R/S ANALYSIS AND OPTION PRICING: APPLICATION TO WORLD MOST IMPORTANT STOCK INDICES

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Abstract: In order to develop new and more efficient predictive modelsin the World Stock Markets, in this paper we consideran option pricing mechanism related to the LRD (the Long Range Dependence) analysis.Following Morimoto (2016), who analyzed the intraday returns of the Tokyo Stock Exchange, and Skaperda (2013),in conflict with the dominant theories, related to the assumption of market efficiency, we focuses in a new form of option pricing based on the LRD.Some numerical results are given.

Keywords: The Long Range Dependence, Option Pricing.

INTRODUCTION

In this paper, a simulation for the valuation of European call options on different types of assets is considered. It is well known that the pricing of derivative securities (such as options and futures) is inconsistent with long-term memory. Following (1991) and Morimoto (2016), we compare the results obtained with the classic Black-Scholes formula, with those obtained using the formula based on the exponent of Hurst.The goal of this paper is to understand the behavior of the price of the financial derivatives with the LRD analysis. This is done with the use of the Hurst index. The empirical study presented in this work has been conducted on the dataset used by Skaperda (2013), which consists of the daily prices of 22 stock indexes, over a period from January 2000 to September 2013. The number of observations varies from 3361 to 3586, because of different closing days of the stock markets in the various states.We focus attention in equity indices Greek and Italian: GD.BATH and FTSE.MIB. Their standard deviation and the average values of the returns appear to be very close to 0.In order to improve our knowledge on rates and yields distributions, we did the BDS test (Brock, Dechert and Scheinkman, 1987).

The empirical results show very high values of z-statistic, this implies that none of the indexes has independent and identically distributed (IID), neither in prices or in returns. This result means that the BDS test is not able to identify the type of nonlinearity of the process, which could be hence chaotic or stochastic.This study investigates the found nonlinearity in the yields of the various indexes under the use of different specifications of R/S analysis.The results are compared through a three-dimensional graph capable of emphasizing the differences in the financial derivatives instruments as a function of the changes in the input parameters.The plain of this paper is as follow. Section 2 briefly explains the LRD (long range dependence) analysis, in discrete time. In section 3, we expose some typical topics of quantitative finance, describing the mathematicalmodel who represents the simulations of evolution of equity values and the different option pricing methods.In section 4 we investigate the presence of long-range dependence in the yields of the major world stock indices. We introduce, a new option pricing method who has been tested over financial derivatives about the same indices. The procedure has been run first with the classic method, then with the LRD. We compare the different results obtained. Finally, the concluding section presents a brief summary of this work, drawing conclusions on the basis of the empirical results obtained.

II. RESULTS AND DISCUSSION

2.1. R/S STATISTIC

The R/S statistic is calculated using the maximum and minimum values assumed by the sum resulting from deviations from the mean of a given sub-sample of the time series considered. The result is then divided by the standard deviation of the same sub-sample. Indicating a return-series as $\{r_1, r_2, \ldots, r_n\}$ and with $\bar{r}_n$ its arithmetic average, we obtain:

$$(R/S)_n = \frac{1}{S_n}\left[\max_{1\le t\le n}\sum_{k=1}^{t}(r_n - \bar{r}_n) - \min_{1\le t\le n}\sum_{k=1}^{t}(r_n - \bar{r}_n)\right], \quad (1)$$

where S_n indicates the standard deviation, calculated as follows:

$$S_n = \left[\frac{1}{n}\sum_{t}(r_t - \bar{r}_n)^2\right]^{\frac{1}{2}}. \quad (2)$$

Hurst found that the R/S statistic was proportional to the length of time the exponent nincreased to H exponent:

$$(R/S)_n = c^*(n)^H , \quad (3)$$

where c is a constant. Taking logarithms:

$$\log(R/S)_n = \log_c + H \log(n). \quad (4)$$

By using a simple Ordinary Least Squares (OLS) regression on (4) we are able to calculate the index H. This is the so called Hurst exponent, Its value indicates presence or absence of the LRD in a series.Some authors, such as Morimoto (2016), approximate this parameter not with a regression but with this simple relationship:

$$H = \frac{\log(R_T/S_T)}{\log(T)} \quad (5)$$

Following Skaperda (2013), based on the value assumed by H, it is possible to have the following scenarios:

1. $H = \frac{1}{2}$: the process is independent, past events are not correlated;
2. $0 < H < \frac{1}{2}$: the system is anti-persistent, it reverses its trend with more frequency than an independent one;
3. $\frac{1}{2} < H < 1$: the system is persistent and characterized by LRD.

An important contribution to this methodology was given by Lo (1991), who underlined how the classical formula of Hurst-Mandelbrot was not able to correctly distinguish between short memory and long memory, running the risk of exchange the first one for the second. For this reason, he proposed a modification to the procedure, by modifying the way of calculate the standard deviation of the model.Lo's work had great following and his methodology is commonly inserted into the LRD analysis in order to make comparisons with the classic one. According Baillie (1996), at a theoretical level, the following relationship is obtained between the parameter d of the ARFIMA model and the Hurst exponent:

$$d = H - \frac{1}{2}$$

2.2. The Options price

This final part of the empirical study focuses on a simulation of pricing for European call options, who has been done with the previously analyzed indices.The results obtained with the classic Black-Scholes formula, , are compared with those obtained with the formula based on Hurst exponent. In order to price financial derivatives in a way different and satisfactory, we have exploiting the study of LRD. This is done by using the H parameters obtained with the first method of analysis exposed in the previous paragraph and just testing indexes with a high Hurst exponent. They are: S&P TSX Composite, NASDAQ Composite, DAX, S&P Sensex, AEX, FTSE MIB, Hang Seng, TSEC, ATX, SMI Swiss, Euronext BEL 20, OMX, GD Athens and SSE Shanghai.Then the results are compared through a three-dimensional graph capable of emphasizing the differences among the different values taken by the derivative in function of changes in the input parameters. Besides the already mentioned H exponent detected empirically, as volatility input it is used the simple standard deviation calculated on the returns of each stock index.It is also assumed that the price of all the indices is equal to 100 and the risk free market rate is 10%. After the identification of the first four input parameters, the option value is calculated by letting vary the other two,: the time to maturity and the strike price.Figure 1 and Figure 2, here inserted as an example, refer to the Greek and Italian indices. In all of them the differences are observable exclusively, and increasingly, when the value of the strike price is closer to 100, which is the actual valuation of the underlying asset.

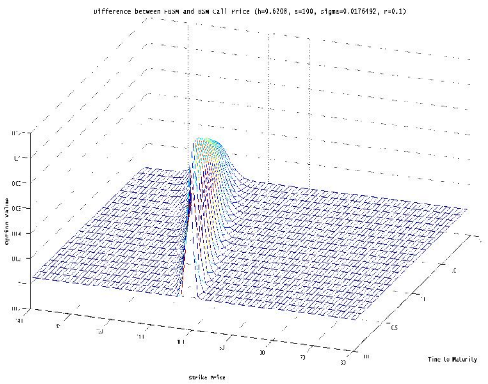

Fig.1

Furthermore, the discrepancy increases also if the time to maturity of the option decreases. All indices exhibit a rather similar behavior among them and the differences are greater as the parameter H increases.

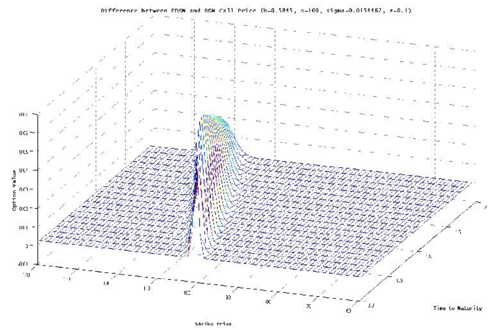

Fig.2

It is easy to understand the differences increase with the increase of the parameter H. However, the topic that deserves more attention is the volatility. It is the most sensitive input, the results change depending on its value. In this case, all the indices yields have very low standard deviations, about 1.5%. This similarity explains why the graphs do not show big differences among them. Furthermore, if the volatility is high, the two pricing models lead to more different results than those obtained in this case. As an example, it is attached a graph representing a simulation designed using as input parameters $H = 0{,}60$ and $\sigma = 0{,}50$.

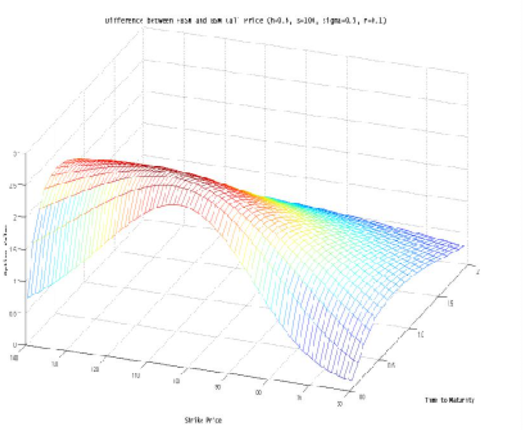

Fig.3

Figure 3 shows that, although the greatest differences are once again near the strike price of the option, they are greatly amplified and present in the whole graph.This result exhibits how the alternative pricing model can be used in the future because capable of giving different and interesting results. However, it is first necessary to carry out more research about what is the most suitable variable toindicate the volatility of the underlying to be included in the model. Maybe it is possible to find better proxies than the standard deviation or methods in order to change its value before using it as an input datum.

CONCLUSIONS

In the present work, it has been developed an empirical analysis on 22 stock indices from different countries and a simulation on how to price of financial derivatives instruments are related to them.We have established that the returns of the indices were stationary but not identically and independently distributed.The R/S analysis has established that this phenomenon is mainly present in emerging markets, probably because they are not yet fully efficient. In contrast, mature markets do not present evidence of LRD.This study has focused on pricing models for European call options, taking into account the presence of the LRD. For this reason, only indices from developing markets have been studied in this section.The aim has been to compare the values obtained through the classical Black-Scholes model with those obtained through the revised formula proposed by Norros, Valkeila, and Virtamo, able to consider also the LRD. Future researches could focus on possible gains by operating on emerging inefficient markets and about the possibility of using the modified Black-Scholes formula after having studied how to indicate better the volatility for the modelThe results of the two models diverge almost exclusively near the strike price and these differences increase proportionally with H exponent and reduction in the time to maturity of the option.However, the variable which exacerbates this gap more than the others is the volatility. In the present study it was represented through the simple standard deviation of stock indices returns. Since the latter has always low values, the differences between the two models have remained relatively small.

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