between two CFTs of different central charge through a singularity. The interpretation of this flow in the large \mathcal{N} description is problematic not only because of the spacetime singularity in the bulk but also owing to the presence of ghosts naturally tied to instabilities in the dual quantum field theory. On the other hand, it is widely believed that string theory could resolve gravitational singularities [60–66]. Some progress about the holographic interpretation of black hole singularities [61, 62] and more recently cosmological singularities [63–66] has been achieved. These results give some hope that in the near future one could have some understanding of the holographic flow between two CFTs through a spacetime singularity.

- (2) The thermodynamic behaviour of charged GB black brane is universal, i.e., when expressed in terms of effective mass and temperature is indistinguishable from that of the RN black brane.
- (3) Higher curvature terms modify the asymptotics (the AdS length) of the 5d AdS-RN gravity leaving unchanged the $\text{AdS}_2 \times R_3$, extremal near-horizon geometry of the RN black brane. At thermodynamic level, when expressed in terms of M_{ADM} and T_H a dependence on the normalization factor N of the metric is introduced but not for the extremal, near-horizon geometry $\text{AdS}_2 \times R_3$. In terms of the dual CFTs, this property can be described as a deformation of the CFT which changes the UV behaviour but leaves unchanged the IR. This behaviour is very similar to the attractor mechanism found in supergravity theories [59, 67–69], where the $\text{AdS}_2 \times R_n$ (or $\text{AdS}_2 \times S_n$) geometry is always the same irrespectively from the asymptotic values of the scalar fields.

We have also computed the shear viscosity to entropy density ratio for the GB charged black brane both for the non-extremal and the extremal case. We have found that consistently with the geometrical and thermodynamic picture, universality of η/s is lost in the UV but is restored in the IR. The ratio η/s has a non-universal temperature-dependent behaviour for non-extremal black branes but attains the universal $1/4\pi$ value at extremality. This result implies that η/s is completely determined by the IR behaviour and is completely insensitive to the UV regime of the dual QFT. This is largely expected because transport features in the hydrodynamic regime should be determined by IR physics (This fact has been also clearly pointed out in ref. [70]). However, it is not entirely clear if this result has a general meaning or it is just a consequence of the peculiarities of the charged GB black brane (higher curvature corrections vanish on the $\text{AdS}_2 \times R_3$ background).

Although the lesson to be drawn from our results is that probably it is not wise to look at the UV physics to infer about bounds on η/s , the question about the possible existence of bounds on η/s lower than the KSS one remains still open. We have found that η/s is a smooth monotonic function of the temperature. Going to small temperatures, it always flows to the universal value $1/4\pi$ but this value is a *minimum* for $\lambda < 0$ and *maximum* for $\lambda > 0$. Thus, the QFT dual to GB-Maxwell gravity with $\lambda < 0$ gives a nice example of temperature-flow of η/s always bounded from below by $1/4\pi$. On the other hand, the KSS-bound-violating flow we obtain in the theory for $0 < \lambda < 1/4$ remains open to further investigations.

A The black hole case

This paper has been focused on the charged black brane solutions of GB gravity. However we would conclude with some comments on the black hole solutions of the theory, i.e., solutions with $\kappa = 1$ in eq. (2.5). In the case of spherical black holes the discussion considerably changes. In fact, in $5d$, from eq. (2.5) we find that the metric function can be written as

$$f(r) = 1 + \frac{r^2}{2\alpha_2} \left[1 \mp \sqrt{1 - 4\alpha_2 \left(\alpha_0 - \frac{\omega_5 M}{r^4} + \frac{4\pi G_5 Q^2}{3 r^6} \right)} \right], \quad (\text{A.1})$$

where $\omega_5 = \frac{16\pi G_5}{3\Sigma_3}$ and Σ_3 is the volume of the 3-sphere. We have two branches of solutions, but similarly to the BB case, the only one admitting horizon solutions is f_- with α_0, α_2 constrained by (4.2). The black hole mass, can be expressed in terms of the horizon radius r_+ [2]

$$M = \frac{r_+^4}{\omega_5} \left[\alpha_0 + \frac{\alpha_2}{r_+^4} + \frac{1}{r_+^2} + \frac{4\pi G_5 Q^2}{3r_+^6} \right]. \quad (\text{A.2})$$

Due to the presence of the curvature ($\kappa = 1$), now the mass depends explicitly both on the AdS radius, $L^2 = \alpha_0^{-1}$ and on the GB coupling constant, α_2 .

The other important aspect which makes black holes different from black branes is that also temperature and entropy depend explicitly from α_2 through the coupling with the curvature since all the higher curvature corrections (like the Gauss Bonnet one) enter in the expression for the temperature trough a coupling with κ . As found by Cai [2], for a charged $5d$ GB black hole one gets

$$T = \frac{1}{4\pi r_+(r_+^2 + 2\alpha_2)} \left[4\alpha_0 r_+^4 + 2r_+^2 - \frac{4\pi G_5 Q^2}{3r_+^4} \right], \quad S = \frac{\Sigma_3 r_+^3}{4G_5} \left(1 + \frac{6\alpha_2}{r_+^2} \right). \quad (\text{A.3})$$

We see that since M, T, S depend explicitly on the GB coupling constant α_2 , differently from the black branes case, it is not anymore true that the thermodynamic behaviour of the Reissner-Nordström and Gauss-Bonnet black hole is the same. From the previous equation one can also realize that for the entropy, the area law no longer holds and that it receives a correction from α_2 .

Let us now consider the extremal of the GB black hole. In the BB case we have found the remarkable property that the extremal, near-horizon solution of the charged GB black

brane is exactly the same as the RN black brane. One can easily show that this is not the case for the extreme, near-horizon GB black hole. In the RN case the extremal, near-horizon, solution, which actually is an exact solution of the field equation is the $\text{AdS}_2 \times S_3$ geometry (S_3 is the three sphere), i.e the direct product of two maximally symmetric spaces, respectively with negative curvature $R^{(2)} = -2/l^2$ and positive curvature $R^{(3)} = \Lambda$, where l and Λ can be written in terms of the $5d$ cosmological constant and the $U(1)$ charge Q .

Using eqs. (4.21) one can show that the individual contributions of the AdS_2 and S_3 spaces, to the two terms in eq. (4.20) that are quadratic in the curvature tensors vanish. Nevertheless there are still some cross-product contributions arising from the mixing of AdS_2 and S_3 terms. Splitting the $5d$ indices (a, b) into $\mu, \nu = 0, 1$ (running on AdS_2) and $i, j = 1, 2, 3$ (running on S_3) we find a contribution to the μ, ν components of the field equations given by $2\alpha_2\Lambda/l^2g_{\mu\nu}$ and a contribution $4\alpha_2\Lambda/3l^2g_{ij}$ to the ij components of the field equations.

We see that the $\text{AdS}_2 \times S_3$ solution of the RN field equations cannot be also solution of the GB field equations. Obviously, this not prevents the existence of a *different* $\text{AdS}_2 \times S_3$ solution, *i.e* a solution with different curvatures for AdS_2 and S_3 . However, from the structure of the field equations and from eqs. (4.21) one can infer that these solutions, if existing, imply a dependence of l and/or Λ not only from the $5d$ cosmological constant and from the black hole charge Q but also from the GB coupling constant α_2 .

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