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Effective fluid description of the dark universe

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ABSTRACT

We propose an effective anisotropic fluid description for a generic infrared-modified theory of gravity. In our framework, the additional component of the acceleration, commonly attributed to dark matter, is explained as a radial pressure generated by the reaction of the dark energy fluid to the presence of baryonic matter. Using quite general assumptions, and a microscopic description of the fluid in terms of a Bose–Einstein condensate of gravitons, we find the static, spherically symmetric solution for the metric in terms of the Misner–Sharp mass function and the fluid pressure. At galactic scales, we correctly reproduce the leading MOND-like $\log(r)$ and subleading $(1/r)\log(r)$ terms in the weak-field expansion of the potential. Our description also predicts a tiny (of order 10^{-6} for a typical spiral galaxy) Machian modification of the Newtonian potential at galactic scales, which is controlled by the cosmological acceleration.

1. Introduction

One of the most intriguing puzzles of contemporary fundamental physics is the origin of the dark components of matter and energy in our universe [1–4]. The most conservative approach to this problem, the ΛCDM model [6,7], explains the experimental data about the present accelerated expansion of the universe [5], the structure formation, the galaxy rotation curves and gravitational lensing effects [8–10], by assuming that about 95% of the matter of our universe is exotic.

Despite the extensive agreement with large scale structure and cosmic microwave background observations, the ΛCDM model is not completely satisfactory, not only from a conceptual point of view, but also because there is some tension at the level of the phenomenology of galaxies and galaxy clusters. Concerning the Milky way galaxy, for example, three problems arise: the *missing satellite problem* [11,12] (N-body simulations predict too many dwarf galaxies within the Milky Way virial radius), the *cusp-core problem* [13] (too much dark matter in the innermost regions of galaxies w.r.t. observations) and the *too-big-to-fail problem* [14,15] (the dynamical properties of the most massive satellites in the Milky way are not correctly predicted by simulations). In particular, these problems become more and more evident when one tries to study galaxy rotation curves. Typically, the rotational velocity in galaxies approaches a non-zero asymptotic value with increasing distance from a galaxy's centre. This asymptotic value satisfies an empirical relationship with the galaxy's total luminosity known as the Tully–Fisher relation [16]. Rephrased as a relation between the asymptotic velocity v and the total baryonic mass $m_{\rm B}$, it takes the form $m_{\rm B} \sim v^4$ (baryonic Tully–Fisher relation) [17,18]. With adjusted units, it is equivalent to[1]

$$v^2 \approx \sqrt{a_0\, G_{\rm N}\, m_{\rm B}}\,, \tag{1.1}$$

where a_0 denotes an empirically determined factor with dimensions of an acceleration. The surprising fact is that the value of a_0

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[1] We use units with $c = 1$, while the Newton and Planck constants are expressed in terms of the Planck length and mass as $G_{\rm N} = \ell_{\rm p}/m_{\rm p}$ and $\hbar = \ell_{\rm p} m_{\rm p}$, respectively.

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appears to be $a_0 \approx H/(2\pi) \approx H/6$, where H is the current value of the Hubble constant. This coincidence begs for a deeper physical explanation and points to a deep connection between the dark matter and dark energy (DE) phenomena.

To explain the Tully–Fisher relation within a ΛCDM model, one must assume that the dark matter halos of all galaxies contain just the right amount of dark matter, which is obviously not a physically motivated assumption. For this reason, the Tully–Fisher relation has been used to argue in support of modified theories of gravity, where the standard description of the gravitational interaction given by Einstein's general relativity (GR) is modified at large scales. The departure from GR in such alternative approaches may involve modifications of the Einstein–Hilbert action, like in $f(R)$ theories [19,20], string inspired brane-world scenarios [21], or a change of the paradigm that describes gravity by means of a metric and covariant theory. To this last class of approaches belongs Milgrom's Modified Newtonian dynamics (MOND) [22,23]. In the MOND framework, in which the acceleration a_0 is promoted to a fundamental constant, the gravitational acceleration is modified with respect to its Newtonian form. At distances outside a galaxy's inner core, it reads

$$a_{\rm MOND}(r) = \sqrt{a_0\, a_{\rm B}(r)}\,, \tag{1.2}$$

where $a_{\rm B}(r) = G_{\rm N} m_{\rm B}(r)/r^2$ is the Newtonian radial acceleration that would be caused by the baryonic mass $m_{\rm B}(r)$ inside the radius r. Phenomenologically, the simple formula (1.2) turns out to explain the rotation curves of galaxies surprisingly well [24,18], although it cannot explain the mass deficit in galaxy clusters [25]. More recently, Verlinde [26] has given a controversial [27,28] derivation of the MOND formula (1.2), proposing that the dark matter phenomena can be attributed to an elastic response of the DE medium permeating the universe.

One common problem of these approaches is the difficulty of performing a "metric-covariant uplifting" of the theory [29,28]. In fact, such theories are usually formulated in the weak-field regime, whereas we know that gravity must allow for the metric-covariant description given by GR, at least at solar system scales. Fluid space–time models may provide a simple way to perform such an uplifting. For example, it is well known that de Sitter space is equivalent to the space–time of an isotropic fluid with constant energy density and equation of state $p = -\varepsilon$. Phenomenologically, galaxy rotation curves and gravitational lensing have been described using two-fluid [30,31] and anisotropic fluid models [32–34]. It is also possible to extend such models to contain DE [35], although the physical nature of these fluid models has yet to be established.

In this letter, we propose a way to describe the infrared modification of gravity remaining in a GR framework, by codifying it in terms of an effective (anisotropic) fluid acting as a source in the Einstein equations. We focus on the dark matter phenomenology within a single galaxy in a background de Sitter space–time. We will not address the problem of explaining the origin of DE, the existence of which we take for granted and which we describe by means of a DE fluid component with vacuum equation of state $\varepsilon_{\rm DE} = -p_{\rm DE} = 3\,H^2/(8\pi\, G_{\rm N})$. For small velocities, such a system is approximately described by a static, spherically symmetric geometry, whose physical content is effectively represented by an anisotropic fluid. The spherical symmetry should be considered as a first rough approximation for spiral and elliptical galaxies, which we adopt in the light of the fact that more realistic anisotropic fluid solutions with rotational symmetry are not known explicitly at the moment.

This model easily accommodates the observed deviation of the galaxy rotation curves from the Newtonian prediction at a typical infrared scale $r_0 \sim \sqrt{G_{\rm N}\, m_{\rm B}/H}$. These deviations are commonly attributed to dark matter, but in fact, they only imply the existence of some *dark force*. Taking a viewpoint similar to Verlinde's [26], we propose that this dark force can be entirely ascribed to the back-reaction of the DE fluid to the presence of baryonic matter and, therefore, is completely determined by the distribution of the latter. In contrast to Verlinde's, however, we explore the possibility that this back-reaction leads to an effective pressure term and not an effective mass density in the anisotropic fluid description. Since this pressure term is, *a priori*, an arbitrary function of the radial distance from the galactic centre, we need some input from an underlying microscopic theory in order to make our model predictive. We will employ a microscopic description in terms of a corpuscular picture, in which the DE fluid component is given by a Bose–Einstein condensate of gravitons [36,37], whereas the back-reaction effects are carried by gravitons in the non-condensed phase of the fluid. This will give us a way to relate the pressure causing the dark force to the Newtonian acceleration originating from the presence of baryonic matter.

Finally, we will calculate the effective metric components for our model and show that they contain, in the weak-field approximation, the typical $\log(r/r_0)$ MOND gravitational potential at galactic scales. Moreover, at such scales, we will also find a tiny Machian correction to the Newtonian potential depending on the position of the cosmological horizon.

2. Anisotropic fluid space–time

We start by considering a static, spherically symmetric system, for which one can employ the Schwarzschild-like metric

$$ds^2 = -f(r)\, e^{\gamma(r)}\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\,. \tag{2.1}$$

It is known that this metric is, in all generality, a solution to Einstein's equations with the energy-momentum tensor of an anisotropic fluid [38,39],

$$T^{\mu\nu} = (\varepsilon + p_\perp)\, u^\mu u^\nu + p_\perp g^{\mu\nu} - \left(p_\perp - p_\parallel\right) v^\mu v^\nu\,, \tag{2.2}$$

where the vectors u^μ and v^μ satisfy $u^\mu u_\mu = -1$, $v^\mu v_\mu = 1$, and $u^\mu v_\mu = 0$. Explicitly, the fluid velocity is $u^\mu = \left(f^{-1/2}\, e^{-\gamma/2}, 0, 0, 0\right)$ and $v^\mu = \left(0, f^{1/2}, 0, 0\right)$ points radially outwards. The energy density is given by ε, and $p_\perp$ and $p_\parallel$ denote the pressures perpendicular and parallel to the space-like vector v^μ, respectively. Energy-momentum conservation is equivalent to the hydrostatic equilibrium condition, and imposes constraints on these quantities.

The Einstein equations with the energy-momentum tensor (2.2) are solved by

$$f(r) = 1 - \frac{2\, G_{\rm N}\, m(r)}{r}\,, \tag{2.3a}$$

$$\gamma'(r) = \frac{8\pi\, G_{\rm N}\, r}{f(r)} \left(\varepsilon + p_\parallel\right)\,, \tag{2.3b}$$

where primes denote differentiation with respect to r, and

$$m(r) = 4\pi \int_0^r d\tilde{r}\, \tilde{r}^2\, \varepsilon(\tilde{r}) \tag{2.3c}$$

is the Misner–Sharp mass function representing the total energy inside a sphere of radius r. Finally, the tangential pressure follows from energy-momentum conservation,

$$p_\perp = p_\parallel + \frac{r}{2}\left[p_\parallel{}' + \frac{1}{2}\left(\varepsilon + p_\parallel\right)\left(\frac{f'}{f} + \gamma'\right)\right]. \tag{2.4}$$

Let us then consider a test particle comoving with the fluid, so that its four-velocity is u^μ. The four-acceleration necessary to keep it at a fixed coordinate radius r is given by $a^\mu = u^\nu \nabla_\nu u^\mu$. In the frame of Eq. (2.1), only the radial component of this acceleration does not vanish and is given by

$$a^r \equiv a = \frac{1}{2}\left(f\,\gamma' + f'\right) = \frac{G_N\, m(r)}{r^2} + 4\pi\, G_N\, r\, p_\parallel(r)\,. \tag{2.5}$$

In Newtonian language, the first term has the obvious interpretation as the acceleration that counters the gravitational pull of the central mass. The second term may be interpreted as the acceleration caused by the radial pressure. The same result can be obtained by considering the geodesic motion along a circular orbit of radius r, with $\theta = \pi/2$ and constant angular velocity $\Omega = \mathrm{d}\phi/\mathrm{d}t$. Of course, this is the physically relevant situation for the motion of stars within a galaxy. Starting with the four-velocity $u^\mu = C(r)\,(1,0,0,\Omega)$, with $C(r)$ such that $u_\mu u^\mu = -1$, and solving the geodesic equation at fixed r and $\theta = \pi/2$, one obtains $\Omega^2 = \mathrm{e}^\gamma\, a/r$, with a again given by Eq. (2.5).

The above equations can describe a variety of physical situations. De Sitter space is equivalent to an isotropic DE fluid with the constant energy density $\varepsilon_{\rm DE}$ and pressure $p_{\parallel \rm DE} = p_{\perp \rm DE} = p_{\rm DE}$ satisfying

$$\varepsilon_{\rm DE} = -p_{\rm DE} = \frac{3\,H^2}{8\,\pi\, G_N}\,. \tag{2.6}$$

This yields

$$f(r) = 1 - H^2\, r^2\,, \tag{2.7}$$

with $\gamma = 0$, and

$$a_{\rm DE}(r) = -H^2\, r\,. \tag{2.8}$$

Being maximally symmetric, de Sitter space does not allow for circular geodesics, which is confirmed by the fact that $a_{\rm DE}$ is negative. This acceleration describes the accelerating cosmological expansion of the universe. Notice that, because of its vacuum equation of state (2.6), the DE fluid component does not contribute to γ but enters only in f via the de Sitter term.

Pressureless baryonic matter can be easily added to de Sitter space,

$$\varepsilon = \varepsilon_B + \varepsilon_{\rm DE}\,, \tag{2.9}$$

where $\varepsilon_{\rm DE}$ is again given in Eq. (2.6). The Misner–Sharp mass function will split correspondingly,

$$m(r) = m_B(r) + m_{\rm DE}(r) = m_B(r) + \frac{H^2\, r^3}{2\, G_N}\,, \tag{2.10}$$

and the metric function f turns out to be

$$f(r) = 1 - \frac{2\, G_N\, m_B(r)}{r} - H^2\, r^2. \tag{2.11}$$

This leads to a Newtonian acceleration term

$$a_B(r) = \frac{G_N\, m_B(r)}{r^2}\,, \tag{2.12}$$

in addition to (2.8). If the baryonic matter is localized within a radius R_B then, for $r > R_B$, the space–time is identical to the Schwarzschild–de Sitter solution.

The observed galaxy rotation curves imply that, in addition to $a_{\rm DE}$ (which, in this context, is actually negligible) and a_B, there is an acceleration caused by a dark force,

$$a = a_B + a_{\rm DE} + a_{\rm DF}\,. \tag{2.13}$$

We think that dark matter does not exist as an independent form of matter, but rather that the phenomena usually attributed to it are a consequence of the interaction between the baryonic matter and the DE fluid. We therefore assume the energy density and the Misner–Sharp in the cosmos are given respectively by Eqs. (2.9) and (2.10). Taking the baryonic matter as approximately pressureless, we write

$$p_\parallel = p_{\parallel \rm DE} + p_{\parallel \rm DF}\,, \tag{2.14}$$

where $p_{\parallel \rm DF}$ is the pressure that generates the dark force. In the next section, we will derive $p_{\parallel \rm DF}$ from the point of view of a corpuscular interpretation of gravity in general, and of the de Sitter space in particular.

At galactic scales, we can neglect the DE terms $p_{\rm DE}$ and $\varepsilon_{\rm DE}$. Splitting the total radial gravitational acceleration into the baryonic acceleration a_B and the dark acceleration $a_{\rm DF}$, Eq. (2.5) now gives

$$a_B + a_{\rm DF} \simeq \frac{G_N\, m_B(r)}{r^2} + 4\pi\, G_N\, r\, p_{\parallel \rm DF}(r)\,. \tag{2.15}$$

The first term on the right hand side is exactly a_B, thus the dark acceleration is completely due to the pressure of the anisotropic fluid. This is an important point, because it implies that the modifications to GR at galactic scales commonly attributed to dark matter can be generated by the pressure $p_\parallel$ in our effective fluid description. Since this pressure term can be thought of as a reaction of the DE fluid to the presence of baryonic matter, it is conceptually very similar to Verlinde's description of dark forces as the elastic response of the DE medium to the presence of baryonic sources [26]. Note also that $p_{\parallel \rm DF}$ will necessarily give rise to an anisotropic component $p_{\perp \rm DF}$ according to the conservation Eq. (2.4).

3. Corpuscular dark force

In this section, we review the fundamentals of the corpuscular picture of the de Sitter space, express the accelerations $a_{\rm DE}$ (DE without matter) and a_B (Newtonian acceleration) in terms of corpuscular quantities and derive $a_{\rm DF}$ from the corpuscular picture of de Sitter in the presence of baryonic matter. We anticipate that the result will be the MOND formula (1.2) (up to a multiplicative constant). Throughout this section, numerical factors of order unity will mostly be omitted.

The basis of the corpuscular picture of gravity [36,37] is that the classical gravitational field of an (isolated) object of mass m is in fact a quantum coherent state of gravitons with occupation number [40–42]

$$N \sim \frac{m^2}{m_p^2}\,. \tag{3.1}$$

These gravitons are closely bound to the source and interact with other objects nearby, *e.g.*, a test particle. If r is the distance between the test particle and the massive object, the effective interaction energy for each graviton is $\omega(r) = \hbar/r$. Therefore, we can express the Newtonian gravitational acceleration felt by the test particle in terms of $\omega(r)$ and N as

$$a(r) = \frac{G_N\, m}{r^2} \sim \frac{\omega^2(r)}{m_p^2\, \ell_p}\,\sqrt{N}\,. \tag{3.2}$$

The argument generalises straightforwardly to a spherically symmetric distribution of mass. In this case, however, not all gravitons can contribute to the acceleration of the test particle, but only those that are bound to the mass inside the radius r. Henceforth, let us denote by $N_{\rm eff}(r)$ the effective number of gravitons which contribute to the acceleration of a test particle at radius r.[2] In the case at hand, it is $N_{\rm eff}(r) = m^2(r)/m_{\rm p}^2$, and (3.2) becomes

$$a(r) = \frac{G_{\rm N}\, m(r)}{r^2} \sim \frac{\omega^2(r)}{m_{\rm p}^2\, \ell_{\rm p}} \sqrt{N_{\rm eff}(r)}\ . \tag{3.3}$$

In the above argument it is important that the gravitons are in the normal (non-condensed) phase, for which we can use the effective law $\omega(r) = \hbar/r$.

We shall call *corpuscular acceleration* the quantity

$$a(r) \sim \frac{\omega^2(r)}{m_{\rm p}^2\, \ell_{\rm p}} \sqrt{N_{\rm eff}(r)}\ . \tag{3.4}$$

Although we have derived this formula for the non-condensed gravitons, which generate the Newtonian acceleration, it turns out to hold also for the acceleration caused by the condensed gravitons, as we will verify in the following. Therefore, every population of gravitons, with an effective number of contributing gravitons $N_{\rm eff}(r)$ and a mean interaction energy $\omega(r)$, will contribute an acceleration $a(r)$ given by (3.4) to the total acceleration of a test particle.

The DE fluid of the pure de Sitter space–time (2.7) is described in the corpuscular picture [37,43,44] as a Bose–Einstein condensate of N (very soft and virtual) gravitons with typical energy $\omega = \hbar\, H$. Since the total energy of the DE fluid inside the de Sitter horizon of radius $1/H$ is given by $m_H = 1/(2\, G_{\rm N}\, H)$, one has[3]

$$N \sim \frac{m_H^2}{m_{\rm p}^2} \sim \frac{1}{\ell_{\rm p}^2\, H^2}\ , \tag{3.5}$$

with $N\,\omega = m_H$. It is important that the number of gravitons scales holographically with the horizon size. Consider a test particle at a fixed distance r. As we recalled in the previous section, such a particle is not in geodesic motion, but feels the acceleration (2.8) caused by the DE condensate. Let us check whether the corpuscular acceleration formula (3.4) reproduces this result. In order to estimate $N_{\rm eff}(r)$, *i.e.*, the effective number of gravitons in the condensate that contribute to the interaction with the test particle, we use the fact that the graviton number scales holographically (with area) and all gravitons contribute to the acceleration of a test particle, when it is at the horizon. In other words, $N_{\rm eff}(r)$ must match Eq. (3.5) for $r = 1/H$. This leads to

$$N_{\rm eff}(r) \sim \frac{r^2}{\ell_{\rm p}^2}\ . \tag{3.6}$$

Moreover, since the gravitons are now in the condensed phase, the interaction energy $\omega(r) = \omega = \hbar\, H$ is constant. Therefore, Eq. (3.4) yields

$$|a_{\rm DE}(r)| \sim \frac{\omega^2}{m_{\rm p}^2\, \ell_{\rm p}} \sqrt{N_{\rm eff}(r)} = H^2\, r\ , \tag{3.7}$$

which reproduces the expected result (2.8).

Putting the above arguments together leads to a new effect. Let us consider baryonic matter present in a relatively small amount (say $m_{\rm B} \ll m_H$) and localised within some radius $R_{\rm B}$. The space–time will be given by the Schwarzschild–de Sitter solution with $f(r)$ in Eq. (2.11) for $r > R_{\rm B}$. Note, in particular, that the horizon radius L is now determined by the corresponding $f(L) = 0$, *i.e.*,

$$H^2\, L^2 = 1 - \frac{2\, G_{\rm N}\, m_{\rm B}}{L} = 1 - \frac{m_{\rm B}}{m(L)}\ , \tag{3.8}$$

where $m(L)$ denotes the total (Misner–Sharp) mass of the space–time,

$$m(L) = \frac{L}{2\, G_{\rm N}}\ . \tag{3.9}$$

From the corpuscular point of view, the DE fluid will react to the presence of baryonic matter, but, since $m_{\rm B} \ll m(L)$, most of the gravitons will remain in the condensed phase and retain an energy $\omega \sim \hbar/L$. From Eqs. (3.1) and (3.8), their number is given by

$$N_{\rm DE} \sim \frac{[m(L) - m_{\rm B}]^2}{m_{\rm p}^2} \sim \frac{H^4\, L^4\, m^2(L)}{m_{\rm p}^2} \sim \frac{H^4\, L^6}{\ell_{\rm p}^2}\ . \tag{3.10}$$

Using the same reasoning as above, the number of the condensed gravitons that effectively contribute to the cosmological acceleration of a test particle at radius r is $N_{\rm eff,DE}(r) = H^4\, L^4\, r^2/\ell_{\rm p}^2$, and Eq. (3.7) remains valid.

However, according to Eq. (3.1), the total number of gravitons in the system is given by

$$N \sim \frac{m^2(L)}{m_{\rm p}^2}\ . \tag{3.11}$$

This implies that there are $N - N_{\rm DE}$ gravitons, which are not in the condensed phase and, therefore, behave differently from the condensate. Since, from Eqs. (3.10) and (3.11),

$$N - N_{\rm DE} \sim \frac{L\, m_{\rm B}}{\ell_{\rm p}\, m_{\rm p}} - \frac{m_{\rm B}^2}{m_{\rm p}^2}\ , \tag{3.12}$$

there must be many more non-condensed gravitons than those that are closely bound to the baryonic mass. In fact, the number of the latter is simply

$$N_{\rm B} \sim \frac{m_{\rm B}^2}{m_{\rm p}^2}\ , \tag{3.13}$$

and their contribution to the acceleration is the Newtonian term (2.12). The remaining non-condensed gravitons, with total number[4]

$$N_{\rm DF} \sim \frac{L\, m_{\rm B}}{\hbar}\ , \tag{3.14}$$

mediate the interaction between the baryonic matter and the DE condensate.

What we have just shown is that the quantum field of gravitons that arises when baryonic matter is placed within a DE fluid comprises three types of gravitons: First, those in the condensed phase forming the cosmological DE fluid, second, the non-condensed gravitons closely bound to the baryonic matter responsible for the Newtonian acceleration and, third, the non-condensed gravitons permeating space–time, which have been "pulled out" of the condensate by the baryonic mass. Each of these graviton populations contributes an acceleration (3.4) to the total acceleration of the test

[2] The number $N_{\rm eff}(r)$ is not a good classical observable and must not be confused with the number of gravitons inside the radius r. Such a number does not exist, because, relativistically, there is no notion of a local number density.

[3] Similar relations hold for the case of a Schwarzschild black hole [36].

[4] The hierarchy of the graviton numbers is $N_{\rm B} \ll N_{\rm DF} \ll N_{\rm DE}$. There must also be corrections, sub-leading in $G_{\rm N} m_{\rm B}/L$, to account for the second term with the negative sign in (3.12).

particle, which correspond precisely to the three contributions to the acceleration in Eq. (2.13).

In order to estimate the effective number of the third type of gravitons that contribute to the acceleration of a test particle at the radius r, $N_{\rm eff,DF}(r)$, we note that the overall scaling is again holographic, but we must also take into account that only those gravitons that are "pulled out" of the condensate by the baryonic mass inside the radius r can contribute (if $m_{\rm B}$ were constant, it would be simply holographic). Hence,

$$N_{\rm eff,DF}(r) \sim \frac{r^2\, m_{\rm B}(r)}{\hbar\, L} \ . \tag{3.15}$$

Finally, from Eq. (3.4) with $\omega(r) = \hbar/r$ (for non-condensed gravitons), we obtain

$$|a_{\rm DF}(r)| \sim \sqrt{\frac{G_{\rm N}\, m_{\rm B}(r)}{L\, r^2}} \sim \sqrt{\frac{a_{\rm B}(r)}{L}} \ , \tag{3.16}$$

which is precisely the MOND acceleration (1.2) up to a numerical factor. Therefore, the corpuscular picture naturally explains the presence of a dark force and the approximate coincidence of the MOND acceleration a_0 with the Hubble constant $H \approx 1/L$. This is our main result in this section. Moreover, the pressure necessary to sustain the dark force is given by

$$p_{\parallel\,\rm DF} \sim \frac{1}{4\,\pi\, r^2}\sqrt{\frac{m_{\rm B}(r)}{G_{\rm N}\, L}} \ , \tag{3.17}$$

as follows from Eqs. (2.15) and (3.16).

Let us conclude this section with a few remarks. First, the previous arguments give order-of-magnitude estimates only, without precise numerical factors and without information on the directions of the various contributions to the acceleration. Second, all expressions must receive higher order corrections in $G_{\rm N}\, m_{\rm B}/L$, as can be seen, *e.g.*, from the different signs of the two terms in Eq. (3.12). Presumably, these corrections will be responsible for the cross-over between the Newtonian and the MOND regimes as well as between the MOND and the de Sitter regimes.

4. Metric at galactic scales

Starting from Eqs. (2.9), (2.14) and (3.17), we will now evaluate the metric of the anisotropic fluid space–time. For any given distribution of baryonic matter $\varepsilon_{\rm B} = \varepsilon_{\rm B}(r)$, Eqs. (2.3a)–(2.3c) determine the metric function $f = f(r)$ and

$$\gamma' = \frac{2}{r\, f(r)}\left[G_{\rm N}\, m'_{\rm B}(r) + \sqrt{a_0\, G_{\rm N}\, m_{\rm B}(r)}\right] \ . \tag{4.1}$$

We examine for simplicity the case of baryonic matter localised inside a sphere of radius $R_{\rm B} \ll r_0$, so that the baryonic mass has a constant profile $m_{\rm B}(r) = m_{\rm B}$, for $r > R_{\rm B}$. This approximation is good when we consider a galaxy at distances much bigger than its bulk. Since we are now interested in scales $r \sim r_0 \ll L$, we again neglect the DE terms, and the metric functions can be easily obtained from Eqs. (4.1) and (2.3a)–(2.3c),

$$\begin{aligned} f(r) &= 1 - \frac{2\, G_{\rm N}\, m_{\rm B}}{r} \\ \gamma_{\rm DF} &= 2\, K\left[\ln\left(\frac{r}{r_0}\right) + \ln\left(1 - \frac{2\, G_{\rm N}\, m_{\rm B}}{r}\right)\right] \ , \end{aligned} \tag{4.2}$$

where $K = \sqrt{a_0\, G_{\rm N}\, m_{\rm B}}$ and the integration constant was set in terms of the infrared scale r_0, which now represents the typical radius at which the "dark force" effects take place.

The non-vanishing function $\gamma_{\rm DF}$ represents the metric effects in our fluid description of the dark force. Since our effective fluid description holds only for $r_0 \lesssim r \ll L$, we neglect $\gamma_{\rm DF}$ for $r \lesssim r_0$ and $r \sim L$. Most of the physical information about the rotation curves of the galaxies is contained in the weak-field approximation of the metric component $g_{00} = -f\, e^{\gamma}$. At galactic scales, this corresponds to the regime $G_{\rm N}\, m_{\rm B} \ll r \sim r_0 \ll L$, which also implies $\gamma_{\rm DF} \sim 0$. Keeping only terms up to $\log^2(r/r_0)$ and $1/r^2$, we have

$$\begin{aligned} -g_{00} \simeq{}& 1 - (1 + 2\, K)\, \frac{2\, G_{\rm N}\, m_{\rm B}}{r} + 2\, K\, \ln\left(\frac{r}{r_0}\right) \\ &- K\, (1 + 2\, K)\, \frac{4\, G_{\rm N}\, m_{\rm B}}{r}\ln\left(\frac{r}{r_0}\right) \ , \end{aligned} \tag{4.3}$$

where we exactly find the logarithmic corrections to the gravitational potential one expects at galactic scales, as MOND (or the Tully–Fisher relation) suggests [22,17,45]. Moreover, it contains the subleading $(1/r)\log(r/r_0)$ corrections, which have also been observed in galactic rotation curves [46,47]. A third feature of the above metric element is the presence of a small correction to the Newtonian potential, which can be seen as a modification of $G_{\rm N}\, m_{\rm B}$, and depends on a_0 in K. This correction is therefore of Machian character, but is tiny because K is of order 10^{-6} for a spiral galaxy with $m_{\rm B} \sim 10^{11}\, m_{\odot}$, and of order 10^{-9} for a dwarf galaxy with $m_{\rm B} \sim 10^7\, m_{\odot}$. This effect is hence not detectable presently, owing to the uncertainties in the determination of the baryonic mass of the galaxies.

Because of the competition between $\log(r/r_0)$ and $1/r$ terms (and also the dS term r^2/L^2 if one goes to distances comparable with the cosmological horizon) in the weak-field expansion, it is useful to introduce, beside r_0, the scales r_1 and r_2 representing the distances at which the MOND acceleration term equals respectively the Newtonian and the dS term. Hence, our effective fluid description holds for $r_0 < r < r_2$. The IR scale r_0 is the typical distance at which the rotation curves of galaxies deviate from the Newtonian prediction, $r_0 \sim \sqrt{G_{\rm N}\, m_{\rm B}\, L}$. In Verlinde's model of Ref. [26], the IR scale r_0 is determined by the competition between area and volume terms in the entropy, and is given by $r_0 = \sqrt{2\, G_{\rm N}\, m_{\rm B}\, L}$. In our case, we have $r_1 = \sqrt{3}\, r_0$ and $r_2 = \sqrt{r_0\, L/(2\sqrt{3})}$. Notice that, as expected, $r_1 \sim r_0$. The window in which the Newtonian contribution to the potential is not obscured by the logarithmic term is therefore very narrow. As specific examples, let us take the typical spiral and dwarf galaxies discussed above. For the spiral galaxy, we have $r_0 \simeq 6$ Kpc, $r_1 \simeq 10$ Kpc, $r_2 \simeq 10^3$ Kpc. For the dwarf galaxy we have instead $r_0 \simeq 80$ pc, $r_1 \simeq 130$ pc, $r_2 \simeq 300$ pc.

We have considered here only the case of a constant profile for the baryonic mass function outside a sphere of radius $R \ll r_0$. However, Eqs. (4.1) and (2.3a)–(2.3c) in principle allow for the determination of the metric for every given distribution of baryonic matter $m_{\rm B} = m_{\rm B}(r)$. For instance, one can consider Jaffe's profile [48] for the baryonic energy density $\varepsilon_{\rm B} = \tilde{A}/r^4$, which corresponds to $m_{\rm B}(r) = m_0 - A/r$. We have checked that this profile reproduces the results for the case of a constant baryonic mass at large distances, as expected. A detailed discussion of Jaffe's model will be presented in a forthcoming paper.

5. Conclusions and outlook

In this letter, we have proposed an effective fluid description in a GR framework for an infrared-modified theory of gravity. Using quite general assumptions and a microscopic description of the fluid in terms of a Bose–Einstein condensate of gravitons, we have found the static, spherically symmetric solution for the metric in terms of the Misner–Sharp mass function of baryonic matter and the fluid pressure. In particular, we have shown that the additional component of the acceleration at galactic scales can be completely attributed to the radial pressure of the fluid, whose interpretation in the corpuscular model is that this is part of the reaction of the condensate of gravitons to the presence of baryonic matter. Moreover, we have shown that our model correctly reproduces the leading MOND $\log(r)$ and subleading $(1/r)\log(r)$ terms at galactic scales in the weak-field expansion of the potential. Our model also predicts a tiny modification of the Newtonian potential at galactic scales which is controlled by the cosmological acceleration.

The next step in our analysis should be to test the model with observational data. Of particular interest are the situations where the predictions of our model are expected to differ from those of MOND and/or ΛCDM. For what concerns the dynamics of galactic systems, our model is testable for an isolated, spherically symmetric system. The most promising candidates are therefore spherical galaxies or isolated spherical dwarf galaxies and dwarf spheroidal satellite galaxies. On the other hand, as we have already seen, the point mass case leads to the same results of MOND. To have a first nontrivial test, *i.e.* to look for significant differences between our model, MOND and ΛCDM we need to consider finite-size galaxies with a specific baryonic mass profile $m_B(r)$.

At the present stage of development, the dynamics of galaxy clusters and of systems that exhibit peculiar features as the external field effect of MOND [49] (like the Crater II dwarf satellite galaxy [50]) does not seem a suitable arena for testing the model. In order to do that, extensions for composite systems and beyond the spherical symmetric approximation appear necessary.

A different, but equally important challenge is represented by the study of the weak lensing effect at the level of galaxies and galaxy clusters. The well-defined form of the spacetime metric in (4.1) allows us to make predictions and, eventually, a direct comparison with results from both MOND and the ΛCDM model about the weak gravitational lensing measurements in galactic systems with static, spherically symmetric and isolated mass distributions. In order to do so, we need to choose our gravitational lenses to satisfy these criteria and to know the baryonic mass profile $m_B(r)$ of the system.

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