Determining the earthquake ductility demand through a rigid-plastic approach

Maria Cristina Porcu
Department of Structural Engineering, University of Cagliari, Italy
e-mail: mcporcu@unica.it

Giorgio Carta
Department of Structural Engineering, University of Cagliari, Italy
e-mail: gcarta@unica.it

Keywords: earthquake ductility demand, rigid-plastic approach, seismic inelastic response

SUMMARY: Assessing the earthquake ductility of seismic resistant structures usually requires a non-linear dynamic analysis involving both elastic and plastic motion of the structure. A simpler way to estimate the inelastic displacements can be neglecting the elastic motion altogether and referring to a rigid-plastic model. The latter may in fact give a good estimate of the maximum plastic displacement for elastoplastic oscillators of a comparatively short period in the elastic range. The same model also bounds the plastic response for sufficiently high periods of the oscillators. For medium-period oscillators, however, the rigid-plastic approximation needs to be corrected. Recently, the authors presented a simple procedure to predict in which ranges of periods the rigid-plastic approximation can be adopted as it stands. Subsequently, they also provided an empirical formula to obtain a suitable correction to apply outside these ranges. Both contributions make the rigid-plastic approach ready to be applied in practice. By referring to some real earthquakes, the present paper applies this approach to various elastoplastic oscillators. The results found show that the rigid-plastic approach proposed by the authors gives quite good –and almost always conservative- predictions of the maximum inelastic displacements of the elastoplastic oscillators.

1. INTRODUCTION

To withstand strong earthquakes, structures are usually designed so that they can undergo large plastic deformations. In order to provide such structures with adequate ductility, their maximum plastic displacements must be determined. This usually involves quite a lengthy non-linear dynamic analysis. To simplify the problem, approximate approaches are often adopted in practice, some of which perform a non-linear static analysis of the structure, see e.g. [Freeman, 1998], [Kim and D'Amore, 1999].

A viable alternative is to neglect the elastic contribution altogether and refer, therefore, to the rigid-plastic model. This was proposed in [Paglietti and Porcu, 2001], where the rigid-plastic oscillator was shown to model adequately the plastic response of rigid enough elastoplastic oscillators or of any elastoplastic oscillator under strong enough earthquakes. Whatever the earthquake, moreover, the rigid-plastic model was also shown to give a good estimate of the peak plastic response of quite flexible oscillators too, see [Porcu and Carta, 2007]. This happens when the natural period of the elastoplastic oscillator exceedes a characteristic value, say T^*, in which case the rigid-plastic approximation is even a conservative one. A still greater value of *T*, say $\overline{T}$ marks the end of the range in which the elastoplastic oscillators can deform plastically. Actually, the values of T^* and $\overline{T}$ depend on the earthquake and on the ratio between the yield strength and the mass of the oscillator, i.e. on the oscillator's *yield acceleration*. However, these values can be predicted easily from the elastic response spectrum of the earthquake by means of the simple procedure provided in [Porcu and Carta, 2007]. That procedure makes it possible to know in advance whether and when the rigid-plastic response can be adopted as it stands and when, on the contrary, it needs to be corrected.

As recalled in Section 2, the rigid-plastic approximation can be adopted as it stands for two different ranges of natural period, namely $0 \leq T \leq 0.1T^*$ and $T^* \leq T \leq \bar{T}$. Within the first range, the peak rigid-plastic displacement gives a very good estimate of the actual peak plastic response, while in the second range it always provides an upper bound the elastoplastic response. On the other hand, for $0.1T^* < T < T^*$, the rigid-plastic response usually underestimates the inelastic displacement demand of the earthquake. In this range an appropriate correction is, therefore, required.

The correction to apply in the range $0.1T^* < T < T^*$ can be estimated by means of some semi-empirical formulae given in [Paglietti and Porcu, 2001] and successively improved in [Porcu and Carta, 2005]. Yet, these formulae, which are in fact rather heavy, depend on an empirical parameter which should be determined for each earthquake and for each value of yield acceleration. This, of course, strongly limits their practical use. By referring to three bands of earthquakes violence defined in [Porcu and Carta, 2005], some curves were given which lead to find graphically the right correction to the rigid-plastic response for three different values of yield acceleration a_y. Unfortunately, however, those curves cannot be exploited to obtain the suitable correction for any value of a_y.

In order to overcome this shorthcoming, a simple empirical formula was finally proposed in [Porcu and Carta, to appear], which provides an effective correction once the characteristics of the oscillator and the couple of values T^* and $\bar{T}$ are given. Thanks to this formula, the rigid-plastic approach can be successfully applied also in the range $0.1T^* < T < T^*$. The results obtained in Section 4 for different earthquakes and different oscillators show that the rigid-plastic approach is quite easy to apply in practice and gives a good and almost always conservative estimate of the seismic inelastic peak displacement for any elastoplastic oscillator.

Of course, applying the rigid-plastic approach requires that the rigid-plastic peak displacement be known. This displacement can be easily obtained from the *rigid-plastic pseudo-spectrum* of the earthquake. This is a single curve diagram, giving the rigid-plastic peak displacement as a function of the oscillator yield acceleration. Simpler than the elastoplastic response spectrum, this diagram was firstly proposed in [Paglietti and Porcu, 2001] and then studied in detail in [Porcu and Mascia, 2006]. It should be noted that the approach here presented is cinfined to simple oscillators and no attempt will made to apply the same approach to multi-degree of freedom systems. Actually, some attempts to apply rigid-plastic models to seismic design of multi-degree of freedom structures were done in the '70s by [Nunziante and Augusti, 1970, 1971]. Recently, however, a simplified rigid-plastic design method was proposed in [Dominguez and Costa, 2007]. Based on the rigid-plastic pseudo-spectrum, this method estimates the earthquake inelastic demand on multi-storey building by means of an equivalent rigid-plastic oscillator.

2. THE RIGID-PLASTIC PREDICTION

Let's consider an elastoplastic oscillator of mass M and elastic stiffness k. The behaviour of the oscillator in the plastic range is assumed to be perfectly plastic and the same absolute value of yield force, say F_y, is supposed to apply to positive and negative loading. Consequently, the oscillator will exhibit the same absolute value of the elastic displacement at yield, that is $x_y = F_y / k$. It is also assumed that in the plastic range the only source of dissipation is due to plastic deformation, while in the elastic range a viscous damping ratio ξ applies.

The ductility required to the above oscillator to withstand to a given earthquake (*earthquake ductility demand*), can be expressed as:

$$\propto = \frac{x_y + x^p_{\max}}{x_y} = 1 + \frac{4\pi^2}{T^2 a_y} x^p_{\max} , \tag{1}$$

cf. [Chopra, 2001], [Porcu and Carta, to appear]. Here $x^p_{\max}$ is the peak plastic displacement of the oscillator, $T = 2\pi\sqrt{M/k}$ is its natural vibration period, while $a_y = F_y / M$ will be referred to as the oscillator *yield acceleration*. Relation (1) shows that, for a given oscillator, the earthquake ductility demand only depends on $x^p_{\max}$. On the other hand, for a given earthquake, we have that:

$$x^p_{\max} = x^p_{\max}(T, \xi, a_y) . \tag{2}$$

Throughout this paper the displacements of the mass are supposed to be relative to the fixed support of the oscillator.

To estimate the value of $x^p_{\max}$, let us now consider a rigid-plastic oscillator possessing the same yield acceleration a_y of the above elastoplastic oscillator. We can refer to it as the *corresponding rigid-plastic oscillator*. For a given ground motion, the peak displacement of this oscillator, which in this case is a purely plastic one, only depends upon a_y, cf. [Paglietti and Porcu, 2001]. That is:

$$x^{RP}_{\max} = x^{RP}_{\max}(a_y) . \tag{3}$$

This displacement is simpler to calculate than $x^p_{\max}$. Moreover, for each given value of a_y, a unique value of $x^{RP}_{\max}$ should be calculated relevant to the considered earthquake.

As a matter of fact, $x^{RP}_{\max}$ was shown to well approximate $x^p_{\max}$ for short-period elastoplastic oscillators ($T \leq 0.1T^*$) and to bound $x^p_{\max}$ for comparatively high-period oscillators ($T^* \leq T \leq \overline{T}$). However, $x^{RP}_{\max}$ is generally lower than $x^p_{\max}$ within the range $0.1T^* < T < T^*$; cf. [Porcu and Carta, 2007], [Porcu and Carta, to appear]. What is clamed above is supported by Figure 1, where the peak plastic displacements of oscillators with different period T and $a_y=0.2g$, (g being the gravity acceleration) are plotted versus the peak displacement of the corresponding rigid-plastic oscillator (heavy horizontal line). The figure refers to the Loma Prieta (CLS000) 1989 earthquake.

It should be observed that T^* and $\overline{T}$ are a couple of characteristic periods, the value of which depend upon the earthquake and, for each earthquake, upon the oscillator yield acceleration. However, the simple graphical procedure presented in [Porcu and Carta, 2007] can predict these two characteristic values directly from the elastic response spectrum of the earthquake. For any given value of a_y, this can be done by intercepting the earthquake elastic spectrum with the following two curves:

$$x^*(T) = \frac{a_y T^2}{4\pi^2}\sqrt{\frac{8\pi^2 x_{\max}^{RP}}{a_y T^2}+1}\,, \tag{4}$$

$$x_y\left(T\right) = \frac{a_y}{4\pi^2}\;T^2\,. \tag{5}$$

In particular, curve (5) singles out the exact value of $\overline{T}$, while curve (4) gives an approximate value of T^*. Should curve (4) intercept more than once the elastic spectrum, the interception point nearest to $\overline{T}$ should conservatively be chosen. Note that plotting curve (4) implies knowing the value of $x_{\max}^{RP}$. This value can be extracted from the rigid-plastic pseudo spectrum of the earthquake, which should in fact be at disposal when adopting a rigid-plastic approach. Both curves (4) and (5) are plotted in Figure 2 together with the elastic response spectrum of the Loma Prieta (CLS000) 1989 earthquake. A comparison with Figure 1 shows that the exact value of $\overline{T}$ is found. Moreover, a rather good prediction of T^* is in fact obtained from Figure 2. It should be noted that Figures 1-2 refer to *ξ=10%*, which is a recommended value for most structural materials when stress level is at yield. This value is assumed in what follows.

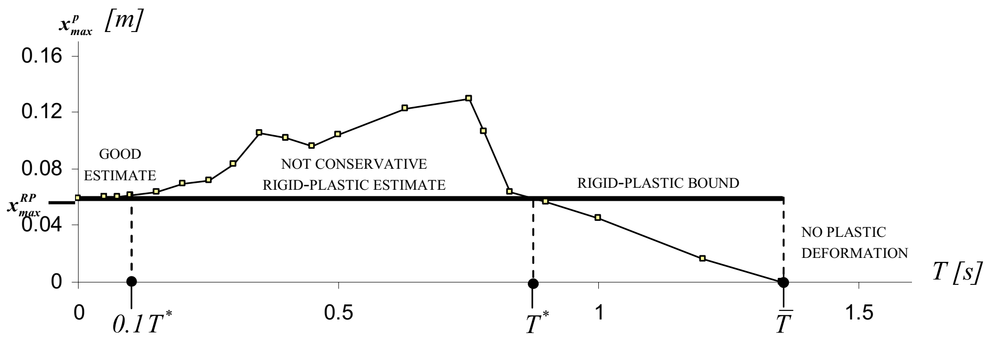

Fig. 1. Elastoplastic versus rigid-plastic peak plastic displacements. The diagram refers to the Loma Prieta (CLS000) 1989 earthquake and to oscillators possessing a_y=0.2g (ξ=10%).

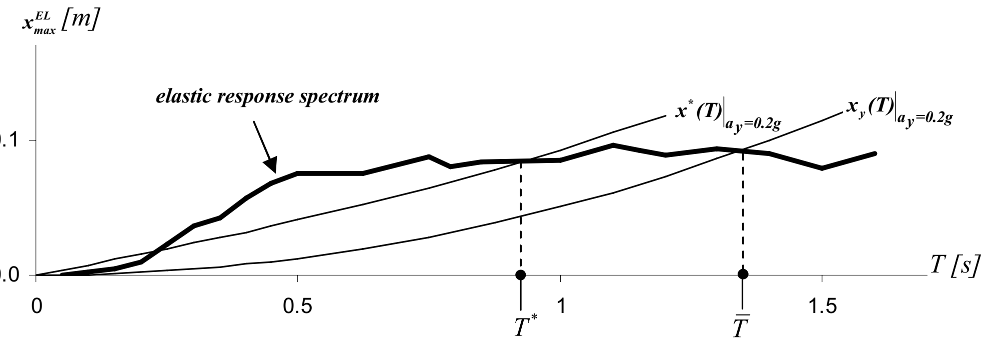

Fig. 2. Determining the values of T^* and $\bar{T}$ from the elastic response spectrum of the Loma Prieta (CLS000) 1989 earthquake (a_y=0.2g and ξ=10%).

3. CORRECTING THE RIGID-PLASTIC RESPONSE IN THE RANGE $0.1T^* < T < T^*$

As quoted in the previous section, for $0 < T < 0.1T^*$ the rigid-plastic oscillator provides a reasonable - though generally not conservative- estimate of the maximum plastic displacements of the elastoplastic oscillator. The same is not true for $0.1T^* < T < T^*$, in which range the rigid-plastic estimate is in need of some significant correction. In this range, in order to obtain a conservative estimate of $x^p_{\max}$, the correction $\overline{\Delta x}^p$ should be such that:

$$x^{RP}_{\max} + \overline{\Delta x}^p \geq x^p_{\max} \quad . \tag{6}$$

On the other hand, the value of $\overline{\Delta x}^p$ should also comply with the following conditions:

$$\begin{aligned} &for\ T = 0.1T^* \quad \overline{\Delta x}^p = 0 \\ &for\ T = T^* \quad \overline{\Delta x}^p = 0 \end{aligned} \tag{7}$$

[Porcu and Carta, to appear] found, through an empirical analysis of a large set of numerical data, that a formula that meets the above requirements is the following:

$$\overline{\Delta x}^p = -11a_y\left(0.15\sqrt{\tau}+0.02\tau\right)^2\left[\left(\frac{T}{T^*}\right)^2 - 1.1\left(\frac{T}{T^*}\right)+0.1\right] \qquad [cm] \quad , \tag{8}$$

which applies whatever the earthquake and whatever a_y. The quantity τ appearing in this formula is given by:

$$\tau = \bar{T}\left(1-\sqrt{\frac{a_y}{g}}\right) \quad [s] \tag{9}$$

4. RIGID-PLASTIC APPROACH TO PREDICT THE EARTHQUAKE INELASTIC DISPLACEMENT DEMAND

From the elastic response spectrum and the rigid-plastic pseudo-spectrum of the earthquake the value of $x^{p}_{\max}$ can be easily obtained through the following steps First of all, the value of $x^{RP}_{\max}$ should be obtained from the rigid-plastic pseudo-spectrum at the considered yield acceleration a_y, see e.g. Fig. 3. Secondly, by using curves (4) and (5), the values of T^* and $\overline{T}$ should be obtained from the elastic response spectrum, as described in Fig. 2. Finally, the quantity $x^{p}_{\max}$ should be estimated as follows:

$$\text{for } T \le 0.1T^* \qquad x^{RP}_{\max} \tag{10a}$$

$$\text{for } 0.1T^* < T < T^* \qquad x^{RP}_{\max} + \overline{\Delta x}^{\,p} \tag{10b}$$

$$\text{for } T^* \le T \le \overline{T} \qquad x^{RP}_{\max} \tag{10c}$$

The value of $\overline{\Delta x}^{\,p}$ to insert in eq. (10)b should be calculated from formula (8).

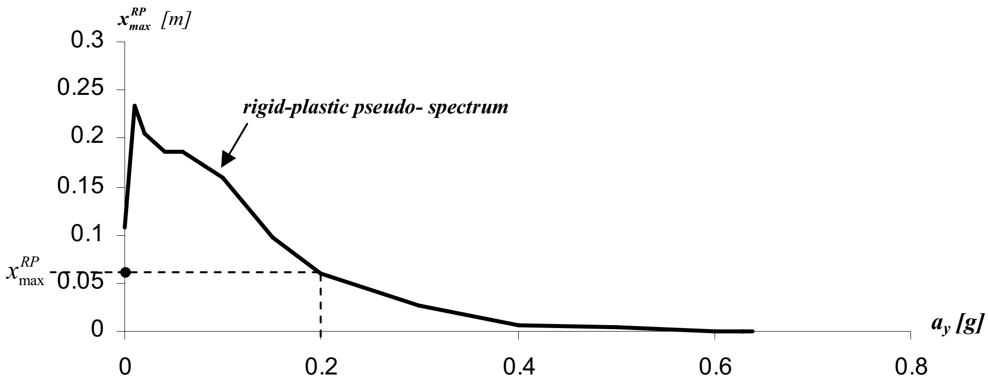

Fig. 3. Determining the value of $x^{RP}_{\max}$ relevant to a_y=0.2g from the rigid-plastic pseudo-spectrum of the Loma Prieta 1989 earthquake (taken from [Porcu and Mascia, 2006]).

The rigid-plastic prediction relevant to the instance considered in Figures 1-3 is plotted in Fig. 4. Relevant to oscillators with $a_y=0.3g$ a similar prediction is plotted in Figure 5. Similarly, the rigid-plastic predictions obtained for different earthquakes and different values of a_y are plotted in Figures 6-10. It should be apparent from Figs 3 to 10 that the present approach provides a rather good and almost always conservative prediction of the maximum plastic displacement of an elastoplastic oscillator. This is true even if the value of T^* is only roughly estimated, as happens, for instance, in the cases reported in Figures 7 and 8.

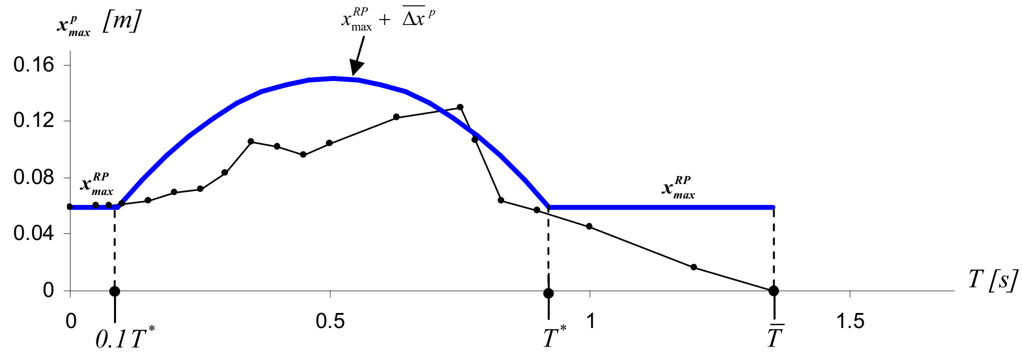

Fig. 4 Rigid-plastic prediction (heavy line) for the Loma Prieta (CLS000) 1989 earthquake and a_y=0.2g (ξ=10 The light dotted line represents the results from a complete non linear analysis (ξ=10%)

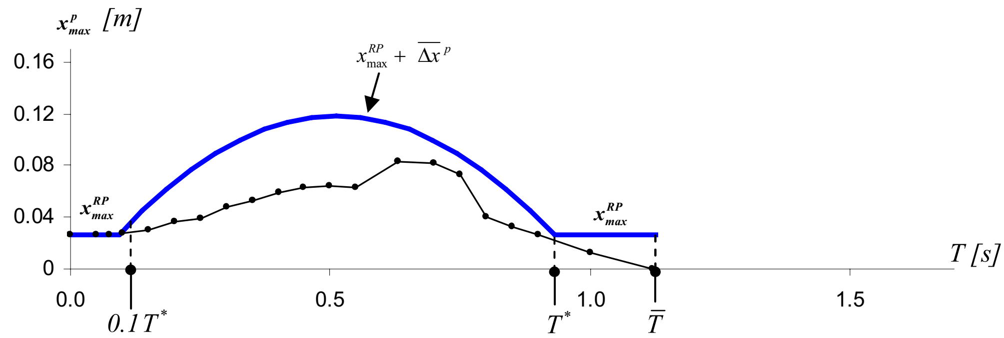

Fig. 5 As in the previous figure but for a_y=0.3g.

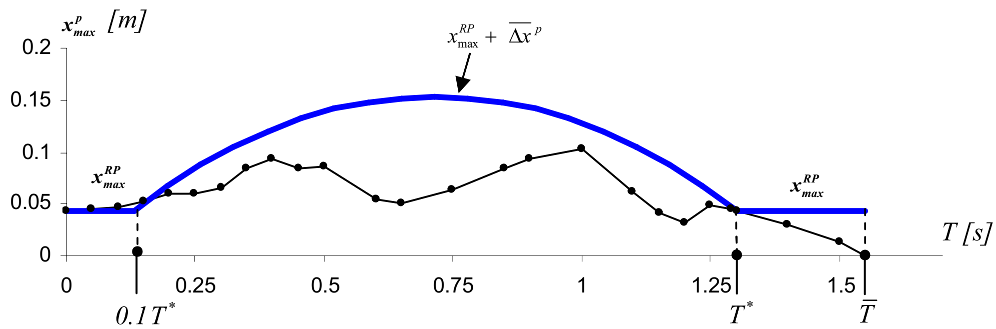

Fig. 6 Rigid-plastic prediction (heavy line) for the Victoria (Mexico), CPE045, 1980 earthquake and oscillators possessing a_y=0.2g (ξ=10%).

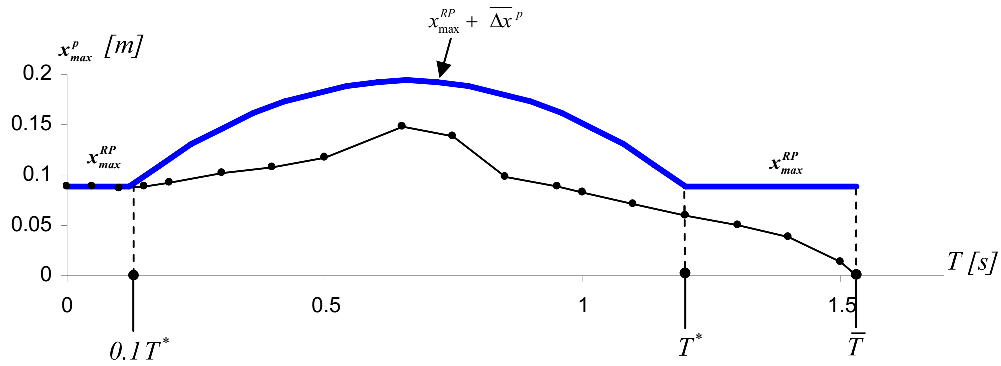

Fig. 7 As in the previous figure, but referring to the Morgan Hill (Cal), CYC195, 1984 earthquake and to oscillators possessing a_y=0.2g, (ξ=10%).

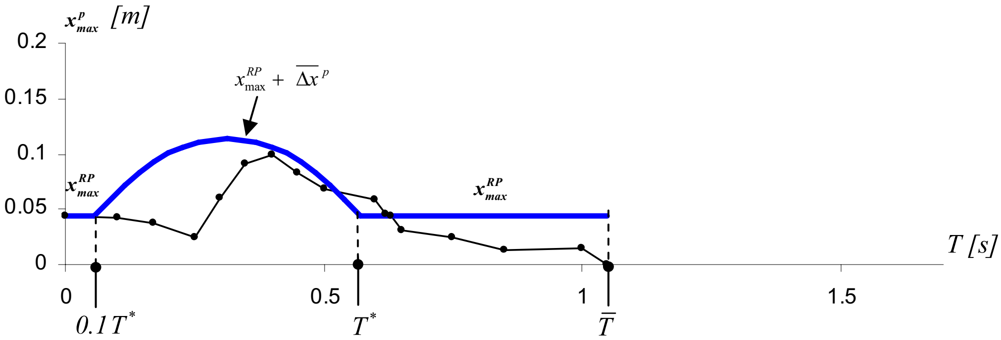

Fig. 8 As in the previous figure, but referring to the El Salvador, LONG, 2001 earthquake and to oscillators possessing a_y=0.2g, (ξ=10%).

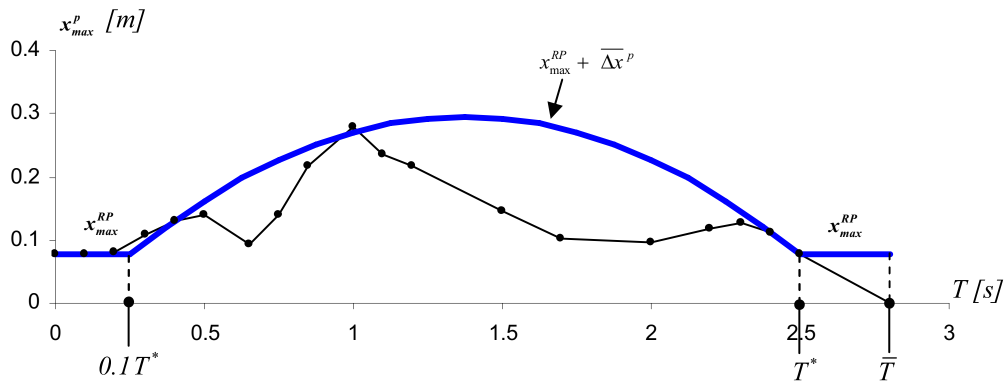

Fig. 9 As in the previous figure, but referring to the Tabas (Iran), N74E, 1978 earthquake and to a_y=0.25g, (ξ=10%).

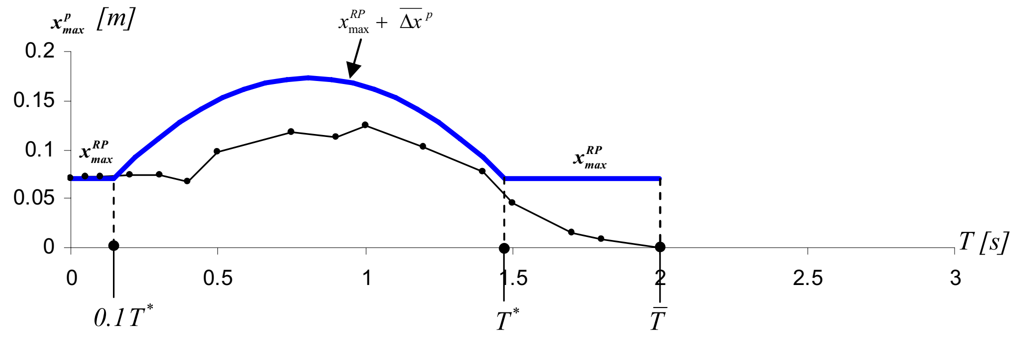

Fig. 10 As in the previous figure, but referring to the Parkfield (Cal) 90, 2004 earthquake and to a_y=0.2g (ξ=10%).

CONCLUSIONS

The paper shows that the rigid-plastic model can successfully be exploited to assess the earthquake ductility demand of any elastoplastic oscillator. To do this, both the rigid-plastic pseudo-spectrum and the elastic response spectrum of the earthquake should be available.

The earthquakes considered in the present paper have been obtained from:
ESD – The European Strong-Motion Database: *http://www.isesd.cv.ic.ac.uk*
PEER Strong Motion Database: *http://peer.berkeley.edu/smcat*
COSMOS Virtual Data Center: *http://db.cosmos-eq.org*

References:

Chopra, A. K. (2001) *Dynamics of Structures: Theory and Applications to Earthquake Engineering* Prentice Hall, Englewood Cliffs, N. J., 2nd ed.

Dominguez Costa JL, Bento R, Vsevolod Levchitch and Nielase MP. (2007) "Rigid-plastic seismic design of reinforced concrete structures", *J. Earthquake Engineering & Structural Dynamics*; **36**: 55-76.

Freeman, SA. (1998) "Development and use of capacity spectrum method", Proc. Of the 6th U.S. National Conference on Earthquake Engineering, CD-ROM, EERI, Oakland.

Kim, B. and D'Amore, E. (1999) "Pushover analysis procedure in earthquake engineering", *Earthquake Spectra* **13**(2): 417-434.

Nunziante, L. and Augusti, G. (1970) "Dynamic behaviour of rigid-plastic frames subject to horizontal repeated loads" *Proc. of the 13th Polish Mechanics Conference*, Jaszowiec, Poland.

Nunziante, L. and Augusti, G. (1971) "Dynamic behavior of multi-storey rigid-plastic frames subject to horizontal motions" MEXANEKA **6**, 153-172.

Paglietti, A., Porcu, M.C. (2001) “Rigid-plastic approximation to predict plastic motion under strong earthquakes”, *J. Earthquake Engineering & Structural Dynamics*, **30**: 115-126.

Porcu, MC, Carta, G. (2007) “Rigid-plastic bound to the seismic inelastic response of flexible elastoplastic oscillators”, *European Earthquake Engineering,* **3**.

Porcu, MC, Carta, G. (2005) “Curve di maggiorazione dell’errore per l’analisi sismica di oscillatori elasto-plastici attraverso il modello rigido-plastico”, Proc. XVII Congresso AIMETA, Firenze, University Press,.

Porcu, MC, Carta, G. (to appear) “Rigid-plastic dynamics to assess the earthquake ductility demand”.

Porcu MC, Mascia M. (2006) “Rigid-Plastic Pseudo-Spectra: Peak Response Charts for Seismic Design, *European Earthquake Engineering*”, **3**: 37-47.