Introducing a relocation service for one-way carsharing with a first-in first-served policy

Abstract

In one-way carsharing users are allowed to return cars to locations different from those where they were picked up, but directional imbalances in their requests result in the accumulation of unnecessary cars in some areas, whereas other areas face car shortages. To correct this situation, we investigate the introduction of a new relocation service by a staff equipped with foldable motorcycles: they are driven to move to unused cars and are put inside cars, which are driven by the staff where they are requested. Although the relocation staff size can be determined by a state-of-the-art model, it tends to overestimate the manpower maximizing the overall system profitability in a first-in first served policy. This paper presents an optimization model correcting this drawback. This model can be used to investigate how different manpower levels change the fraction of satisfied user bookings and determine the most profitable staff size configuration.

Keywords: One-way carsharing, relocation, manpower, integer programming

1. Introduction

Private transport has produced long-term congestion problems, which have resulted in high consumption of time and energy. In addition, there are increasing ownership costs and the net use of private vehicles is typically very low, in fact they are often parked for most of the time. These drawbacks may be limited by carsharing services, which consist of a number of vehicles used several times a day by a relatively large group of members[1], [2].

*Email address of the corresponding author: mdifrance@unica.it

In traditional carsharing systems, users are required to return cars to the same station from which they were picked up. A more attractive and flexible service for users is the so-called one-way carsharing, which provides them with the flexibility in return stations and, sometimes, in return times [3]. Nowadays, in large urban areas there is a trend toward the so-called free-floating service, which is a one-way carsharing from any point to any point [4].

The effectiveness of all one-way systems depends on the possibility for customers to find cars where and when they are requested. However, this requirement is not automatically guaranteed: as trips may not end where there they start, cars tend to get stuck in areas of low individual demand, while they are in shortage in high demand zones [5]. Therefore, it is important to adopt proper relocation mechanisms and evaluate their impacts on the overall service profits.

A first relocation mechanism is to shift this task to the users. It is based on the intuition that a group of people having common origins, destinations and travel times may be split in departure areas with an excess of unused vehicles or joined in those with few vehicles [6], [7], [8], [9], [10]. Since user-based relocation may not be attractive for users, it may not work in practice.

A second relocation mechanism is controlling users' accessibility and accepting only the bookings favoring the return of cars in popular departure areas, while refusing those which are not enough profitable [11], [12], [13]. Therefore, some user bookings may end up being rejected.

A third relocation mechanism is the employment of specific staff in charge of picking up unused cars and moving them where they may be requested. This mechanism was investigated by Bath and Todd [14] and Kek et al. [15], who presented simulation models based on lower and upper inventory thresholds: when the number of cars is not between these thresholds, cars are repositioned by the staff. The model proposed in [15] was calibrated by Kek et al. [16], who developed an optimization phase setting the staff size, the number of relocations and car stocks. The optimal staff size of the relocation personnel was investigated by [17], who also determined the optimal fleet size, the number and location of the required stations of a one-way non-floating car-sharing system.

In this paper we investigate the introduction of a particular staff-based relocation mechanism by foldable motorbikes and infer conclusions on its profitability. In order to guarantee parking areas, some stations with a number of parking spots are already arranged. Despite the emerging interest in carsharing services without reservation [18], [19], users are recommended to book their transportation requests in advance, because they are served according to a first-in first-served policy. The relocation is performed by staff equipped with foldable motorcycles: they are used to move to unused cars and are put inside the cars, which are driven by the staff to the stations where they are demanded.

Our case study is a medium size city, where the decisions on station locations and car fleet size have already been made, whereas the number of relocation personel has not been determined yet. Although the size of the relocation staff can be planned in our problem setting by adapting the approach of [16], it tends to add a new relocation worker whenever a booking cannot be met. However, additional relocation workers increase the system fixed costs, which may not be covered by the revenues of the new bookings. Therefore, the manpower determined by [16] is expected to be an overestimate with respect to that maximizing the overall system profitability. To correct this drawback, we propose an optimization model, which can be used to evaluate the profitability of different manpower configurations, taking into account the revenues, relocation costs and fixed costs, which depend on the relocation manpower.

To summarize, the objectives of this paper are:

- To present a relocation service for one-way carsharing systems between stations with a first-in first-served policy, required reservations, and required return times. The service is performed by relocation workers equipped with foldable motorcycles;
- To formulate an optimization model for planning car relocation, in order to evaluate why it is important to introduce this relocation activity, how different manpower levels change the fraction of satisfied bookings and which is the most profitable staff size configuration.

This paper is organized as follows. The investigated problem is presented in Section 2 and modeled in Section 3. The profitability of different staff size configurations is discussed in Section 4. Finally, Section 5 presents a summary of our conclusions and describes future research perspectives.

2. Problem description

Consider a carsharing system where cars can be picked up and returned in a set of predefined stations. Users can reserve cars by bookings, each of which has four attributes:

- where the car must be picked up by the user at the beginning of the trip (i.e. the departure station);
- when the car must be picked up by the user at the beginning of the trip (i.e. the departure period);
- where the car must be returned by the user at the end of the trip (i.e. the arrival station);
- when the car must be returned by the user at the end of the trip (i.e. the arrival period).

Due to directional imbalances in the bookings of users, some stations tend to accumulate unnecessary cars, while other stations face car shortages. To correct this situation, carsharing providers must periodically relocate cars between stations.

Car surpluses and shortages can be described in terms of supply and demand. The supply is defined as the number of cars which can be picked up in a station at any given time. These cars (which are also called available cars) can be kept in stations to meet future user bookings or relocated to other stations. The demand is defined as the number of cars requested in a station at any given time. The demand in a station must be met by the supply of cars kept in the same station or relocated from other stations.

In this paper, we consider a particular relocation service performed by workers equipped with foldable motorcycles. A relocation worker moves by his foldable motorcycle to a station where a car is in supply, puts the motorcycle inside the car, drives the car to a station where there is a demand and takes the motorcycle from the car. Next, the worker can either wait in this station or move by the motorcycle to another station in order to relocate another car. Thus, the main challenge in this relocation service is to determine the sequence of moves by motorcycles and cars for each worker.

In addition, the relocation staff is charge of checking cars and perform a bit of maintenance, such as cleaning, substituting lights, blowing tyres up, etc. As these problems decrease the number of cars available for users, the maintenance plays an important role to turn unavailable cars into available ones, which may help serve additional bookings and generate more profits.

In this paper, we consider a first-in first-served policy to serve bookings within the considered planning horizon. Whenever a new booking arrives, the values of demand at the departure station and the supply at the arrival station are increased by one. If these values become larger than the number of parking slots in these stations, the booking is rejected before planning any car relocation. If this is not the case and the number of available cars is sufficient, the carsharing provider plans the relocation and checks if

- cars can be provided where and when they are requested;
- free parking slots can be provided where and when cars must be returned.

If both these requirements are met, the booking is accepted, otherwise it is rejected. An optimal relocation plan penalizes the storage of unavailable cars, minimizes the costs of driving motorcycles and cars, and aims at serving as many bookings as possible.
In the following section, an optimization model is proposed to deal with this problem.

3. Modeling

3.1. Graph

This problem can be modeled by a time–space graph $G(N, A)$, where the nodes of a set N represent the stations replicated in every period of the planning horizon. For instance, Figure 1 shows a time-extended network with circular shaped nodes. They represent four stations, which are denoted by the letters from A to D and replicated over six periods, which are denoted by the integers from 1 to 6. The sequence of activities performed by each relocation worker is represented by a path in $G(N, A)$.
The arcs from a station in a period to the same station in the next period model a relocation worker who waits in this station between these periods or performs maintenance in this station between these periods. Waiting and maintenance arcs are represented in Figure 1 by discontinuous of equal length and dotted lines, respectively. For example, in Figure 1, a relocation worker waits at station B from period 1 to period 2 and performs maintenance at station D from period 3 to period 4.
Arcs from a station in a period to another station in another period represent relocation workers moving between these stations by a motorcycle or a car. These arcs are denoted in Figure 1 by discontinuous lines of different length

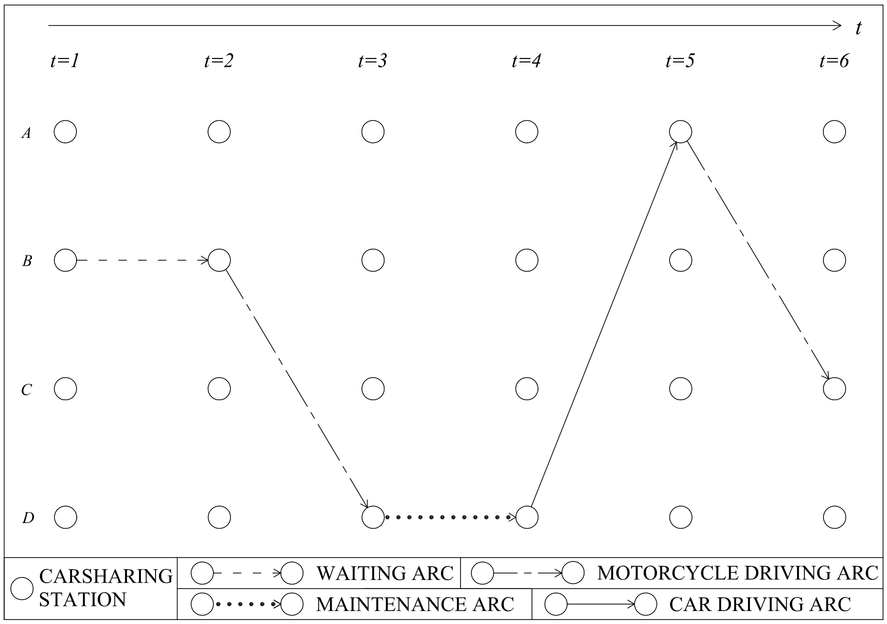

Figure 1: A time-space network for planning car relocation

and continuous lines, respectively. Therefore, Figure 1 shows a relocation worker moving from station B at time 2 to station D at time 3 by his motorcycle and from station D at time 4 to station A at time 5 by car and, next, to station C at time 6 by the motorcycle.
Determining the sequence of activities of each staff member is not sufficient for this problem: in fact, one must also decide how many available and unavailable cars are stored in each station and each period. In addition, whenever a new booking arrives, it is important to check if the carsharing provider is able to serve all bookings arrived so far. If this is not the case, the last booking must be rejected, owing to the first-in first-served policy.

3.2. Optimization model

Let I be the set of stations and T the set of contiguous time periods representing the planning horizon. The time period index $t \in T$ takes values from 1 to $|T|$. Let $i_t \in N$ be the node representing station $i \in I$ at time $t \in T$, A_y the set of waiting arcs, A_z the set of maintenance arcs, A_u the set of motorcycle driving arcs, and A_v the set of car driving arcs. Let K be the set of relocation workers, who may perform waiting, maintenance, or

motorcycle or car driving activities.
Let d_{i_t} be the number of cars required in station $i \in H$ between periods $t-1 \in T$ and $t \in T$, and let s_{i_t} be the number of cars returned to station $i \in H$ between periods $t-1 \in T$ and $t \in T$. We denote by p_i the maximum number of cars that can be stored at station $i \in H$. Since each car must be picked up from and returned to a parking slot, d_{i_t} and s_{i_t} cannot be larger than p_i. Let τ be the time required to maintain a car and m_{i_t} be the number of cars requiring maintenance, which are returned in station i between periods $t-1$ and t. Thus, $m_{i_t} \leq s_{i_t}$.
The model decision variables are defined as follows:

- The variable $y^k_{i_t,i_{t+1}}$ takes the value 1 if relocation worker $k \in K$ waits at station $i \in I$ from time $t \in T$ to time $t+1 \in T$, 0 otherwise.

- The variable $z^k_{i_t,i_{t+\tau}}$ takes the value 1 if relocation worker $k \in K$ performs maintenance in station $i \in I$ from time $t \in T$ to time $t+\tau \in T$, 0 otherwise.

- The variable $u^k_{i_t,j_{t+t_{ij}}}$ takes the value 1 if relocation worker $k \in K$ moves by a motorcycle from station $i \in I$ at time $t \in T$ to station $j \in I$ at time $t+t_{ij} \in T$, 0 otherwise; $c^{uk}_{i_t,j_{t+t_{ij}}}$ represents the related unit cost.

- The variable $v^k_{i_t,j_{t+t_{ij}}}$ takes the value 1 if relocation worker $k \in K$ drives a car from station $i \in I$ at time $t \in T$ to station $j \in I$ at time $t+t_{ij} \in T$, 0 otherwise; $c^{vk}_{i_t,j_{t+t_{ij}}}$ represents the related unit cost.

- The variable $x^a_{i_t}$ represents the number of cars which are available to be picked up by users and kept in stock at station $i \in I$ at the beginning of period $t \in T$.

- The variable $x^n_{i_t}$ represents the number of cars in need of maintenance at station $i \in I$ at the beginning of period $t \in T$; c^n represents the related unit cost that penalizes the storage of unavailable cars, to turn them into available cars by maintenance.

- The variable $x^d_{i_t}$ represents the number of cars that cannot be provided to customers at station $i \in I$ from period $t-1 \in T$ to period $t \in T$ owing to relocation understaffing. Let c^d be the related unit cost penalizing car shortages.

- The variable $x^s_{i_t}$ represents the number of cars that cannot be returned to station $i \in I$ from period $t-1 \in T$ to period $t \in T$ owing to the lack of free parking slots and/or relocation understaffing. Let c^s be the related unit cost penalizing the saturation of stations.

When the relocation worker $k \in K$ is used in the first period of the planning horizon, he must perform only one of these four activities: waiting, doing maintenance or driving a motorcycle or a car. This constraint is formalized as follows:

$$\sum_{(i_1,i_2)\in A_y} y^k_{i_1,i_2} + \sum_{(i_1,i_{1+\tau})\in A_z} z^k_{i_1,i_{1+\tau}} + \sum_{(i_1,j_{1+t_{i,j}})\in A_u} u^k_{i_1,j_{1+t_{ij}}} + \\ + \sum_{(i_1,j_{1+t_{i,j}})\in A_v} v^k_{i_1,j_{1+t_{ij}}} = 1 \qquad \forall k \in K \quad (1)$$

When an activity is completed, a new one must be started by each relocation worker $k \in K$ at any node $i_t \in N$ such that $t \neq 1$ and $t \neq |T|$. This constraint can be enforced as follows:

$$y^k_{i_{t-1},i_t} + z^k_{i_{t-\tau},i_t} + \sum_{j_{t-t_{j,i}}\in N} u^k_{j_{t-t_{ji}},i_t} + \sum_{j_{t-t_{j,i}}\in N} v^k_{j_{t-t_{ji}},i_t} = y^k_{i_t,i_{t+1}} + z^k_{i_t,i_{t+\tau}} + \\ + \sum_{j_{t+t_{i,j}}\in N} u^k_{i_t,j_{t+t_{i,j}}} + \sum_{j_{t+t_{i,j}}\in N} v^k_{i_t,j_{t+t_{i,j}}} \qquad \forall i_t \in N, t = 2,\dots,|T|-1, \forall k \in K \quad (2)$$

An relocation worker $k \in K$ can perform only one activity in the last period of the planning horizon. This constraint can be formalized as follows:

$$\sum_{(i_{|T|-1},i_{|T|})\in A_y} y^k_{i_{|T|-1},i_{|T|}} + \sum_{(i_{|T|-\tau},i_{|T|})\in A_z} z^k_{i_{|T|-\tau},i_{|T|}} + \\ + \sum_{(i_{|T|-t_{i,j}},j_{|T|})\in A_u} u^k_{i_{|T|-t_{i,j}},j_{|T|}} + \sum_{(i_{|T|-t_{i,j}},j_{|T|})\in A_v} v^k_{i_{|T|-t_{i,j}},j_{|T|}} = 1 \qquad \forall k \in K \quad (3)$$

The number of available cars at each station is adjusted in each period by the cars relocated into and out of the station, cars returned to the station after maintenance, cars returned and picked up by users. Moreover, some cars may become unavailable for users and may need maintenance. More formally, the number of available cars in each station and each period is updated as follows:

$$x^a_{i_t} = x^a_{i_{t-1}} + \sum_{k \in K}(\sum_{j_{t-t_{j,i}} \in N} v^k_{j_{t-t_{ji}}, i_t} - \sum_{j_{t+t_{i,j}} \in N} v^k_{i_t, j_{t+t_{i,j}}} + z^k_{i_{t-\tau}, i_t}) +$$
$$+ s_{i_t} - x^s_{i_t} - d_{i_t} + x^d_{i_t} - m_{i_t} \qquad \forall i \in I, \forall t \in T \quad (4)$$

The number of unavailable cars at each station in each period is updated by cars under maintenance and requiring maintenance, when they are returned by users, that is:

$$x^n_{i_t} = x^n_{i_{t-1}} - \sum_{k \in K} z^k_{i_{t-1}, i_{t-1+\tau}} + m_{i_t} \qquad \forall i \in I, \forall t \in T \quad (5)$$

The number of available and unavailable cars at a station must not exceed that station's capacity $i \in I$ from any time $t \in T$ to $t+1 \in T$:

$$x^a_{i_t} + x^n_{i_t} + s_{i_{t+1}} - x^s_{i_{t+1}} \leq p_i \qquad \forall i \in I, \forall t \in T \quad (6)$$

The number of unsatisfied car restitutions must be lower than the total number of cars that must be returned at any node:

$$x^s_{i_t} \leq s_{i_t} \qquad \forall i \in I, \forall t \in T \quad (7)$$

The number of unsatisfied car requests must be lower than the total number of cars requested at any node:

$$x^d_{i_t} \leq d_{i_t} \qquad \forall i \in I, \forall t \in T \quad (8)$$

We minimize the transportation costs of motorcycles and cars, as well as the costs of shortages, parking saturation, and delayed maintenance:

$$z = min \sum_{k \in K} \sum_{i_t \in N} \sum_{j_{t+t_{ij}} \in N} \left(c^{uk}_{i_t, j_{t+t_{ij}}} u^k_{i_t, j_{t+t_{ij}}} + c^{vk}_{i_t, j_{t+t_{ij}}} v^k_{i_t, j_{t+t_{ij}}} \right) + c^d \sum_{i_t \in N} x^d_{i_t} +$$
$$+ c^s \sum_{i_t \in N} x^s_{i_t} + c^n \sum_{i_t \in N} x^n_{i_t} \qquad (9)$$

The proposed model can be turned into a staff sizing model by adding a binary variable x^k that takes the value 1 if relocation worker k is employed, 0 otherwise. In this model, if we denote by c^k the fixed cost of relocation worker k, the objective function is

$$z' = min \sum_{k\in K} \sum_{i_t\in N} \sum_{j_{t+t_{ij}}\in N} \left(c^{uk}_{i_t,j_{t+t_{ij}}} u^k_{i_t,j_{t+t_{ij}}} + c^{vk}_{i_t,j_{t+t_{ij}}} v^k_{i_t,j_{t+t_{ij}}} \right) + c^d \sum_{i_t\in N} x^d_{i_t} + \\ + c^s \sum_{i_t\in N} x^s_{i_t} + c^n \sum_{i_t\in N} x^n_{i_t} + \sum_{k\in K} c^k x^k = z + \sum_{k\in K} c^k x^k \quad (10)$$

Moreover, contraint (1) is modified as follows:

$$\sum_{(i_1,i_2)\in A_y} y^k_{i_1,i_2} + \sum_{(i_1,i_{1+\tau})\in A_z} z^k_{i_1,i_{1+\tau}} + \sum_{(i_1,j_{1+t_{i,j}})\in A_u} u^k_{i_1,j_{1+t_{ij}}} + \\ + \sum_{(i_1,j_{1+t_{i,j}})\in A_v} v^k_{i_1,j_{1+t_{ij}}} = x_k \qquad \forall k \in K \quad (11)$$

This sizing model is similar to that of [16]. The main difference is the penalization in the objective function of the storage of unavailable cars, which are forced to be turned into available cars by a maintenance process. This novelty results in a larger number of available cars for users and, hence, it can put carsharing providers in the position of meeting more user bookings. In the following section, we will discuss the drawbacks deriving from the use of the sizing model and show that they will be corrected by the proposed model.

4. Experimentation

If the number of relocation workers is determined by the sizing model, the overall system profitability is unlikely to be maximized. In fact, the sizing model increases the staff size as soon as the current number of workers is no longer sufficient for the considered set of user bookings. Although additional workers can increase the demand satisfaction rate, they also result in additional fixed costs, which may not be covered by the revenues of the new bookings. Therefore, it is worth investigating how many relocation workers must be employed to maximize the overall system profitability. The particular staff size configuration with no worker is also investigated to understand which drawbacks occur when the relocation service is not provided and, thus, why it is beneficial to introduce the relocation service.

In addition, the sizing model is much more difficult to solve than the proposed one, where the number of available workers is a datum. As a result, the sizing model is expected to return more rapidly low-quality solutions, to meet only a part of the bookings and, thus, to lose some potential revenues. To shed light on these intuitions, we run the proposed model to plan car relocation using different manpower levels and compare their profitability to that obtained by the sizing model.

In order to carry out the experimentation, we generate a set of customer bookings, each of which involves the request of one car. Each booking has four associated attributes: the departure station, the departure period, the arrival station, and the arrival period. In this experimentation, the four attributes are generated by a uniform distribution, enforcing the restriction that the arrival periods must be larger than the related departure periods.

For a specific number of relocation workers, the proposed model is run for the first time using the data of the first booking only; it is run for the second time using the data of the first booking and of the second one; it is executed for the third time using the data of the first booking, the second and the third, and so on.

As a new booking is considered, two conditions must be checked: if the updated values for supply and demand are lower than the station's capacity and the minimum number of cars required to serve all users is larger than the number of available cars in the last model solution, this booking must be automatically rejected without undertaking any optimization. If this is not the case, a problem instance is generated and solved by a mathematical programming solver. In this experimentation, the solver is Ilog Cplex Optimization Studio 12.6, which employs state-of-the-art algorithms to solve mixed integer programming problems. Experiments are performed on a laptop with 2.60 Ghz and 8 Gb, running with default parameter settings. The maximum running time is 5 minutes for the proposed model, whereas it is 30 minutes in the case of the sizing model, because it is much more difficult to solve. If all variables $x_{i_t}^d$ and $x_{i_t}^s$ take the value 0 in the solution, the last booking is accepted, otherwise it is rejected, because at least one user does not have an available car in the departure station or a free packing slot in the arrival station.

More formally, let B be the set of bookings and C_b the minimum number of cars assigned to users after the arrival of booking $b \in B$. Since each booking is supposed to involve one car only, C_b is computed as the maximum number of bookings that can be served over all periods:

$$C_b = max_{t \in T}(\sum_{j=1}^{b} \delta_{jt} - \sum_{j=1}^{b-1} \gamma_j), \tag{12}$$

where $\delta_{jt} = 1$ if a booking $j \in B$ using one car at time $t \in T$ has arrived, 0 otherwise; γ_j takes the value 1 if a booking $j \in B$ that has arrived before $b \in B$ was served, but 0 if not.

The experimentation is carried out on a network with 30 cars and 30 stations denoted by letters from S_1 to S_{30}, each of which can store up to 2 cars. At the beginning of the planning horizon, there is 1 car in each station. We consider 48 periods of 10 minutes each in a planning horizon of 8 hours. The following unit costs are taken from the case study:

- Motorcycle driving per kilometer: €0.08 (i.e. $c^{uk}_{it,j_{t+t_{ij}}} = 0.08$ multiplied by the number of kilometers from station $i \in I$ to station $j \in I$);
- Car driving per kilometer: €0.12 (i.e. $c^{vk}_{it,j_{t+t_{ij}}} = 0.12$ multiplied by the number of kilometers from station $i \in I$ to station $j \in I$);
- Penalization for failed car provision (c^d): €500;
- Penalization for failed car restitution due to saturation (c^s): €400;
- Penalization for the storage of an unavailable car (c^n): €300.

To set the values of d_{it} and s_{it} for each station $i \in I$ in each period $t \in T$, we aggregate across the number of cars demanded and returned in each station and period, respectively. These values are checked to be lower than the capacity of each station. In addition, $m_{it} = 0$ for each station $i \in I$ in each period $t \in T$.

Table 1 and Table 2 show how a growth in staff size reduces relocation costs and increases demand satisfaction, while taking into account the ability of Ilog Cplex Optimization Studio to solve problem instances within the maximum running time. Columns indicate the arriving transportation request in the booking set B, the related departure station DS, the departure period DP, the arrival station AS, and the arrival period AP. Each row is associated with an arriving booking and a problem instance built as described

above. For example, the instance associated with the value 5 in column B considers all bookings from 1 to 5. Booking 5 consists of the request of one car that must picked up at station S_{20} at period 34 and returned to station S_{22} at period 48. For the sake of space, Table 1 reports bookings 1 to 50 and Table 2 bookings 51 to 100. The generation of bookings is stopped after 100 customer requests, because the minimum number of requested cars for these bookings would become larger than the total number of available cars.
The proposed model is run according to three manpower configurations:

- no relocation worker, denoted by $|K|$=0;
- one relocation worker, denoted by $|K|$=1;
- two relocation workers, denoted by $|K|$=2;

The configuration $|K|$=0 is obtained from the operational model by removing the variables $y^k_{i_t,i_{t+1}}$, $z^k_{i_t,i_{t+\tau}}$, $u^k_{i_t,j_{t+t_{ij}}}$, and $v^k_{i_t,j_{t+t_{ij}}}$, and removing the constraints (1), (2) and (3).
For each size configuration, three results are shown:

- The minimum number of cars that can be assigned to users after the arrival of booking b—this has been denoted by C_b;
- The objective function z[€] of the proposed model;
- The solution optimality gap, which is denoted by *Gap*[%].

These results are also reported when the sizing model is run, in order to point out the drawbacks emerging in its utilization.
We put in boldface those bookings that cannot be served, either because at least one of the variables $x^d_{i_t}$ and $x^s_{i_t}$ takes a positive value in the problem solution. For example, when $|K|$=0, the booking 8 cannot be served, because, even if C_b is much lower than the number of available cars, one of the $x^s_{i_t}$s takes the value 1, due to the lack of available cars in station S_{16}. Hence, booking 8 is rejected and, when booking 9 is considered, in the generated problem instance there are only bookings $1, 2, \ldots, 7$ and 9. Moreover, the string *oom* shows which instances cannot be solved because the solver runs out of memory.
Table 1 and Table 2 show that the worst booking satisfaction rate is obtained when there is no relocation worker: in fact, just 39 out of 100 bookings can

		Data				Sizing model			|K|=0			|K|=1			|K|=2		
B	DS	DP	AS	AP	C_b	$\bar{K}$	z[€]	Gap[%]	C_b	z[€]	Gap[%]	C_b	z[€]	Gap[%]	C_b	z[€]	Gap[%]
1	S_1	17	S_{29}	29	1	0	0.00	0.00	1	0.00	0.00	1	0.00	0.00	1	0.00	0.00
2	S_{22}	23	S_1	30	2	0	0.00	0.00	2	0.00	0.00	2	0.00	0.00	2	0.00	0.00
3	S_5	41	S_1	44	2	0	0.00	0.00	2	0.00	0.00	2	0.00	0.00	2	0.00	0.00
4	S_{19}	28	S_{18}	39	3	0	0.00	0.00	3	0.00	0.00	3	0.00	0.00	3	0.00	0.00
5	S_{20}	34	S_{22}	48	3	0	0.00	0.00	3	0.00	0.00	3	0.00	0.00	3	0.00	0.00
6	S_{16}	12	S_4	21	3	0	0.00	0.00	3	0.00	0.00	3	0.00	0.00	3	0.00	0.00
7	S_{28}	6	S_{15}	18	3	0	0.00	0.00	3	0.00	0.00	3	0.00	0.00	3	0.00	0.00
8	S_{16}	27	S_4	34	4	1	0.89	0.00	3	**900.00**	**0.00**	4	0.89	0.00	4	0.75	0.00
9	S_8	27	S_9	37	5	1	0.89	0.00	4	0.00	0.00	5	0.89	0.00	5	0.75	0.00
10	S_4	30	S_{16}	39	5	1	0.69	0.00	4	0.00	0.00	5	0.69	0.00	5	0.69	0.00
11	S_{14}	27	S_8	30	6	1	0.69	0.00	5	0.00	0.00	6	0.69	0.00	6	0.69	0.00
12	S_{12}	29	S_{15}	40	7	1	1.06	0.00	6	**400.00**	**0.00**	7	1.06	0.00	7	0.89	0.00
13	S_1	27	S_{26}	34	8	1	1.68	0.00	6	**500.00**	**0.00**	8	1.68	0.00	8	1.15	0.00
14	S_{28}	11	S_1	22	8	1	1.63	0.00	5	**900.00**	**0.00**	8	1.63	0.00	8	1.15	0.00
15	S_2	21	S_{10}	34	9	1	1.63	0.00	6	0.00	0.00	9	1.63	0.00	9	1.15	0.00
16	S_{29}	8	S_{23}	22	9	1	1.63	0.00	6	0.00	0.00	9	1.63	0.00	9	1.15	0.00
17	S_{23}	11	S_8	25	9	1	1.63	0.00	6	0.00	0.00	9	1.63	0.00	9	1.15	0.00
18	S_8	13	S_{22}	20	9	1	1.63	0.00	6	0.00	0.00	9	1.63	0.00	9	1.15	0.00
19	S_{28}	43	S_{23}	48	9	1	1.81	0.00	6	**500.00**	**0.00**	9	1.81	0.00	9	1.34	0.00
20	S_9	30	S_{11}	43	10	1	1.81	0.00	7	0.00	0.00	10	1.81	0.00	10	1.34	0.00
21	S_{22}	43	S_3	47	10	1	1.81	0.00	7	0.00	0.00	10	1.81	0.00	10	1.34	0.00
22	S_{14}	42	S_{17}	48	10	1	2.68	0.00	7	**500.00**	**0.00**	10	2.68	0.00	10	1.90	0.00
23	S_{26}	7	S_{22}	12	10	1	2.86	0.00	7	**400.00**	**0.00**	10	2.86	0.00	10	2.09	0.00
24	S_{11}	30	S_{13}	42	11	1	2.86	0.00	8	0.00	0.00	11	2.86	0.00	11	2.09	0.00
25	S_{21}	28	S_3	44	12	1	3.91	0.27	9	**900.00**	**0.00**	12	3.91	8.69	12	2.93	0.00
26	S_{20}	3	S_{12}	11	12	1	4.56	0.21	8	**500.00**	**0.00**	12	4.56	0.00	12	3.88	0.00
27	S_{27}	23	S_{18}	27	12	1	4.60	66.44	8	**400.00**	**0.00**	12	4.60	0.00	12	3.89	0.00
28	S_6	26	S_{28}	39	13	1	4.51	68.39	9	0.00	0.00	13	4.51	0.00	13	3.86	4.71
29	S_{15}	8	S_{27}	22	13	1	4.51	46.55	9	0.00	0.00	13	4.51	0.00	13	3.89	3.97
30	S_{13}	35	S_3	45	13	1	5.54	0.20	9	**400.00**	**0.00**	13	5.54	0.00	13	4.77	9.76
31	S_6	26	S_{11}	41	14	1	5.63	0.18	10	**500.00**	**0.00**	14	5.63	0.00	14	4.69	5.50
32	S_{20}	5	S_{17}	14	14	1	6.69	0.33	9	**500.00**	**0.00**	14	6.69	0.00	14	5.73	6.15
33	S_{16}	34	S_{19}	40	14	1	8.11	0.00	9	**500.00**	**0.00**	14	8.11	0.00	14	7.45	9.68
34	S_1	21	S_8	32	15	1	8.46	0.81	10	**500.00**	**0.00**	15	8.44	6.19	15	7.71	10.37
35	S_{26}	2	S_2	13	15	1	8.70	0.67	9	0.00	0.00	15	8.69	4.52	15	7.99	10.94
36	S_{27}	17	S_{14}	20	15	1	7.60	0.17	9	0.00	0.00	15	7.60	0.00	15	6.63	6.49
37	S_{17}	29	S_5	36	16	1	7.78	0.00	10	0.00	0.00	16	7.78	0.00	16	6.93	8.21
38	S_{25}	34	S_{24}	48	16	1	7.78	0.00	10	0.00	0.00	16	7.78	0.00	16	6.82	6.47
39	S_8	19	S_{11}	26	16	1	9.09	0.24	10	**500.00**	**0.00**	16	9.09	0.00	16	7.60	1.05
40	S_{15}	4	S_{29}	11	16	1	9.97	0.59	10	**500.00**	**0.00**	16	9.93	0.00	16	7.96	0.75
41	S_{18}	4	S_4	19	16	1	10.09	0.42	10	**400.00**	**0.00**	16	10.09	0.00	16	8.09	1.92
42	S_{24}	20	S_7	33	17	1	10.09	0.67	11	0.00	0.00	17	10.09	3.50	17	8.09	1.45
43	S_{17}	45	S_{20}	48	17	1	10.09	0.58	11	**500.00**	**0.00**	17	10.14	4.21	17	8.10	0.00
44	S_8	46	S_{26}	48	17	1	10.25	1.00	11	0.00	0.00	17	10.09	3.71	17	8.10	0.00
45	S_{28}	19	S_{20}	27	17	1	9.23	0.00	11	**500.00**	**0.00**	17	9.23	0.00	17	7.46	1.27
46	S_1	24	S_{18}	33	18	1	10.65	0.17	12	**900.00**	**0.00**	18	10.29	8.10	18	8.36	4.63
47	S_{20}	26	S_3	42	19	1	12.34	1.83	12	**900.00**	**0.00**	19	11.58	2.56	19	9.25	0.00
48	S_{10}	18	S_{21}	33	20	1	11.72	12.88	12	0.00	0.00	20	11.01	3.23	20	8.48	0.00
49	S_{10}	9	S_9	12	20	1	11.78	6.87	12	**500.00**	**0.00**	20	11.55	5.74	20	9.11	6.81
50	S_{29}	6	S_{20}	16	20	1	11.43	1.00	12	**500.00**	**0.00**	20	11.18	0.00	20	8.54	6.15

Table 1: Model solutions for bookings from 1 to 50

	Data				Sizing model				$\|K\|$=0			$\|K\|$=1			$\|K\|$=2		
B	DS	DP	AS	AP	C_b	$\bar{K}$	z[€]	Gap[%]	C_b	z[€]	Gap[%]	C_b	z[€]	Gap[%]	C_b	z[€]	Gap[%]
51	S_{2}	45	S_{21}	48	20	1	11.67	0.97	12	0.00	0.00	20	11.48	0.00	20	8.48	2.81
52	S_{16}	35	S_{9}	47	20	1	13.62	1.01	12	**500.00**	**0.00**	20	13.27	0.00	20	10.12	2.60
53	S_{16}	24	S_{27}	36	21	1	16.01	0.95	13	**500.00**	**0.00**	21	15.64	0.00	20	11.88	0.00
54	S_{17}	30	S_{17}	39	22	1	16.74	0.00	13	**500.00**	**0.00**	22	16.74	0.00	21	12.40	3.74
55	S_{4}	10	S_{21}	17	22	1	17.19	0.00	12	**400.00**	**0.00**	22	17.19	0.00	22	12.58	1.98
56	S_{16}	11	S_{6}	23	22	**1**	**1093.74**	**91.33**	12	**500.00**	**0.00**	22	**517.18**	**82.16**	22	14.18	0.00
57	S_{15}	30	S_{3}	41	23	1	17.50	0.00	13	**400.00**	**0.00**	23	17.50	0.00	22	14.57	0.00
58	S_{5}	15	S_{20}	20	23	1	16.13	0.38	12	0.00	0.00	23	17.13	0.00	23	14.43	0.00
59	S_{1}	6	S_{8}	16	23	1	17.11	0.00	12	**500.00**	**0.00**	23	17.11	0.00	23	14.27	0.00
60	S_{21}	29	S_{11}	35	24	1	18.28	0.00	13	0.00	0.00	24	18.28	0.00	23	15.80	1.88
61	S_{5}	32	S_{6}	39	24	1	19.54	0.00	13	**500.00**	**0.00**	24	19.54	0.00	24	16.42	4.61
62	S_{13}	11	S_{7}	15	24	2	17.53	42.28	13	**400.00**	**0.00**	24	**516.22**	**64.13**	24	16.20	2.73
63	S_{12}	38	S_{28}	48	24	**0**	**12500.00**	**99.22**	13	0.00	0.00	24	**418.80**	**0.00**	24	17.24	6.16
64	S_{30}	13	S_{17}	26	24	1	19.27	0.00	13	0.00	0.00	24	19.26	0.00	24	16.97	4.61
65	S_{30}	33	S_{22}	45	24	2	16.14	41.47	13	**900.00**	**0.00**	24	**516.44**	**0.00**	24	17.24	4.26
66	S_{25}	3	S_{8}	16	24	2	15.65	38.33	13	**500.00**	**0.00**	24	**418.11**	**0.00**	24	17.50	4.45
67	S_{29}	24	S_{18}	36	25	2	18.06	37.99	14	**900.00**	**0.00**	25	**420.54**	**0.00**	24	18.64	6.37
68	S_{3}	14	S_{20}	24	25	2	17.44	37.87	13	**400.00**	**0.00**	24	**419.62**	**0.00**	25	18.47	5.80
69	S_{6}	24	S_{23}	28	25	2	7.05	0.46	13	**500.00**	**0.00**	24	**517.48**	**0.00**	25	19.31	5.16
70	S_{26}	24	S_{26}	34	26	2	18.34	0.40	14	**500.00**	**0.00**	25	**517.69**	**0.00**	25	20.56	5.07
71	S_{18}	42	S_{15}	48	26	2	18.36	0.44	13	0.00	0.00	24	19.26	0.00	26	20.75	5.80
72	S_{5}	30	S_{11}	35	27	2	20.17	0.22	14	**900.00**	**0.00**	25	**517.05**	**0.00**	27	22.34	3.38
73	S_{2}	3	S_{3}	13	27	2	19.93	0.29	13	**900.00**	**0.00**	24	19.74	0.00	27	22.09	2.61
74	S_{16}	14	S_{8}	17	27	2	21.84	0.35	13	**500.00**	**0.00**	24	**518.98**	**0.00**	27	23.88	2.15
75	S_{22}	16	S_{27}	26	27	2	21.91	0.31	13	0.00	0.00	24	**418.98**	**0.00**	27	24.10	1.87
76	S_{6}	3	S_{11}	11	27	2	24.17	0.67	13	**900.00**	**0.00**	24	**519.66**	**0.00**	27	25.48	0.00
77	S_{3}	35	S_{16}	39	27	2	25.46	0.61	13	0.00	0.00	24	**419.77**	**0.00**	27	27.16	1.87
78	S_{17}	1	S_{18}	15	27	2	24.87	0.15	13	**400.00**	**0.00**	24	**419.64**	**0.00**	27	27.30	1.49
79	S_{22}	6	S_{3}	9	27	2	24.63	0.15	13	**500.00**	**0.00**	24	19.58	0.00	27	27.08	1.32
80	S_{1}	6	S_{26}	9	27	2	24.68	0.10	13	**500.00**	**0.00**	24	**518.04**	**0.00**	27	27.05	0.27
81	S_{18}	46	S_{21}	48	27	2	24.68	0.00	13	0.00	0.00	24	19.58	0.00	27	27.08	1.43
82	S_{18}	7	S_{2}	21	27	2	25.78	0.14	13	**900.00**	**0.00**	24	**517.54**	**0.00**	27	27.91	0.70
83	S_{23}	32	S_{30}	44	27	3	22.00	0.06	13	0.00	0.00	24	**419.17**	**0.00**	27	**525.25**	**0.03**
84	S_{19}	14	S_{2}	18	27	3	23.45	0.10	13	**900.00**	**0.00**	24	**419.24**	**0.00**	27	28.69	1.14
85	S_{26}	27	S_{20}	40	28	4	22.99	0.00	14	**500.00**	**0.00**	25	**517.28**	**0.00**	28	**528.29**	**0.26**
86	S_{6}	10	S_{12}	17	28	4	24.01	0.00	13	**500.00**	**0.00**	24	**517.49**	**0.00**	27	30.63	3.58
87	S_{16}	21	S_{27}	26	28	4	26.03	0.00	13	**900.00**	**0.00**	24	**518.37**	**0.00**	27	33.45	0.00
88	S_{18}	29	S_{27}	42	29	5	30.89	0.00	14	**900.00**	**0.00**	25	**419.13**	**0.00**	28	**529.38**	**0.00**
89	S_{11}	11	S_{1}	25	29	*oom*	*oom*	*oom*	13	**900.00**	**0.00**	24	16.78	0.00	27	**527.80**	**0.16**
90	S_{28}	15	S_{4}	27	29	*oom*	*oom*	*oom*	13	**900.00**	**0.00**	24	17.71	0.00	27	35.30	0.00
91	S_{2}	32	S_{23}	35	29	*oom*	*oom*	*oom*	13	**500.00**	**0.00**	24	**419.02**	**0.00**	27	**531.99**	**0.12**
92	S_{14}	4	S_{23}	8	29	*oom*	*oom*	*oom*	13	0.00	0.00	24	20.10	0.00	27	35.85	0.00
93	S_{28}	44	S_{20}	48	29	*oom*	*oom*	*oom*	13	0.00	0.00	24	20.60	0.00	27	36.17	0.00
94	S_{11}	19	S_{6}	23	29	*oom*	*oom*	*oom*	13	**500.00**	**0.00**	24	**518.03**	**0.00**	27	33.41	0.00
95	S_{12}	15	S_{28}	19	29	*oom*	*oom*	*oom*	13	**500.00**	**0.00**	24	19.31	0.00	27	32.82	0.00
96	S_{19}	23	S_{26}	31	29	*oom*	*oom*	*oom*	14	**500.00**	**0.00**	25	20.67	0.00	28	**529.47**	**0.00**
97	S_{14}	45	S_{28}	48	29	*oom*	*oom*	*oom*	13	**500.00**	**0.00**	25	**520.99**	**18.99**	27	34.24	0.00
98	S_{15}	42	S_{25}	48	29	*oom*	*oom*	*oom*	13	0.00	0.00	25	20.67	0.00	27	35.11	0.00
99	S_{16}	15	S_{8}	28	29	*oom*	*oom*	*oom*	13	**500.00**	**0.00**	25	**520.51**	**0.00**	27	**533.42**	**0.00**
100	S_{11}	19	S_{13}	22	29	*oom*	*oom*	*oom*	13	**900.00**	**0.00**	25	**520.09**	**15.47**	27	**24834.25**	**98.87**

Table 2: Model solutions for bookings from 51 to 100

be served and, thus, $100 - 39 = 61$ solutions are typed in bold. All instances in this staff size configuration are solved to optimality, as shown by the optimality gaps. The fraction of served bookings is much larger when 1 worker is deployed to perform car relocation, in fact 72 out of 100 bookings are satisfied and $100 - 72 = 28$ solutions are typed in bold. The impossibility to meet these transportation requests can be disclosed by the analysis of optimality gaps, which can be utilized to compute the lower bounds as:

$$l = z(100 - Gap)/100, \tag{13}$$

In case of $|K|$=1, several entries of Table 2 are reported in bold with null optimality gaps, thus one can argue that customers are not served due to understaffing. In case of bookings 56, 62, 97, 100, optimality gaps are high and lower bounds amount to 92.26, 185.17, 422.05 and 439.63, respectively. As a result, the optimal solutions of these instances will have positive values of $x^d_{i_t}$ and $x^s_{i_t}$. Hence, even these solutions disclose understaffing issues.

Generally speaking, the case $|K|$=2 is the best one in terms of booking satisfaction rates, in fact only 8 bookings cannot be met: the first 7 cannot be evidently served due to understaffing, as shown by the tight optimality gap. Yet, the last booking is not served because of understaffing, since the lower bound is 280.63. In addition, the comparison between the values of the objective functions z shows that adding the second worker reduces the relocation costs, but this lessening seems to be not so relevant.

Table 1 and Table 2 also provide details on the sizing model, in fact they report the number $\bar{K}$ of recommended workers, the sum z of transportation and penalization costs and the solution optimality gap, which is denoted by Gap[%]. The experimentation shows that this model can be easily utilized to determine the number of workers up to booking 55, even of sometimes optimality gaps are quite high (bookings 27, 28 and 29). In the case of booking 56, the solution recommends utilizing 1 worker, but the optimality gap is huge and 2 workers must be used to serve all these requests, as shown when $|K|$=2. A similar problem also occurs for booking 63, in fact in this solution no worker is used, 17 requests are not met at the departure stations and 10 at arrival ones. In addition, the sizing model cannot be utilized after booking 88, because it becomes too difficult to be solved and the solver systematically runs out-of-memory.

We are now ready to determine which is the most profitable staff size configuration. Generally speaking, a larger staff size means paying larger fixed costs, which are not considered in Table 1 and Table 2, as well as the service

profitability. These results are reported in Table 3 and Table 4, where each row is associated with a booking and a problem instance, as done in Table 1 and Table 2. The column denoted by Rev represents the revenue produced by a booking. Table 3 and Table 4 report for each staff size configuration the cumulative revenue $TRev$ generated up to this booking, the total cost $TCost$, which is computed as the sum of z and the fixed costs, and the profits $Profit$, which are computed as the difference between the cumulative revenues and the total costs. For example, booking 18 will produce a revenue of €9.47 if served and the sizing model recommends using 1 worker in this case. In this experimentation, the fixed cost generated by the employment of 1 worker in the planning horizon is supposed to be €120.00. Nonetheless, the experimentation can be carried out with any value for the fixed cost.
When $|K|$=0, we are able to serve booking 18 and the total cumulative revenue becomes €152.28, which is computed as the revenue obtained before the arrival of this booking (i.e. €142.81) plus €9.47. The total cost is €0.00, because there are no fixed and relocation costs, hence the total profit is €152.28. In addition, since booking 19 cannot be served in case $|K|$=0, the cumulative revenue is again €152.28, as well as the profit.
When $|K|$=1, we are also able to serve booking 18 and the total cumulative revenue becomes €194.11, which is computed as the revenue obtained before the arrival of this booking (i.e. €184.63) plus €9.47. The total cost is €121.63, which is computed as the sum of the fixed cost (i.e. €120.00) and the relocation cost taken from Table 1 (i.e. €1.63). Therefore, the total profit is €194.11 minus €121.63, that is €72.48. Since in cases $|K|$=2 the service profit is €$-$47.05, the most profitable staff size configuration at this stage is $|K|$=0, even if it serves only 14 bookings out of 18. More important, in this case the staff size returned by the sizing model is an overestimate compared to the size configuration returning the maximum profit.
At the end of Table 4, one notices that the most profitable size configuration is $|K|$=2, because it results in a profit of €668.69, whereas in case $|K|$=1, the associated profit is €609.26. In addition, if the third relocation worker was introduced and all bookings were served, the total revenue would amount to €1027.89, the fixed costs would be €360.00 and the profit would be lower be €$1027.89 - 360.00 = 667.89$, because transportation costs are not considered yet.
To conclude, Table 3 and Table 4 show that the staff size returned by the sizing model is overestimated with respect to the size configuration maximizing the service profit. Such a criticality is clustered in the values of B in these

Data			Sizing model			$\|K\|$=0			$\|K\|$=1			$\|K\|$=2		
B	Rev [€]	$\bar{K}$	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]
1	11.93	0	11.93	0.00	11.93	11.93	0.00	11.93	11.93	120.00	-108.07	11.93	240.00	-228.07
2	9.47	0	21.40	0.00	21.40	21.40	0.00	21.40	21.40	120.00	-98.60	21.40	240.00	-218.60
3	7.51	0	28.91	0.00	28.91	28.91	0.00	28.91	28.91	120.00	-91.09	28.91	240.00	-211.09
4	11.44	0	40.35	0.00	40.35	40.35	0.00	40.35	40.35	120.00	-79.65	40.35	240.00	-199.65
5	12.91	0	53.26	0.00	53.26	53.26	0.00	53.26	53.26	120.00	-66.74	53.26	240.00	-186.74
6	10.46	0	63.72	0.00	63.72	63.72	0.00	63.72	63.72	120.00	-56.28	63.72	240.00	-176.28
7	11.93	0	75.65	0.00	75.65	75.65	0.00	75.65	75.65	120.00	-44.35	75.65	240.00	-164.35
8	9.47	1	85.12	120.89	-35.76	**75.65**	**0.00**	**75.65**	85.12	120.89	-35.76	85.12	240.75	-155.63
9	10.95	1	96.07	120.89	-24.82	86.60	0.00	86.60	96.07	120.89	-24.82	96.07	240.75	-144.68
10	10.46	1	106.53	120.69	-14.17	97.05	0.00	97.05	106.53	120.69	-14.17	106.53	240.69	-134.17
11	7.51	1	114.03	120.69	-6.66	104.56	0.00	104.56	114.03	120.69	-6.66	114.03	240.69	-126.66
12	11.44	1	125.47	121.06	4.41	**104.56**	**0.00**	**104.56**	125.47	121.06	4.41	125.47	240.89	-115.42
13	9.47	1	134.94	121.68	13.27	**104.56**	**0.00**	**104.56**	134.94	121.68	13.27	134.94	241.15	-106.21
14	11.44	1	146.38	121.63	24.76	**104.56**	**0.00**	**104.56**	146.38	121.63	24.76	146.38	241.15	-94.77
15	12.42	1	158.81	121.63	37.18	116.98	0.00	116.98	158.81	121.63	37.18	158.81	241.15	-82.35
16	12.91	1	171.72	121.63	50.09	129.90	0.00	129.90	171.72	121.63	50.09	171.72	241.15	-69.43
17	12.91	1	184.63	121.63	63.01	142.81	0.00	142.81	184.63	121.63	63.01	184.63	241.15	-56.52
18	9.47	1	194.11	121.63	72.48	152.28	0.00	152.28	194.11	121.63	72.48	194.11	241.15	-47.05
19	8.49	1	202.60	121.81	80.78	**152.28**	**0.00**	**152.28**	202.60	121.81	80.78	202.60	241.34	-38.74
20	12.42	1	215.02	121.81	93.21	164.71	0.00	164.71	215.02	121.81	93.21	215.02	241.34	-26.32
21	8.00	1	223.02	121.81	101.20	172.70	0.00	172.70	223.02	121.81	101.20	223.02	241.34	-18.32
22	8.98	1	232.00	122.68	109.32	**172.70**	**0.00**	**172.70**	232.00	122.68	109.32	232.00	241.90	-9.90
23	8.49	1	240.49	122.86	117.63	**172.70**	**0.00**	**172.70**	240.49	122.86	117.63	240.49	242.09	-1.60
24	11.93	1	252.42	122.86	129.56	184.63	0.00	184.63	252.42	122.86	129.56	252.42	242.09	10.33
25	13.90	1	266.32	123.91	142.40	**184.63**	**0.00**	**184.63**	266.32	123.91	142.40	266.32	242.93	23.38
26	9.96	1	276.28	124.56	151.72	**184.63**	**0.00**	**184.63**	276.28	124.56	151.72	276.28	243.88	32.40
27	8.00	1	284.28	124.60	159.68	**184.63**	**0.00**	**184.63**	284.28	124.60	159.68	284.28	243.89	40.39
28	12.42	1	296.70	124.51	172.19	197.06	0.00	197.06	296.70	124.51	172.19	296.70	243.86	52.84
29	12.91	1	309.61	124.51	185.10	209.97	0.00	209.97	309.61	124.51	185.10	309.61	243.89	65.73
30	10.95	1	320.56	125.54	195.02	**209.97**	**0.00**	**209.97**	320.56	125.54	195.02	320.56	244.77	75.80
31	13.41	1	333.97	125.63	208.34	**209.97**	**0.00**	**209.97**	333.97	125.63	208.34	333.97	244.69	89.27
32	10.46	1	344.42	126.69	217.73	**209.97**	**0.00**	**209.97**	344.42	126.69	217.73	344.42	245.73	98.69
33	8.98	1	353.40	128.11	225.29	**209.97**	**0.00**	**209.97**	353.40	128.11	225.29	353.40	247.45	105.95
34	11.44	1	364.84	128.46	236.38	**209.97**	**0.00**	**209.97**	364.84	128.44	236.40	364.84	247.71	117.14
35	11.44	1	376.28	128.70	247.59	221.41	0.00	221.41	376.28	128.69	247.59	376.28	247.99	128.29
36	7.51	1	383.79	127.60	256.19	228.92	0.00	228.92	383.79	127.60	256.19	383.79	246.63	137.16
37	9.47	1	393.26	127.78	265.48	238.39	0.00	238.39	393.26	127.78	265.48	393.26	246.93	146.33
38	12.91	1	406.18	127.78	278.39	251.30	0.00	251.30	406.18	127.78	278.39	406.18	246.82	159.35
39	9.47	1	415.65	129.09	286.56	**251.30**	**0.00**	**251.30**	415.65	129.09	286.56	415.65	247.60	168.04
40	9.47	1	425.12	129.97	295.15	**251.30**	**0.00**	**251.30**	425.12	129.93	295.19	425.12	247.96	177.16
41	13.41	1	438.53	130.09	308.44	**251.30**	**0.00**	**251.30**	438.53	130.09	308.44	438.53	248.09	190.43
42	12.42	1	450.95	130.09	320.86	263.73	0.00	263.73	450.95	130.09	320.86	450.95	248.09	202.86
43	7.51	1	458.46	130.09	328.37	**263.73**	**0.00**	**263.73**	458.46	130.14	328.32	458.46	248.10	210.35
44	7.01	1	465.47	130.25	335.22	270.74	0.00	270.74	465.47	130.09	335.38	465.47	248.10	217.37
45	9.96	1	475.43	129.23	346.20	**270.74**	**0.00**	**270.74**	475.43	129.23	346.20	475.43	247.46	227.97
46	10.46	1	485.89	130.65	355.24	**270.74**	**0.00**	**270.74**	485.89	130.29	355.60	485.89	248.36	237.53
47	13.90	1	499.79	132.34	367.45	**270.74**	**0.00**	**270.74**	499.79	131.58	368.21	499.79	249.25	250.54
48	13.41	1	513.19	131.72	381.48	284.15	0.00	284.15	513.19	131.01	382.19	513.19	248.48	264.71
49	7.51	1	520.70	131.78	388.92	**284.15**	**0.00**	**284.15**	520.70	131.55	389.15	520.70	249.11	271.59
50	10.95	1	531.65	131.43	400.22	**284.15**	**0.00**	**284.15**	531.65	131.18	400.47	531.65	248.54	283.11

Table 3: Profitability of bookings from 1 to 50

Data			Sizing model			|K|=0			|K|=1			|K|=2		
B	Rev [€]	$\bar{K}$	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]	$TRev$ [€]	$TCost$ [€]	$Profit$ [€]
51	7.51	1	539.15	131.67	407.49	291.65	0.00	291.65	539.15	131.48	407.67	539.15	248.48	290.67
52	11.93	1	551.08	133.62	417.46	**291.65**	**0.00**	**291.65**	551.08	133.27	417.82	551.08	250.12	300.97
53	11.93	1	563.01	136.01	427.00	**291.65**	**0.00**	**291.65**	563.01	135.64	427.38	563.01	251.88	311.13
54	10.46	1	573.47	136.74	436.73	**291.65**	**0.00**	**291.65**	573.47	136.74	436.73	573.47	252.40	321.07
55	9.47	1	582.94	137.19	445.75	**291.65**	**0.00**	**291.65**	582.94	137.19	445.75	582.94	252.58	330.36
56	11.93	**1**	**582.94**	**137.19**	**445.75**	**291.65**	**0.00**	**291.65**	**582.94**	**137.19**	**445.75**	594.87	254.18	340.70
57	11.44	1	594.38	137.50	456.88	**291.65**	**0.00**	**291.65**	594.38	137.50	456.88	606.31	254.57	351.74
58	8.49	1	602.87	136.13	466.74	300.14	0.00	300.14	602.87	137.13	465.74	614.80	254.43	360.37
59	10.95	1	613.82	137.11	476.71	**300.14**	**0.00**	**300.14**	613.82	137.11	476.71	625.75	254.27	371.49
60	8.98	1	622.80	138.28	484.52	309.12	0.00	309.12	622.80	138.28	484.52	634.73	255.80	378.93
61	9.47	1	632.27	139.54	492.73	**309.12**	**0.00**	**309.12**	632.27	139.54	492.73	644.20	256.42	387.79
62	8.00	2	640.27	257.53	382.74	**309.12**	**0.00**	**309.12**	**632.27**	**139.54**	**492.73**	652.20	256.20	396.00
63	10.95	**0**	**640.27**	**257.53**	**382.74**	320.07	0.00	320.07	**632.27**	**139.54**	**492.73**	663.15	257.24	405.91
64	12.42	1	652.69	139.27	513.43	332.49	0.00	332.49	644.70	139.26	505.43	675.57	256.97	418.60
65	11.93	2	664.62	256.14	408.49	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	687.50	257.24	430.26
66	12.42	2	677.05	255.65	421.40	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	699.93	257.50	442.43
67	11.93	2	688.98	258.06	430.92	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	711.86	258.64	453.21
68	10.95	2	699.93	257.44	442.49	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	722.80	258.47	464.33
69	8.00	2	707.92	247.05	460.87	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	730.80	259.31	471.49
70	10.95	2	718.87	258.34	460.54	**332.49**	**0.00**	**332.49**	**644.70**	**139.26**	**505.43**	741.75	260.56	481.19
71	8.98	2	727.85	258.36	469.49	341.47	0.00	341.47	653.68	139.26	514.41	750.73	260.75	489.98
72	8.49	2	736.34	260.17	476.17	**341.47**	**0.00**	**341.47**	**653.68**	**139.26**	**514.41**	759.22	262.34	496.88
73	10.95	2	747.29	259.93	487.35	**341.47**	**0.00**	**341.47**	664.62	139.74	524.88	770.17	262.09	508.08
74	7.51	2	754.79	261.84	492.96	**341.47**	**0.00**	**341.47**	**664.62**	**139.74**	**524.88**	777.67	263.88	513.79
75	10.95	2	765.74	261.91	503.84	352.42	0.00	352.42	**664.62**	**139.74**	**524.88**	788.62	264.10	524.52
76	9.96	2	775.71	264.17	511.54	**352.42**	**0.00**	**352.42**	**664.62**	**139.74**	**524.88**	798.58	265.48	533.10
77	8.00	2	783.70	265.46	518.24	360.42	0.00	360.42	**664.62**	**139.74**	**524.88**	806.58	267.16	539.43
78	12.91	2	796.62	264.87	531.75	**360.42**	**0.00**	**360.42**	**664.62**	**139.74**	**524.88**	819.50	267.30	552.20
79	7.51	2	804.12	264.63	539.49	**360.42**	**0.00**	**360.42**	672.13	139.58	532.55	827.00	267.08	559.93
80	7.51	2	811.63	264.68	546.95	**360.42**	**0.00**	**360.42**	**672.13**	**139.58**	**532.55**	834.51	267.05	567.46
81	7.01	2	818.64	264.68	553.96	367.43	0.00	367.43	679.14	139.58	539.56	841.52	267.08	574.44
82	12.91	2	831.56	265.78	565.78	**367.43**	**0.00**	**367.43**	**679.14**	**139.58**	**539.56**	854.44	267.91	586.53
83	11.93	3	843.49	382.00	461.49	379.36	0.00	379.36	**679.14**	**139.58**	**539.56**	**854.44**	**267.91**	**586.53**
84	8.00	3	851.49	383.45	468.04	**379.36**	**0.00**	**379.36**	**679.14**	**139.58**	**539.56**	862.43	268.69	593.74
85	12.42	4	863.91	502.99	360.92	**379.36**	**0.00**	**379.36**	**679.14**	**139.58**	**539.56**	**862.43**	**268.69**	**593.74**
86	9.47	4	873.38	504.01	369.37	**379.36**	**0.00**	**379.36**	**679.14**	**139.58**	**539.56**	871.91	270.63	601.28
87	8.49	4	881.87	506.03	375.84	**379.36**	**0.00**	**379.36**	**679.14**	**139.58**	**539.56**	880.40	273.45	606.94
88	12.42	5	894.29	630.89	263.40	**379.36**	**0.00**	**379.36**	**679.14**	**139.58**	**539.56**	**880.40**	**273.45**	**606.94**
89	12.91	5	**894.29**	**630.89**	**263.40**	**379.36**	**0.00**	**379.36**	692.06	136.78	555.28	**880.40**	**273.45**	**606.94**
90	11.93	5	**894.29**	**630.89**	**263.40**	**379.36**	**0.00**	**379.36**	703.99	137.71	566.28	892.33	275.30	617.03
91	7.51	5	**894.29**	**630.89**	**263.40**	**379.36**	**0.00**	**379.36**	**703.99**	**137.71**	**566.28**	**892.33**	**275.30**	**617.03**
92	8.00	5	**894.29**	**630.89**	**263.40**	387.36	0.00	387.36	711.99	140.10	571.89	900.33	275.85	624.47
93	8.00	5	**894.29**	**630.89**	**263.40**	395.36	0.00	395.36	719.98	140.60	579.38	908.32	276.17	632.15
94	8.00	5	**894.29**	**630.89**	**263.40**	**395.36**	**0.00**	**395.36**	**719.98**	**140.60**	**579.38**	916.32	273.41	642.91
95	8.00	5	**894.29**	**630.89**	**263.40**	**395.36**	**0.00**	**395.36**	727.98	139.31	588.68	924.32	272.82	651.50
96	9.96	5	**894.29**	**630.89**	**263.40**	**395.36**	**0.00**	**395.36**	737.95	140.67	597.28	**924.32**	**272.82**	**651.50**
97	7.51	5	**894.29**	**630.89**	**263.40**	**395.36**	**0.00**	**395.36**	**737.95**	**140.67**	**597.28**	931.82	274.24	657.58
98	11.98	5	**894.29**	**630.89**	**263.40**	407.34	0.00	407.34	749.93	140.67	609.26	943.80	275.11	668.69
99	12.42	5	**894.29**	**630.89**	**263.40**	**407.34**	**0.00**	**407.34**	**749.93**	**140.67**	**609.26**	**943.80**	**275.11**	**668.69**
100	7.51	5	**894.29**	**630.89**	**263.40**	**407.34**	**0.00**	**407.34**	**749.93**	**140.67**	**609.26**	**943.80**	**275.11**	**668.69**

Table 4: Profitability of bookings from 51 to 100

ranges:

- from 8 to 31, where the maximum profit is obtained with $|K|$=0 instead of $|K|$=1;
- from 65 to 75, where the maximum profit is obtained with $|K|$=1 instead of $|K|$=2;
- from 83 to 100, where the maximum profit is obtained with $|K| = 2$ instead of $|K| > 2$;

Therefore, the experimentation shows why it is important to perform the profitability evaluation, as the sizing model overestimates the workforce demand in the 50 percent of cases. Hence, the proposed model is preferable for setting the manpower in charge of the relocation service.

5. Conclusion

In this paper we have investigated the introduction of a relocation service in one-way carsharing system between stations with first-in first served policy. It is performed by a dedicated staff using foldable motorcycles to travel to unused cars, putting the motorcycles inside the cars, and then driving the cars toward the stations where they are demanded. The service has been tested in the case study of a medium size city, where the decisions on station locations and car fleet size were already made, but the number of relocation workers must be determined.

Although the relocation staff size can be planned by the model of [16], it adds a new worker whenever the current manpower is no longer able to serve the current set of bookings without paying attention to the overall system profitability. Therefore, additional work was required to investigate how different manpower levels change the number of served bookings, in order to select the most profitable staff size configuration. This paper has proposed a mathematical programming model to carry out this study.

Several insights can be derived from the experimentation. Firstly, it shows that the relocation introduction provides a crucial leverage for the profitability of one-way carsharing, because it increases remarkably the number of served bookings. Secondly, our model usually returns solutions with tight optimality gaps, whereas the model of [16] cannot solve the largest instances, because the solver runs out-of-memory. Finally, the experimentation in a real case study shows that the employment of two relocation workers lead to the maximum profit. If a third worker was introduced, the demand satisfaction rate would increase, but the overall system profitability would be suboptimal. Future research will be carried out to compare at the operational planning level the fixed manpower determined in this study and variable contractors, who may become active on demand. In addition, it is possible to integrate these options: fixed workers could be deployed and on-demand contractors could be added during peak times. To improve computation time, we will investigate heuristic methods and compare the results with the proposed model.

References

[1] S. A. Shaheen, A. P. Cohen, Growth in worldwide carsharing an international comparison, Transportation Research Record (1992) (2007) 81–89.

[2] D. Jorge, G. Correia, Carsharing systems demand estimation and defined operations: a literature review, European Journal of Transport and Infrastructure Research (13) (2013) 201–220.

[3] M. Nourinejad, M. Roorda, A dynamic carsharing decision support system, Transportation Research part E (66) (2014) 36–50.

[4] F. Ciari, B. Bock, M. Balmer, Modeling station-based and free-floating carsharing demand, Transportation Research Record (2416) (2014) 37–47.

[5] S. Weikl, K. Bogenberger, Relocation strategies and algorithms for free-floating car sharing systems, IEEE Intelligent transportation systems magazine (5) (2013) 100–111.

[6] K. Uesugi, N. Mukai, T. Watanabe, Optimization of vehicle assignment for car sharing system, Knowledge-Based Intelligent Information and Engineering Systems, , (4693) (2007) 1105–1111.

[7] M. Barth, M. Todd, L. Xue, User-based vehicle relocation techniques for multiple-station shared-used vehicle systems, Transportation Research Board 80th Annual Meeting (04-4161) (2004) 1–16.

[8] G. Correia, D. Jorge, D. Antunes, The added-value of accounting for users' flexibility and information on the potential of a station-based one-way carsharing system: An application in lisbon, portugal, Journal of Intelligent Transportation Systems: Technology, Planning, and Operationsdoi:10.1080/15472450.2013.836928.

[9] A. Di Febbraro, N. Sacco, M. Saeednia, One-way carsharing solving the relocation problem, Transportation Research Record (2319) (2012) 113–120.

[10] M. Clemente, M. Fanti, A. Mangini, W. Ukovich, The vehicle relocation problem in car sharing systems: Modeling and simulation in a petri net framework, in: J.-M. Colom, J. Desel (Eds.), Application and Theory of Petri Nets and Concurrency, Lecture Notes in Computer Science, Springer Berlin Heidelberg, 2013, pp. 250–269.

[11] W. Fan, R. B. Machemehl, N. E. Lownes, Carsharing dynamic decision-making problem for vehicle allocation, Transportation Research Record (2063) (2008) 97–104.

[12] G. H. A. Correia, A. P. Antunes, Optimization approach to depot location and trip selection in one-way carsharing systems, Transportation Research part E (48) (2012) 233–247.

[13] D. Jorge, G. Correia, C. Barnhart, Testing the validity of the mip approach for locating carsharing stations in one-way systems, Procedia - Social and Behavioral Sciences (0) (2012) 138–148.

[14] M. Barth, M. Todd, Simulation model performance analysis of a multiple station shared vehicle system, Transportation Research part C (7) (1999) 237–259.

[15] A. G. H. Kek, R. L. Cheu, M. L. Chor, Relocation simulation model for multiple-station shared-use vehicle systems, Transportation Research Record (1868) (2006) 81–88.

[16] A. G. H. Kek, R. L. Cheu, Q. Meng, C. H. Fung, A decision support system for vehicle relocation operations in carsharing systems, Transportation Research part E (45) (2009) 149–158.

[17] Z. K. G. Boyac, B., N. Geroliminis, An optimization framework for the development of efficient one-way car-sharing systems, European Journal of Operational Research (240) (2015) 718–733.

[18] R. Nair, E. Miller-Hooks, Fleet management for vehicle sharing operations, Transportation Science (45) (2011) 524–540.

[19] E. M. Cepolina, A. Farina, A new shared vehicle system for urban areas, Transportation Research part C (21) (2012) 230–243.