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Scale evolution of gluon TMDPDFs

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Abstract.
By applying the effective field theory machinery we factorize the transverse momentum spectrum of Higgs boson production, where the main hadronic quantities are the gluon transverse momentum dependent parton distribution functions (TMDPDFs). We properly define those quantities, showing explicitly, in the case of an unpolarized hadron, that they are free from rapidity divergences, and extract their evolution properties. It turns out that the evolution for all eight (un-)polarized leading-twist gluon TMDPDFs is driven by the same evolution kernel, for which we derive the necessary ingredients to obtain a resummation of large logarithms at next-to-next-to-leading-logarithmic accuracy. We make predictions for the contribution of linearly polarized gluons to the Higgs boson q_T-spectrum.

1 Factorization theorem in terms of well-defined TMDPDFs

The derivation of the factorization theorem for the Higgs-boson q_T-spectrum is done by applying the following set of consecutive matchings between effective theories [1]:

$$\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{\rm QCD}} .$$

First we integrate out the top quark mass to build the effective coupling ggH. Then we integrate out the mass of the Higgs boson, m_H, obtaining a factorization theorem which holds for $q_T \ll m_H$. Finally, for $\Lambda_{\rm QCD} \ll q_T \ll m_H$ we can refactorize the gluon TMDPDFs in terms of the collinear gluon/quark collinear PDFs, integrating out the intermediate scale q_T.

The effective local interaction that describes the gluon-gluon fusion process is [2–6]

$$\mathcal{L}_{\rm eff} = C_t(m_t^2,\mu)\,\frac{H}{v}\,\frac{\alpha_s(\mu)}{12\pi}\,F^{\mu\nu,a}\,F^a_{\mu\nu}\,, \tag{1}$$

where $v \approx 246\,\mathrm{GeV}$ is the Higgs vacuum expectation value. The Wilson coefficient C_t is known up to NNNLO [7, 8]. At NNLO it is [9, 10]

$$\begin{aligned}C_t(m_t^2,\mu) &= 1 + \frac{\alpha_s(\mu)}{4\pi}\,(5C_A - 3C_F)\\ &+ \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left[\frac{27}{2}\,C_F^2 + \left(11\ln\frac{m_t^2}{\mu^2} - \frac{100}{3}\right)C_F C_A\right.\\ &- \left(7\ln\frac{m_t^2}{\mu^2} - \frac{1063}{36}\right)C_A^2 - \frac{4}{3}\,C_F T_F - \frac{5}{6}\,C_A T_F\\ &\left.- \left(8\ln\frac{m_t^2}{\mu^2} + 5\right)C_F T_F n_f - \frac{47}{9}\,C_A T_F n_f\right], \end{aligned} \tag{2}$$

which anomalous dimension is given solely by the QCD β-function,

$$\gamma^t(\alpha_s(\mu)) = \frac{d\ln C_t(m_t^2,\mu)}{d\ln\mu} = \alpha_s^2\,\frac{d}{d\alpha_s}\,\frac{\beta(\alpha_s(\mu))}{\alpha_s(\mu)}\,. \tag{3}$$

Using the effective lagrangian just introduced, the differential cross section for Higgs production is factorized as

$$d\sigma = \frac{1}{2s}\left(\frac{\alpha_s(\mu)}{12\pi v}\right)^2 C_t^2(m_t^2,\mu)\,\frac{d^3q}{(2\pi)^2 2E_q}\int d^4y\,e^{-iq\cdot y}$$
$$\times\sum_X \left\langle PS_A,\bar{P}S_B\right|F^a_{\mu\nu}F^{\mu\nu,a}(y)\left|X\right\rangle\left\langle X\right|F^b_{\alpha\beta}F^{\alpha\beta,b}(0)\left|PS_A,\bar{P}S_B\right\rangle\,, \tag{4}$$

where $s = (P+\bar{P})^2$.

In a second step, the effective QCD operator is matched onto the SCET-q_T one by

$$F^{\mu\nu,a}\,F^a_{\mu\nu} = -2q^2 C(-q^2,\mu)\,g^\perp_{\mu\nu}\mathcal{B}^{\mu,a}_{n\perp}\left(\mathcal{S}_n^\dagger\mathcal{S}_{\bar{n}}\right)^{ab}\mathcal{B}^{\nu,b}_{\bar{n}\perp}\,, \tag{5}$$

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where $q^2 = m_H^2$ and $\mathcal{S}_{n(\bar{n})}$ represent the soft Wilson lines [1]. The $\mathcal{B}^{\perp\mu}_{n(\bar{n})}$ operators, which stand for gauge invariant gluon fields, are given by

$$\mathcal{B}^{\mu}_{n\perp} = \frac{1}{g}[\bar{n}\cdot\mathcal{P}W_n^\dagger iD_n^{\perp\mu}W_n] = i\bar{n}_\alpha g^{\mu}_{\perp\beta}W_n^\dagger F_n^{\alpha\beta}W_n$$
$$= i\bar{n}_\alpha g^{\mu}_{\perp\beta}t^a(\mathcal{W}_n^\dagger)^{ab}F_n^{\alpha\beta,b}\,, \tag{6}$$

and the Wilson lines are

$$W_n(x) = P\exp\left[ig\int_{-\infty}^{0}ds\,\bar{n}\cdot A_n^a(x+\bar{n}s)t^a\right],$$
$$S_n(x) = P\exp\left[ig\int_{-\infty}^{0}ds\,n\cdot A_s^a(x+ns)t^a\right]. \tag{7}$$

The calligraphic typography means that the Wilson lines are written in the adjoint representation, i.e., the color generators are given by $(t^a)^{bc} = -if^{abc}$. Gauge invariance among regular and singular gauges is guaranteed by the inclusion of transverse gauge links (see Refs. [11, 12] for more details). The Wilson matching coefficient $C(-q^2,\mu)$, which corresponds to the infrared finite part of the gluon form factor, is given at one-loop by

$$C(-q^2,\mu) = 1 + \frac{\alpha_s C_A}{4\pi}\left[-\ln^2\frac{-q^2+i0}{\mu^2} + \frac{\pi^2}{6}\right]. \tag{8}$$

The cross-section at leading order can be then written as

$$d\sigma = \sigma_0(\mu)\,C_t^2(m_t^2,\mu)H(m_H^2,\mu)\,\frac{m_H^2}{\tau s}\,\frac{dy\,d^2q_\perp}{(2\pi)^2}\int d^2y_\perp\,e^{-iq_\perp\cdot y_\perp}$$
$$\times 2J_n^{\mu\nu}(x_A,y_\perp;\mu)\,J_{\bar{n}\mu\nu}(x_B,y_\perp;\mu)\,S(y_\perp;\mu) + O(q_T/m_H)\,, \tag{9}$$

where $H(m_H^2,\mu) = |C(-q^2,\mu^2)|^2$, $x_{A,B} = \sqrt{\tau}\,e^{\pm y}$, $\tau = (m_H^2 + q_T^2)/s$ and

$$\sigma_0(\mu) = \frac{m_H^2\,\alpha_s^2(\mu)}{72\pi(N_c^2-1)sv^2}\,. \tag{10}$$

The *pure* collinear and soft matrix elements are defined as

$$J_n^{\mu\nu}(x_A,y_\perp,\mu) = -\frac{x_AP^+}{2}\int\frac{dy^-}{2\pi}\,e^{-i\frac{1}{2}x_Ay^-P^+}$$
$$\times\sum_{X_n}\langle PS_A|\,B^{\mu,a}_{n\perp}(y^-,y_\perp)\,|X_n\rangle\,\langle X_n|\,B^{\nu,a}_{n\perp}(0)\,|PS_A\rangle\,,$$
$$S(y_\perp,\mu) = \frac{1}{N_c^2-1}$$
$$\times\sum_{X_s}\langle 0|\,(\mathcal{S}_n^\dagger\mathcal{S}_{\bar{n}})^{ab}(y_\perp)\,|X_s\rangle\,\langle X_s|\,(\mathcal{S}_{\bar{n}}^\dagger\mathcal{S}_n)^{ba}(0)\,|0\rangle\,. \tag{11}$$

As it is shown in Ref. [1] by performing an explicit NLO perturbative calculation, the collinear and soft matrix elements defined above contain un-cancelled rapidity divergences and thus are ill-defined. Thus, they need to be properly combined to obtain well-defined hadronic quantities. Extending the work done in Refs. [13, 14] for the quark case to the gluon case, it can be easily shown that the soft function is split as

$$\tilde{S}(b_T;m_H^2,\mu^2) = \tilde{S}_-\left(b_T;\zeta_A,\mu^2;\Delta^-\right)\tilde{S}_+\left(b_T;\zeta_B,\mu^2;\Delta^+\right),$$
$$\tilde{S}_-\left(b_T;\zeta_A,\mu^2;\Delta^-\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+},\frac{\Delta^-}{\bar{p}^-}\right)},$$
$$\tilde{S}_+\left(b_T;\zeta_B,\mu^2;\Delta^+\right) = \sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+},\frac{\Delta^+}{\bar{p}^-}\right)}, \tag{12}$$

where $\zeta_A\zeta_B = m_H^4$. Using this splitting, the gluon TMDPDFs are then defined as

$$G^{\mu\nu}_{g/A}(x_A,\boldsymbol{k}_{n\perp},S_A;\zeta_A,\mu^2;\Delta^-) =$$
$$\int d^2\boldsymbol{b}_\perp\,e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_{n\perp}}\,\tilde{J}_n^{\mu\nu}(x_A,\boldsymbol{b}_\perp,S_A;\mu^2;\Delta^-)\,\tilde{S}_-(b_T;\zeta_A,\mu^2;\Delta^-)\,,$$
$$G^{\mu\nu}_{g/B}(x_B,\boldsymbol{k}_{\bar{n}\perp},S_B;\zeta_B,\mu^2;\Delta^+) =$$
$$\int d^2\boldsymbol{b}_\perp\,e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_{\bar{n}\perp}}\,\tilde{J}_{\bar{n}}^{\mu\nu}(x_B,\boldsymbol{b}_\perp,S_B;\mu^2;\Delta^+)\,\tilde{S}_+(b_T;\zeta_B,\mu^2;\Delta^+)\,. \tag{13}$$

We emphasize the fact the the hadronic quantities defined above are free from rapidity divegences and thus have well-behaved evolution properties and can be extracted from experiment.

Now, we can write the cross-section in terms of well-defined TMDPDFs:

$$d\sigma = 2\sigma_0(\mu)\,C_t^2(m_t^2,\mu)H(m_H^2,\mu)\,\frac{dy\,d^2q_\perp}{(2\pi)^2}\int d^2y_\perp\,e^{-iq_\perp\cdot y_\perp}$$
$$\times\tilde{G}^{\mu\nu}_{g/A}(x_A,y_\perp,S_A;\zeta_A,\mu)\,\tilde{G}_{g/B\mu\nu}(x_B,y_\perp,S_B;\zeta_B,\mu)$$
$$+ O(q_T/m_H)\,, \tag{14}$$

where the twiddle refers to impact parameter space (IPS). This factorized cross-section is valid for $q_T \ll m_H$. The involved gluon TMDPDFs can be separated in terms on the polarization of the hadron, i.e., unpolarized (O), longitudinally polarized (L) and transverse polarized (T). In Ref. [15] the authors obtained the decomposition of collinear correlators at leading-twist. Extending that decomposition to well-defined TMDPDFs, as given in Eq. (13), we have

$$G^{\mu\nu[O]}_{g/A}(x,\boldsymbol{k}_{nT}) = g_\perp^{\mu\nu}f_1^g(x,k_{nT}^2)$$
$$+\left(\frac{k^\mu_{n\perp}k^\nu_{n\perp}}{M_A^2} + g_\perp^{\mu\nu}\frac{k_{nT}^2}{2M_A^2}\right)h_1^{\perp g}(x,k_{nT}^2)\,,$$
$$G^{\mu\nu[L]}_{g/A}(x,\boldsymbol{k}_{nT}) = i\epsilon_\perp^{\mu\nu}\lambda\,g_{1L}^g(x,k_{nT}^2) + \frac{\epsilon_\perp^{k_T\{\mu}k^{\nu\}}_{n\perp}}{2M_A^2}\lambda\,h_{1L}^{\perp g}(x,k_{nT}^2)\,,$$
$$G^{\mu\nu[T]}_{g/A}(x,\boldsymbol{k}_{nT}) = -g_\perp^{\mu\nu}\frac{\epsilon_\perp^{k_TS_T}}{M_A}f_{1T}^{\perp g}(x,k_{nT}^2)$$
$$- i\epsilon_\perp^{\mu\nu}\frac{\boldsymbol{k}_{nT}\cdot\boldsymbol{S}_T}{M_A}g_{1T}^g(x,k_{nT}^2)$$
$$+\frac{\epsilon_\perp^{k_T\{\mu}k^{\nu\}}_{n\perp}}{2M_A^2}\frac{\boldsymbol{k}_{nT}\cdot\boldsymbol{S}_T}{M_A}h_{1T}^{\perp g}(x,k_{nT}^2)$$
$$+\frac{\epsilon_\perp^{k_T\{\mu}S_T^{\nu\}} + \epsilon_\perp^{S_T\{\mu}k^{\nu\}}_{nT}}{4M_A}h_{1T}^g(x,k_{nT}^2)\,. \tag{15}$$

[1] A generic vector v^μ is decomposed as $v^\mu = \bar{n}\cdot v\frac{n^\mu}{2} + n\cdot v\frac{\bar{n}^\mu}{2} + v_\perp^\mu = (\bar{n}\cdot v, n\cdot v, v_\perp^\mu) = (v^+, v^-, v_\perp^\mu)$, with $n = (1,0,0,1)$, $\bar{n} = (1,0,0,-1)$, $n^2 = \bar{n}^2 = 0$ and $n\cdot\bar{n} = 2$. We also use $v_T = |\boldsymbol{v}_\perp|$, so that $v_\perp^2 = -v_T^2 < 0$.

2 Evolution of gluon TMDPDFs

The evolution of all (un-)polarized gluon TMDPDFs is governed by the same anomalous dimension,

$$\frac{d}{d\ln\mu}\ln\tilde{G}^{[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu)\equiv\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right),\quad(16)$$

which is fixed by the renormalization group (RG) equation applied to the cross-section:

$$0=2\frac{\beta(\alpha_s(\mu))}{\alpha_s(\mu)}+2\gamma^t(\alpha_s(\mu))+\gamma_H\left(\alpha_s(\mu),\ln\frac{m_H^2}{\mu^2}\right)+\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)+\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_B}{\mu^2}\right).\quad(17)$$

Thus we have

$$\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)=-\Gamma^A_{\rm cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2}-\gamma^{nc}(\alpha_s(\mu)),$$
$$\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_B}{\mu^2}\right)=-\Gamma^A_{\rm cusp}(\alpha_s(\mu))\ln\frac{\zeta_B}{\mu^2}-\gamma^{nc}(\alpha_s(\mu)),\quad(18)$$

where the non-cusp piece is

$$\gamma^{nc}(\alpha_s(\mu))=\gamma^g(\alpha_s(\mu))+\gamma^t(\alpha_s(\mu))+\frac{\beta(\alpha_s(\mu))}{\alpha_s(\mu)}.\quad(19)$$

The perturbative coefficients of $\Gamma_{\rm cusp}$ and γ^V are known up to three loops and can be found in Ref. [1].

On the other hand, we also have

$$\frac{d}{d\ln\zeta_A}\ln\tilde{G}^{[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu)=-D_g(b_T;\mu^2),\quad(20)$$

where the RG-equation implies also that

$$\frac{dD_g}{d\ln\mu}=\Gamma^A_{\rm cusp}.\quad(21)$$

Regardless how the non-perturbative contribution of the D_g-term at large b_T is parametrized, the evolution of all leading-twist gluon TMDPDFs can be consistently performed up to NNLL:

$$\tilde{G}^{[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,f},\mu_f)=\tilde{G}^{[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,i},\mu_i)\,\tilde{R}^g\left(b_T;\zeta_{A,i},\mu_i,\zeta_{A,f},\mu_f\right),\quad(22)$$

where the evolution kernel $\tilde{R}^g$ is given by

$$\tilde{R}^g(b_T;\zeta_{A,i},\mu_i,\zeta_{A,f},\mu_f)=\exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_G\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_{A,f}}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_g(b_T;\mu_i)}.\quad(23)$$

The evolution equation of the D_g-term can be solved analytically and obtain the resummed D_g (as done in Ref. [16] in the quark case). Setting $a=\alpha_s(\mu_i)/(4\pi)$ and

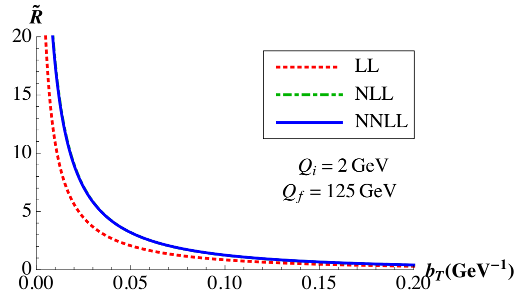

Figure 1. *Evolution kernel with $Q_i = 2$ GeV, $Q_f = 125$ GeV and fixed number of flavours $n_f = 4$. The difference between NLL and NNLL curves is unappreciable.*

$X=a\beta_0L_\perp$ we get

$$D_g^R(b_T;\mu_i)=-\frac{\Gamma_0^A}{2\beta_0}\ln(1-X)+\frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0^A}{\beta_0^2}(X+\ln(1-X))+\frac{\Gamma_1^A}{\beta_0}X\right]+\frac{1}{2}\left(\frac{a}{1-X}\right)^2\left[2d_2(0)+\frac{\Gamma_2^A}{2\beta_0}(X(2-X))+\frac{\beta_1\Gamma_1^A}{2\beta_0^2}(X(X-2)-2\ln(1-X))+\frac{\beta_2\Gamma_0^A}{2\beta_0^2}X^2+\frac{\beta_1^2\Gamma_0^A}{2\beta_0^3}(\ln^2(1-X)-X^2)\right]+\ldots,\quad(24)$$

The coefficient $d_2^g(0)$ is:

$$d_2^g(0)=C_AC_A\left(\frac{404}{27}-14\zeta_3\right)-\left(\frac{112}{27}\right)C_AT_Fn_f.\quad(25)$$

In Fig. 1 we show the evolution kernel for fixed initial and final scales ($\zeta=\mu^2=Q^2$), implementing the resummed D_g. For those scales the role of the non-perturbative contribution to the D_g term at large b_T is *numerically* irrelevant, given the perfect convergence between different resummation orders. However, this does not mean that for other scale choices that contribution would not be relevant.

3 Gluon TMDPDFs in an unpolarized hadron

The content of gluons inside an unpolarized hadron is parametrized in terms of two distributions:

$$G^{\mu\nu[O]}_{g/A}(x,\boldsymbol{k}_{nT})=-\frac{1}{2}g_\perp^{\mu\nu}f_1^g(x,k_{nT}^2)+\frac{1}{2}\left(g_\perp^{\mu\nu}-\frac{2k_{n\perp}^\mu k_{n\perp}^\nu}{k_{n\perp}^2}\right)h_1^{\perp g}(x,k_{nT}^2).\quad(26)$$

In impact parameter space we write

$$\tilde{G}^{\mu\nu[O]}_{g/A}(x,\boldsymbol{b}_T) = \int d^2\boldsymbol{k}_{n\perp}\, e^{i\boldsymbol{k}_{n\perp}\cdot\boldsymbol{b}_\perp}\, G^{\mu\nu[O]}_{g/A}(x,\boldsymbol{k}_{nT})$$
$$= -\frac{1}{2}g_\perp^{\mu\nu}\tilde{f}_1^g(x,b_T^2) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right)\tilde{h}_1^{\perp g}(x,b_T^2)\,, \quad (27)$$

and thus the relations between the distributions in momentum and IPS are

$$\tilde{f}_1^g(x,b_T^2) = 2\pi\int dk_{nT}\,k_{nT}\,J_0(k_{nT}b_T)\,f_1^g(x,k_{nT}^2)\,,$$
$$\tilde{h}_1^{\perp g}(x,b_T^2) = -2\pi\int dk_{nT}\,k_{nT}\,J_2(k_{nT}b_T)\,h_1^{\perp g}(x,k_{nT}^2)\,. \quad (28)$$

For $b_T \ll \Lambda_{\rm QCD}^{-1}$ we can refactorize the (renormalized) gluon TMDPDFs of an unpolarized hadron A in terms of (renormalized) collinear quark/gluon distributions:

$$\tilde{f}_1^{g/A}(x,b_T;\zeta_A,\mu) =$$
$$\sum_{j=q,\bar{q},g}\int_x^1 \frac{d\bar{x}}{\bar{x}}\tilde{C}^f_{g\leftarrow j}(\bar{x},b_T;\zeta_A,\mu)\,f_{j/A}(x/\bar{x};\mu) + O(b_T\Lambda_{\rm QCD})\,,$$
$$\tilde{h}_1^{\perp g/A}(x,b_T;\zeta_A,\mu) =$$
$$\sum_{j=q,\bar{q},g}\int_x^1 \frac{d\bar{x}}{\bar{x}}\tilde{C}^h_{g\leftarrow j}(\bar{x},b_T;\zeta_A,\mu)\,f_{j/A}(x/\bar{x};\mu) + O(b_T\Lambda_{\rm QCD})\,, \quad (29)$$

where the unpolarized collinear quark and gluon PDFs are defined as

$$f_{q/A}(x;\mu) = \frac{1}{2}\int\frac{dy^-}{2\pi}\,e^{-i\frac{1}{2}y^- xP^+}$$
$$\times\langle PS_A|\left[\bar{\xi}_n W_n\right](y^-)\frac{\bar{n}\!\!\!/}{2}\left[W_n^\dagger\xi_n\right](0)\,|PS_A\rangle\,,$$
$$f_{g/A}(x;\mu) = -\frac{xP^+}{2}\int\frac{dy^-}{2\pi}\,e^{-i\frac{1}{2}y^- xP^+}$$
$$\times\langle PS_A|\,\mathcal{B}^{\mu,a}_{n\perp}(y^-)\mathcal{B}^a_{n\perp\mu}(0)\,|PS_A\rangle\,. \quad (30)$$

Since the soft function can be written to all orders as

$$\ln\tilde{S} = \mathcal{R}_s(L_T,\alpha_s(\mu)) + D_g(L_T,\alpha_s(\mu))\ln\frac{\Delta^+\Delta^-}{m_H^2\mu^2}\,, \quad (31)$$

with some generic function $\mathcal{R}_s$ and where the D_g term is related to the cusp anomalous dimension in the adjoint representation as in Eq. (21), given Eqs. (13) and (31) one can exponentiate the ζ dependence of the TMDPDFs in the following way:

$$\tilde{f}^g_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)}$$
$$\times\sum_{j=q,\bar{q},g}\int_x^1\frac{d\bar{x}}{\bar{x}}\tilde{C}^{\mathcal{Q}}_{f,g\leftarrow j}(\bar{x},b_T;\mu)\,f_{j/A}(x/\bar{x};\mu)\,,$$
$$\tilde{h}^{\perp g}_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)}$$
$$\times\sum_{j=q,\bar{q},g}\int_x^1\frac{d\bar{x}}{\bar{x}}\tilde{C}^{\mathcal{Q}}_{h,g\leftarrow j}(\bar{x},b_T;\mu)\,f_{j/A}(x/\bar{x};\mu)\,. \quad (32)$$

Notice that the exponentiation of the ζ-dependence above applies in the same way to all (un)-polarized TMDPDFs defined before, through the same factor and D_g term.

In Ref. [1] we perform an explicit one-loop calculation of the kernel functions $\tilde{C}^{\mathcal{Q}}_{g\leftarrow i}(x,b_T;\mu)$, which are given by

$$\tilde{C}^{\mathcal{Q}}_{f,g\leftarrow j}(x,b_T;\mu) = \delta(1-x)\,\delta_{gi}$$
$$+\frac{\alpha_s}{2\pi}\left[\delta(1-x)\,\delta_{gi}\left(\Gamma_0^A\frac{L_T^2}{8}+\frac{\beta_0}{2}L_T\right) - \mathcal{P}_{g\leftarrow j}(x)L_T + \mathcal{T}^f_{g\leftarrow j}(x)\right],$$
$$\tilde{C}^{\mathcal{Q}}_{h,g\leftarrow j}(x,b_T;\mu) = \frac{\alpha_s}{2\pi}\mathcal{T}^h_{g\leftarrow j}(x)\,, \quad (33)$$

where $L_T = \ln(\mu^2 b_T^2 e^{2\gamma_E}/4)$, the one-loop DGLAP splitting kernels are

$$\mathcal{P}_{g\leftarrow g}(x) = 2C_A\left[\frac{x}{(1-x)_+}+\frac{1-x}{x}+x(1-x)\right]+\frac{\beta_0}{2}\delta(1-x)\,,$$
$$\mathcal{P}_{g\leftarrow q}(x) = C_F\frac{1+(1-x)^2}{x}\,, \quad (34)$$

and the functions $\mathcal{T}^f_{g\leftarrow j}(x)$ and $\mathcal{T}^h_{g\leftarrow j}(x)$ are given by

$$\mathcal{T}^f_{g\leftarrow g}(x) = -C_A\frac{\pi^2}{12}\delta(1-x)\,, \qquad \mathcal{T}^f_{g\leftarrow q}(x) = C_F\,x\,,$$
$$\mathcal{T}^h_{g\leftarrow g}(x) = -2C_A\frac{1-x}{x}\,, \qquad \mathcal{T}^h_{g\leftarrow q}(x) = -2C_F\frac{1-x}{x}\,. \quad (35)$$

Under RG-equation one has

$$\frac{d}{d\ln\mu}\tilde{C}^{\mathcal{Q}}_{g\leftarrow j}(x,b_T;\mu) = (\Gamma^A_{\rm cusp}L_T - \gamma^{nc})\tilde{C}^{\mathcal{Q}}_{g\leftarrow j}(x,b_T;\mu)$$
$$-\sum_i\int_x^1\frac{d\xi}{\xi}\tilde{C}^{\mathcal{Q}}_{g\leftarrow i}(\xi,b_T;\mu)\mathcal{P}_{i\leftarrow j}(x/\xi)\,, \quad (36)$$

so that we can partially exponentiate the double logarithms by (see Ref. [17] for the quark case):

$$\tilde{C}^{\mathcal{Q}}_{g\leftarrow j}(x,b_T;\mu) \equiv \exp\left[h_\Gamma(b_T;\mu) - h_\gamma(b_T;\mu)\right]\tilde{I}_{g\leftarrow j}(x,b_T;\mu)\,, \quad (37)$$

where

$$\frac{dh_\Gamma}{d\ln\mu} = \Gamma^A_{\rm cusp}L_T\,, \qquad \frac{dh_\gamma}{d\ln\mu} = \gamma^{nc}\,. \quad (38)$$

Thus, the TMDPDFs can be written as

$$\tilde{f}^g_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} e^{h_\Gamma(b_T;\mu)-h_\gamma(b_T;\mu)}$$
$$\times\sum_{j=q,\bar{q},g}\int_x^1\frac{d\bar{x}}{\bar{x}}\tilde{I}_{f,g\leftarrow j}(\bar{x},b_T;\mu)\,f_{j/A}(x/\bar{x};\mu)\,,$$
$$\tilde{h}^{\perp g}_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} e^{h_\Gamma(b_T;\mu)-h_\gamma(b_T;\mu)}$$
$$\times\sum_{j=q,\bar{q},g}\int_x^1\frac{d\bar{x}}{\bar{x}}\tilde{I}_{h,g\leftarrow j}(\bar{x},b_T;\mu)\,f_{j/A}(x/\bar{x};\mu)\,. \quad (39)$$

When $\alpha_s L_T$ is of order 1, the expression above still contains large logarithms L_T that need to be resummed.

Solving the evolution equations for h_Γ and h_γ, in the same way we have done previously for D_g, we get

$$h_\gamma^R(b_T;\mu) = D_g^R(b_T;\mu)\Big|_{\Gamma^A_{\rm cusp}\to\gamma^{nc}} \tag{40}$$

and

$$\begin{aligned} h_\Gamma^R(b_T;\mu) &= \frac{\Gamma_0^A(X-(X-1)\ln(1-X))}{2a_s\beta_0^2} \\ &+ \frac{\beta_1\Gamma_0^A\left(2X+\ln^2(1-X)+2\ln(1-X)\right)}{4\beta_0^3} \\ &- \frac{2\beta_0\Gamma_1^A(X+\ln(1-X))}{4\beta_0^3} \\ &+ \frac{a_s}{4\beta_0^4(1-X)}\Big[\beta_0^2\Gamma_2^A X^2 - \beta_0(\beta_1\Gamma_1^A(X(X+2)+2\ln(1-X)) \\ &+\beta_2\Gamma_0^A((X-2)X+2(X-1)\ln(1-X)))+\beta_1^2\Gamma_0^A(X+\ln(1-X))^2\Big]. \end{aligned} \tag{41}$$

4 Phenomenology

So far, the resummation techniques discussed previously only make sense in the perturbative region of small b_T. For the large b_T region we need to parametrize the non-perturbative contribution of TMDPDFs with a model:

$$\begin{aligned} &\tilde f_1^g(x,b_T;Q^2,Q) = \\ &\exp\left\{\int_{Q_i}^{Q}\frac{d\bar\mu}{\bar\mu}\gamma_G\left(\alpha_s(\bar\mu),\ln\frac{Q^2}{\bar\mu^2}\right)\right\}\left(\frac{Q^2b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g^R(b_T;Q_i)} \\ &\times e^{h_\Gamma^R(b_T;Q_i)-h_\gamma^R(b_T;Q_i)}\sum_{j=q,\bar q,g}\int_x^1\frac{d\bar x}{\bar x}\tilde I_{f,g\leftarrow j}(\bar x,b_T;Q_i)\,f_{j/N}(x/\bar x;Q_i) \\ &\times \tilde F_f^{\rm NP}(x,b_T;Q)\,, \\ &\tilde h_1^{\perp g}(x,b_T;Q^2,Q) = \\ &\exp\left\{\int_{Q_i}^{Q}\frac{d\bar\mu}{\bar\mu}\gamma_G\left(\alpha_s(\bar\mu),\ln\frac{Q^2}{\bar\mu^2}\right)\right\}\left(\frac{Q^2b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g^R(b_T;Q_i)} \\ &\times e^{h_\Gamma^R(b_T;Q_i)-h_\gamma^R(b_T;Q_i)}\sum_{j=q,\bar q,g}\int_x^1\frac{d\bar x}{\bar x}\tilde I_{h,g\leftarrow j}(\bar x,b_T;Q_i)\,f_{j/N}(x/\bar x;Q_i) \\ &\times \tilde F_h^{\rm NP}(x,b_T;Q)\,, \end{aligned} \tag{42}$$

where $F_{f,h}^{\rm NP}$ account for the non-perturbative contribution at large b_T. For the Higgs distribution we have $Q = m_H$, and we fix the resummation scale $Q_i = Q_0+q_T$ (we choose $Q_0 = 2$ GeV). In Ref. [17] it was shown that for Z-boson production it was enough to take simple scale-independent exponentials to parametrize the non-perturbative contribution of quark-TMDPDFs. Following the same logic, let us consider scale-independent exponentials for the Higgs boson production:

$$\begin{aligned} \tilde F_f^{\rm NP}(x,b_T;Q) &= e^{-\lambda_1 b_T}\,, \\ \tilde F_h^{\rm NP}(x,b_T;Q) &= e^{-\lambda_2 b_T}\,. \end{aligned} \tag{43}$$

With these models, one can quantify the contribution of linearly polarized gluons to the Higgs boson q_T-spectrum [18–20]), taking into account the resummation scheme presented above and analyzing the impact of the non-perturbative parameters $\lambda_{1,2}$. We define the ratio between the contributions of unpolarized and linearly polarized gluons to the cross-section by

$$\begin{aligned} &\mathcal{R}(x_A,x_B,q_T;Q) = \\ &\frac{\int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\tilde h_1^{\perp g}(x_A,b_T;Q^2,Q)\,\tilde h_1^{\perp g}(x_B,b_T;Q^2,Q)}{\int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\tilde f_1^{g}(x_A,b_T;Q^2,Q)\,\tilde f_1^{g}(x_B,b_T;Q^2,Q)}\,. \end{aligned} \tag{44}$$

In Fig. 2 we show the results, where the band comes from varying the resummation scale from $2Q_i$ to $Q_i/2$.

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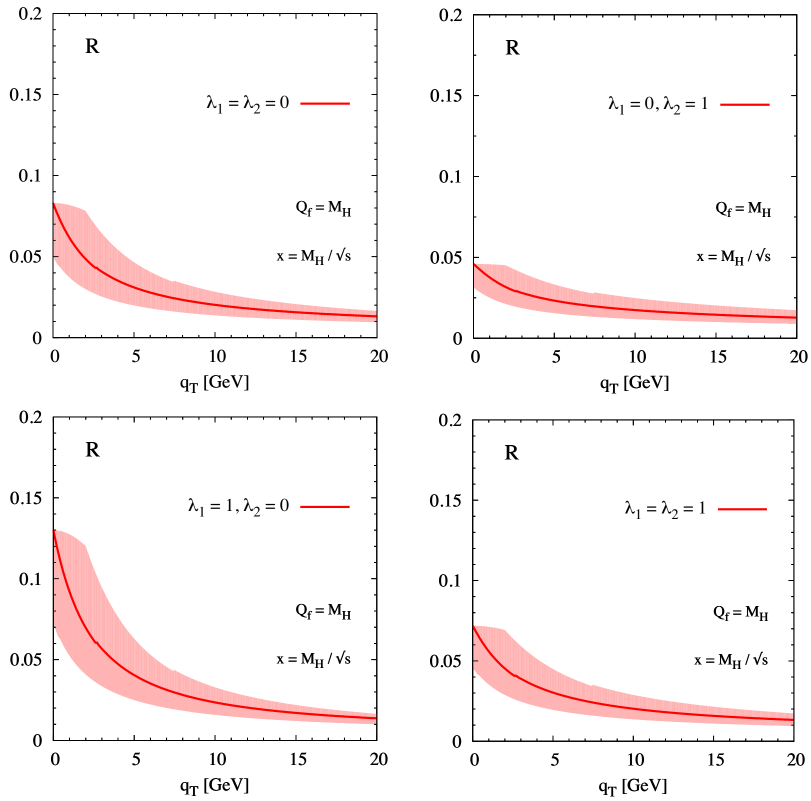

Figure 2. *Ratio of the contributions of linearly polarized and unpolarized gluons to the Higgs boson q_T-spectrum at* $\sqrt{s} = 8$ TeV *and* $x_A = x_B$ *for different values of the non-perturbative parameters* $\lambda_{1,2}$.

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