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Time, tenses and modalities

Diodorus' *κυριεύων*, Prior's *Master Argument* and his heritage

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Abstract

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by Fabio Corpina

The present dissertation, combining historical and theoretical methodologies, spans several centuries of logical thought with its double focus on Diodorus Cronus and Arthur Prior. As to the former author, the Megaric fragments are duly examined; concerning the latter, we could benefit from a first-hand participation in the *Virtual Lab for Prior Studies*. We start from Diodorus' *κυριεύων λόγος* and some ancient doctrines, and then we examine Prior's analysis and formalisation of the *Master Argument*, his Diodorean system and the birth of temporal logics. One goal of the thesis is to enhance Prior's heritage through an analysis of unpublished manuscripts from the *Prior Collection*.

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Special thanks go to Giuseppe Roccaro and Nicholas Denyer, respectively. The first introduced me to ancient and medieval logic when I was an undergraduate at the *University of Palermo*, the second because suggested me to link Diodorus' account and Arthur Prior's approach about logic and reality during a visit at the *University of Cambridge*.

I do not know counterexamples to say that $p \rightarrow HFp$ is not a theorem in life. I can say I was happy during this marvelous 'journey' in Sardinia, despite occasionally feeling somehow homesick, especially when thinking the view from Tempio d'Ercole to Aeolian Islands. At Viale Merello, 87, I met fantastic friends and, above all, I met Maria's love. I hope there will be a long road ahead with her.

Lastly, I would thank my family for their constant support and encouragement: in particular, my parents Gisella and Carlo, and my brother Giovanni. This work is dedicated to them.

Contents

List of Tables

Abbreviations of ancient texts

Alex.Aphr. *in APr.* = Alexander Aphrodisiensis, *In Aristotelis Analyticorum Priorum*

Arist. *APr.* = Aristotle, *Analytica Priora*

Cael. = *De Caelo*

Cat. = *Categorias*

Int. = *De Interpretatione*

Metaph. = *Metaphysica*

Ph. = *Physica*

S.E. = *Sophistici Elenchi*

Arr. *Epict.* = Arrianus, *Epicteti Dissertationes*

Boeth. *in Int.* = Boethius, *In Aristotelis De Interpretatione*

Cic. *Luc.* = Cicero, *Lucullus*

Fat. = *De Fato*

Fam. = *Epistulae ad familiares*

Top. = *Topica*

D.L. = Diogenes Laertius, *Vitae Philosophorum*

Epicur. *Nat.* = Epicurus, *De rerum natura*

Epict. *Ench.* = Epictetus, *Enchiridion*

Euseb. *Praep. ev.* = Eusebius of Cesarea, *Praeparatio Evangelica*

Eust. *Ad Hom. Od.* = Eustathius Thessalonicensis, *Commentarii ad Omeri Odysseam*

Gal. = Galenus, *Institutio Logica*

Gell. = Aulus Gellius, *Noctes Actiacae*

Hieron. *Adv. Jovin.* = Jerome, *Adversus Jovinianus*

Adv. Pelag. = *Adversus Pelagium*

Oc. *Praed.* = Occam, *Tractatus de Praedestinatione*

Phlp. *in APr.* = Philoponus, *In Aristotelis Analyticorum Priorum*
Pl. *Cra.* = Plato, *Cratylus*
Men. = *Meno*
Plu. *Mor.* = Plutarch, *Moralia*
Stoic. rep. = *De Stoicorum repugnantiis*
Simp. *in Cat.* = Simplicius, *In Aristotelis Categorias Commentarium*
S.E. *M.* = Sextus Empiricus, *Adversus Mathematicos*
P.H. = *Pirroneion Hypotyposeon*
Steph. *in Int.* = Stephanus, *In Aristotelis De Interpretatione*
Stob. *Ecl.* = Stobaeus, *Eclogae*
Str. = Strabo, *Geographica*

Preamble

The **conceptual genesis** of this thesis has its roots in several generic questions: *What is time? How do we live and describe it? What we mean by the necessary? How are necessity and possibility related? Are time and modalities linked?* Furthermore, all these are approached from the perspective induced by the following question:

> How can we use formal methods in order to account for the basic flux of life and things?

A possible answer to this query is suggested in a brilliant manuscript by Prior [nda], kept in the *Prior Collection*. It is an undated manuscript, in rather poor condition, written in a faded ink, with a barely legible handwriting. Nevertheless, it is worthwhile paying attention to its content, both because of its intrinsic philosophical interest, and also because it provides a reading key for the present thesis, bringing everything together under a coherent vision. The manuscript is concerned with the motivation for adopting formal methods when it comes to interpreting *reality as things acting*. There is a pattern in the flow of things, and Prior suggests that tense logic can be used to grasp it. Prior polemicises against those who refuse to take any interest whatsoever in the *realm of life and time*, letting in become the province of *existentialists and Bergsonians and others who loves darkness*. In truth, the target of the polemic is not the explicitly mentioned philosophers; Prior attacks all those who consider formal techniques to be a mere technical exercise, as the following remarks make it clear:

> Philosophy, including Logic, is not primarily about language, but about the real world. [...] Formalism, i.e. the theory that Logic is

> just about symbols and not things, is false. Nevertheless, it is important to 'formalise' as much as we can, i.e. to state truths about things in a rigorous language with a known and explicit structure [Prior, 1996b, 45].

The last part of [Prior, nda] presents a reconstruction of Diodorus Cronus' Master Argument. This is worth noticing because, (i) it shows a renewed interest in modal and temporal notions which are central for our worldview, while (ii) it puts the Master Argument to the test with the use of modern formal tools. This fits with a time-honoured tradition, since Diodorus' *κυριεύων* seems to have had a similar upshot. The age of early Hellenism, when Diodorus works, and the sixth decade of the 20th century, when Prior devises the first modern reconstruction of the Master Argument, are two millenia apart. Despite this the two philosophers are united by their interest in *time*, *modality* and *life* (because philosophy is not strange to life). This allows to us to safely pursue the two-fold interest of this work.

The research reported in the present thesis follows **three complementary perspectives**:

- It contributes to the classical themes of time and modality from a double viewpoint. First, we have undertaken a careful examination of the fragments of Diodorus, in the logical and philosophical context of the Antiquity. Second, we have analysed Prior's texts, both published and unpublished (*see*, [VLP, 2011] and [NAP, 2014]).
- From a formal perspective, we have investigated apposite formal methods for analysing modal and temporal notions. We have paid special attention to modal logics, particularly to the systems between $S4$ and $S5$. At the same time we have studied the genesis of temporal logics starting from Prior's tense logic.

- Finally, we have strived to discern and to provide an adequate picture of the power of the above-mentioned logical systems, by studying their interrelations, by looking both at Prior's work and at its heritage and subsequent development.

As said before, the main topics of the analysis are several temporal and modal concepts. In what follows, we shall describe the **objectives** and the **structure of the thesis**.

The main objective of the first chapter is the reconstruction of the thought of Diodorus, especially in respects to his logical doctrines and the original version of the Master Argument. As far as the physical doctrines are concerned, we aim to exhibit a pattern which dialectically maps the development of Diodorus' views on language. At the same time we aim to provide a comprehensive image of the conception of modality in the Hellenistic Age, including a novel approach to the concept of *capacity*. We analyse the structure of the Master Argument starting from *Epict.* 2, 19, 1, going through a careful analysis of each of its components.

In the second chapter we focus on Prior's reconstruction of the Master Argument, in relation to the grand themes of modality and temporal logics. Our objective here is to exhibit the inferential sequence that leads to its conclusion and to clarify the Łukasiewicz-style strategy employed by Prior. At the same time we pay attention to Prior's heritage. We wish to bring to light some unpublished material by Prior as well as some works by his closest collaborators in order to reconstruct the birth of tense logic in relation to the Diodorean systems.

In the first chapter (1), in the tradition of philosophy conceived as history of ideas, we explore the thought of Diodorus Cronus and, in general, the Megaric philosophy, as well as the *κυριεύων λόγος*. In section 1.1 we discuss Diodorus' atomism and the arguments against motion. In section 1.2 we deal with Diodorus' thesis of the conventionality of language and we illustrate some of his views regarding the use of tenses in discourse. Section 1.3 is the central

part of this chapter – and it is dedicated to the *κυριεύων λόγος*, *i.e.* Diodorus' most significant attempt to *master* modal and temporal notions from a deterministic perspective. Subsection 1.3.1 discusses the Megarics and Diodorus Cronus in particular with respect to their views about capacity (1.3.1.1), temporal notions and modality (1.3.1.2). Subsection 1.3.1.3 deals with the problem of determinism –and its relation to fatalism– and we argue that Diodorus is a determinist. In subsection 1.3.1.4 we clarify the meaning of the terms *δυνατὸν* and *δύναμις*; this allows us a more careful interpretation of Diodorus' position and of its relation to that of Aristotle. With these preliminaries out of the way, in section 1.3.2 we turn to a sentence-by-sentence analysis of the ancient Master Argument.

In chapter 2 we compare the Diodorean perspective with Prior's modern approach to modal and temporal notions. As a matter of fact, Prior's thinking on these Diodorean themes starts from the opposite perspective: that of indeterminism. As a preliminary, in section 2.1 we present Prior's approach to tense logic, laying the ground for the subsequent discussion. In 2.1.1 we retrace the debate on Diodorean modalities which underscores the search for an adequate intermediate system between $S4$ and $S5$. We procede analogously in section 2.1.2, this time with respect to tense logics. This section reports our findings based on the Hamblin-Prior and Lemmon-Prior correspondence, extremely valuable archival material for tracing the origins of the research on temporal logics. The central section of this chapter is 2.2, in which we give a list of occurrences of the Master Argument in Prior's works, and we deploy modern formal methods, temporal as well as modal, to analyse the Master Argument. In 2.2.1 we present, step by step, Prior's version of the Master Argument and its formalisation. In connection with Prior's strategy against determinism, we explain the approach based on a three-valued logic, its results and problems. In section 2.3 we explore the contemporary reception of Prior's account. First, we compare this with a *Danish* version of the Master Argument (2.3.1). Second, we present the NAP [2014] (2.3.2).

These two chapters are followed by two *appendices*. In appendix A we present a formal semantics applied to Diodorean ontologies. In appendix B we present some documents from the *Prior Collection*, illustrating Prior's role as father of temporal logics.

We wrap up this *Preamble* with some **remarks on composition and notation**. The quotations occurring in this thesis, especially in chapter 1, are translations from ancient fragments. In these cases we give the original text in footnote. The references are standardly made, as per the list of *abbreviation of ancient texts*. For the quotations from unpublished material from *Prior Collection*, in the *Bibliography* we associate to the title of the document the archive index. Logical notation is standard throughout this thesis, despite Prior's preference for the Polish notation.

Chapter 1

An overview on Diodorus Cronus and the *κυριεύων λόγος*

A complete overview of the philosophical context and doctrines in the period between the late Classical age and early Hellenism is beyond the scope of this thesis. We do not aim to give a historical account of philosophy in that period.[1] Nevertheless, as preliminaries, we do need some historical information, at least on the Megaric group, and on a crafty dialectician from an heterodox branch of the Megaric tradition, namely Diodorus Cronus.[2]

The founder of Megara's philosophical school was Euclides (c.435–365 B.C.), a student of Socrates. So a first generation of Megaric philosophers precede Aristotle.[3] Cic. *Luc.* 42, 129, reports that:

[1] For that, we refer the reader to *e.g.* [Reale, 2001], [Algra et al., 1999], [Long and Sedley, 1987, 1989], [Giannantoni, 1977].

[2] The following is an array of five of the most important collections of fragments about the Megarics, from the 19th century till today: [Deycks, 1827], [Döring, 1972], [Montoneri, 1984], as well as [Long and Sedley, 1987, 1989] and the monumental four volumes by Giannantoni [1990]. As regards the history of logic, a good reference for the Megarics and their heritage to the Stoics is [Mates, 1973]. In this thesis, we vindicate Diodorus' status as a dialectician, namely logician. Diodorus Cronus is considered by Mates [1973] as a precursor for Stoic logic. Inspired by the *Stoic logic*, Arthur Prior examined, for the first time, the Diodorean logic, *e.g.* in [Prior, 1955a], starting the search for the Diodorean frame (*see*, [Ciuni, 2009] and section 2.1.1).

[3] This is important seeing that the first reference to the Megarics, that is, *οἱ Μεγαρικοί* occurs in Arist. *Metaph.* 9. 3, 1046b 29-32. Aristotle's passage contains a criticism against the Megarics, probably referring to a group around Euclides, focusing on the distinction between *capacity* and *act*.

> A famous school was that of the Megarics, whose founder, as I see it recorded, was Xenophanes whom I mentioned just now; next he was followed by Parmenides and Zeno (and so the school of thought derived from them the name of Eleatic) and afterwards by Euclides, the pupil of Socrates, a Megaric (from whom the same school obtained the title of Megaric); their doctrine was that the sole good is that which is always one and alike and the same [Rackham, 1933, 635].[4]

Cicero's fragment, which obviously misidentifies the founder of the School, suggests nonetheless an important syncretism between Eleaticism and Socraticism within the Megaric tradition.[5] Although with different results, that syncretism forged the Megaric thought, under its various doctrinal aspects. We will deal with the dialectical tradition represented by Diodorus, dubbed 'socratic' by Hieron. *Adv. Jovin.* I 42.

Euclides' interest in logic (at the time named *dialectic*), is attested by D.L. 2, 106. Further, the same text at 2, 107, also states that the group of Megarics and the one of Dialecticians derived from the same school of Euclides:

> His followers were called Megarics after him, then Eristics, and at a later date Dialecticians. [...] When he impugned a demonstration, it was not the premises but the conclusion that he attacked [Hicks, 1959, 235].[6]

According to D.L. 2, 112, a new direction of Megaricism like Dialecticism was started by Clinomachus from Turi. An interesting outline of the Dialecticians' arguments is presented by Ebert [2008, 275-293]. As already stated,

[4] Megaricorum fuit nobilis disciplina, cuius, ut scriptum video, princeps Xenophanes quem modo nominavi; deinde eum secuti Parmenides et Zeno (itaque ab his Eleatici philosophi nominabantur), post Euclides, Socratis discipulus, Megareus (a quo idem illi Megarici dicti); qui id bonum solum esse dicebant quod esset unum et simile et idem semper.

[5] Aristocles of Messene *apud* Euseb. *Praep. ev.* 14, 17, 1, gives a similar report.

[6] *οἱ ἀπ᾽ αὐτοῦ* Μεγαρικοὶ *προσηγορεύοντο*, *εἶτ᾽ ἐριστικοί*, *ὕστερον* δὲ διαλεκτικοί [...]. Τα*ῖς τε ἀποδείξεσιν ἐνίστατο οὐ κατὰ λήμματα*, *ἀλλὰ κατ᾽ἐπιφοράν*.

Diodorus Cronus was an important figure of this branch of the Megaric School. Diodorus' principal contribution consists in a strong, logically oriented, characterisation of Megaric methaphysics, focused on a critique of capacity, time, and modalities: the *κυριεύων λόγος* or Master Argument is Diodorus' best synthesis of these arguments.

It is not easy to reconstruct a biography of Diodorus Cronus. Indirect reports are not sufficient to decide definitely on many obscure biographical points: most of them are posthumous and fragmentary, and sometimes the information concerning Diodorus is only implied. In some ways, we are forced to deal with guesswork and conjectures.[7] Quite a few surviving fragments help us place Diodorus Cronus as a member of the Megaric school.[8] Whithin this group Diodorus represents the *dialectical tradition* – *i.e.* the part of the circle with a strong interest in logic.[9]

Diodorus was born in Iasus, a city of Caria in Asia Minor, around the middle of the 4th century or perhaps a little later. However, there are no reports to date his birth. According to D.L. 2, 111, Diodorus' father was Aminia, his teacher was Apollonio Cronus, who was a pupil of Eubulides from Miletus.[10] Thus Diodorus may be chronologically collocated between Aristotle and the

[7] Sedley [1977] offers a complete biography of Diodorus – however, we do not fully agree with his account. Sedley maintains that Diodorus was a rival of the Megarics. Following [Sedley, 1977], there were two different schools, namely the Megaric and the Dialectic, and Diodorus was a member of the second. This view is supported by a passage in D.L. 2, 113. However Sedley's interpretation may be contested on the basis of other evidence, e.g., the *Μεγαρικὰ ἐρωτήματα* quoted in Plu. *Mor.* 13, 72, 1036e-f, as shown by Döring [1989, 297], and confirmed by the *σοφίσματα* of Stilpo, Diodorus, and Alexinus in Cic. *Luc.* 24, 75. Four years later [Sedley, 1977], Giannantoni [1981] published an interesting work on Diodorus, which completes the first reports collection of the *Socraticorum Reliquiae* [Giannantoni, 1990].

[8] Diodorus' membership to the Megaric group is also documented by a *neglected* piece of evidence in Eust. *Ad Hom. Od.* 28, 46 - 29, 2, although the same Denyer [2002] has expressed some doubt.

[9] The word 'dialectic' received different interpretations in Antiquity. For Plato, dialectic is the strategy of dividing a term into two further specifications. According to Aristotle dialectic is the syllogism whose premises are in general only probable. Stoics learnt dialectic from Megarics (in particular from Dialectics if that group really existed within the Megaric tradition). They identified dialectic and logic (*see*,[Abbagnano, 1958, 123-130]). This was a flourishing new trend in logic, characterised by an interest in propositions rather than terms, in turn based on the attempt to find an objective criterion for entailment and validity and having also a strong focus on modalities.

[10] Eubulides is known as the author of the seven Megarics paradoxes (D.L. 2, 108). So we have here a further indication or perhaps explanation of the interest Diodorus took in logical matters.

first Hellenism, as further testified by his leaving from Athens to Alexandria, suggesting that by that time, Athens might have begun to (slowly) lose its importance as an intellectual centre.[11]

One of the most important themes of Diodorus Cronus was the truth criterion for conditional sentences, that is the *συνημμένον*. Another central topic in Diodorus' thought were possibility and necessity and the correlation between them. According to the dynamics of time and tenses, modalities were very relevant in order to define *being*. The *κυριεύων λόγος* was supposed to make it possible to explain modalities by means of time notions. This argument was famous at that time, but remains cryptic today: we only know its two premises and the conclusion. There is no complete report on the inferential machinery connecting there and of the proof of the principle that *nothing is possible which is neither true now nor ever will be*. Therefore, it may be useful to go through some of the commentaries before trying to construct the ancient Master Argument. *E.g.*, here is Alex.Aphr. *in APr.* 183, 34 - 184, 6:

> [Aristotle] may possibly be talking also about the issue 'What things are possible?', and about the so-called 'Diodorean' answer, 'What either is or will be'. For Diodorus set down as possible only what either is or, in any event, will be. According to him, for me to be in Corinth was possible if I was in Corinth or if I was, in any event, going to be; if not, it was not even possible. And for the child to become literate was possible if he was, in any event, going to be. It is to establish this that Diodorus' Master Argument is posed [Long and Sedley, 1989, 231].[12]

[11] *See*, S.E. *P.H.* 2, 245, for the quotation by Erophilus, doctor from Alexandria; D.L. 2, 111 and S.E. *M.* 1, 309, for reports by the poet Callimachus from Alexandria about Diodorus.

[12] *δύναται λέγειν καὶ περὶ τῶν Δυνατῶν, τοῦ τε, ὃ Διοδώρειον λέγεται, ὃ ἢ ἔστιν ἢ ἔσται τὸ γὰρ ἢ ὂν ἢ ἐσόμενον πάντως δυνατὸν μόνον ἐκεῖνος ἐτίθετο. τὸ γὰρ ἐμὲ ἐν Κορίνθῳ γενέσθαι δυνατὸν κατ' αὐτόν, εἰ εἴην ἐν Κορίνθῳ, ἢ εἰ πάντως μέλλοιμι ἔσεσθαι εἰ δὲ μὴ γενοίμην, οὐδὲ δυνατὸν ἦν καὶ τὸ τὸ παιδίον γενέσθαι γραμματικὸν δυνατόν, εἰ πάντως ἔσοιτο. οὗ εἰς κατασκευὴν καὶ ὁ Κυριεύων ἠρώτηται λόγος [ὁ] ὑπὸ τοῦ Διοδώρου.*

Even if Aristotle's text never explicitly quote Diodorus, it seems clear that there is a disagreement between the young Megaric and the older Aristotle. Furthermore, it is certain that Diodorus reconsidered some arguments by Eubulides, *e.g.*, the Veiled and the Horned paradoxes (D.L. 2, 111). A solution of the first is given by Arist. *S.E.* 24, 179a 30 - 179b 5, while the second is quoted in *S.E.* 24, 178a 29-30. Nevertheless, a link between Eubulides and Diodorus concerns the topic of conventionality and ambiguity of language. For instance, according to Steph. *in Int.* 9, 20-24, Diodorus did not admit the thesis that names are *per natura* and, therefore he decided to name his daughters μέν and δέ, i.e., by the conjunctive particles of Greek. In this way, Diodorus gave an odd demonstration that names are imposed by convention.

This chapter provides an overview of some relevant doctrines of Diodorus Cronus and a detailed study of the *κυριεύων λόγος*. Section 1.1 is dedicated to Diodorus' atomism, and we analyse some fragments regarding his physics. In particular, we deal with the ἀμερῆ doctrine, by comparing this with other atomistic views such as the Epicurean and the Democritean ones. Then, we present the so-called *Soritic* and *Isomorphic* arguments, which are used by Diodorus as strategies to deny that every motion is like a flux. Section 1.2 is about Diodorus' doctrines on language. This section deals with two themes: linguistic conventionalism and an odd doctrine on tenses. We will see that Diodorus' thesis about speech is based on a diatribe already started, and first traceable, in Plato, *Cratylus* and in a fragment by Parmenides. In relation to tenses, a fragment from Sextus, *Adversus Mathematicos* will elucidate an ancient schema. The core of the first chapter is in section 1.3 in which we will focus on the *κυριεύων λόγος*. In order to investigate accurately the ancient Master Argument, we need a preliminary examination of the history of modal notions. These notions are involved in Diodorus' Master Argument, and are probably derived from a debate about capacities. Therefore, at first, we will discuss, in section 1.3.1.1, the development of the doctrines of capacities; second, in section 1.3.1.2, we review three Hellenistic perspectives on modalities. The aim of section 1.3.1.3 is to eliminate any ambiguity from what we mean by Diodorus' determinism, arguing that it

differs from fatalism. We will make it clear that Diodorus' doctrine does not refer to any causal process, such that facts have to be in a given way – as is the case with fatalism. Following the works of some ancient commentators, namely Simplicius and Boethius, in section 1.3.1.4 we point out that some caution is required to translate δυνατὸν and δύναμις, so that we do not end up conflating the two different terms; later, we show that Diodorus' and Aristotle's accounts of modalities are comparable. At this point, in section 1.3.2, we will have achieved the survey of the prerequisite tools required to investigate every sentence of the *κυριεύων λόγος*, and so we will opt to use philological methods and to introduce a tensed formalism for the Master Argument.

1.1 Atomism and the arguments against motion

Diodorus' physics is built on a conception of capacity opposed to Aristotle's view in *Metaph.* 9, and develops into a particular kind of atomism, and, as a consequence, in some arguments against motion. Here we will briefly review Diodorus' physics, while Diodorus' modalities will be explained in section 1.3.1.2.

The two main themes of physics are the *Soritic* and the *Isomorphic* arguments. Based on two different strategies, they are both crucial in deriving Diodorus thesis against motion.[13] First, we will present the ἀμερῆ doctrine, then the Soritic argument, and finally the Isomorphismus.

Following Eus., *Praep. ev.* 14, 23, 4, Dionysius of Alexandria mentions different kinds of atomism. Dionysius, after discussing both Epicurus and Democritus atomism, turns his attention to Diodorus Cronus:

> But others change the name of the atoms, and say that they are bodies which have no parts, but are themselves parts of the universe, out

[13]Some objection may arise during our explanation, *e.g.* Diodorus physical arguments are dialectical theses only: they are *argumenti ad absurdum*, as the Eleatics. We believe that it is possible to reply this objection according to ancient texts.

> of which in their indivisible state all things are composed, and into which they are resolved. And they say that it was Diodorus who invented the name (*ἀμερῆ*) of these bodies without parts [Gifford, 1903, 772].[14]

The passage above refers to a specific kind of atomism; perhaps, an appropriate name for it would be *amerism*. In fact, Diodorus defines those partless bodies as *τὰ ἀμερῆ*. Any substance in the universe is composed of minimal quanta. According to Stob. *Ecl.* 1, 10, 16a, Diodorus' *τὰ ἀμερῆ* are:

> Partless bodies, said minimals: unlimited in number, but delimited in magnitude.[15]

Diodorus calls the physical principles *ἐλάχιστα*, since they have the smallest existent magnitude. Hence, they all possess the same dimension. Furthermore, they are *ἀμερῆ*, partless, and numerically infinite. So, even if Giannantoni [1980, 127] does not include, *strictu sensu*, Diodorus among the atomists, however, he considers the concept of *ἀμέρεια* a key idea in the history of ancient atomism.

According to S.E. *P.H.* 3, 32, 1-6 there is evidence that *τὰ ἀμερῆ* are to be considered physical elements. Sextus includes Diodorus among those who supported a physical conception of the minimal elements of the world:

> Democritus and Epicurus, atoms; Anaxagoras of Clazomenae, uniform stuffs; Diodorus (surnamed Cronos), minimal and partless bodies; Heraclides of Pontus and Asclepiades of Bithynia, seamless masses [Annas and Barnes, 2000].[16]

[14] *οἱ δὲ τὰς ἀτόμους μετονομάσαντες ἀμερῆ φασιν εἶναι σώματα, τοῦ παντὸς μέρη, ἐξ ὧν ἀδιαιρέτων ὄντων συντίθεται τὰ πάντα καὶ εἰς ἃ διαλύεται. καὶ τούτων φασὶ τῶν ἀμερῶν ὀνοματοποιὸν Διόδωρον γεγονέναι.*

[15] *Διόδωρος ἐπίκλην Κρόνος τὰ ἀμερῆ σώματα ἄπειρα, τὰ δ' αὐτὰ λεγόμενα καὶ ἐλάχιστα ἄπειρα μὲν κατ' ἀριθμόν. ὡρισμένα δὲ κατὰ μέγεθος.*

[16] *Δημόκριτος δὲ καὶ Ἐπίκουρος ἀτόμους, Ἀναξαγόρας δὲ ὁ Κλαζομένιος ὁμοιομερείας, Διόδωρος δὲ ὁ ἐπικληθεὶς Κρόνος ἐλάχιστα καὶ ἀμερῆ σώματα, Ἡρακλείδης δὲ ὁ Ποντικὸς καὶ Ἀσκληπιάδης ὁ Βιθυνὸς ἀνάρμους ὄγκους.*

Some scholars do not consider *τὰ ἀμερῆ* to be physical particles.[17] However, both the discussion on the different kinds of atomism and Sextus' discussion of physical principles, thwarts the above mentioned interpretation. Of course, the philosophers listed by Sextus supported different doctrines. Therefore, a brief comparison of the best known atomistic models – those of Democritus and Epicurus with Diodorus' – will be helpful.

Let us start by considering Democritus' account. According to D.L. 9, 44, atoms are not the unique principle. Principles are atoms and void, *i.e. ἀτόμους καὶ κενόν*. On the other hand, Diodorus seems to deny the existence of voids,[18] as suggested by the following passage referring to the theme of connection, to the judgement on a proposition as a valid implication. S.E. *M.* 8, 333, reports that:

> According to Diodorus, as it begins with the falsehood 'motion exists' and ends in the falsehood 'void exists', it will in itself be true [Bury, 1967, 413].[19]

As reported in D.L. 9, 44, 5, Diodorus claims atoms, as worlds, are infinite in magnitude and number, *i.e. κατὰ μέγεθος καὶ πλῆθος*. So, there are undeniable differences between Diodorus' and Democritus' accounts:

1. Diodorus does not admit voids, while Democritus surely does.
2. Democritus wants *ἀτόμους* to be infinite in number, as does Diodorus for *τὰ ἀμερῆ*. However, only in the latter account those *corpuscula* are minimal.

[17] The interpretation of *τὰ ἀμερῆ* as dialectical hypothesis prevails in the old historiography, *e.g.* in Zeller [1877, 270], and Henne [1843, 176-177]. But, the thesis of *τὰ ἀμερῆ* as dialectical hypothesis is sometimes moderately maintained in recent works too, *e.g.* in Verde [2013, 218-219]. On the contrary, according to Sedley [1999], the topic about *τὰ ἀμερῆ* is properly physical and metaphysical, and it concerns a typical account of the Hellenistic period. Further, the 'neglected' fragment analysed in Denyer [2002] may refer to a vague astronomical interest of Diodorus, confirming, in some way, a kind of physical account for the Megaric Diodorus.

[18] Denyer [1981a, 40-41, 43-45] considers the quotation above as a *verbal Eleaticism* of Diodorus, a kind of dialectical reasoning aimed to the defence of *being*, in its fixity and unalterability. Thus, according to Denyer, the fragment refers *strictu sensu* to physics only secondarily.

[19] *κατὰ δὲ Διόδωρον ἀρχόμενον ἀπὸ ψεύδους τοῦ "ἔστι κίνησις" καὶ λῆγον ἐπὶ ψεῦδος τὸ "ἔστι κενόν" αὐτὸ μὲν ἔσται ἀληθές*.

3. Democritus' theory includes cosmological aspects: he discusses specific *ἀτόμους* that are infinite in magnitude. On the other hand, to the best of our knowledge, there are no fragments to date referring to some *strictu sensu* cosmological theory of Diodorus Cronus. A general interest for astronomy, as noticed in [Denyer, 2002], at most, may be ascertained according to Eust. *Ad Hom. Od.* 28, 46 – 29, 2.

With reference to Epicurus, in S.E. *M.* 8, 332 the statement 'the void exist' is said to be true. However, the discussion there focuses on logical aspects. More relevant evidence as to the existence of void in Epicurus are in the *Letter to Herodotus*, D.L. 10, 39. Sextus Empiricus discusses the consistency between the partless bodies hypothesis and motion. In fact, both Diodorus and Epicurus support some versions of atomism, but Diodorus denies motion – at least as progressive movement – while Epicurus does not. The next fragment is crucial. Referring to a movement in a simple or partless place,[20] S.E. *M.* 10, 142, 3 – 144, 1, states:

> Those who, like Epicurus, have assumed that all things are reducible to indivisibles involves themselves in more formidable difficulties, – such as, firstly, the fact that motion will not exist, as Diodorus showed when treating of indivisible places and bodies. For the indivisible body contained in the indivisible place and fills it up. And again: the body situated in the second place does not move, for it has moved already. But if the moving object neither moves in the first place – inasmuch as it exists in the first – nor yet in the second, and besides

[20] Arist. *Ph.* 4 deals extensively with the concept of place. According to *Ph.* 4, 4, 211a 12-17, the notion of *τόπος* is the condition of local motion for Aristotle. Further, the notion of place is made conceivable by the concept of movement. Modern science takes the notion of place as spatial extension, while Aristotle explores different paths. *E.g.* in *Ph.* 4, 4, 212a, the place is 'the boundary of the containing body at which it is in contact with the contained body' [Ross, 1936].

> these no third place is conceived, then that which is said to move does not move [Bury, 1968, 283].[21]

The passage above highlights a link between the existence of *τὰ ἀμερῆ* and the refusal of movement. This argument is surely about the atomism of place and matter. Later, our claim will be that it implies a full isomorphismus of place, matter and time atoms. So, the thesis about *τὰ ἀμερῆ* is Diodorus' starting point in order to contrive a schema for arguments against motion. This view is one of the most heterodox instances of ancient atomists. For now, we set aside this topic and turn to the Soritic argument proposed by Diodorus.

What is a *sorites*? The term is connotative of the so-defined Soritic argument against motion set forth by Diodorus, taken as an argument schema, rather than one particular instantiation of it. The Sorites paradox was proposed for the first time by the Megaric dialectician Eubulides of Miletus.[22] As explained in Copi et al. [2014, 274-276, 279-280] the Sorites uses a chain of categorical syllogisms, the enthymeme. Namely, the conclusion of the first syllogism is the premiss of the second, and so on up to the conclusion.[23]

Let us examine the passage S.E. *M.* 10, 113-117, subdividing the text into three sections. Here is the first, S.E. *M.* 10, 113-114:

[21] *οἱ δὲ πάντα εἰς ἀμερῆ καταλήγειν ὑπειληφότες, ὡς οἱ περὶ τὸν Ἐπίκουρον, νεανικωτέραις μᾶλλον ἐνέχονται ταῖς ἀπορίαις, καὶ πρῶτον, ὅτι οὐκ ἔσται κίνησις, ὡς ὁ Διόδωρος ἐδίδασκε τῶν ἀμερῶν ἐχόμενος τόπων τε καὶ σωμάτων. τὸ γὰρ ἐν τῷ πρώτῳ ἀμερεῖ τόπῳ περιεχόμενον ἀμερὲς σῶμα οὐ κινεῖται περιείχετο γὰρ ἐν τῷ ἀμερεῖ τόπῳ καὶ ἐκπεπλη ρώκει τοῦτον. καὶ πάλιν τὸ ἐν τῷ δευτέρῳ ὑποκείμενον οὐ κινεῖται κεκίνηται γὰρ ἤδη. εἰ δὲ μήτε ἐν τῷ πρώτῳ τὸ κινούμενον κινεῖται ἐφ' ὅσον ἔστιν ἐν τῷ πρώτῳ, μήτ' ἐν τῷ δευτέρῳ, παρὰ δὲ ταῦτα τρίτος οὐκ ἐπινοεῖται τόπος, οὐ κινεῖται τὸ λεγόμενον κινεῖσθαι.*

[22] Eubulides preceded Diodorus by about thirty years. He invented the seven Megaric paradoxes, the *Liar*, the *Hidden*, the *Electra*, the *Veiled*, the *Sorites*, the *Horned*, the *Bald* (*cf.*, D.L. 2, 108, 4-6). It seems that a direct line links Euclides of Megara (pioneer of the Megaric group) to Eubulides (*cf.*, D.L. 2, 108, 2-3), and Eubulides to Diodorus via Apollonius Cronus. According to D.L. 2, 111, 1-4 and Str. 14, 2, 21, Diodorus was nicknamed Cronus by Apollonius.

[23] Here is Cic. *Luc.* 49 introduction to the Sorites. Cicero accuses the method that 'cum aliquid minutatim et gradatim additur aut demitur. Soritas hoc vocant, qui acervum efficiunt uno addito grano. Vitiosum sane et captiosum genus – By minute steps of gradual addition or withdrawal. They call this class of arguments soritae because by adding a single grain at a time they make a heap. It is certainly an erroneous and captious kind of argument. [Rackham, 1933]'.

> Motion being twofold, – the one sort that of the major portion, the second sort absolute, – and that of the major portion being the sort in which while most parts of the body are in motion a few are at rest, and the absolute sort that in which all the part of the body are in motion, – it seems that of these two motions that of the major portion precedes the absolute kind. For in order that a thing should move absolutely, – that is, as a whole wholly, – it must first be conceived as moving in respect of its major portion; just us, in order that a man may become completely grey-headed he must first become grey as to the major part, and in order that a complete heap may be obtained, the major part of a heap must first be formed; in much the same way motion as to the major part must precede absolute motion; for absolute motion is an intensification of that of the major part [Bury, 1968, 267-269].[24]

In considering motion, Diodorus distinguishes at first the major portion motion (*κατ᾽ ἐπικράτειάν*) from absolute motion (*κατ᾽ εἰλικρίνειαν*), namely pure or total motion. Thereafter, Diodurus explains the meaning of those two kinds of motion. Again, he considers the matter of a thing that has to move as a conglomerate of particles.

Then, we may notice in the text some influence from Eubulides and his paradoxical arguments, the Bald (the Grey-Headed in Diodorus version) and the Sorites (*i.e.* the Heap), which is an expedient way of introducing Diodorus' argument. Absolute motion must be thought of as an intensification of the major

[24] *διττῆς δὲ οὔσης κινήσεως, μιᾶς μὲν τῆς κατ᾽ ἐπικράτειαν, δευτέρας δὲ τῆς κατ᾽ εἰλικρίνειαν, καὶ κατ᾽ ἐπικράτειαν μὲν ὑπαρχούσης ἐφ᾽ ἧς τῶν πλειόνων κινουμένων μερῶν τοῦ σώματος ὀλίγα ἠρεμεῖ, κατ᾽ εἰλικρίνειαν δὲ ἐφ᾽ ἧς πάντα κινεῖται τὰ τοῦ σώματος μέρη, δοκεῖ τούτων τῶν δυεῖν κινήσεων ἡ κατ᾽ ἐπικράτειαν προηγεῖσθαι τῆς κατ᾽ εἰλικρίνειαν. ἵνα γάρ τι εἰλικρινῶς κινηθῇ, τουτέστιν ὅλον δι᾽ ὅλου, πρότερον ὀφείλει νοεῖσθαι κατ᾽ ἐπικράτειαν κινούμενον, ὃν τρόπον ἵνα τις κατ᾽ εἰλικρίνειαν γένηται πολιός, ὀφείλει κατ᾽ ἐπικράτειαν πεπολιῶσθαι, καὶ ἵνα τις κατ᾽ εἰλικρίνειαν ληφθῇ σωρός, ὀφείλει κατ᾽ἐπικράτειαν γεγονέναι σωρός κατὰ τὸν ὅμοιον τρόπον ἡγεῖσθαι δεῖ τῆς κατ᾽ εἰλικρίνειαν κινήσεως τὴν κατ᾽ ἐπικράτειαν ἐπίτασις γὰρ τῆς κατ᾽ ἐπικράτειάν ἐστιν ἡ κατ᾽ εἰλικρίνειαν.*

part motion. So, Diodorus gives a qualitative explanation of motion. A quantitative specification of such a concept would define it as an extension of motion to the whole set of particles.

According to S.E. *M.* 10, 115, the next passage reports the main claim of Diodorus' argument:

> But there does not exist any motion of the major part, as we shall establish; neither, then, will absolute motion exist. – For let us assume the existence of a body composed of three indivisible parts, two being in motion and one motionless; for this is what motion of the major part demands [Bury, 1968, 269].[25]

Diodorus is ruling out absolute motion, namely *εἰλικρίνειαν κίνησις*, as a consequence of avoiding any previous motion of the major part, that is *ἐπικράτειαν κίνησις*. Diodorus' argument is built on the assumption that a body's structure is a conglomerate of parts. So, the motion of the major part is both the minimal condition to obtain the body motion, and, at the same time, the necessary condition for absolute motion. Subsequently, Sextus Empiricus refers to Diodorus' proof: the next passage shows the Sorites of Diodorus. The composition of a body by three partless particles, two being in motion and one motionless, is already on hand as an assumption at S.E. *M.* 10, 115. Finally, according to S.E. *M.* 10, 116-117, Diodorus' soritic proof of the inconsistency of the hypothesis of motion is:

> If, then, we were to add to this body a fourth indivisible which is motionless, there will again be motion. For if the body composed from three indivisibles, two in motion and one motionless, moves, it will also move when a fourth indivisible is added; for the three indivisibles, with which he was moving before, are stronger than the

[25] *οὐχὶ δέ γε ἔστι τις κατ᾽ἐπικράτειαν κίνησις, ὡς παραστήσομεν τοίνυν οὐδ᾽ἡ κατ᾽ εἰλικρίνειαν γενήσεται. ὑποκείσθω γὰρ ἐκ τρι ὧν ἀμερῶν συνεστὼς σῶμα, δυεῖν μὲν κινουμένων ἑνὸς δὲ ἀκινητίζοντος τοῦτο γὰρ ἡ κατ᾽ ἐπικράτειαν ἀπαιτεῖ κίνησις.*

> one indivisible which is added. But if the body composed of four indivisibles moves, that composed of five will also move; for the four indivisibles, with which it was moving before, are stronger than the added indivisible. And if that which is composed of five moves, it will certainly move also when a sixth indivisible is added, the five being stronger than the one. And in this way Diodorus proceeds up to 10,000 [*μυρίων*] indivisibles, by way of proving that motion of the major part is non-existent; for it is absurd, he says, to assert that, a body moves as to its major part when it has 9,998 [*ἐνακισχίλια ἐνακόσια ἐνενήκοντα ὀκτὼ*] of its indivisibles motionless and 2 [δύο] only in motion. So that nothing moves as to its major part. And if so, neither does anything move absolutely; from which it follows that nothing moves [Bury, 1968, 269].[26]

In order to contextualize the argument, we notice that Diodorus has two famous predecessors in the Megaric and dialectical tradition, Zeno Eleates and Eubulides. Importantly, Diodorus uses his sorites to pursue different aims rather than the other two philosophers. Their arguments are very similar, and have identical form to Eubulides paradox.[27]

[26] *οὐκοῦν εἰ προσθείημεν τέταρτον ἀμερὲς ἀκινητίζον τούτῳ τῷ σώματι, πάλιν γενήσεται κίνησις. εἴπερ γὰρ τὸ ἐκ τριῶν ἀμερῶν συγκεί μενον σῶμα, δυεῖν μὲν κινουμένων, ἑνὸς δὲ ἀκινη τίζοντος, κινεῖται, καὶ τετάρτου προστεθέντος ἀμεροῦς κινήσεται ἰσχυρότερα γὰρ τὰ τρί᾽ ἀμερῆ, μεθ᾽ ὧν πρότερον ἐκινεῖτο, τοῦ προστεθέντος ἑνὸς ἀμεροῦς. ἀλλ᾽ εἴπερ τὸ ἐκ τεσσάρων ἀμερῶν συγκείμενον σῶμα κινεῖται, κινήσεται καὶ τὸ ἐκ πέντε ἰσχυρό τερα γάρ ἐστι τὰ τέσσαρ᾽ ἀμερῆ, μεθ᾽ ὧν πρότερον ἐκινεῖτο, τοῦ προστεθέντος ἀμεροῦς. καὶ εἰ τὸ ἐκ τῶν πέντε συγκείμενον κινεῖται, πάντως καὶ ἕκτου προσελθόντος ἀμεροῦς κινήσεται, ἰσχυροτέρων ὄντων τῶν πέντε παρὰ τὸ ἕν. καὶ οὕτω μέχρι μυρίων ἀμερῶν προέρχεται ὁ Διόδωρος δεικνύς, ὅτι ἀνυπόστατός ἐστιν ἡ κατ᾽ ἐπικράτειαν κίνησις ἄτοπον γάρ, φησί, τὸ λέγειν κατ᾽ ἐπικράτειαν κινεῖσθαι σῶμα ἐφ᾽ οὗ ἐνακισχίλια ἐνακόσια ἐνενήκοντα ὀκτὼ ἀκινητίζει ἀμερῆ καὶ δύο μόνον κινεῖται. ὥστε οὐδὲν κατ᾽ ἐπικράτειαν κινεῖται. εἰ δὲ τοῦτο, οὐδὲ κατ᾽ εἰλικρίνειαν, ᾧ ἕπεται τὸ μηδὲν κινεῖσθαι.*

[27] Zeno Eleates denies movement *sine exceptione*. Zeno admits as true only Parmenides *being*, characterised by immobility as one of its feature. Zeno's soritic argument may be found in Arist. *Ph.* 7, 5, 250a 20-25. By his sorites, both in *Bald* and in *Heap paradox* (*cf.*, Cic. *Luc.* 49; S.E. *M.* 1, 69; D.L. 7, 82; Arist. *S.E.* 25, 179b, 34-37), Eubulides takes position against becoming processes, and their conceptualization. Eubulides denies *tout court* movement, while Diodorus rules out only specific motion based on a *dense* flowing. As a matter of fact, Diodorus admits movements by *discrete* times and places.

Let us turn our attention on S.E. *M.* 10, 116-117. In the first section of the passage, Diodorus considers the case of a body composed by more particles, which for their major part are in motion. Therefore, by adding only a particle to the whole conglomerate in motion, time by time, we again obtain the motion of a composite body. In fact, the whole conglomerate of particles is in motion (by the motion of his major part). Then, how a single minimal particle could be able to stop the process of motion? Diodorus' hypothesis is deliberately built on an *absurdum* scheme: the ending point is the paradox of a body as a conglomerate of $10,000$ particles, of which $9,998$ are motionless, and only 2 are in motion.[28] And yet, from the premises of the argument we obtain a body in motion in any case. The same paradoxical form is manifest starting from two single grains. Two grains are definitely not a heap: by adding a further single grain, time by time and little by little, we never obtain a heap. That is the reason why Diodorus concludes his *sorites against motion* by denying the process of movement. In fact, if motion of the major part is not possible, then nothing can move absolutely.

Now, let us consider *isomorphismus* again. The correspondence of place, matter and time, is not a unique feature of Diodorus' account. Even if some philosophers might not agree with such a view,[29] there are different examples of isomorphismus both in the sense of structural indivisibility and divisibility.[30]

[28] The Greek word for $10,000$ unit is *μυρίοι*. In common parlance, the number is used to indicate a very large quantity. So, the term *μυρίοι* has a qualitative value as well as a quantitative meaning. It means both *strictu sensu* $10,000$ and at the same time 'greatly' or 'very much'. S.E. *M.* 7, 418-421 explains very well the value number of $10,000$, considered in relation to the *sorites*. Moreover, in Sextus' passage, there is a comparison between the propositions '50 is few' and '$10,000$ is few'. In fact, by the Sorites reasoning, Sextus may state that the *apprehensive* [*καταληπτική*] appearance '50 is few' will become equal to the *non-apprehensive* appearance '$10,000$ is few'. And, of course, it is a paradox. Even more, if we consider the suggestive power of *μυρίοι* for the Greeks. In addition to confirming how soritical reasoning works, Sextus relates '$10,000$ is few' to the Stoic non-apprehensive appearance. Obviously, admitting a conglomerate of $10,000$ particle in motion, when only 2 particles having motion and the rest are motionless (that is how the Sorites works) may also be considered as a non-apprehensive fact. Therefore, Diodorus' step is denying this possibility, *i.e.* motion of the major part, by his sorites. And so, as a consequence, Diodorus rules out absolute motion, too. *Cf.*, [Sillitti, 1977, 78], and [Wieland, 1992], for the Sorites and the number $10,000$ in Hellenistic culture.

[29] *E.g.*, Strato of Lampsacus theorizes minute interstitial pockets of void causing matter discontinuity and, at the same time, divisibility ad infinitum of the structure. This should be the case of places where matter is located also.

[30] In the case of isomorphismus of structural divisibility, we may refer to *e.g.* the Stoic account of physics: there is no void, and matter is infinitely divisible.

It is important to consider that Diodorus inherited his background both from Eleatic and Socratic philosophy. So, Zeno's arguments, in particular the *arrow* (*see* Arist. *Ph.* 6, 9, 239b 5 - 240b 8), are closely related with arguments against motion in Diodorus. Sedley [1999, 356] explains in Pl. *Prm.* 138d 2 - e 7, and Arist. *Ph.* 6, the context of Diodorus' arguments, and the most relevant reports are definitely in Sextus Empiricus. Further, as suggested by Sorabji [1983, 17], Diodorus evaluates the objections of Arist. *Ph.* 6, then, cunningly replies by putting forward his original atomistic doctrine and its consequences on motion. One of the most striking differences between Aristotle and Diodorus Cronus is in their views of physical reality. Aristotle's model is *dense*, Diodorus' is *discrete*.[31]

According to Arist. *Ph.* 4, 11, 219a 10-13, if motion exists, it exists as an action in progress. Therefore, every movement is from an initial to an ending point:[32]

> Since anything that moves moves from a 'here' to a 'there', and magnitude as such is continuous [συνεχές], movement is dependent on magnitude; for it is because magnitude is continuous that movement is so also, and because movement is continuous so is time [Wicksteed and Cornford, 1934].[33]

Therefore, in some way, Aristotle sustained that magnitude, movement

[31] This topic is important in ancient physics. In fact, Aristotle does not use the term density. Aristotle speaks of continuity, *συνεχὲς*, mostly in a physical rather than mathematical context. Wieland [1992] points out that Aristotle discusses the fundamental structure of the world. On this approach, continuity is a physical feature evident *per se*. In modern terms, one would say that in his account the physical world is dense. Density is certainly admissible for Zeno too. Actually, several of Zeno paradoxes stem from the notion of density.

[32] This is only in principle so. Aristotle is aware of the difficulties involved in such a view (*see*, Arist. *Ph.* 236a 14 and *ff.*). However Aristotle starts from bare natural facts, while Diodorus, following the Eleatic approach, pursues the falsity of appearance.

[33] *ἐπεὶ δὲ τὸ κινούμενον κινεῖται ἔκ τινος εἴς τι καὶ πᾶν μέγεθος συνεχές, ἀκολουθεῖ τῷ μεγέθει ἡ κίνησις διὰ γὰρ τὸ τὸ μέγεθος εἶναι συνεχὲς καὶ ἡ κίνησίς ἐστιν συνεχής, διὰ δὲ τὴν κίνησιν ὁ χρόνος.*

and time[34] are physically continuous, since they are in a kind of mutual analogy of structure.

Let us now turn to Diodorus approach to time place and matter. We may find in S.E. *M.* 10, 85, 1 – 86, 8, the reason why Diodorus claims that nothing is moving, but something moved – namely, the composition of matter from partless and minimal size bodies (*ἀμερῆ*). In the following passage we may also find a reason for a correspondence between place and matter structure in Diodorus:

> Another weighty argument for the non-existence of motion is adduced by Diodorus Cronus, by means of which he establishes that not a single thing is in motion, but has been in motion. And the fact that nothing is in motion follows from his assumptions of indivisibles. For the indivisible bodies [*ἀμερὲς σῶμα*] must be contained in an indivisible place [*ἐν ἀμερεῖ τόπῳ*] and therefore must not move either in it (for it fills it up, but a thing which is to move must have a larger place) or in the place in which it is not; for as yet it is not in this place so as to be moved therein; consequently it is not in motion. But, according to reason, it has been in motion; for that which was formerly observed in this place is now observed in another place, which would not have occurred if it had not been moved [Bury, 1968].[35]

[34] In Aristotle, time is a metaphysical category, conferring reality different meanings. To avoid misunderstandings, we observe that the notion possesses a twofold character, described as flowing, and enumerating. Time as *flowing* is comparable with Diodorus' account, and it is considered in relation with the infinite divisibility of local motion, *cf.* Arist. *Ph.* 4, 11, 219a 10-15: time is physically continuous, since it is never interrupted or suspended. Further, it is also dense, in a mathematical sense, since it is infinitely and proportionally divisible with respect to matter and place. Time as *enumerating* traces back to the same section of *Physics*. According to Arist. *Ph.* 4, 11, 219b 1-2 is: *τοῦτο γάρ ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον* – 'time – is just this – number of motion in respect of *before* and *after*'. And in 219b 5-8 is: *ἐπεὶ δ' ἀριθμός ἐστι διχῶς (καὶ γὰρ τὸ ἀριθμούμενον καὶ τὸ ἀριθμητὸν ἀριθμὸν λέγομεν, καὶ ᾧ ἀριθμοῦμεν), ὁ δὴ χρόνος ἐστὶν τὸ ἀριθμούμενον καὶ οὐχ ᾧ ἀριθμοῦμεν.* – 'Number, we must note, is used in two senses, both of what is counted or the countable and also of that with which we count. Time obviously is what is counted, not that with which we count' [Ross, 1936]. In the latter case, time has the same form of number, *i.e.* it is mathematically discrete, although it is physically continuous. In this work, in comparing Aristotle and Diodorus' accounts, we take Aristotle's idea of time as dense flowing, while Diodorus' as discrete.

[35] *Κομίζεται δὲ καὶ ἄλλη τις ἐμβριθὴς ὑπόμνησις εἰς τὸ μὴ εἶναι κίνησιν ὑπὸ Διοδώρου τοῦ Κρόνου, δι' ἧς παρίστησιν, ὅτι κινεῖται μὲν οὐδὲ ἕν, κεκίνηται δέ. καὶ μὴ κινεῖσθαι μέν, τοῦτο ἀκόλουθόν ἐστι ταῖς κατ' αὐτὸν τῶν ἀμερῶν ὑποθέσεσιν. τὸ γὰρ ἀμερὲς σῶμα ὀφείλει ἐν*

Bury translated the *ἀμερὲς σῶμα* as 'indivisible bodies'. Since it is important to distinguish Diodorus' different approach to atomism, we would rather use 'partless bodies', which is more adequate to the original Greek text. Further, it refers directly to Diodorus' doctrine, since *ἀμερὲς* means without-parts, *i.e.* partless.[36]

On the one hand, traditional atomism considers 'uncuttable' particles: *ἀτόμους*. Whereas Diodorus prefers referring to 'uncuttable' particles which, as a matter of fact, are minimal and partless, *ἀμερὲς*. Namely, *ἀτόμους* are in principle uncuttable, while *ἀμερὲς* are minimal and singulars. They could never be infinite in magnitude, whilst atoms may. Moreover, even if they are uncuttable *ἀτόμους* may present particles. But *ἀμερὲς* are uncuttable since by their *nature* they possess no parts.

Today we can speak of the notion of *space*. However, before the modern age, this notion of space as a void, homogeneous portion of universe, arbitrarily chosen, where matter behaves coherently was completely different. *ἀμερὲς τόπος* are always non empty: void does not exist. Moreover, the place may affect the behavior of matter. Also, space is not related to matter. But according to S.E. *M.* 10, 85-86 *ἀμερὲς τόπος* are.

Diodorus takes place to be *partless and simple* just like bodies are partless and simple. Partless and simple place is a limit 'container' of simple body, defined as contiguous to the partless body it contains. Therefore, partless and simple place does not allow any movement of the particle.

Diodorus accepts only minimal places and partless bodies of dimension appropriate to the former. There is remarkable difference between Diodorus' and Aristotle's accounts. White [1992, 263] notes that 'either (a) that Diodorus, in explicit opposition to Aristotle, decomposed *kinesis* into *kinemata*, or (b) that

ἀμερεῖ τόπῳ περιέχεσθαι, καὶ διὰ τοῦτο μήτε ἐν αὐτῷ κινεῖται (ἐκπεπλήρωκε γὰρ αὐτόν, δεῖ δὲ τόπον ἔχειν μείζονα τὸ κινησόμενον), μήτε ἐν ᾧ μὴ ἔστιν οὕπω γὰρ ἔστιν ἐν ἐκείνῳ, ἵνα καὶ ἐν αὐτῷ κινηθῇ. ὥστε οὐδὲ κινεῖται. κεκίνηται δὲ κατὰ λόγον τὸ γὰρ πρότερον ἐν τῷδε τῷ τόπῳ θεωρούμενον, τοῦτο ἐν ἑτέρῳ νῦν θεωρεῖται τόπῳ.

[36] Furley [1967, 131-135] confirms our translation and interpretation of *ἀμερὲς* as 'partless'.

he connected this doctrine to his doctrine of minimal and partless quanta of spatial magnitude'.

Diodorus refuses movement as a process *in acto*, not *in se*. As seen in Sextus Empiricus' text, he does this because he recognises that what was seen in one place, now is seen in another place. In fact, Diodorus' real target is to highlight the paradox of motion, as an evolving process. Diodorus tries to set up new categories to frame a notion of movement as a succession of static and discontinuous stages. Certainly, when something moves, it moves from a position to another. However, according to the assumptions, it is rationally absurd to consider motion as a becoming process.

So far, we discussed the correspondence between place and the structure of matter. Does Diodorus physical isomorphismus work correctly for time also?

Following Denyer [1981a, 38] we can consider the passage in Arist. *Ph.* 6, 3, 234a 3-25, where Aristotle proves the indivisibility of the present moment: if the present could be divided in past and future, then it would never be present.

> If it [the *νῦν*] is once shown that is is essentially of this character and one and the same, it will at once be evident also that it is indivisible. Now the present that is the extremity of both times must be one and the same: for if each extremity were different, the one could not be in succession to the other, because nothing continuous can be composed of things having no parts: and if the one is apart from the other, there will be time intermediate between them, because everything continuous is such that that there is something intermediate between its limits and described by the same name as itself. But if the intermediate thing is time, it will be divisible: for all time has been shown to be divisible. Thus on this assumption the present is divisible. But if the present is divisible, there will be part of the past in the future and part of the future in the past: for past time will be marked off from future time at the actual point of division. Also

> the present will be a present not in the proper sense but in virtue of something else: for the division which yields it will not be a division proper. Furthermore, there will be a part of the present that is past and a part that is future, and it will not always be the same part that is past or future: in fact one and the same present will not be simultaneous: for the time may be divided at many points. If, therefore, the present cannot possibly have these characteristics, it follows that it must be the same present that belongs to each of the two times. But if this is so it is evident that the present is also indivisible: for if it is divisible it will be involved in the same implications as before. It is clear, then, from what has been said that time contains something indivisible, and this is what we call a present.[37]

Therefore, the present, the *νῦν* (namely, the *now*), is indivisible. On the other hand, it is impossible to think of the present as an interval. So, at least two possibilities are available: the present is a kind of limit or it is a kind of atom.

In Aristotle, the present time is a limit. One may interpret the present, as a zipper joining between past and future: as an intant. However, in Diodorus, the present time is a partless time: a quantum, an *ἀμερές* time. What does this point of view involve? How to get it? A strategy is from Sextus Empiricus.

According to S.E. *M.* 10, 119-120, Diodorus Cronus expressed his doctrine as in the following passage:

[37] *τοῦτο δὲ ἐὰν δειχθῇ ὅτι τοιοῦτόν ἐστιν [καθ' αὑτὸ] καὶ ταὐτόν, ἅμα φανερὸν ἔσται καὶ ὅτι ἀδιαίρε τον. ἀνάγκη δὴ τὸ αὐτὸ εἶναι τὸ νῦν τὸ ἔσχατον ἀμφοτέρων τῶν χρόνων εἰ γὰρ ἕτερον, ἐφεξῆς μὲν οὐκ ἂν εἴη θάτερον θατέρῳ διὰ τὸ μὴ εἶναι συνεχὲς ἐξ ἀμερῶν, εἰ δὲ χωρὶς ἑκάτερον, μεταξὺ ἔσται χρόνος πᾶν γὰρ τὸ συνεχὲς τοιοῦτον ὥστ' εἶναί τι συνώνυμον μεταξὺ τῶν περάτων. ἀλλὰ μὴν εἰ χρόνος τὸ μεταξύ, διαιρετὸν ἔσται πᾶς γὰρ χρόνος δέδεικται ὅτι διαιρετός. ὥστε διαιρετὸν τὸ νῦν. εἰ δὲ διαιρετὸν τὸ νῦν, ἔσται τι τοῦ γεγονότος ἐν τῷ μέλλοντι καὶ τοῦ μέλλοντος ἐν τῷ γεγονότι καθ' ὃ γὰρ ἂν διαιρεθῇ, τοῦτο διοριεῖ τὸν παρήκοντα καὶ τὸν μέλλοντα χρόνον. ἅμα δὲ καὶ οὐκ ἂν καθ' αὑτὸ εἴη τὸ νῦν, ἀλλὰ καθ' ἕτερον ἡ γὰρ διαίρεσις οὐ καθ' αὑτό. πρὸς δὲ τούτοις τοῦ νῦν τὸ μέν τι γεγονὸς ἔσται τὸ δὲ μέλλον, καὶ οὐκ ἀεὶ τὸ αὐτὸ γεγονὸς ἢ μέλλον. οὐδὲ δὴ τὸ νῦν τὸ αὐτό πολλαχῇ γὰρ διαιρετὸς ὁ χρόνος. ὥστ' εἰ ταῦτα ἀδύνατον ὑπάρχειν, ἀνάγκη τὸ αὐτὸ εἶναι τὸ ἐν ἑκατέρῳ νῦν. ἀλλὰ μὴν εἰ ταὐτό, φανερὸν ὅτι καὶ ἀδιαίρετον εἰ γὰρ διαιρετόν, πάλιν ταὐτὰ συμβήσεται ἃ καὶ ἐν τῷ πρότερον. ὅτι μὲν τοίνυν ἔστιν τι ἐν τῷ χρόνῳ ἀδιαίρετον, ὅ φαμεν εἶναι τὸ νῦν, δῆλόν ἐστιν ἐκ τῶν εἰρημένων ὅτι δ' οὐθὲν ἐν τῷ νῦν κινεῖται, ἐκ τῶνδε φανερόν ἐστιν.*

> If a thing moves, it moves now; if it moves now, it moves in the present time; and if it moves in the present time, it moves, therefore, in a indivisible time. For if the present time is divided, it will be certainly be divided into the past and the future, and thus it will no longer be present. And if a thing moves in an indivisible time, it passes through indivisible places. And if it passes through indivisible places, it does not move. For when it is in the first indivisible place it does not move; for it is still in the first indivisible place. And when it is in the second indivisible place, again it does not move but it has moved. Therefore nothing moves [Bury, 1968].[38]

In the first fragment the argument follows a strategy already used by Aristotle and leads to the same result: the present time, namely *νῦν*, is indivisible. What does this mean?

As well as for Aristotle, the answer is in the isomorphismus of matter, place and time. So, the second part of the fragment relates partless place and time, *εἰ δ' ἐν ἀμερεῖ χρόνῳ τι κινεῖται, ἀμερίστους τόπους διέρχεται*, by concluding that particles are immobile. In fact, we have shown previously that Diodorus takes an ameristic view of the structure of place. Therefore, time too should have such a structure. So, contrary to Aristotle, Diodorus takes time structure as discrete rather than dense. In this way, Diodorus gets an ameristic isomorphismus on matter, place and time. He declares that what is passing through indivisibles places does not move. Sorabji [1982, 62] confirms this interpretation. Sorabji argues that 'Aristotle had used the same proof that the present must be indivisible (otherwise it would overlap with past and future). But he had taken the indivisible present to be an instant[39] not an atom', as opposed to Diodorus.

[38] *εἰ κινεῖταί τι, νῦν κινεῖται εἰ νῦν κινεῖται, ἐν τῷ ἐνεστῶτι χρόνῳ κινεῖται εἰ δὲ ἐν τῷ ἐνεστῶτι χρόνῳ κινεῖται, ἐν ἀμερεῖ χρόνῳ ἄρα κινεῖται. εἰ γὰρ μερίζεται ὁ ἐνεστὼς χρόνος, πάντως εἰς τὸν παρῳχημένον καὶ μέλλοντα μερισθήσεται, καὶ οὕτως οὐκέτ' ἔσται ἐνεστώς. εἰ δ' ἐν ἀμερεῖ χρόνῳ τι κινεῖται, ἀμερίστους τόπους διέρχεται. εἰ δὲ ἀμερίστους τόπους διέρχεται, οὐ κινεῖται. ὅτε γὰρ ἔστιν ἐν τῷ πρώτῳ ἀμερεῖ τόπῳ, οὐ κινεῖται ἔτι γὰρ ἔστιν ἐν τῷ πρώτῳ ἀμερεῖ τόπῳ. ὅτε δὲ ἔστιν ἐν τῷ δευτέρῳ ἀμερεῖ τόπῳ, πάλιν οὐ κινεῖται, ἀλλὰ κεκίνηται. οὐκ ἄρα κινεῖταί τι.*

[39] Of course, Aristotle does not say that the present is *strictu sensu* an instant in the way of a substantial object. In Aristotle view, the present has an instantaneous duration. But according

In the last section of the fragment of S.E. *M.* 10, 120, Diodorus explains properly the paradox of a movement on a discrete scheme, a kind of *jump* or *jerk motion*, namely a static states sequence. The particle never is in progression from a place to another one, however it has moved. Nothing is flowing from a first to a second place. But, rationally, we should admit that something has moved.[40]

Diodorus does not deny the existence of the present. Diodorus maintains that the present time has the same structure as the past and future. Namely, the present is an atom. Present time is as a part of minimal but positive size. In this way, Sorabji [1983, 21] comments, Diodorus is able to avoid any time overlapping. As in a sequence, any time atom occupies its position. Any atom is both the *immediately next* in relation to its predecessor and the *immediately previous* in relation to its successor. Diodorus' account, therefore, does not deny motion at all. Diodorus denies motion as a process of flowing on a dense scheme. If, instead, Diodorus' account is discrete, then motion can be represented as a sequence. So, Diodorus avoids motion as dense progression, but he admits atomic transit from a place to another place. Diodorus' conclusion is that something has moved.

The scheme for Diodorus physics seems analogue to his language doctrine. Some ancient reports testify that Diodorus maintained the truth of past tenses, but the falsehood of what we would name the present continuous tenses, in relation to the former ones. In fact, Diodorus would say that 'Some ἀμερές has moved' is true, but the corresponding present tense 'Some ἀμερές is moving' is false. We will explore this theme in the next paragraph, focusing on some linguistic doctrines.

to Arist. *Metaph.* 3. 5, 1001b 26 and *ff.*, instants, by analogy with mathematical points, may be thought as aporetical objects. In the knotty passage of *Ph.* 4, 11, 220a 18 and *ff.*, Aristotle makes two relevant considerations: (i) the *now* is no part of time nor the section of any part of the movement; (ii) the *now* is a boundary, it is not time, but an attribute of it; in so far as it numbers, it is number.

[40] Denyer [1981a, 33-34] explains the odd ancient Diodorean account of motion, by a *naive* contemporary analogy, that is, *Leicester Square device*. It is a dynamic switched on and switched off sequence of light bulbs in the device. Letters in the display seem to be flowing to the pedestrians in the square. But the motion process is only an illusion. Bulbs are for *places*, light of bulb is for *ameres body*, and the switching on and switching off times are in the same way atomics and *partless*.

1.2 On conventionality of language and tenses

The roots of Diodorus' outlook lie in Parmenides [Diels and Kranz, 1951] 28B 8, 7-10, and in Ermogenes' thesis from Pl. *Cra.* 384d, although Diodorus is an innovator within the Eleatic tradition, and despite the fact that he comes across as a conservative by comparison with Aristotle's doctrines on language. We can reconstruct Diodorus' tenets starting from Epicurus' attack on them.

According to Sedley [1973, 21], Epicurus opposed the extreme conventionalism of Diodorus Cronus in his *Nat.* 28, in which the thesis of the Megaric is delivered by Metrodorus, who says, essentially, that a thing can be referred to by any name at all, and no name is better than any other.

A summary of Diodorus' view on naming is in Gell. 11, 12, 2-3:

> But Diodorus surnamed Cronus, says: 'No word is ambiguous, and no one speaks or receives a word in two sense; and it ought not to seem to be said in any other sense than that which the speaker feels that he is giving it. But when I,' said he, 'meant one thing and you have understood another, it may seem that I have spoken obscurely rather than ambiguously; for the nature of an ambiguous word should be such that he who speaks it expresses two or more meanings. But no man expresses two or more meanings who has felt that he is expressing but one [Rolfe, 1927].[41]

It follows that Diodorus embraces a hard conventionalism about language: naming an object means assigning it a reference, stipulating arbitrarily the link between word and object. Naming does not concern sound, but what the speaker

[41] Diodorus autem, cui Crono cognomentum fuit: 'nullum' inquit 'verbum est ambiguum, nec quisquam ambiguum dicit aut sentit, nec aliud dici videri debet, quam quod se dicere sentit is, qui dicit. At cum ego' inquit 'aliud sensi, tu aliud accepisti, obscure magis dictum videri potest quam ambigue; ambigui enim verbi natura illa esse debuit, ut, qui id diceret, duo vel plura diceret. Nemo autem duo vel plura dicit, quise sensit unum dicere'.

wants to say, by arbitrary choosing some conventional term. There exists a biunique link between what the speaker asserts and the denoted thing. In Diodorus' linguistic doctrine there is no chance for ambiguity.

Sometimes names may appear as odd, but once they are given, they get their evocative grip on both things and persons. Here is the passage by Steph. *in Int.* 9, 20-24:

> Thus, Cratylus asserted that names follow nature according to their first meaning, on the other hand Diodorus did not assert that names follow nature but names follow application, and that one by according to the second meaning, namely for what accidentally occurs. And so, he nicknamed his children by the same names of subjunctive particles, Μὲν and Δὲ.[42]

According to Diodorus, names are result from the purpose of the speaker and the listener's understanding of this *intentio*. Names are not *per natura* but only *per accidens*, *i.e.*, established by convention. A kind of bizarre but eloquent practical proof of Diodorus' thesis are the names imposed to his doughters, namely Μὲν and Δὲ.

The issue of time is constant in Diodorus' arguments with significant consequences for his modal, physical and linguistic doctrines. In particular, a debate on tenses concerning Diodorus' view was complementary to his arguments against movement. As reported in S.E. *M.* 10, 97-101, Diodorus in his analysis of the proposition 'the ball touches the roof' relies on:

1. The assumption that all the sentences that contain given occurrences in a present progressive tense form are false, despite their corresponding past tenses sentences, *e.g.* 'it has touched the roof', may be true.

[42] *ὁ μὲν οὖν Κράτυλος φύσει ἔλεγεν τὰ ὀνόματα κατὰ τὸ πρῶτον σημαινόμενον, ὁ δὲ Διόδωρος φύσει μὲν οὐκ ἔλεγεν ἀλλὰ θέσει, καὶ τοῦτο κατὰ τὸ δεύτερον σημαινόμενον τὸ ἁπλῶς καὶ ὡς ἔτυχεν. ὅθεν καὶ τοὺς ἰδίους παῖδας τοῖς τῶν συνδέσμων ὀνόμασιν ἐκάλει, Μὲν' καὶ 'Δὲ' προσαγορεύων αὐτούς.*

2. The admission that motion exists only as jump or jerk, such that something has moved but nothing is in movement, in the sense of progression.

Let us consider (some preliminary steps of) Sextus' report in *M.* 10, 97 in more detail. According to Crivelli [1994], Stoics are the opponents in the polemical response to Diodorus Cronus in *M.* 10, 97. A first example is about propositions regarding Elene's marriages with Menelaus, Paris, and Deiphobus, and relative scheme for tenses, in S.E. *M.* 10, 97.

> Diodorus seems to have answered [...] that when preterites are true their presents admit of being false [Bury, 1968, 259].[43]

The above claim may appear counterintuitive, or at least unclear, in the absence of further clarifications. It means that for a true proposition in the past tense, its corresponding one at present is false. What is the sense of Diodorus' view? Sextus proposes two arguments justifying Diodorus' claim in *M.* 10, 98 and *M.* 10, 101. Then, he expounds also two replies in *M.* 10, 99, and *M.* 10, 102 and *ff.*. However, if Diodorus' first argument is a sort of sophism, the second one works against both the concept of progressive motion and against the standard view on tenses: it deals with a position not corresponding to the common intuition but correct nonetheless.

Let us start from the case of a proposition as 'Helen had three husbands', and Sextus' analysis of an odd corresponding proposition at the present tense, in *M.* 10, 98:

> Neither when she [Helen] had Menelaus as her husband in Sparta, nor when she had Paris in Ilium, nor when, after his death, she married Deiphobus, is the present – 'she has three husbands' – true,

[43] δοκεῖ δὲ Διόδωρος πρὸς τὴν πρώτην εὐθὺς ὑπηντηκέναι διδάσκων, ὅτι ἐνδέχεται τῶν συντελεστικῶν ἀληθῶν ὄντων τὰ τούτων παρατατικὰ ψευδῆ τυγχάνειν.

> though the preterite – 'she had three husbands' – is true [Bury, 1968, 259].[44]

As Crivelli [1994, 491] hypothesises on account of complementary passages in S.E. *M.* 11, 11-14, the Stoics, in particular Chrysippus, solved the dilemma by distinguishing between singular and plural propositions, and for those last, between collective and distributive senses of utterances. Below is *M.* 10, 99-100, in which Diodorus' opponent shows his well formulated reply.

> For the proposition 'these men married' has two senses, of which the one is plural and equivalent to 'these men married together', which is false, but the other is formed by the combination of one singular proposition 'this man married', and another singular proposition 'that man married', and of these singulars, again, the presents are true, namely, 'this man is marrying' and 'that man is marrying'; for these statements are true in both cases. It is, then, impossible, if the presents are false, that their preterites should be found to be true; and of necessity both of them must either be abolished together or co-exist along with each other [Bury, 1968, 259-261].[45]

On the other hand, Diodorus Cronus' thesis on tenses seems to have the same structure as his argument against movement (the case of the ball thrown on to an overhanging roof). Referring to S.E. *M.* 10, 101:

> Let a ball, he says, be thrown on to an overhanging roof. Then, at the point of time that is midway in the throw, the proposition 'the

[44] "Ελένη τρεῖς ἔσχεν ἄνδρας" οὔτε γὰρ ὅτε Μενέλαον εἶχεν ἐν Σπάρτῃ ἄνδρα οὔθ' ὅτε Πάριν ἐν Ἰλίῳ, οὔθ' ὅτε θανόντος τούτου Δηιφόβῳ ἐγαμήθη, ἀληθές ἐστι τὸ παρατατικὸν τὸ "τρεῖς ἔχει ἄνδρας", ἀληθοῦς ὄντος τοῦ συντελεστικοῦ τοῦ "τρεῖς ἔσχεν ἄνδρας".

[45] τὸ γὰρ "οὗτοι ἔγημαν" δύο σημαίνει, ἓν μὲν πληθυντικὸν καὶ ἴσον τῷ "οὗτοι συνέγημαν", ὅπερ ἐστὶ ψεῦδος, ἕτερον δὲ τὸ κατὰ περίληψιν ἑνικοῦ πράγματος ἐγκεκλιμένου ἀπὸ τοῦ "οὗτος ἔγημεν" καὶ ἑτέρου ἑνικοῦ τοῦ "οὗτος ἔγημεν", ὧν πάλιν ἑνικῶν τὰ παρατατικά ἐστιν ἀληθῆ, τὸ "οὗτος γαμεῖ" καὶ τὸ "οὗτος γαμεῖ" ἐπ' ἀμφοτέρων γὰρ ἀληθῆ γέγονε ταῦτα. ἀμήχανον οὖν ἐστι τῶν παρατατικῶν ψευδῶν ὄντων ἀληθῆ εὑρίσκεσθαι τὰ τούτων συντελεστικά, ἀλλ' ἀνάγκη συναναιρεῖσθαι ἢ συνυπάρχειν τὰ ἕτερα τοῖς ἑτέροις.

> ball touches the roof' is false; for it is still on its way. But when it has touched the roof, the preterite 'the ball has touched the roof' becomes true; therefore it is possible for the preterite to be true when the present is false, and therefore possible for a thing not 'to be moving' in the present but 'to have moved' in the preterite [Bury, 1968, 261].[46]

Diodorus' argument confirms the so-called (1) *isomorphismus* of place and time as discrete, (2) marks Diodorus' critics against motion in progress which follows a dense account on time, (3) establishes a linguistic link with a theory of tenses.

1. When Diodorus says 'the point of time', he means a discrete point, in which there is a ball, during a discrete time. Further, he analyses time following a static view, in the sense of a static sequence of states.

2. Diodorus does not accept as possible to individuate an instant between the moment of time in which the ball is in its last atomic place of throw and the first moment of its falling back to earth, namely when is already in the first discrete place to return.

3. As a consequence, Diodorus may affirm that 'it is possible for the preterite to be true when the present is false' in order to examine the case in which 'the ball has touched the roof' is already true and its present tense form 'the ball touches the roof' was never true, nor will be after the return of the ball is coming. Therefore, Diodorus does not approve any tense describing an action in the present, when it indicates an instantaneous progression. So,

[46] *βαλλέσθω γάρ, φησί, σφαῖρα εἰς τὸν ὑπερκείμενον ὄροφον. οὐκοῦν ἐν τῷ μεταξὺ τῆς βολῆς χρόνῳ τὸ μὲν παρατατικὸν ἀξίωμα "ἅπτεται ἡ σφαῖρα τῆς ὀροφῆς" ψεῦδός ἐστιν ἔτι γὰρ ἐπιφέρεται. ὅταν δὲ ἅψηται τῆς ὀροφῆς, γίνεται ἀληθὲς τὸ συντελεστικόν, τὸ "ἥψατο ἡ σφαῖρα τῆς ὀροφῆς". ἐνδέχεται ἄρα ψεύδους ὄντος τοῦ παρατατικοῦ ἀληθὲς ὑπάρχειν τὸ συντελεστικόν, καὶ διὰ τοῦτο μὴ κινεῖσθαι μέν τι παρατατικῶς, κεκινῆσθαι δὲ συντελε στικῶς. μήποτε δὲ κἀνταῦθα πλανᾶται.*

> according to Diodorus *'a thing is not moving'* in the present, and yet it is the case that *'it has moved'* in the preterite.[47]

Going back to the text, the argument reported by S.E. *M.* 10, 102, this time against the second presentation of Diodorus' thesis is not persuasive, or at least, it misses the mark:

> The present – 'the ball touches the roof' – is true not when the ball is travelling in mid air but when it begins to touch the roof. But when it comes down again, after ending its contact, then the preterite becomes true – 'the ball touched the roof'. Therefore it is absurd of Diodorus to accept 'to have moved' as true and to reject 'to move' as false, when he ought either to assent to both or to reject both.

We suspect that Sextus does not take Diodorus' argument at face value: he considers a dense account of time and place, while Diodorus' account is clearly discrete. Sextus is asserting the so defined *false* Diodorean proposition 'the ball touches the roof' but with a different and ambiguous meaning, in place of what Diodorus meant. Then, it is not coherent to accept Sextus' reply: if he reinterprets what Diodorus wanted to assert by the proposition 'the ball touches the roof', we deal with a different and not Diadorean sentence. S.E. *M.* 10, 102, specifies by periphrasis that 'the ball touches the roof' stands for such a ball that *begins to touch the roof*. In fact, *'when the ball begins to touch the roof'* has no valid equivalent formula in Diodorus' account of speech. Following Diodorus' view, there exists an exact discrete time, atomic, like ἀμερὲς time, at which 'the ball touched the roof' refers to an already completed action.

A Diodorean periphrasis of 'the ball touched the roof' suggests a precise scheme, which today we would translate by assigning the time connotation that is *from the first moment in which the ball is touching the roof and hereafter*. However,

[47] We make clear Diodorus' rejection of actions in progress, in relation to every present tense, and even if those ones have occurred during some past, by the tense formula $\neg (Pp \rightarrow p)$, where P is the past operator on an atomic sentence, and p is the corresponding present continuous tense sentence, related to action in progress at now.

the present continuous tense in the sentence 'the ball is touching the roof' (which lacks an exact tense correspondence in ancient Greek), or any other sentence in relation to an instantaneous discrete present, without any minimal temporal magnitude, states a nonsense and, then, Diodorus can consider it to be false.

1.3 The *κυριεύων λόγος*

The *κυριεύων λόγος* is the heart of Diodorus' doctrines and his most famous topic. It is an enigmatic argument, and we would not to be able to reconstruct exactly Diodorus' strategy to obtain its conclusion, without any complete ancient reports.

We would attempt to discover the meaning of premises, to reconstruct their logical form, and then to infer the conclusion of *κυριεύων λόγος* in order to reveal its strength as well as its philosophical and logical target. Furthermore, the argument is very relevant for Diodorus' conception of time and modalities. It is almost certain that the *κυριεύων* is at first formulated both in relation with Aristotle's doctrines on capacity (in Arist. *Metaph.* 9) and modalities (in Arist. *Int.* 9; *APr.* 1, 13; *Cael.* 1, 12, 283b). Nevertheless, both its wide resonance in the Hellenistic period and some ancient textual evidence lead us to believe that it continued to interest quick witted logicians, *e.g.* Cleanthes and Chrysippus.

According to Alex.Aphr. *in APr.* 183, 34 - 184, 6, the *κυριεύων* is intended to found Diodorus' thesis on the modal notion of *possible*. Some scholars, *e.g.*, Mignucci [1966], suppose the *κυριεύων λόγος* is best understood in relation to the theme of *implication* and *conditional sentences*. On the basis of Boeth. *in Int.* 234, 22-26, and some similar passages, we want to clarify whether Diodorus formulated modal notions depending on time, or *vice versa* modal notions gave the schema for time notions and tenses.

Let us begin by reviewing the most complete ancient report of the *κυριεύων λόγος*, in Arr. *Epict.* 2, 19, 1:

> These seem to be the sort of starting-points from which the Master Argument is posed. The following three propositions mutually conflict:
> *Every past truth is necessary*;
> *Something impossible does not follow from something possible*;
> *There is something possible which neither is nor will be true*.
> Diodorus saw this conflict and exploited the convincingness of the first two to establish the conclusion that
> *Nothing which neither is nor will be true is possible*
> [Long and Sedley, 1987, 230].[48]

However, before analysing theoretically the ancient Master Argument, we need some preliminary clarifications. The general perspective of Diodorus links possibility to actuality by interdefining modal and temporal notions. Dealing with the modalities in Diodorus Cronus requires that we clarify some aspects in the history of modal concepts insomuch as they involve the Megarics. Diodorus' modalities are remarkable and they are sometimes charged with fatalism.

In the Megaric tradition, there exists no fragment concerning modalities before Diodorus. But, it is certain that Diodorus' modalities remain an important attempt to construct a theory of modal notions. Only Philo of Megara, Diodorus' most famous pupil, talks about modalities by engaging in an interesting debate with him.

Then, modalities and the doctrine of capacity are connected, even if these are not the same thing, as some scholars took them to be.[49] As a matter of fact, there is a real difficulty here, since we might interpret as possible either that someone has the capability to do such and such or that he is able to make use

[48] Ὁ *κυριεύων λόγος ἀπὸ τοιούτων τινῶν ἀφορμῶν ἠρωτῆσθαι φαίνεται κοινῆς γὰρ οὔσης μάχης τοῖς τρισὶ τούτοις πρὸς ἄλληλα, τῷ* [τὸ] *πᾶν παρεληλυθὸς ἀληθὲς ἀναγκαῖον εἶναι καὶ τῷ* [ἀ]*δυνατῷ ἀδύνατον μὴ ἀκολουθεῖν καὶ τῷ δυνατὸν εἶναι ὃ οὔτ᾽ ἔστιν ἀληθὲς οὔτ᾽ ἔσται, συνιδὼν τὴν μάχην ταύτην ὁ Διόδωρος τῇ τῶν πρώτων δυεῖν πιθανότητι συνεχρήσατο πρὸς παράστασιν τοῦ μηδὲν εἶναι δυνατόν, ὃ οὔτ᾽ ἔστιν ἀληθὲς οὔτ᾽ ἔσται.*

[49] *E.g.*, to name but one, Reale and Radice [2000] in their translation of *book 9*. See also Beere [2009, 119-137].

of some capacity (cf. [Weidemann, 2008, 131]). At any rate, in order to avoid misunderstandings, we will opt to give special attention to original Greek and Latin ancient texts. Furthermore, we will preliminarly focus on the theme of capacities, in order to understand their effect on the modal and temporal notions involved also in the *κυριεύων λόγος* and, more in general, in the Hellenistic debate.

After having done this, we will specify the central concepts structure, and purpose of the *κυριεύων λόγος*. We will give an interpretation of the meaning of the argument in relation to some complementary texts.

The *κυριεύων λόγος* is composed of three propositions, and a further one, being deduced by Diodorus ruling out the third one on reason of incompatibility. That is, the argument would be not correct by admitting the third proposition. So, Diodorus probably began his proof from the first two premises. However, no report transmits the strategy leading to the fourth sentence, *i.e.* the *κυριεύων* conclusion, as confirmed by Epictetus.

Diodorus' starting points for his argument are the notions of necessity, possibility, truth, and time. According to Montoneri [1984, 179] terms as *ἀληθὲς*, *ἀναγκαῖον*, *δυνατόν*, *ἀδύνατον*, are *supreme genera* of *being*, and *universal predicates* of judgment when we state propositions. Of course, this admits a twofold interpretation, logical as well as ontological. This would complicate our investigation, but it would preserve the ancient and medieval point of view on the *κυριεύων λόγος*.

Our proposal follows the schema below:

1. Hypothesising a given doctrine of capacity by Diodorus Cronus, as a reply to Aristotle;

2. Focusing on the development of modal notions in the Megaric tradition and examining logical interdefinitions with time notions, with particular attention to Diodorus, Philo of Megara, and the Stoics;

3. Analysing every proposition of the *κυριεύων λόγος*.

1.3.1 Megaricism and Diodorus: capacities, modalities, and temporal notions

When Arist. *Metaph.* 9. 3, 1046b 29-32 quotes, for the first time, the Megarics and their doctrine, he refers to *capacity* and *actuality*, and does not allude to modalities. Moreover, Aristotle does not cite Diodorus or Philo, instead he credits the tenets generically to the Megarics. However, in later passages, Aristotle mentions the modalities as well, presumably aiming to dispute Diodorus' claims. Thus, echoes of that are present in the famous book 9 of *Int.*

The characterisation of capacity (as well as that of actuality), as single state description, considers strictly one point in its temporal collocation. Sometimes, it has contrived a wrong interpretation, leading to the assimilation of possibility and capacity, necessity and actuality. Instead, the descriptions for a state are not inevitably related to a temporal schema. On the other hand, we also have a temporal dimension in the interpretation of the modalities (*see*, appendix A). Nevertheless, the two question are not equivalent.

One of the aims of this paragraph is to explain *capacities* and *modalities* respectively by the concepts of *property* and *operator*. From a logical point of view, properties are linked to an individual (a given *substance*), either simply as possession, or like a given activity which is attributed to the individual. For all occurrences in time, instead, the operator may express the linguistic mood translated in (temporal) location. Furthermore, modal operators are not compulsorily linked to capacities, but might be the prefix of some very simple sentence. Deep analysis on the ancient sources will be preparatory and of utmost importance for this work.

1.3.1.1 Developments of doctrines of capacity

Here is the *incipit* of Arist. *Metaph.* 9. 3, 1046b 29-32:

> There are some – such as the Megarics – who say that something is capable only when it is acting, and when it is not acting it is not capable. For example, someone who is not building is not capable of building, but someone who is building is capable when he is building; and likewise too in other cases. It is not hard to see the absurd consequences of this [Makin, 2006].[50]

Aristotle deals with capacity, not with modality. Aristotle is disapproving the consequences of a Megaric thesis about capacity, since it entails some absurdities. The Megaric view would admit the case of a builder who would have the capacity of building if and only if he were building a house. Following the meticulous commentary of *Metaph.* 9, by Makin [2006, 60], the Megaric position presented by Aristotle may be stated as follows:

M Something possesses a capacity at t iff it is exercising the capacity at t.

Rewriting **M** as **M*** in formal terms, it becomes:

M* $(\forall x)(C_B x_t \longleftrightarrow Bx_t)$.

Let C be the property 'to have some capacity'; and B the property 'to be a builder', **M*** says that for every individual x, x has the capacity to have the property B (*i.e.*, to be a builder) at the time t, if and only if x is exercising B at t. We consider properties of individuals to describe capacities, and **M*** is the formal rendering of what Aristotle takes as the Megarics's view on capacity: *ὅταν ἐνεργῇ μόνον δύνασθαι, ὅταν δὲ μὴ ἐνεργῇ οὐ δύνασθαι.*

[50] Εἰσὶ δέ τινες οἵ φασιν, οἷον οἱ Μεγαρικοί, ὅταν ἐνεργῇ μόνον δύνασθαι, ὅταν δὲ μὴ ἐνεργῇ οὐ δύνασθαι, οἷον τὸν μὴ οἰκοδομοῦντα οὐ δύνασθαι οἰκοδομεῖν, ἀλλὰ τὸν οἰκοδομοῦντα ὅταν οἰκοδομῇ ὁμοίως δὲ καὶ ἐπὶ τῶν ἄλλων. οἷς τὰ συμβαίνοντα ἄτοπα οὐ χαλεπὸν ἰδεῖν.

In spite of Aristotle's charge against Megarics, we will propose an account in which the Stagirite does not appear as the toughest adversary of Diodorus Cronus; it is rather Philo who plays that role. We will examine some Aristotelian texts and commentators in order to reconstruct some paths to Diodorus' doctrine of capacity, modalities, and time; finally, we will dwell briefly on the difference between determinism and fatalism by showing what view Diodorus really held.

Aristotle gives four arguments in favor of the alleged absurdity of the Megaric thesis (Arist. *Metaph.* 9. 3, 1046b 33 - 1047a 17):

1. It cannot be the case that one who is unable to build something at a given time suddenly becomes able to do so at a later time, and conversely, one who has learnt some art does not lose it when he does not exercise this art.[51]

2. The same holds, suggests Aristotle, of the abilities of perception, as far as awareness of things is concerned.

3. Likewise, the Megaric view entails that by merely not exercising a sense, one is deprived of that sense (for instance, if someone closes his eyes, then he would become blind).

4. Furthermore, if something lacks a capacity, then, according to the Megarics, this thing is not exercising that capacity, nor will it do it. Therefore, the Stagirite denies that what is impossible to happen, since something is deprived of a capacity, is or will be.

Thereafter, Aristotle proposes his replies.[52] For now, we go on to report the lines at 1047a 17-24:

[51] On the background of this argument is the *slave's aporia* in Pl. *Men.* 80c-d. *Cf.* Arist. *Metaph.* 5. 20, 1022b 4-14 in relation to possession and disposition of something.

[52] However, some caution has to be exercised with respect to the fourth item, since it might not be aimed *in toto* against a Megaric of the second generation, such as Diodorus Cronus. Both Aristotle in *Metaph.* 1047a 12-13, and Diodorus by ruling the third premise of the *κυριεύων λόγος* out, deny that what is or will be is impossible to happen.

> So if these things cannot be said it is plain that capacity and actuality are different (for those arguments make capacity and actuality the same, and so it is no small thing that they try to abolish), so that it can be possible to be something and yet not be that and possible not to be something and yet be that, and likewise too in the case of the other categories – it is possible for something not walking to walk, and possible for something walking not to walk [Makin, 2006].[53]

However, in this way Aristotle does not produce a real proof against the Megarics, but his argument appears to be outcome of practical thinking. The next passage is more rigorous, but in addition Aristotle introduces some modal notions. Arist. *Metaph.* 9. 3, 1047a 24-29, suggests a criterion in order to establish what is meant by possibility:

> And this is what is possible that for which, if the actuality of which it is said to have the capacity obtains, there will be nothing impossible. I mean, for example, if it is possible for it to sit and it can sit, should sitting belong to it, there will be nothing impossible. And likewise in the case of being changed or changing or standing or making stand or being or coming to be or not being or not coming to be [Makin, 2006].[54]

In this passage Aristotle's view is compatible to Diodorus', and at a later stage we would explain their complementarity. Before that, we should, however, investigate the assumptions behind 1047a 24-26.

Aristotle holds that it is possible something whose taking place does not imply any impossibility, provided that the actuality of that which it is said to

[53] *εἰ οὖν μὴ ἐνδέχεται ταῦτα λέγειν, φανερὸν ὅτι δύναμις καὶ ἐνέργεια ἕτερόν ἐστιν (ἐκεῖνοι δ' οἱ λόγοι δύναμιν καὶ ἐνέργειαν ταὐτὸ ποιοῦσιν, διὸ καὶ οὐ μικρόν τι ζητοῦσιν ἀναιρεῖν), ὥστε ἐνδέχεται δυνατὸν μέν τι εἶναι μὴ εἶναι δέ, καὶ δυνατὸν μὴ εἶναι εἶναι δέ, ὁμοίως δὲ καὶ ἐπὶ τῶν ἄλλων κατηγοριῶν δυνατὸν βαδίζειν ὃν μὴ βαδίζειν, καὶ μὴ βαδίζειν δυνατὸν ὃν βαδίζειν.*

[54] *ἔστι δὲ δυνατὸν τοῦτο ᾧ ἐὰν ὑπάρξῃ ἡ ἐνέργεια οὗ λέγεται ἔχειν τὴν δύναμιν, οὐθὲν ἔσται ἀδύνατον. λέγω δὲ οἷον, εἰ δυνατὸν καθῆσθαι καὶ ἐνδέχεται καθῆσθαι, τούτῳ ἐὰν ὑπάρξῃ τὸ καθῆσθαι, οὐδὲν ἔσται ἀδύνατον καὶ εἰ κινηθῆναι ἢ κινῆσαι ἢ στῆναι ἢ στῆσαι ἢ εἶναι ἢ γίγνεσθαι ἢ μὴ εἶναι ἢ μὴ γίγνεσθαι, ὁμοίως.*

have the capacity obtains. Aristotle's claim seems an instance of the second proposition of the *κυριεύων λόγος*, *i.e.* 'Something impossible does not follow from something possible'. For, the proposition suffixed by modality deals with the capacity of something which becomes actual. Thus, the difference between Aristotle and Diodorus is not in the mere conception of possibility but in their view of capacity.

We restate Aristotle's statement as **M****, in the form of an instance of the second proposition of the Master Argument:

M** $\neg\Diamond Bx \rightarrow (\Box(C_B x \rightarrow Bx) \rightarrow \neg\Diamond C_B x)$.

Therefore, a difference with the Megaric account, *e.g.* by Diodorus, is the following: the implication $C_B x \rightarrow Bx$ might be false by Aristotle's lights, while it is always true for a strong supporter of **M***.

It may be useful to look at some relevant points in Makin [2006, 60-81]. He claims that his **M** (and hence **M***) follows validly from **NP**, *i.e.*, the *necessity of the present*, in conjunction with **S**, namely the *synchronicity* thesis.[55] Those are in our notation the formulas:

NP* $(\forall t)(\neg Bx_t \rightarrow \neg C_B x_t)$

S* $(\forall t)(\forall t^*)(C_{B_{t^*}} x_t \rightarrow t = t^*)$

Thus, Makin [2006, 62-63] has highlighted how some opponent of Aristotle, in order to hold up against *Metaph.* 9. 3, would affirm a further position,

[55] We replace **NP** by **NP***, **S** by **S***. Makin [2006, 61] clarifies, respectively **NP** and **S**. **NP** is the following thesis *if something does not act in a certain way at t, it does not at t have the capacity to act in that way at t*, that is expressed by $(\forall t)(A$ does not ϕ at $t \rightarrow A$ does not at t have the capacity to ϕ at $t)$; **S** is the claim that *all capacities are really synchronous. A synchronic capacity is one such that if something possesses at a time the capacity for acting at a time, the time at which it possesses the capacity is identical to the time specified in the content of the capacity* by $(\forall t)(\forall t^*)(A$ has at t the capacity to ϕ at $t^* \rightarrow t = t^*)$.

which is named *diachronicity*, that is **D**,[56] presented in this work as **D***, and $\mathbf{T}_t$ is for 'it is true at t that-'.

D* $(\forall t)(\forall t^*)(t < t^*)(C_{B_{t^*}} x_t \rightarrow \mathbf{T}_t C_{B_{t^*}} x_{t^*})$

According to Makin [2006, 62-63], (**S*** **+** **NP*** **+** **D***) establishes Diodorus' doctrine of capacity. It is compatible with and can be restated by means of Diodorus' perspective on modal notions: 'Diodorus Cronus, writing probably the generation after Aristotle, defined the possible in temporal terms, as what either is or will be the case. The Diodoran definition was well known in antiquity, and might be seen as a development of a Megaric position in response to Aristotelian criticism'.

We would explain what is it to opt for Makin's formalism, or something similar deduced from that: it is the best attempt that we know in order to hypothesise on the strength of a Diodorean doctrin for capacity. In fact, Diodorus' definitions of modal notions are renowned in the ancient texts. This is not the case with his views on capacity. Still, the direct quotation from the Megarics in Aristotle's incipit of *Metaphysics* concerns a reply to some presumed absurdities of their doctrine of capacity, rather than modal notions, and it precedes Diodorus. Therefore, in order to better grasp the Diodorean notion of capacity, we need something analogous to Makin's hypothesis. Makin [2006, 62-63]'s hypothesis is that Diodorus Cronus endorsed both **S***, **NP***, and **D***. The acceptance of **NP***, that is the logical *necessity of the present*, might provide support to the fatalist thesis. On the other hand, if Diodorus referred to the necessity of the

[56] **D** is the stance *for distinct t and t^*, such that A is capable at t of ϕ-ing-at-t^* only if it is true at t that A will be capable at t^* of ϕ-ing-at-t^**. In fact, Makin's **D** is also subsuming the modalities in the form of capacities; indeed, the diachronic property means that some capacity can be already possessed even if not yet exercised. However, in order that a given capacity may be started, it needs some interval of time to have passed. By **D**, we are again able to describe diachronic capacities by postponing synchronic powers, not vice versa.

present in the sense that, what occurs is irrevocably given, then Diodorus can be said to be a determinist but not necessarily a fatalist.[57]

1.3.1.2 The modalities: Diodorus, Philo, and the Stoics

It is useful to read ancient texts with the help of modern logical tools. On the other hand, indisputable and categorical judgments are inappropriate if referring to some philosophical doctrines not well bequeathed to the contemporary world. So, our aim here is to illustrate some conjectures. Our next point of focus are the modalities, at first, without any crossbreed with capacities and other properties of individuals. The first fragment to introduce the topic is in Boeth. *in Int.* 234, 22-26. Boethius refers to Diodorus' definitions of the possible and the necessary, and their respective negations, *i.e.*, the impossible and the not-necessary.

> Diodorus delimits the possible as that which either is or will be; the impossible as that which when it is false will not be true; the necessary as that which when it is true will not be false; the non-necessary as that which either now is or will be false [Blank and Kretzmann, 1998].[58]

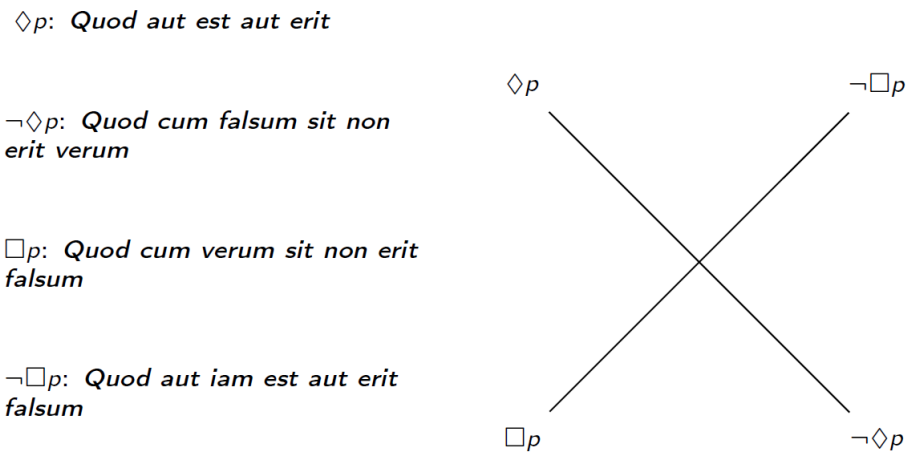
[57] On the debate on determinism in Diodorus Cronus, at first, *see* [Denyer, 1981b, 52-53] and [Bobzien, 1998, 102-108]. **NP*** can be interpreted as inferred by the modal necessitation rule $\vdash p \Rightarrow \vdash \Box p$, or maybe by an instance of the first proposition of the *κυριεύων λόγος* or Master Argument, $Pp \rightarrow \Box Pp$, by substitution of p for Pp, in which P is the tense operator for the past. In some way, an interpretation of **NP*** would be linked to the meaning of the first premise of the *κυριεύων λόγος*. Scholars who translate the first premise as 'Every proposition true about the past is necessary' [Mates, 1973, 38] or 'When anything has been the case, it cannot not have been the case' [Prior, 1955a, 210] allude to the truth value of some proposition. They mean the logical necessity, leading to a supposed strong determinism, or fatalism. On the other hand, those who translates the first premise as 'Everything that is past and true is necessary' [Hintikka, 1973, 180] might interpret what is necessary as something irrevocable, freeing Diodorus of ascribing necessity to past tense statements [Sorabji, 1980, 108]. *See*, section 1.3.2 about the Master Argument and, in particular, its first premises.

[58] Diodorus possibile esse determinat, quod aut est aut erit; inpossibile, quod cum falsum sit non erit verum; necessarium, quod cum verum sit non erit falsum; non necessarium, quod aut iam est aut erit falsum.

Boethius' account is one of the most detailed texts about Diodorus' modalities.[59] The fragment considers possibility and necessity in order to interdefine those with temporal notions. Furthermore, Boethius focuses on those notions dealing with their duals.

In order to have a visual representation of the modalities and their interrelations (between: possibility $\Diamond$, impossibility $\neg\Diamond$, necessity $\Box$, non-necessity $\neg\Box$), we present below the classical *square of opposition*.

Table 1.1: Modal square of Diodorus



The graphic is the square of opposition applied to Diodorus' modalities. It highlights relations and contrapositions in terms of temporal notions. The strongest opposition is contradiction, between what is necessary and what is not-necessary, and between what is possible and what is impossible: following the schema of Diodorus, *e.g.*, 'Fabius will run henceforth' (that is, the interdefinition for 'It is necessary that Fabius is running') and 'Sometimes Fabius is not running' (that is, the interdefinition for 'It is not-necessary that Fabius is running').

Formalising the modalities can be useful in order to analyse their relations and strength. Ancient texts do not refer to dated propositions, they deal with

[59] Other important sources being Boeth. *in Int.* 412, 16-17; Arr. *Epict.* 2, 19, 1-5; Cic. *Fat.* 12, 13, 17, and *Fam.* 9, 4; Plu. *Stoic. rep.* 1055e-f; Alex.Aphr. *in APr.* 183, 34 - 184, 6; Phlp. *in APr.* 169, 17-19; Simp. *in Cat.* 196, 4-6.

tensed sentences: propositions may be about the past, present, or future. We would refer to Bobzien [1998]'s formalism, and then, more generally, we would also consider the modalities using the adequate tools in order to obtain an account in tense logic aimed at chapter 2.

Table 1.2: Formalisation of Diodorus' modalities

$$(D/M)\ M_n[p] =_{df} \exists_t(t \geq n \wedge \mathbf{T}_t[p]), \text{ that is, } \Diamond p \equiv p \vee Fp$$
$$(D/\neg M)\ \neg M_n[p] =_{df} \forall_t(t \geq n \rightarrow \mathbf{F}_t[p]), \text{ that is, } \neg\Diamond p \equiv \neg p \wedge \neg Fp$$
$$(D/L)\ L_n[p] =_{df} \forall_t(t \geq n \rightarrow \mathbf{T}_t[p]), \text{ that is, } \Box p \equiv p \wedge Gp$$
$$(D/\neg L)\ \neg L_n[p] =_{df} \exists_t(t \geq n \wedge \mathbf{F}_t[p]), \text{ that is, } \neg\Box p \equiv \neg p \vee \neg Gp$$

In the age of Diodorus, modalities do not merely state in some technical sense the logical operators for propositions.[60] Some ancient texts, namely, S.E. *M.* 8, 70, 71, 73, 74; D.L. 7, 63, 65 - 66; Gell. 16, 8, deal with the theme of *λεκτὰ* and *ἀξιώματα*; these last are subclasses of what we define as proposition.

In some mature texts of the Stoics, propositions in which modalities occur are defined as *ἀξιώματα*. However, the term does not occur as strictly Diodorean. In fact, there was a clear evolution from sentences, assertions, to propositions or *ἀξιώματα* and, as indicated above, it concerns the history of *λεκτὰ*.[61] For instance, about the propositions of the ancient Master Argument, Arr. *Epict.* 2, 19, 1, never uses the word *ἀξιώματα*, but refers to *ἀφορμῶν*, namely 'starting points' like theoretical presuppositions and conditions of the *κυριεύων λόγος*.

Diodorus' modalities concern, at first, things, such the denotations of tensed sentences. Diodorus' account does not reject contingent propositions. Later, we will see how Diodorus' system is deterministic but nonetheless safeguards contingencies. This is strictly related to a logic of tense, rather than a tenseless logic, as the majority of modern accounts of logic are.

[60] Letters at the beginning of lines denote the author of the modality, *e.g.*, D for Diodorus. The view for possibility is labelled by M, by L is the notion of necessity; bold letters are to mark the metalinguistic 'It is *true/false* at a given time that ...'; $\forall$ and $\exists$ quantifies respectively on every instant, and on singular instants; n is the 'now' or present instant. In the second, Priorean, notation F and G are future tense operators. F stands for the future as 'It will be the case that...', and G for the strong future as 'It will always be the case that...'.

[61] *See* [Mates, 1973, 11-26].

As a preamble before to compare the modalities, in the way that they were interpreted during the Hellenistic age, is D.L. 7, 65, which explains the use of a proposition:

> When you say 'It is day', you seem to accept the fact that it is day. Now, if it really is day, the judgement before us is true, but if not, it is false [Hicks, 1925, 175].[62]

This text gives an example of the form of a typical temporalised but dateless sentence. Hence, to affirm that 'it is day' is correct *if and only if* it is daytime and you are uttering that sentence. In fact, in the Hellenistic age, the truth-criteria for atomic propositions about the past, present, or the future, defined as *enuntiationes* by Boethius, always include, implicitly, quantification over time [Bobzien, 1998, 100].

Boethius' definitions of Diodorus' modalities are part of a more general comparison on modalities. In particular, Boeth. *in Int.* 234, 10, discusses 'three theories of possibilities'. In addition to the Diodorean, the other two *doctrines on possibilities* are by Diodorus' best pupil, Philo, and by the Stoics.

Let us begin to compare Diodorus' modalities with some others. Here is Philo's definitions of modal notion in Boeth. *in Int.* 234, 10-22:

> Philo says that the possible is this: that the very nature of the statement is supportive of truth – as when I say that I am going to reread Theocritus' *Bucolica* today, [since] considered in itself, if nothing external prevents it, this can be truthfully asserted. At the same time, Philo defines the necessary in the same way to be [this:] that [the very nature of the statement is such that] when it is true, then considered in itself it can never be supportive of falsity. And he delimits the non-necessary as that which considered in itself can be supportive of falsity. But the impossible as that which in accordance with its

[62] ὁ γὰρ λέγων Ἡμέρα ἐστίν, ἀξιοῦν δοκεῖ τὸ ἡμέραν εἶναι. οὔσης μὲν οὖν ἡμέρας, ἀληθὲς γίνεται τὸ προκείμενον ἀξίωμα μὴ οὔσης δέ ψεῦδος.

> own nature can never be supportive of truth. But he confirms that the contingent and the possible are one [Blank and Kretzmann, 1998].[63]

A characteristic of Philo's account is the identity between the contingent and the possible, as Boethius remarks in the last part of the text. Philo's modalities, can be comparatively presented by the following square of opposition.

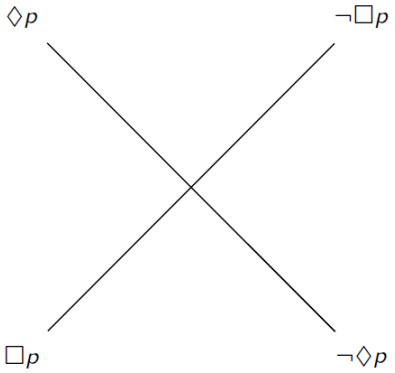
Table 1.3: Modal square of Philo

$\Diamond p$: *Quod natura propria enuntiationis suscipiat veritatem*

$\neg\Diamond p$: *Quod secundum propriam naturam numquam possit sucipere veritatem*

$\Box p$: *Quod cum verum sit, quantum in se est, numquam possit susceptivum esse mendacii*

$\neg\Box p$: *Quod quantum in se est possit suscipere falsitatem*



In order to understand the meaning of Philo's notion of possible, Alex.Aphr. *in APr.* 184, 6-10, provides a good track:[64]

> And likewise about Philo's answer. This was: 'That which is predicated in accordance with the bare fitness of the subject, even if it is prevented from coming about by some necessary external factor.' On this basis he said that it was possible for chaff in atomic dissolution

[63] Philo enim dicit possibile esse, quod natura propria enuntiationis suscipiat veritatem, ut cum dico me hodie esse Theocriti Bucolica relecturum. Hoc si nil extra prohibeat, quantum in se est, potest veraciter praedicari. Eodem autem modo idem ipse Philo necessarium esse definit, quod cum verum sit, quantum in se est, numquam possit susceptivum esse mendacii. Non necessarium autem idem ipse determinat, quod quantum in se est possit suscipere falsitatem. Inpossibile vero, quod secundum propriam natura numquam possit suscipere falsitatem. Inpossibile vero, quod secundum propriam naturam numquam possit sucipere veritatem. Idem tamen ipse contingens et possibile unum esse confirmat.

[64] *See* also Simp. *in Cat.* 196, 1-2.

> to be burnt, and likewise chaff at the bottom of the sea, while it was there, even though the circumstances necessarily prevented it [Long and Sedley, 1987, 231].[65]

The above text prompts the following considerations. The Philonian clarification of the possible looks at propositions which can be evaluated without reference to any specific time or circumstance: *e.g.*, we can rightly say that chaff or maybe some piece of wood possibly burns, even if it would be at the bottom of the sea, that is, though it never actually burn. Clearly, it comes to light that Philo's notion of possible is less strong than the respective Diodorean notion. In fact, Diodorus established that for every assertion that we may classify as possible, there will exist some instant of time in which the claim it makes is or will be true. Namely, what the proposition claims will occur. So, following Diodorus, a sentence prefixed by the modal notion of possibility may change its truth value, while following Philo, a proposition prefixed by possibility does not change its truth value.[66] The schema below represents formally Philo's modalities. Philo's modal notions state mere dispositions or natural aptitudes of a thing. At first, Philo's modalities are relating to the interpretation for possibility. It is useful to follow Todd [1972, 25-29], which used the suggestion of *ἐπιτηδειότης*.

[65] *ὁμοίως καὶ περὶ τοῦ κατὰ Φίλωνα ἦν δὲ τοῦτο τὸ κατὰ ψιλὴν λεγόμενον τὴν ἐπιτηδειότητα τοῦ ὑποκειμένου, κἂν ὑπό τινος ἔξωθεν ἀναγκαίου ᾖ γενέσθαι κεκωλυμένον. οὕτως τὸ ἄχυρον τὸ ἐν τῇ ἀτόμῳ ἢ τὸ ἐν τῷ βυθῷ δυνατὸν ἔλεγε καυθῆναι ὃν ἐκεῖ, καίτοι κωλυόμενον ὑπὸ τῶν περιεχόντων αὐτὸ ἐξ ἀνάγκης.*

[66] Some further analysis makes clear the issue. According to Diodorus' view: to say 'possible that a given piece of wood burns' means that there is now or in some instant of the future a piece of wood which is burning. Hence, if that piece of wood really burns in some moment of time, then the quoted proposition is true before that fact and when it is burning. However, we would hypothesise that if a piece of wood is sinking at the bottom of sea and therefore it never burns, then the previous proposition would change its truth value from true to false. According to Philo if that piece of wood has a particular natural disposition to burn, and of course this assertion is true, Philo takes the above mentioned proposition to be always true, even in the case that the piece of wood is at the bottom of sea. Ultimately, we can deduce that Philonean truth values do not change.

Table 1.4: Formalisation of Philo's modalities

$$(P/M)\ M_n[p] =_{df} \exists_t(\mathbf{T}_t[p^\epsilon])$$
$$(P/\neg M)\ \neg M_n[p] =_{df} \forall_t(\mathbf{F}_t[p^\epsilon])$$
$$(P/L)\ L_n[p] =_{df} \forall_t(\mathbf{T}_t[p^\epsilon])$$
$$(P/\neg L)\ \neg L_n[p] =_{df} \exists_t(\mathbf{F}_t[p^\epsilon])$$

The Philonian schema of modality is unusual, since it weighs the aptitude of a subject rather than the effective and temporal definition.[67] This was already remarked by [Kneale and Kneale, 1962, 122]. The authors refer to external circumstances, while the subject is confirmed to be the possessor of a kind of aptitude. So, *e.g.*, according to Diodorus, something possible concerns a present or future occurrence, while according to Philo, the occurrence of something defined as possible is not taken for granted in time. The criterion of possibility in Philo is just the *internal consistency*, that is the not-contradictoriness of a proposition [Bobzien, 1998, 109].

Boethius also takes into account the Stoic treatment of modalities.[68] We would quote D.L. 7, 75, as an alternative source, since it gives the complete list of four modalities, alongside some examples of propositions:

> Again, some things are possible, others impossible; and some things are necessary, others are not necessary. A proposition is possible which admits of being true, there being nothing in external circumstances to prevent it being true, *e.g.*, 'Diocles is alive'. Impossible is one which does not admit of being true, as *e.g.*, 'The earth flies'. That is necessary which besides being true does not admit of being false

[67] The ϵ-index on p denotes an internal aptitude to fulfill what the proposition announces, ϵ is in relation to *ἐπιτηδειότης*.

[68] Boeth. *in Int.* 234, 27 - 235, 9: The Stoics have set up the possible as that which would be supportive of true predication when [other things] which can happen together with it (even though they are external to it) do not in any way prevent it; the impossible, as that which is never supportive of any truth with other things external to its own outcome preventing it; the necessary as that which when it is true is not supportive of false predication for any reason [Blank and Kretzmann, 1998] – Stoici vero possibile quidem posuerunt, quod susceptibile esset verae praedicationis nihil his prohibentibus, quae cum extra sint cum ipso tamen fieri contingunt. Inpossibile autem, quod nullam umquam suscipiat veritatem aliis extra eventum ipsius prohibentibus. Necessarium autem, quod cum verum sit falsam praedicationem nulla ratione suscipiat.

> or, while it may admit of being false, is prevented from being false by circumstances external to itself, as 'Virtue is beneficial'. Not necessary is that which *is true and opened to be false, if external circumstances do not hinder it*,[69] *e.g.*, 'Dion is walking'. A reasonable proposition is one which has to start with more chances of being true than not, *e.g.*, 'I shall be alive tomorrow' [Hicks, 1925].[70]

Here is the square representing the Stoics's view of modalities:[71]

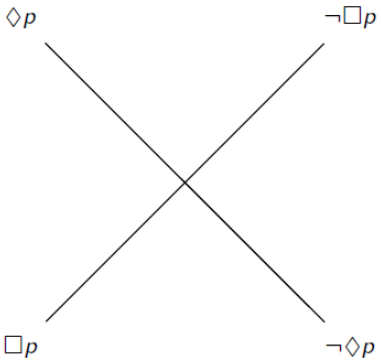
Table 1.5: Modal square of Stoics

◇*p*: δυνατὸν μὲν τὸ ἐπιδεκτικὸν τοῦ ἀληθὲς εἶναι, τῶν ἐκτὸς μὴ ἐναντιουμένων εἰς τὸ ἀληθὲς εἶναι

¬◇*p*: ἀδύνατον δὲ ὃ μή ἐστιν ἐπιδεκτικὸν τοῦ ἀληθὲς εἶναι [ἢ ἐπιδεκτικὸν μέν ἐστι, τὰ δ'ἐκτὸς αὐτῷ ἐναντιοῦται πρὸς τὸ ἀληθὲς εἶναι]

□*p*: ἀναγκαῖον δέ ἐστιν ὅπερ ἀληθὲς ὂν οὐκ ἔστιν ἐπιδεκτικὸν τοῦ ψεῦδος εἶναι, ἢ ἐπιδεκτικὸν μέν ἐστι, τὰ δ'ἐκτὸς αὐτῷ ἐναντιοῦται πρὸς τὸ ψεῦδος εἶναι

¬□*p*: οὐκ ἀναγκαῖον δέ ἐστιν ὃ καὶ [εἰ] ἀληθές ἐστιν καὶ ψεῦδος οἷόν τε εἶναι, τῶν ἐκτὸς μηδὲν ἐναντιουμένων [εἰ τὸ ψεῦδος εἶναι]



[69] We substitute the emphasised text of the last part of Hicks' translation with our own translation. Hicks [1925, 185] says 'which, while true, yet is capable of being false if there are no external conditions to prevent'. Our translation is more appropriate because it avoids every misinterpretation related to the theme of *capacity* by keeping more closely to Diogenes Laertius's text.

[70] Ἔτι τε τὰ μέν ἐστι δυνατά, τὰ δ' ἀδύνατα καὶ τὰ μὲν ἀναγκαῖα, τὰ δὸὐκ ἀναγκαῖα. δυνατὸν μὲν τὸ ἐπιδεκτικὸν τοῦ ἀληθὲς εἶναι, τῶν ἐκτὸς μὴ ἐναντιουμένων εἰς τὸ ἀληθὲς εἶναι, οἷον "ζῇ Διοκλῆς" ἀδύνατον δὲ ὃ μή ἐστιν ἐπιδεκτικὸν τοῦ ἀληθὲς εἶναι, οἷον "ἡ γῆ ἵπταται." ἀναγκαῖον δέ ἐστιν ὅπερ ἀληθὲς ὂν οὐκ ἔστιν ἐπιδεκτικὸν τοῦ ψεῦδος εἶναι, ἢ ἐπιδεκτικὸν μέν ἐστι, τὰ δ'ἐκτὸς αὐτῷ ἐναντιοῦται πρὸς τὸ ψεῦδος εἶναι, οἷον "ἡ ἀρετὴ ὠφελεῖ." οὐκ ἀναγκαῖον δέ ἐστιν ὃ καὶ ἀληθές ἐστιν καὶ ψεῦδος οἷόν τε εἶναι, τῶν ἐκτὸς μηδὲν ἐναντιουμένων, οἷον τὸ "περιπατεῖ Δίων." εὔλογον δέ ἐστιν ἀξίωμα τὸ πλείονας ἀφορμὰς ἔχον εἰς τὸ ἀληθὲς εἶναι, οἷον "βιώσομαι αὔριον."

[71] We follow [Bobzien, 1998, 112, n. 40] in order to reformulate the four propositions of the square.

One can see that the Stoics – almost certainly Chrysippus – mediate between Diodorus' view and Philo's view. In particular, the Stoics adopt Philo's modal definitions and add clauses to underscore some *de facto* potential fulfilment of given occurrences. At the same time, they use temporal parameters, just like Diodorus.[72]

Table 1.6: Formalisation of Stoic modalities

$$
\begin{array}{c}
(S/M)\ M_n[p] =_{df} \exists_t(t \geq n \wedge (\mathbf{T}_t[p^\epsilon] \wedge \neg H_t, \mathbf{T}_t[\neg p])) \\
(S/\neg M)\ \neg M_n[p] =_{df} \forall_t(t \geq n \rightarrow (\mathbf{F}_t[p^\epsilon] \vee (\mathbf{T}_t[p^\epsilon] \wedge (H_t, \mathbf{F}_t[p])))) \\
(S/L)\ L_n[p] =_{df} \forall_t(t \geq n \rightarrow (\mathbf{T}_t[p^\epsilon] \vee (\mathbf{F}_t[p^\epsilon] \wedge (H_t, \mathbf{T}_t[p])))) \\
(S/\neg L)\ \neg L_n[p] =_{df} \exists_t(t \geq n \wedge (\mathbf{F}_t[p^\epsilon] \wedge \neg H_t, \mathbf{T}_t[\neg p]))
\end{array}
$$

Diodorus', Philo's, and Stoics' modalities, like Hellenistic modalities, share the same general traits:

1. Possibility and impossibility are mutually contradictory and so are necessity and non-necessity;
2. Necessity and possibility are interdefinable;
3. Necessity entails truth, and truth entails possibility;
4. Every proposition is either necessary or impossible or contingent.

There are also significant differences between the three doctrines on modalities:

1. The Diodorean modalities concern notions in relation to time and, *e.g.*, they are suitable to be dealt with the tensed logical approach of Prior;
2. The Philonean modalities concern dispositions or ἐπιτηδειότης of individuals, and time is considered only *in secunda facie*;

[72] In the Stoic schema of modalities H is a characteristic operator on p, translating the Greek ἐκτὸς, and denotes external circumstances or events which might *hinder* the realisation of some potential internal aptitude, namely ϵ.

3. The Stoics opt for a compromise between Diodorus' and Philo's doctrines: they embrace Philo's definitions, but add a further proposition, by conjunction[73] or disjunction[74], in order to specify the condition about the prospective concrete and temporal completion of the internal disposition, also referring to external circumstances.

1.3.1.3 Why Diodorus is a determinist, not a fatalist

Let us try to follow the Diodorean view. For instance, we might suppose some alleged soothsayer, his prophesy about the future, and define propositions only by possibility. However, we do not heed to him, since Diodorus did not take interest in predictions, contrary to Chrysippus.[75] Instead, we – and also Diodorus – might talk about events or occurrences, which are supposed to be possible. If these correspond to the truth, what is said to be possible is supposed to be fulfilled, maybe now or in some instant of the future. In that case, a proposition is both possible and not-necessary at the same time: that is what contingent means.

Furthermore, every proposition can state something about the past, present or future, and sometimes a given proposition is used for different events. *E.g.*, 'there will be a storm' may refer to the storm that will occur tomorrow, or to that which will occur on January 17, 2061. Every Hellenistic proposition is tensed and, according to its canonical form, expressed without any temporal index. Hellenistic dialecticians do not consider dated propositions [Bobzien, 1998, 99-100].

The three conceptions of modality examined present some differences. In particular, the Diodorean has no room for possibilities that are *counterfactuals*, whereas both the Philonian and the Stoic allow it. The Philonian criterion seems moulded exactly on counterfactuals: we usually agree that it is possible to burn

[73] These are the cases of modal possibility, $M[p]$, and not-necessity, $\neg L[p]$.
[74] These are the cases of modal necessity, $L[p]$, and not-possibility, $\neg M[p]$.
[75] *See*, Cic. *Fat.* 7, 13

a piece of wood. What could be a counterfactual example to this possibility, namely a case in which a piece of wood would not burn? It may be the case of driftwood in the sea. Therefore, if, according to Simp. *in Cat.* 196, 1-2, we define as possible that a piece of wood burns, even if it is in the sea, then we are in the exact case of a counterfactual possibility.

Also the Stoic criterion is open to counterfactual possibilities.[76] Following D.L. 7, 75, the realisation of what is declared as possible might depend on external circumstances. Sometimes, these prevent that (i) occurrences be fulfilled, and (ii) that the respective sentences be evaluated as true.

With respect to counterfactual possibilities, the case of Diodorus is not alike that of Philo or of the Stoics. Diodorus' modal system does not admit that occurrences may take place differently in relation to the reality. We affirm, with Bobzien [1998, 106], that Diodorus' theory allows no counterfactual possibilities and no factual non-necessities. This does not concern propositions nor, for that matter, does it *strictu sensu* concern a modal logic. Diodorus' theory refers to a specific conception of the world, namely determinism.

On theoretical grounds, Diodorus is among those philosophers, as Williamson [2013, 1] reports, who deny that the contingent is what there is, holding that *what things are, are by necessity*, while allowing that *how* those things are is contingent.

In Didorus's system 'Paul is going to the beach' is a contingent proposition, since it is both possible and not-necessary. But, when Paul is going to the beach, we should say that necessarily Paul is going to the beach, and absolutely it is not the case that Paul is not doing it. The previous utterance does not mean that 'it is necessary that Paul is going to the beach'. In fact, the last sentence, considered as a proposition, and following the Diodorean meaning, would be that Paul is going to the beach from now and hereafter; paradoxically, if he is always going to the beach, then he will never be at the beach.

[76]The difference of opinion between Diodorus and Chrisyppus is remarked also by Hieron. *Adv. Pelag.* 1, 702. Moreover, Plut. *Stoic. rep.* 46, 1055 d-e, expresses some doubts about the way in which Chrisyppus holds two conflicting theses, namely, counterfactual possibilities and fate, while he focuses on the debate that opposed the Stoic and, in particular, Diodorus' view.

Let us consider again the notions of possibility and impossibility, starting from the standpoint of the Megaric ontology. Diodorus admits as possible what is or will be, otherwise we deal with an impossibility. But, what by definition we call 'impossible', ontologically is said as *non ens*. Furthermore, from a logical perspective it would be stated by prefixing the sentence with the connective ¬. This produces a negated formula. Then, the resulting formula denotes a non-object, something that does not become aught, neither occurs as an existent. According to [Hartmann, 1937], there is no way to affirm something as an existent by referring to a negative-possible, because it would be lacking in *real-conditions* in order to be. If being is real, then also its possibility has to be real. In this way Diodorus' view would entail the Megaric conception of being, ruling out every feature of that which is not real.

Let us return to propositions: in relation to the impossible, and also to the necessary, following Bobzien [1998, 104-105], once a proposition has assumed one of these modalities, it cannot change. So, as we will argue in the next section, as a consequence of that, Diodorus maintains that all true propositions about what happened in the past are necessary. On the contrary, it is not the case that every false proposition about what happened in the past deals with the impossible. Nevertheless, all the false propositions about what will happen in the future are impossible. Therefore, we agree with Bobzien [1998] that even if Diodorus' modal system does not preclude contingency, his modal theory is not indeterministic.

Diodorus' modal theory is strictly correlated with his doctrine of time. And although contingencies have only a marginal role in the whole, this is decisive in order to transmit to us the knowledge of the passage of time. In fact, according to [Denyer, 1981b, 51], the passage of time consists in the elimination of the contingencies, giving definitive judgment to open question. Then, Diodorus may hypothesise a halt of the time when there is no change, *i.e.*, no unrealised possibilities remain.

We should interpret Cic. *Fam.* 9, 4 not as fatalism, but in light of determinism. Furthermore, according to Cicero, we can also focus on Diodorus' modalities and their interdefinition with temporal notions. Below is the passage of Cic. *Fam.* 9, 4:

> About things *possible* you must know that I judge according to Diodorus. And therefore if you are about to come, you must know that your coming is *necessary*; if you are not, then your coming is *impossible* [Bailey, 2001].[77]

This fragment has been written in a confidential style, being addressed to Cicero's friend, Varro; nonetheless, *Fam.* 9, 4, contains technical terms pertaining to modalities. The authenticity of Cicero's language confirms that he is referring to the original schema of Diodorus. At any rate, it is certain that Cicero's topic concerns things, not propositions. Ultimately, nothing else is possible in relation to what Varro will accomplish. And, if 'no alternative to tomorrow's events is possible, then tomorrow's events are necessary' [Sorabji, 1980, 105].

At first blush, *Fam.* 9, 4, might seem used by Cicero, in order to validate a fatalistic outlook,[78] but, on the contrary, we argue that it is not so. Cicero is not examining any proposition: he is just announcing to Varro the chance that a given event happens, namely the supposed occurrence that Varro will visit him. But Cicero is not referring to the sentence 'it is necessary that Varro visits Cicero'. If that were the case, Cicero would be committing an error in relation to the modal notion of necessary, that is, following Boeth. *in Int.* 234, 22-26, what is and will be true henceforth. Actually, Cicero is asserting that, for every event, if something occurs, then it necessarily happens; that is, it cannot be different, since it *de facto* occurs in a given way. Therefore, according to Diodorus' determinism, if events follow the metaphysical necessity of *being*, then it is not the

[77] Περὶ δυνατῶν me scito κατὰ Διόδωρον κρίνειν. Quapropter, si venturus es, scito necesse esse te venire; sin autem non es, ἀδύνατον est te venire.

[78] A close examination of this view on the topic is introduced in [White, 1985] at the third chapter.

case of counterfactual occurrences, worlds, or possibilities; further, if anything is possible, it either occurs or will occur.

We agree with Beere [2009, 96] when he says that 'there may well be no sufficient causal conditions for an event, short of the actual occurrence of the event. This version of Megaricism focuses not on causation, but on the temporal structure of can-claims'.[79] In fact, the determinism of Diodorus Cronus is not a sort of fatalism. Speaking on this very topic, Cic. *Fat.* 9, 20, explains that:

> Nor do those who say that the things that are going to be are unchangeable, and that a future truth cannot be turned into a falsehood, establish the necessity of fate; rather, they are explaining the meaning of words. It is those who introduce an eternal series of causes who rob the mind of free will and bind it in the necessity of the fate [Sharples, 1991, 71].[80]

On the basis of Cic. *Fat.* 9, 20, we can hypothesise that Diodorus has demonstrated how his system embraces contingent propositions in a deterministic and non-counterfactual world. In fact, by using contingent propositions we can debate and hypothesise also in connection with what is said to be possible, and we can interpret our world by modal notions in relation to time. For instance, 'John has finished reading this thesis chapter' is a contingent proposition according to the Hellenistic tradition. It is both possible and non-necessary, and it refers to the exact occurrence uttered by the previous quoted sentence. But when modalities as logical operators are in question, it means that some of those should be added as *incipit* of the previous sentence. *E.g.*, (i) 'It is possible that John has finished reading this thesis chapter', or (ii) 'It is necessary that John has finished reading this thesis chapter'. Following Diodorus' doctrine this

[79] On the contrary, we do not agreee when Beere [2009, 97] assumes the interpretation of a causal version of Megaricism, because the concept of causation is derived by the Stoics.

[80] Nec ii qui dicunt immutabilia esse quae futura sint, nec posse verum futurum convertere in falsum, fati necessitatem confirmant, sed verborum vim interpretantur; at qui introducunt causarum seriem sempiternam, ii mentem hominis voluntate libera spoliatam necessitate fati devinciunt.

means that: (i) John has just now finished reading this thesis chapter, or there will be some given instant in the future in which he will surely do it; or (ii) John has finished reading this thesis chapter, and according to the principle that *everything that is past and true is necessary*,[81] the sentence 'John has finished reading this thesis chapter' is true now and hereafter.

1.3.1.4 Δυνατὸν and δύναμις: Diodorus, the Hellenistic debate, and a comparison with Aristotle

We wrap up this historical analysis by comparing the Hellenistic theories of modalities with Aristotle's perspective on δυνατῶν. For this purpose, let us begin by considering an ancient text treating each of the already mentioned doctrines – Philo's, Diodorus', and the Stoics. This longer text is Simp. *in Cat.* 195, 31 - 196, 24:

> [About the δυνατῶν] they say, 'how do we judge (κρινοῦμεν) the perceptible (αἰσθητὸν) and the knowable (ἐπιστητὸν)? Is it by aptitude (ἐπιτηδειότης) alone, as Philo says, even if there is no knowledge of it, and no likelihood of any, just as the piece of driftwood in the middle of the Atlantic is combustible in itself and according to its nature? Or must we then judge such things by unhindered aptitude, in so far as they are naturally suited to be subjects of knowledge or perception *per se* provided no evident hindrance prevents it? Or is it neither of these, and is the knowable spoken of when there is knowledge of it or when there is likely to be, and when the δυνατῶν is likely to be judged by the outcome?' But the commentators now reject the judgement of these matters on the grounds that it depends on a very difficult view περὶ δυνατῶν, and they discuss Aristotle's statement here in terms of the philosophical school it fits and which it does not fit. For when Aristotle [*Cat.* 7b 30-31, 8a 3-4] declares that even if knowledge does not exist the knowable does, and even

[81] That is, the first principle of the Master Argument.

> if every animal is removed the perceptible is not removed with it, the judgement concerning δυνατοῦ is transferred to mere aptitudes; but when it is said by some to be entirely unknowable unless there is some knowledge of it, it is then according to them that the potential is judged by the outcome. It is agreed that if the knowable does not exist then there is no knowledge; for it is from the knowable that any activity concerning it comes-to-be; but that it is *possible* for the knowable to exist if knowledge does not, some agree by judging τὸ δυνατὸν only according to aptitude (for it has a nature appropriate for being known), while others who test by the outcome do not agree, unless it is undoubtedly going to reach an evident result. Consider how absurd is the position of those who on the one hand judge τὸ δυνατὸν in the way the ancients did, in terms of any sort of aptitude, as Philo did, but on the other hand now find it problematic according to the view of Diodorus who judges τὸ δυνατὸν by the outcome, and who use the fact that the knowable exists when knowledge does not, as an objection against this view [Fleet, 2014].[82]

[82] Ἀποροῦσι δὲ καὶ ἄλλας ἀπορίας οἰκείας τῷ περὶ δυνατῶν λεγομένῳ σκέμματι καὶ ἐξ ἐκείνου δυναμένας διακρίνεσθαι. "πῶς γὰρ δή, φασίν, τὸ αἰσθητὸν καὶ τὸ ἐπιστητὸν κρινοῦμεν; πότερον τῇ ἐπιτηδειότητι μόνῃ, ὥσπερ Φίλων ἔλεγεν, κἂν μήτε ᾖ μήτε μέλλῃ γίνεσθαι αὐτοῦ ἐπιστήμη, ὥσπερ τὸ ἐν τῷ Ἀτλαντικῷ πελάγει ξύλον καυστόν ἐστιν ὅσον ἐφ' ἑαυτῷ καὶ κατὰ τὴν ἑαυτοῦ φύσιν; ἢ ἄρα τῇ ἀκωλύτῳ ἐπιτηδειότητι κριτέον τὰ τοιαῦτα, καθ' ὅσον πέφυκεν ὑποπίπτειν ἐπιστήμῃ καὶ αἰσθήσει καθ' ἑαυτὰ μηδενὸς φανεροῦ κωλύματος ἐνισταμένου; ἢ τούτων μὲν οὐδέτερον, τὸ δὲ ἐπιστητὸν λέγεται, ὅταν ἐπιστήμη αὐτοῦ ᾖ ἢ μέλλῃ ἔσεσθαι καὶ μέλλῃ τῇ ἐκβάσει κρίνεσθαι τὸ δυνατόν;". ἀλλὰ τὴν μὲν ἐπίκρισιν τούτων ὡς τῆς περὶ δυνατῶν χαλεπωτάτης θεωρίας ἐχομένην παραιτοῦνται νῦν οἱ ἐξηγηταί, τοσοῦτον δὲ μόνον τὰ νῦν λεγόμενα διακρίνουσιν, ποίᾳ αἱρέσει συνᾴδει τῶν εἰρημένων καὶ πρὸς ποίαν ἀναρμοστεῖ. καὶ γὰρ ὅταν μὲν ὁ Ἀριστοτέλης διορίζηται ὡς ἐπιστήμης μὴ οὔσης τὸ ἐπιστητὸν ἔστιν καὶ παντὸς ζῴου ἀναιρεθέντος οὐ συναναιρεῖται τὸ αἰσθητόν, ἐπὶ τὰς ψιλὰς ἐπιτηδειότητας ἡ τοῦ δυνατοῦ κρίσις ἀποφέρεται ὅταν δὲ μηδὲ ὅλως εἶναι ἐπιστητὸν λέγηται ὑπό τινων, ἐὰν μὴ ᾖ ἐπιστήμη αὐτοῦ, τότε τῇ ἐκβάσει κρίνεται τὸ δυνατὸν κατὰ τούτους. καὶ ὅτι μὴ ὄντος ἐπιστητοῦ οὐκ ἔστιν ἐπιστήμη, συγχωρεῖται ἀπὸ γὰρ τοῦ ἐπιστητοῦ γίνεται καὶ ἡ περὶ αὐτοῦ ἐνέργεια ὅτι δὲ μὴ οὔσης ἐπιστήμης δυνατὸν εἶναι τὸ ἐπιστητόν, οἱ μὲν κατὰ τὴν ἐπιτηδειότητα μόνην κρίνοντες τὸ δυνατὸν συγχωροῦσιν (ἔχει γὰρ τὴν οἰκείαν φύσιν πρὸς τὸ γινώσκεσθαι), οἱ δὲ τῇ ἐκβάσει δοκιμάζοντες οὐ συγχωροῦσιν, ἐὰν μὴ πάντως μέλλῃ εἰς ἔργον ἐναργὲς προχωρήσειν. ὅρα οὖν ὅπως ἄτοπον πάσχουσιν οἱ κρίνοντες μὲν τὸ δυνατὸν κατὰ τὸν αὐτὸν τοῖς ἀρχαίοις τρόπον, κατὰ τὴν ὁποιανοῦν ἐπιτηδειότητα, ὥσπερ ὁ Φίλων, ἀποροῦντες δὲ πρὸς αὐτὸν νῦν κατὰ τὴν Διοδώρου ἔννοιαν † ταύτῃ τῇ ἐκβάσει τὸ δυνατὸν κρίνοντος καὶ ὡς ἔνστασιν πρὸς ταύτην κομίζοντες τὸ τὸ ἐπιστητὸν ὡς ἐπιστητὸν εἶναι μὴ οὔσης ἐπιστήμης.

This quotation is very relevant since it discloses in a specific way the theme of δυνατῶν. In fact, the category of the possible is discussed in relation to *judgment*, *perception*, and *scientific knowledge*. From Simplicius' account we recover in a certain sense the doctrine of Arist. *Metaph.* 9. 3. At this stage Simplicius both reveals the name of the putative interlocutor and opponent of the Stagirite and considers as well the development of his doctrine.

When we are talking about chance of changing or realisation of something, we do not merely consider *capacity*, but modal notions as well. In turn, these last are definable by means of temporal notions. However, scholars often use indiscriminately the notion of potency or *capacity* and the notion of *possibility*, as Fleet [2014, 52-53] does.[83]

The first dialectician quoted by Simplicius is Philo. Philo's interpretation of the possible highlights the specific characterisation of ἐπιτηδειότης. Hence, what is possible subsists independently of both the perception and the scientific knowledge of the thing that we are evaluating. Thereafter, Simplicius introduces, without quoting them, the Stoic view about the notion of possible: while according to Philo things have to be evaluated by the criterion of aptitude or disposition, for the Stoics one must also consider the actual knowledge, which assumes perception or intellection of thing, this means, by adding evaluation of external circumstances which entail or hinder that something occurs. And so, the third author to which Simplicius alludes is Diodorus Cronus. The approach of Simpl. *in Cat.* 196, 4-6, is very original in relation to other fragments about the Megaric dialectician: scientific knowledge gains the same temporal connotation as that which is defined as the possible. In fact, what is now or will be in some future instant is classified as possible by Diodorus. Similarly, there exists knowledge when knowing is or will be (actual). Thus, according to Simplicius's account, the possible is likely to be judged by the outcome.

[83] In the previous English translation of Simp. *in Cat.* 195, 31 - 196, 24, we recognise the term δυνατὸν as an equivalent for *possible* and do not embrace the term potency or potential. We consider these last to refer to capacity or δύναμις. On the contrary, in his translation, Fleet [2014, 53] uses the term *possible* only one time out of seven occurrences of δυνατὸν, while *potential* turns out to be preferred in six case.

This fragment can unveil some essential differences about the interpretation of the concepts of possible and potential in Diodorus. And, for instance, we would recall the formalisation of the first notion as the modal operator for possibility; for the second, in relation to a given property, in that case, *e.g., knowing*.

Another important remark in Simpl. *in Cat.* 196, 4-6, is the link with *Metaph.* 9. 3, 29-33: does Simplicius allude to the Diodorean doctrine of capacity, in reply to Aristotle's doctrine against the Megarics? We think so, but, of course, this remains only a fascinating supposition.

Further, Simpl. *in Cat.* 196, 4-6, can also testify for Diodorus' determinism, ruling out insinuations about fatalism. What is defined as possible, is evaluated on its outcome, not on predictions, nor on divination or fate.

Sometimes, Aristotle and Diodorus have been radically counterposed in respect to their views on the possible, and about modalities in general. However, their strongest divergence is on the topic of *capacity*. Boeth. *in Int.* 412, 8-30, 413, 1-16, gives a useful analytic comparison between Aristotle and Diodorus on possibility:

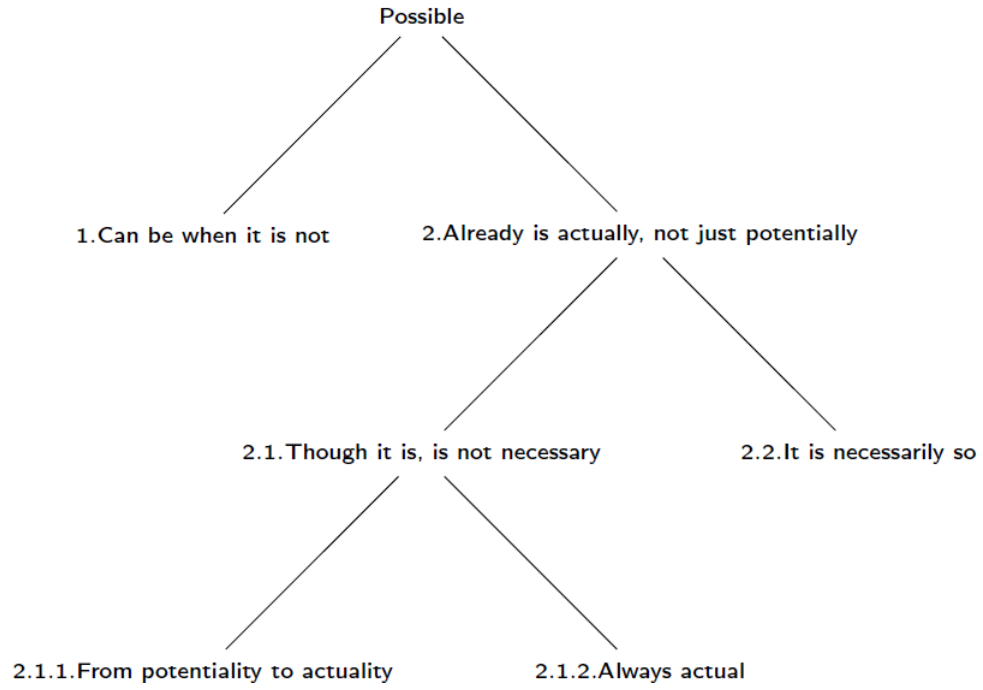
> Thus the possible has in principle two aspects, one whereby something can be when it is not, the other which is said of what already is something actually and not just potentially. And the latter kind of possibility which is already actual yields from itself two species: one which, though it is, is not necessary, the other which, though it is, has also the characteristic that it is necessarily so. And it is not just Aristotle's subtle mind that discovered this. In fact, Diodorus also defined the possible as 'what is or will be'. Hence Aristotle thinks that Diodorus' 'will be' is the aspect of possibility which can be when it is not, and his 'is' is what is said to be possible because it already actually is. We have laid down that this latter kind of possibility has two aspects, one we called necessary, the other we described as not necessary. But the not necessary kind also has two aspects, one which

> moves from potentiality to actuality, the other which was always actual from the first moment of existence of the thing which possesses possibility. And the one which moves from potentiality to actuality is open to contradiction on both sides, *e.g.* I, who am now writing, have moved from potentiality to actuality and whilst actually writing can write. For before I was writing, the potentiality of writing was in me, but I came from the potentiality of writing to the actuality of writing. Thus both, not writing and writing, fit my situation; for I can not write and I can also write, which is a sort of contradiction. And so whatever has come from potentiality to actuality, can both do and do not, be and not be. *E.g.* take a man who speaks; because he was able to speak before he does speak and now can speak because he is speaking, he both can speak and can not speak. But the other kind of possibility, which was never in potentiality beforehand but always actual from the first moment of existence of what is said to be something potentially, is suited for one thing only. *E.g.* fire was never potentially hotso that it afterwards was felt to be actually hot, nor was snow potentially cold before, and then actually so afterwards, but fire was actually hot from the time it came into being, and snow actually cold from its first existence [Smith, 2014, 96].[84]

[84]Cum igitur principaliter possibilis duae sint partes: una quae secundum id dicitur quod cum non sit esse tamen potest, altera quae secundum id praedicatur quod iam est aliquid actu non solum potestate, huiusmodi possibile quod iam sit actu duas ex se species profert: unam quae cum sit non est necessaria, alteram quae cum sit illud quoque habet ut eam esse necesse sit. Nec hoc solius Aristotelis subtilitas deprehendit, verum Diodorus quoque possibile ita definit: quod est aut erit. Unde Aristoteles id quod Diodorus ait erit illud possibile putat quod cum non sit fieri tamen potest, quod autem dixit Diodorus est id possibile Aristoteles interpretatur quod idcirco dicitur esse possibile, quia iam actu est. Cuius possibilitatis | modi duas partes esse docuimus: unam quam necessariam dicimus, alteram quam non necessariam praedicamus. Huius autem non necessariae duae rursus partes sunt: una quae a potestate pervenit ad actum, altera quae semper actu fuit, a quando res illa quae susceptibilis ipsius est fuit. Et illa quidem quae a possibilitate ad actum venit utriusque partis contradictionis susceptibilis est, ut nunc ego qui scribo ex potestate ad actum veni et agens possum scribere. Ante enim quam scriberem erat mihi scribendi potentia, sed ex potestate scribendi veni ad actum scribendi. Quare utraque mihi couveniunt et non scribere et scribere. Possum enim et non scribere, possum et scribere, quae est quodammodo contradictio. Atque ideo quaecumque ex potestate ad actum venerunt, ea et facere possum et non facere et esse et non esse, ut qui loquitur, quia autea potuit loqui quam loqueretur et nunc ideo potest loqui quia loquitur, et potest loqui et potest non loqui. Alia vero quae numquam aute potestate fuit, sed semper actu, a quando res ipsa fuit quae aliquid potestate esse diceretur, ad unam rem tantum apta est, ut ignis numquam fuit potestate calidus,

If Boethius report is accurate, then Diodorus Cronus and Aristotle share a very similar account of possibility; indeed, they would held different views on capacity, but not on the modal notion of possible. The tree-diagram below explicates what Boeth. *in Int.* 412, 8-30, 413, 1-16, means to say.

Table 1.7: The notion of possible: Boeth. *in Int.* 412-413



The Boethian tree of the possible clarifies in important ways the complementarity between Aristotle and Diodorus on the notion of possible. As we can see, the differences are restricted to the specific case represented by node 2.1.1, namely 'from potentiality to actuality'. The principle of *becoming* would be validated in virtue of the doctrine of capacity by Aristotle. But Diodorus does not admit of *changing*, ruling it out both in physics, logic, metaphysics, and in particular also for modalities. It is plausible that Diodorus proposed some alternative doctrine of capacity, counterposing that, as a Megaric, to the Aristotelian view.

According to Boethius, both 1, and 2, there are common principles appealed to by Diodorus and Aristotle in order to describe the notion of possible. And so, the crucial issue results from Diodorus' definition of the possible,

ut postea actu calidus sentiretur, nec nix ante frigida potestate, post actu, sed a quando fuit ignis actu calidus fuit, a quaudo nix actu frigida.

as referred by Boeth. *in Int.* 234, 23-24: to affirm that *possible* is 'quod aut est aut erit' means that, by substitution, *possible* is 'something already actual and not just potential (that is, the node 2), or something that can be when it is not (that is, the node 1). Therefore, a first level definition of the notion of possible is completely shared by Aristotle and Diodorus. 2.1 and 2.2 are the two direct outcomes of the branching of node 2. They still represent communal tenets of both Diodorus' and Aristotle's doctrines. Namely, 2.2 in reference to something possible, already actual and not just potential; 2.1 in reference to some existing thing, even if is not necessary.

However, a real disagreement between Aristotle and Diodorus is given by their traditionally counterposed doctrine of capacity. In fact, according to Boethius' report, Aristotle would conceive some non-necessary existing as either something which is susceptible of *becoming*, by passage from a potentiality state to an actuality state (2.1.1.), or something which is always actual from its coming into being (2.1.2.). Since Diodorus embraces only the 2.1.2. and not 2.1.1., here is the real point of separation with Aristotle. That is, Aristotle and Diodorus disagreement reveals itself only when the topic of modal possibility (δυνατῶν) is associated to the doctrine of capacity (δύναμις).[85]

In the second part of Simplicius's report are examined some cases concerning *capacity*, in relation to the passage from *potentiality* to *actuality*. It is open to contradiction on both sides, in the sense that: 'whatever has come from potentiality to actuality, can both do and do not, be and not be'. But, this point takes implicitly into consideration that, before the phase in which the passage happens, an interval of time will tick away. The existence of this interval is not the real discriminating factor between Aristotle and Diodorus (it might be easily described by modal or temporal notions). The point of contention reveals itself in the subsequent passage in virtue of capacity: from potentiality to actuality. For

[85]There are many papers and works about the dispute between Aristotle and Megarics, in particular Diodorus Cronus. However, since we know only second hand testimonies on the dialectic of Diodorus, there exist several good but mutually inconsistent interpretations. Some important and worthy works are Weidemann [2008], Hintikka [1973], Beere [2009], Makin [1996], Gaskin [1995].

Diodorus, contrary to Aristotle, it is not a flowing-dense passage but a *jump* or *jerk motion* to actuality.

1.3.2 An analysis of the Hellenistic *κυριεύων λόγος*

Several logicians evaluate the ancient Master Argument as the best rejoinder to Aristotle and, in particular, against Arist. *Int.* 9.[86] The main aim of this section is to provide a careful examination on the report of Arr. *Epict.* 2, 19, by weighing every sentences, key words and concepts, of the ancient *κυριεύων λόγος*. In order to reach that target, we need to worry about:

1. The analysis of the original text, including the precise meaning of premises and conclusion of Diodorus' argument;
2. The formalisation of the *κυριεύων λόγος* theses, such that, in the end, we will be able to hypothesise as to the correctness of its conclusion;
3. The recognition of its logical and philosophical targets, so that we will be able to assess whether the Master Argument reaches its goals.

Let us begin by analysing *Epict.* 2, 19, 1, *κυριεύων λόγος*. Since it is the most important topic by Diodorus Cronus, we must deal with the original Greek fragment. At this point, we opt to privilege completeness of sentences rather than respecting the correct articulation of the lines in the manuscript:

> Ὁ *κυριεύων λόγος ἀπὸ τοιούτων τινῶν ἀφορμῶν ἠρωτῆσθαι φαίνεται*

[86] For instance, Jarmużek and Pietruszczak [2009] introduce to a linear model within a semantics for linear future; Gaskin [1995] deals with a metaphysics of the future. It reviews the texts by Aristotle and Diodorus in light of the formalism and tools of modern logic. Many researchers and scholars suggest interesting reconstructions. *E.g.*, Vuillemin [1996] presents an account in which the Diodorean argument is closely related to Arist. *Cael.* 1, 283b 6-17. However we are completely unaware of what was the deduction process to obtain the main thesis: *Nothing is possible which is neither true now nor ever will be*. The conclusion will be proved also as consequence of an alternative view about the capacity, by the Megaric Diodorus.

This is Epictetus' introduction to Diodorus' argument. The text is a report, the most complete one on the issue, but not the original by Diodorus, which has been lost. Epictetus tells us that: *the argument* named *the dominant*, and better known today as *the Master Argument*, *appears to have been propounded on the basis of these following starting-points*.

> κοινῆς γὰρ οὔσης μάχης τοῖς τρισὶ τούτοις πρὸς ἄλληλα, τῷ

The previous starting-points, or principles, to which Epictetus alludes, are not consistent each other. The report makes clear that: *there exists a strong mutual opposition between those three*. The term μάχης, in both the Stoic context and for Epict. *Ench.* 52, 1, and S.E. *M.* 7, 392, is technically used for *contradiction* in a logical sense; however, μάχης also retains the meaning of 'battle', which presupposes the contraries to be one to one, namely πρὸς ἄλληλα. So, the three starting-points are enunciated as follows.

The first one is:

> [τὸ] πᾶν παρεληλυθὸς ἀληθὲς ἀναγκαῖον εἶναι

That is, *everything truly accomplished*, namely, *everything that is past and true* at the same time, *is necessary*. Regarding this sentence, that is the first premise, we focus on two elements: the necessity (ἀναγκαῖον), and the past (it is not the notion of past, but with the past is at stake an ontology, since the past is in things that have been *accomplished*). The principle inherent to the first premise is maintained by Diodorus, in opposition to both Philo and the Stoics. It is confirmed also in other fragments, *e.g.* in Boeth. *in Int.* 235, 6-9:

> He thought that if someone were to die at sea, he could not have met his death on land. Neither Philo nor the Stoics say this [Smith, 2014, 141].[87]

[87] Ille enim arbitratus est, si quis in mari moretur, eum in terra mortem non potuisse suscipere. Quod neque Philo neque Stoici dicunt.

Determining the past as necessary alludes to its characteristic *irrevocability*. And so, if someone died at sea, of course, this fact does not change anymore, and he could not have met his death on land. However, this principle does not concern only facts that occur one time and never again; *e.g.*, if someone passes away, then, it is sure that he does not die again. At first, the irrevocability of the past regards those occurrences implicitely expressed by contingent propositions – for instance, by suffixing by the past operator P the proposition 'Jill is going to the cinema', in the case that Jill went to the cinema. In fact, if Jill was to the cinema last night, we avoid the case that she was dancing. Some caution may be useful in order to rule out some possible misunderstandings. We agree with Denyer [1981b, 33-37] when he affirms that Michael [1976]'s interpretation of the first premise is not correct:[88] if it was true that a proposition which once was true is necessarily true thereafter, following Diodorus' construal of modalities as temporal notions, we should admit that, if Jill was at the cinema last night, then she will always be at the cinema. However, if our interest is to translate this into formulas, we do not accept $Pp \rightarrow \Box p$, but we will opt for $Pp \rightarrow \Box Pp$: namely, *every past truth is necessary*. With this clarification, we approach again Boethius' last remark. By focusing on his example, we learn that what is both past and true could not have been different. It is here that the Diodorean account melds the logical and the ontological side. If we consider the Eleatic heritage of the Megaric Diodorus, by introduction of modal notions, what according to Parmenides is the eternal present, now, according to Diodorus, acquires the temporalised characterisation of phenomenas. From a metaphysical point of view, the eternal present, by the medium of time, actualizes itself in the plurality of what is, was, and will be real.

Necessity too is a key-issue in the first premise of the *κυριεύων λόγος*. A comparison between Aristotle and Diodorus will reveal some interesting differences. For instance, in Arist. *Metaph.* 4. 4, 1006b 31-32, we read:

[88] Stahl [1963] makes the same mistake.

> This is what 'being necessary' means – that is impossible for the thing not to be [Ross and Smith, 1960].[89]

Aristotle in fact is dealing with what is impossible not to be, from eternity; for instance, for human is necessary to be a biped animal. On the contrary, following Diodorus, what is said as necessary, may be interpreted in a so-called 'logic of futurity'[90], namely we define *necessary*, what is and will be true thereafter. At any rate, to consider the necessity of the past, is more undisputed and accepted than to talk about necessity for the future. In that last case, someone may accuse us of embracing (again) fatalist doctrines. Now, we want to return strictly to the first premise of the *κυριεύων λόγος*, *everything true as an event in the past is necessary*, which means that everything that has occurred in the past cannot be different from how it was the case.

Let us to give some example for the first premise: the case of Anny, a volleyball player. Once the coach chooses the squad for the next away game, Anny knows if she will be on the field or not. Let us suppose that (i) Anny is in the squad, or that (ii) Anny is not in the squad. Therefore, if (i) happens, then (i) occurred as non-contradictory in reference to reality, and (i) become a past event. On the other hand, if (ii) has happened, then it is impossible that Anny will be in the squad for the next match. Further, if we are arguing about a given proposition: to state that the fact (i) is the case, means that to utter 'Anny is in the squad' is true; while, the fact that (ii) is the case, means that to utter 'Anny is in the squad' is false: its occurrence is ontologically impossible at that time, though the proposition itself is not impossible. Every proposition both possible and not-necessary is a contingent proposition. Therefore, supposing that the occurrence of (i) is the case, after that, it has been true to state the proposition 'Anny is in the squad'; then, it is necessarily true that it has been. That is, it is and will be true thereafter that, in some given time in the past 'Anny is in the

[89] *τοῦτο γὰρ σημαίνει τὸ ἀνάγκη εἶναι, τὸ ἀδύνατον εἶναι μὴ εἶναι.*

[90] In order to face a *Diodorean logic* the definition of 'logic of futurity' has been borrowed from [Prior, 1955a, 1958c].

squad'. This is the sense of the first Diodorean premise. In fact, it has only one direction and we cannot intervene on past events.

The second principle of the ancient Master Argument maintains that:

τῷ [ἀ]δυνατῷ ἀδύνατον μὴ ἀκολουθεῖν

According to Diodorus, Epictetus reports, *an impossible [event] does not follow a possible [event]*. The ἀ in square bracket usually needs some caution. It indicates a possible reading of the manuscript in which the copyist wrote that 'an impossible does not follow an impossible'. But, of course, this would be a nonsense. Therefore, there is no need for us to question the standard interpretation. The Greek verb ἀκολουθεῖν must be understood in context: *something impossible does not* follow *from something possible*. ἀκολουθεῖν, 'to follow', has different meanings: 'to occur subsequently in time', 'to imply', and 'to be in accordance with', are the most plausible. The range of these meanings is very wide, and the term has a considerable importance in order to interpret properly the second premise. To take the verb ἀκολουθεῖν to mean 'to follow in time/after' (*cf.*, *e.g.* [Zeller, 1882], [Rescher, 1966]), is out of place when it is used by a valuable dialectician as Diodorus.[91] From a logical point of view the most accurate definition of it seems ἀκολουθεῖν as 'to infer', 'to entail', maybe in a Diodorean sense (cf., e.g. [Mates, 1973], [Denyer, 1981b, 40-1]). On the other hand, it would be a mistake to underrate the third solution: 'to be in accordance with', which hints to a kind of modal principle of non-contradiction in relation to the possibility (*cf. e.g.* [Becker, 1956], [Mignucci, 1966, 11-15]), *i.e.* if a proposition is possible, at the same time its impossibility is ruled out. Better yet, this formulation appears to be at least suggested in the wide sense by the second proposition of the Master Argument.

We also need to establish the correct interpretation of 'possible' and 'impossible' (*i.e.*, not-possible) as modal notions, in the tricky endeavour to avoid a

[91] In S.E. M. 8, 112, the explanation for the Diodorean implication, and therefore also for the verb ἀκολουθεῖν, excludes that 'to follow' may be meant on a temporal sequence.

circle with the conclusion of the Master Argument. So what do 'possible' and 'necessary' mean? The definition of the possible is obviously temporal, as 'what is already now or will be in a given future'. The meaning of the necessary is usually interpreted temporally, as 'what is and will be thereafter'.[92] A valid supposition is the connection of the second premise of the *κυριεύων λόγος* with the Eleatic tradition. In order to make it clear we can, for instance, interdefine modalities with temporal notions. Starting from the sentence 'an impossible does not follow a possible', we obtain that: 'something that is not nor will be true, does not follow somenthing that is or will be true'. And then, if it is definitively ruled out that *ἀκολουθεῖν* would mean 'to occur subsequently in time', by eliminating every time characterisation, here is the result, fully compatible with Parmenides: 'not-being does not follow from being'. So, we deal with a sort of law of non-contradiction, such that it is not the case that both being and not-being are. In fact, being determines everything that occurs or will occur, namely, the possibility of events in the reality. Not-being refers to what is not occurring nor will occur in reality, characterising in this way what counts as impossible according to Diodorus. Therefore, the opposition between being and not-being is analogue to the maximum opposition between possible and impossible. This last relation specifies contradiction between modalities.

However, we need to thread carefully here particularly if, according to [Denyer, 1981b, 38], we analyse again the second premise of the *κυριεύων λόγος*. What is the logical form of the proposition asserting that 'something impossible does not follow from something possible'? In fact, taking it literally, as 'the impossible does not follow the possible', following Barreau [2006, 283-301] we can formalise it as: $\neg\Diamond(\Diamond p \rightarrow \neg\Diamond p)$. This formula is a sort of *reductio ad absurdum*. But Diodorus would not agree with this interpretation. Indeed, by supposing something impossible such as the formula $\neg\Diamond p$, and it being the case that a truth is materially implied by anything, it can be inferred that $\Diamond p \rightarrow \neg\Diamond p$, and therefore that $\Diamond(\Diamond p \rightarrow \neg\Diamond p)$. But this is the contradictory of the second

[92] A different suggestion following literally Boeth. *in Int.* 234, 25, is: 'what, being true, will not be (at the same time) false'. However, in this case, the necessity would lose its *strictu sensu* modal value.

premise. This result would entail, anti-Diodoreanly, that it is impossible for a proposition to be impossible. This is not only a non-sense in relation to propositions and not directly to occurrences; it is also a blatant inconsistency, given Diodorus' definition of the modal notion of not-possible. Instead, the formula $(H\Box(p \to q) \wedge \Box(p \to q)) \to (\Diamond p \to \Diamond q)$, suggested by Denyer [1981b, 39], appears a bit more complex, but also more appropriate as a formal rendering of the second premise of the ancient Master Argument. Further, it is a weaker version of the modal thesis $\Box(p \to q) \to (\Diamond p \to \Diamond q)$.

After that, *Epict.* 19, 1, turns to the third principle stated by Diodorus:

> τῷ [**] δυνατὸν εἶναι ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται[93]

Namely, *[the] possible is, what is not, nor will be, true* or, formally, $\Diamond p \wedge \neg(p \vee Fp)$. It is clear that this third principle is inconsistent with the Diodorean definition of possibility, as Boeth. *in Int.* 234, 24, testifies. Nevertheless, once the three principles, or starting-points, have been laid down,

> συνιδὼν τὴν μάχην ταύτην ὁ Διόδωρος τῇ τῶν πρώτων δυεῖν πιθανότητι συνεχρήσατο πρὸς παράστασιν

Diodorus having noticed that opposition, took advantage of the plausibility of the first two [premises] in the proof. Then, we learn that the first two sentences appear to make sense to Diodorus, as correct philosophical tenets, while the third sentence does not present a satisfactory view. This point is obvious to Diodorus, since he justifies his move by the criterion of πιθανότητι or plausibility. As a matter of fact, Diodorus endorses the first two premises, avoiding the third, because he does not approve its philosophical meaning. In fact, how could something possible take some given ontological shape in the way of what never becomes an aught, neither now nor in the future? However, from a logical point of view, there exist no reasons leading Diodorus to choose that strategy; for instance,

[93] As referred by [Oldfather, 1956, 358], [**] stands for an erasure of two letters in Cod. Bodleianus Misc. Graec. 251, s. xi/xii.

Cleanthes and Chrysippus do not agree with him, and combine differently the premises. At any rate, the *κυριεύων λόγος* is less transparent to us: the ancient Master Argument was very well known in the Hellenistic period, and because of this, Arrianus, who copied the report of Epictetus, did not consider it worthwhile to elucidate Diodorus' inferential strategy in order to obtain the following conclusion:

> *τοῦ μηδὲν εἶναι δυνατόν, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται*

This last sentence, namely the conclusion of the *κυριεύων λόγος*, can be translated as: *it is not possible at all, what is not, nor will be true*.

Let us observe that Diodorus' thesis is equivalent to his modal definition of the notion of possible, which in Boeth. *in Int.* 234, 22-24, is formulated as *the possible is that which either is or will be [true]*.

Let us consider now the tense formalisation.
The conclusion of the *κυριεύων* is:
$\neg\Diamond p \equiv \neg p \wedge \neg Fp$
We apply the connective $\neg$ to the above definition on both sides, to obtain:
$\neg\neg\Diamond p \equiv \neg(\neg p \wedge \neg Fp)$.
By a simple application of the *double-negation* to the first part, and by *De Morgan's law* on the second part, it turns out that:
$\Diamond p \equiv p \vee Fp$
which is what Boeth. *in Int.* 234, 22-24, confirms to be Diodorus' definition of possible.

In fact, the already quoted Alex.Aphr. *in APr.* 184, 5-6, stated that:

> It is to establish this [the modal notion of possibility] that Diodorus' Master Argument is posed.

Then, by comparing what has come to light in our investigation about capacity and modalities, **D*** – obtained in the previous section of this work (at

1.3.1.1) – allows us to measure modalities as functions of *diachronic capacities*; and, on the other hand, the *κυριεύων λόγος* conclusion can also explain diachronic capacities in terms of possibility.

Diodorus' *κυριεύων λόγος* was a very relevant topic for those who succeeded the dialecticians of the Megaric group. In fact, the Stoics benefited from this tradition. For instance, they obtained many important results in logic, including first systems of propositional logic and modalities [Mates, 1973]. And so, it comes as no surprise that Cleanthes, Antipater, Chrysippus, and Panthoides, carried on the *κυριεύων λόγος* debate, in order to decide what premise to avoid.

According to *Epict.* 2, 19, 2-6:

> Now some will retain the pair 'there is something possible which neither is nor will be true' and 'something impossible does not follow from something possible', but deny that 'every past truth is necessary'. This seems to have been the line taken by Cleanthes and his circle, and was in general endorsed by Antipater. Whereas others will retain the other pair, that 'there is something which neither is nor will be true', and that 'every past truth is necessary', but hold that something impossible does follow from something possible. To retain all three is impossible because of their mutual conflict. So if someone asks me. 'Which of them do you retain?', I shall answer 'I don't know; but my information is that Diodorus retained the first pair I mentioned, the circles of Panthoides, I think, and Cleanthes the second pair, and Chrysippus and his circle the third pair' [Long and Sedley, 1987, 230-231].[94]

[94] λοιπὸν ὁ μέν τις ταῦτα τηρήσει τῶν δυεῖν, ὅτι ἔστι τέ τι δυνατόν, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται, καὶ δυνατῷ ἀδύνατον οὐκ ἀκολουθεῖ: οὐ πᾶν δὲ παρεληλυθὸς ἀληθὲς ἀναγκαῖόν ἐστιν, καθάπερ οἱ περὶ Κλεάνθην φέρεσθαι δοκοῦσιν, οἷς ἐπὶ πολὺ συνηγόρησεν Ἀντίπατρος. οἱ δὲ τἆλλα δύο, ὅτι δυνατόν τ' ἐστίν, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται, καὶ πᾶν παρεληλυθὸς ἀληθὲς ἀναγκαῖόν ἐστιν, δυνατῷ δ' ἀδύνατον ἀκολουθεῖ. τὰ τρία δ' ἐκεῖνα τηρῆσαι ἀμήχανον διὰ τὸ κοινὴν εἶναι αὐτῶν μάχην. ἂν οὖν τίς μου πύθηται ᾽σὺ δὲ ποῖα αὐτῶν τηρεῖς;᾽ ἀποκρινοῦμαι πρὸς αὐτὸν ὅτι οὐκ οἶδα: παρείληφα δ' ἱστορίαν τοιαύτην, ὅτι Διόδωρος μὲν ἐκεῖνα ἐτήρει, οἱ δὲ περὶ Πανθοίδην οἶμαι καὶ Κλεάνθην τὰ ἄλλα, οἱ δὲ περὶ Χρύσιππον τὰ ἄλλα.

Cleanthes and Chrysippus are undoubtedly the most relevant philosophers quoted in the fragment. As such, we will focus on the schemas which they used in order to avert determinism and defeat the Master Argument of Diodorus. Cleanthes avoids the principle stated by the first premise, 'every past truth is necessary'; Chrysippus admits that 'something impossible does not follow from something possible'; and both of them opt for the third premise of Diodorus' argument, which was ruled out by the Megaric, namely, 'there is something possible which neither is nor will be true'.

According to [Magris, 1995, 35-36], the last principle would make Cleanthes and Chrysippus safe from determinism, because it states that something possible is not given as determined now, nor in the future. In relation to Cleanthes' doctrine, we have not many evidences, but the thesis of Celluprica [1982, 374] appears to be reliable: according to Cleanthes, not everything past and truth is necessary, since some of those things could not happen.

Diodorus has already sided against Philo, and he denied possibilities which would not inherit necessary existence, *e.g.*, the Philonian possibility of burning driftwood. So, Philo was interested also in possibilities which never happen; in turn, Cleanthes focused on possible occurrences that happen even if lacking of ontological necessity. Therefore, Diodorus, as a determinist, would not admit Philo's or Cleanthes' notion of possibility; and, in fact, Diodorus avoids the third premise, namely the sentence 'there is something possible which neither is nor will be true'.

However, Chrysippus, the most important logician of the Stoà, believes that it is not always true that 'an impossible does not follow an impossible'. Nevertheless, Chrysippus' argument against the second premise of the *κυριεύων λόγος* appears to be a sophism. This is quoted by Alex.Aphr. *in APr.* 177, 25-33:

> Nothing prevents something impossible following even from something possible [...]. [Chrysippus] says that in the conditional 'if Dion is dead, this one is dead', which is true when Dion is being demonstratively referred to, the antecedent 'Dion is dead' is possible, since

> it can one day become true that Dion is dead; but 'this one is dead' is impossible. For when Dion has died the proposition 'this one is dead' is destroyed, the object of the demonstrative reference no longer existing. For demonstrative reference is appropriate to, and is said of, a living being. So if 'this one' is no longer possible once he is dead, nor does Dion come to be again, so that 'this one is dead' can be said of him, 'this one is dead' is impossible [Long and Sedley, 1987, 233].[95]

Anyone who wishes to reply to Chrysippus' argument, has to deny that 'this one is dead' is an impossibility. Then, we should confirm that the demonstrative reference or denotation 'this one is dead' results invalidated. In fact, Epictetus has established that, according to the use of language, we refer to someone as alive inasmuch he lives. Thus we cannot say 'this one is dead' even in order to indicate the dead Dion. Therefore, we are not dealing with an impossible proposition, but with an ill-formed one. If we assume Epictetus' supposition, we need to restate it as 'Dion is dead'. Of course, what we obtain is the *identity law* 'If Dion is dead then Dion is dead', that is, a logical tautology. If this is so, then the result is that something possible follows to something possible. This disproves Chrysippus argument.

[95] Χρύσιππος δὲ λέγων μηδὲν κωλύειν καὶ δυνατῷ ἀδύνατον ἕπεσθαι πρὸς μὲν τὴν ὑπ' Ἀριστοτέλους εἰρημένην δεῖξιν οὐδὲν λέγει, πειρᾶται δὲ διὰ παραδειγμάτων τινῶν οὐχ ὑγιῶς συγκειμένων δεικνύναι τοῦτο μὴ οὕτως ἔχον. φησὶ γὰρ ἐν τῷ συνημμένῳ τῷ 'εἰ τέθνηκε Δίων, τέθνηκεν οὗτος' δεικνυμένου τοῦ Δίωνος ἀληθεῖ ὄντι τὸ μὲν ἡγούμενον 'τὸ' 'τέθνηκε Δίων' δυνατὸν εἶναι τῷ δύνασθαί ποτε ἀληθὲς γενέσθαι τὸ τεθνηκέναι Δίωνα, τὸ δὲ τέθνηκεν οὗτος' ἀδύνατον ἀποθανόντος γὰρ Δίωνος φθείρεσθαι τὸ ἀξίωμα τὸ 'οὗτος τέθνηκε' μηκέτ' ὄντος τοῦ τὴν δεῖξιν ἀναδεχομένου ἐπὶ γὰρ ζῶντος καὶ κατὰ ζῶντος ἡ δεῖξις.

Chapter 2

Arthur N. Prior's thought and his *Master Argument*

Arthur Norman Prior was born in New Zealand in 1914. He was an eclectic scholar, with various interests in logic and philosophy. Prior made important contributions in several other philosophical fields (metaphysics, theology, ethics, history of logic) but his main impact was in logic. These results, obtained in the 1950s and 60s of the last century, are in the areas of non-classical logics. His best results in logic were obtained during his stays at the University of Manchester, and at Balliol College in Oxford; but also his visits at Berkeley and UCLA were of utmost importance, in particular for the birth of temporal logics. Some essential references are [Prior, 1967, 2003b], and the posthumous collection [Prior, 2003a]. He died of a heart attack in 1969. A critical study on his unpublished works is underway nowadays in the VLP [2011], aiming to complete the NAP [2014] project. In honour of Prior, in the year of his Centenary (2014), several workshops and conferences have been organised to bring together scholars from all over the world, *e.g.* the *Arthur Prior Centenary Conference*, August 20-22, Balliol College, Oxford (`http://conference.prior.aau.dk/`).

In order to introduce the main topic, that is the Master Argument, at first,

we try to trace Prior road to his modern version of the *κυριεύων*. In fact, someone might ask why Prior was fascinated by this argument. This is an interesting aspect of his intellectual *parcours*. Prior's first works are neither in logic, nor in philosophy, but in theology, *e.g.*, [Prior, 1940], [Prior, 1942], then *cf.* [Jakobsen, 2012] and [Hasle, 2012]. We can recognise two different reasons which moved Prior from an initial theological background to the invention of temporal logics: these are ethical and logical reasons. In relation to the first kind, Prior's Calvinism appeared to entail predestination and, as a consequence of predestination, determinism. Since a crisis of faith led Prior to rule out predestination, and the Master Argument was considered, even from Antiquity, to be an excellent argument for determinism, Prior's attempt was aimed to face the argument of Diodorus and fight off the determinism. The logical reasons for his interest in the Master Argument are connected to a search for an intermediate system between $S4$ and $S5$. In fact, according to Boeth. *in Int.* 234, 22-26, and [Mates, 1973], Prior envisaged that one could obtain a modal logic from the Diodorean modalities [Prior, 1955a, 1958a]. Thus, it could be interesting to combine modal and temporal notions, and a version of the Master Argument – formalised in a modal-tense language in the *PF-calculus* – was well suited Prior's purpose [Prior, 1967, 2003b].

In this chapter, we focus on Arthur Prior's approach to the Master Argument. We will investigate some tense, modal logics and mixed systems. The formalisation and the proof strategies behind Prior's version of the Master Argument make it possible to deal with time, modal notions, and in general with dynamical categories describing the flux of life. We aim to characterise the Priorean realm of things by means of adequate formalisms, and thus, as Prior puts it, to *master* and not *destroy* it [Prior, nda]. We will investigate some unpublished letters and manuscripts from the *Prior Collection*, including correspondence between Prior and Hamblin, Lemmon, Kripke, and some others logicians. Our intention is to rediscover the strength of Prior's formal languages, to give a historical analysis of Prior's Master Argument, and to examine his heritage.

Before giving a detailed survey of the above mentioned themes, we provide the reader with some elementary notions pertaining to tense logics. Section 2.1 is intended to introduce a minimal formalism and underline some Priorean peculiarities in relation to the calculus. Section 2.1.1 focuses on a search for the Diodorean frame, starting an exploration of some unpublished documents from the *Prior Collection* too. We will compare three Diodorean systems, namely a tense-modal, a strictly modal, and a strictly tensed logic. We will present some historical developments of this search, and some important results as, for instance, 'Dummett's formula', which we trace back to Prior [Lemmon, 1958]. Apart from this historical investigation, we will investigate some significant consequences of this formula. Section 2.1.2 is directly linked to the previous and concerns a debate on the birth of tense logics. Prior, Hamblin and Lemmon are the protagonists of this section: K_t, $HAMB$, and some extensions of these logics are compared, both in their syntactic and semantic consequences (*cf.* appendix B for a close examination of some manuscripts and correspondence that are relevant for this section). Next, in section 2.2, we introduce Prior's version of the Master Argument. We emphasise the fact that the Master Argument is a recurring interest of Prior, the topic occurs in many Prior's works from [Prior, 1955a] till [Prior, 1969b]. Section 2.2.1 focuses on Prior's formalisation of the Master Argument and the logical strategies against the deterministic thesis (z). We show that Prior's version is not moulded on the Hellenistic argument, since two additional premises are required to infer the conclusion. However, these additional premises are compatible with Diodorus' perspective. We can interpret the strategy on the Master Argument both as: (i) a proof from an extended classical propositional calculus, with 4 further tense-modal theorems, definition of H and necessitation rule; and as (ii) a very weak derivation in 14 steps, with definition of H and necessitation rule. According to Prior, the argument is valid, but he brings its correctness into question by means a three-valued approach. Nevertheless, everything is not all right with this approach, and we will show some of its flaws. In section 2.3 we will present the contemporary reception of Prior's account. Section 2.3.1 deals with a revised version of the Master

Argument by Øhrstrøm and Hasle [1995, 23-28], which we compare with the Priorean one. Finally, section 2.3.2 deals with the historical genesis of an archive of Arthur Prior's works at the Bodleian (Oxford) and on the web [NAP, 2014]. We will describe the aim of the VLP [2011] as an opportunity of web-multimedia and cooperative work in Priorean fields.

2.1 Prior's approach to tense logic

What is tense logic? It can be interpreted as a kind of scientific description of phenomena in their temporal occurrence. Various approaches have been proposed in order to formalise temporal notions, e.g., past, future, will be, was the case, for instance in [Quine, 1960] and [Prior, 1967, 2003b].

Quine proposed to regiment tensed language extending classical logic and using:

- An extra and quantified argument for time, namely t;
- The Earlier(-than) predicate, E, that is also expressible by the $\prec$ relation;
- A constant for the actual moment or 'now', namely n;
- One-place predicate variables, P, Q, ..., expressing properties on instants.

The following is an example of Quine's formalisation of the statement 'Godot is coming before Vladimir and Estragon will go away', that is uttered before Godot's arrival; and, let P stand for 'Godot comes', and Q for 'Vladimir and Estragon go away'.

$$\exists t \exists t_1 (n \prec t \prec t_1) \wedge P(t) \wedge Q(t_1)$$

In Quine's formalism, we obtain a *tenseless* logic, in which every predicate variable expresses an *eternal property* on a given instant.

Prior, on the other hand, developed a modern approach to *tense logic*.[1] Prior's approach to time and tenses is rather innovative in the 20th century. He takes sentences like p, q, ..., as already tensed, being present continuous tensed sentences. By extending the classical propositional calculus with tense operators, we obtain a *first level tense logic*. Prior formulates four different levels for tense logics. The first grade 'reduces the minimal tense logic to a by-product of the introduction of four definitions into an ordinary first order theory, and richer systems to by-products of conditions imposed on a relation in that theory' [Prior, 2003a, 119-120].[2] Tense operators act on sentences in the same way as functions on their arguments. For every sentence prefixed by one or more tense operators, we obtain an exact indication of some (sentential) occurrence at a given instant of time.

Prior considers four basic tense operators.[3] Two of those act on the past, H for the strong past and P for the weak one; whilst G and F for the strong and the weak future, respectively. Therefore, the previous sentence 'Godot is coming before Vladimir and Estragon will go away', that is uttered before Godot's arrival, by assuming that p stands for 'Godot is coming', and q stands for 'Vladimir and Estragon are going away', may be formalised by the following schema:

$$F(p \wedge Fq).$$

According to Prior, we obtain a *tense* logic.

[1] *See*, 'Prior's Invention of Tense Logic and its Early History' [Copeland, 1996, 15-25] for an introductive presentation outlining the early steps of the development of Prior's tense logic.

[2] Referring to the *second grade*, tenses are not reduced to McTaggart's B-logical notions. They are treated on a par with the earlier-later relation. A proposition without explicit temporal reference is not an incomplete proposition. A- and B-theories are on the same conceptual level. In the *third grade*, instant-variables a, b, c, etc. are also representing propositions. *E.g.*, a is the conjunction of all the propositions true at that instant. We need some interpretation. For instance, $Ta(p)$ might mean that the proposition a at all times implies that p, that is $Ta(p)$ iff $\Box(a \rightarrow p)$. Finally, at the *fourth grade*, the only primitive operators are P and F. Modalities are reduced to tenses. We need that: (i) time is like a single non-branching line, (ii) transitivity of the earlier-later relation, (iii) $\Box p \Leftrightarrow (p \wedge Hp \wedge Gp)$, (iv) there are no distinct and independent time-series.

[3] Further operators may extend tense logic to several temporal logics, and in particular, following Prior [1967, 59-76]'s approach, to several non standard tense-logics. For instance, the binary tense operators S and U, namely 'since' and 'until' discussed by Kamp [1968], or the 'next time' operator, O , used to describe the next atom in temporal sequences.

The Prior-Kripke correspondence shows a stimulating exchange about the formal concept of time. In a letter from Kripke [1958a], dated October 13th, Kripke asks:

> Do you think a tensed logic is needed for scientific discourse? I should think that, for scientific discourse a *tenseless logic* may be preferable. For example, in relativistic physics two events may be simultaneous to one observer but not to another.

On his side, Prior's belief about *indeterminism* justifies his take on *tense logic*, as expressed in the following reply to Kripke:

> I do not see how indeterminism can be expressed in a tenseless language at all. For indeterminism asserts a certain difference between the future and the past [Prior, 1958b].[4]

But even before Quine, Prior and Kripke, McTaggart [1908] proposed two different philosophical perspectives on time, the famous A- and the B-series, respectively, time as *flowing*, and time as *static description* of moments in relation to each other. Today, tense logic has developed and reached an advanced level of technicality. A good account of it can be found in Gabbay and Guenthner [2013].

Looking for an adequate formalisation and guessing (better: reconstructing) the strategy of the ancient argument of Diodorus Cronus was very important in Arthur N. Prior's investigation of the Diodorean frame. The interest for ancient logic, the debate about indeterminism and determinism, a particular attention to C. I. Lewis' modal systems are decisive factors that stimulated Prior in formalising the Master Argument and discussing Diodorean modalities (*see*, [Prior, 1955a, 1967], and the critics [Denyer, 2009], [Ciuni, 2009]). Those researches led to the birth of tense logic (*see*, appendix B.1).

[4] *See* [Ploug and Øhrstrøm, 2012] for a critical examination of Prior-Kripke correspondence.

2.1.1 Between S4 and S5: the search for the Diodorean frame

Many Priorean studies from the 1950s and the 60s of the last century concern tense logic as temporal interpretations of modal logic. Prior was looking for an intermediate system between $S4$ and $S5$, who might prove adequate for formalising the Diodorean modalities.

Let us briefly recapitulate Prior's formalisation of temporal and modal operators. These are directly connected to Diodorus' notions – already discussed, in respect to their historical genesis, in section 1.3.1.2 – used by the Megaric dialectician to depict modalities by means of temporal notions. Here is Prior's formalisation:

Fp. 'It will be the case that p' (Weak future operator)

Gp. 'It will always be the case that p' (Strong future operator)

Pp. 'It has been the case that p' (Weak past operator)

Hp. 'It has always been the case that p' (Strong past operator)

$\Diamond p$. 'Possibly p', *i.e.*, $p \vee Fp$

$\Box p$. 'Necessarily p', *i.e.*, $p \wedge Gp$

Observe that a sentence may be true at a given time, and false at another. Clearly, here we should think in terms of propositional functions: sentences are the arguments of the operators. Furthermore, if a formula is a logical law, then for every substitution of arbitrary formulas for its propositional variables, we obtain a proposition true at all times.

The calculus of tenses was built in [Prior, 1955a], where alethic and temporal operators interact on propositional variables. Today, we know that a minimal logic for Prior's Master Argument is given by the axiomatic system $DIOD$, subsequently presented.

In fact, the Master Argument should be consistent with a Diodorean logic. So, many logicians analysed different schemas for modal and temporal features, both on a syntactical and on a semantical level.

Before dealing with Prior's strategy in relation to the Master Argument (next paragraph 2.2), we summarise the best attempts of building a Diodorean logic. For the reader's convenience, we start by indicating our labels for the Diodorean systems:

- $DIOD$ is a Diodorean modal with tense logic;
- $ModalDIOD$ is the pure modal Diodorean logic;
- K_t4P is Jarmużek and Pietruszczak [2009]'s tense Diodorean logic.

The postulates of $DIOD$, as reported by [Ciuni, 2009], are as follows:

1. $G((p \to q) \wedge Gp) \to Gq$
2. $Gp \to GGp$
3. $(p \wedge Gp) \to PGp$
4. $PGp \to p$
5. $Gp \to Fp$
6. $\Box((p \to q) \wedge \Box q) \to \Box q$
7. $\Box p \to p$
8. $\Box p \to \Box\Box p$

MP. $\vdash p, p \to q \Rightarrow \vdash q$

RG. $\vdash p \Rightarrow \vdash Gp$

R$\Box$. $\vdash p \Rightarrow \vdash \Box p$

Next, we define $modalDIOD$. Sometime we refer to it as $KT4Dum$ (as in [Gabbay and Guenthner, 2012]), because it is given by the axioms of K + T + 4 + *Dummett's formula*. This system is also named D by Prior [1967, 20-31], alluding to Diodorus' modal intuitions;[5] whilst, $modalDIOD$ is among those systems intermediate between $S4$ and $S5$, standardly recognised as $S4.3.1$ (cf. Sobocinski [1964] and, later, *e.g.*, [Zeman, 1973]).

$ModalDIOD$ consists of:

- All instances of theorems of PC (the classical propositional calculus)
- The axiom schemas

  P $\Diamond\alpha \equiv \neg\Box\neg\alpha$

  K $\Box(\alpha \to \beta) \to (\Box\alpha \to \Box\beta)$

  R$\Box$ $\alpha/\Box\alpha$
- The axiom schema T: $\Box\alpha \to \alpha$
- The axiom schema 4: $\Box\alpha \to \Box\Box\alpha$
- Dummmett's formula, *i.e.* Dum: $\Box(\Box(\alpha \to \Box\alpha) \to \alpha) \to (\Diamond\Box\alpha \to \alpha)$[6]

$ModalDIOD$ is intermediate between $S4$ and $S5$. $ModalDIOD$ contains at least two theorems that are not in $S4$. By adding these to $S4$, $S4.2$ and $S4.3$ are obtained as extensions of the previous logic. Here are the formulas which identify the two modal systems:

S4.2 $\Diamond\Box\alpha \to \Box\Diamond\alpha$

S4.3 $\Box(\Box\alpha \to \Box\beta) \vee \Box(\Box\beta \to \Box\alpha)$

[5] Not to be confused with the D standing for deontic logic.

[6] This version by Geach is a simplification of $\Box(\Box(\alpha \to \Box\alpha) \to \Box\alpha) \to (\neg\Box\neg\Box\alpha \to \Box\alpha)$. An alternative to Dum is given by Bull, *i.e.*, $\Box\neg\Box\alpha \vee \Box(\Box(\Box(\alpha \to \Box\alpha) \to \Box\alpha) \to \Box\alpha) \to \Box\alpha)$. The formula produces the $S4.3$-extension ($S4.3.1$) complete with respect to our interpretation when time is taken to be discrete [Bull, 1965, 58].

The temporal interpretation of $S4.3$ gives us the *linear* (not-branching) series. What is said to be possible is what is now or, alternatively, will be at a later instant of time, that is, at a given future moment. On the contrary, if the state of affairs denoted by our sentence does not happen now or in the future, then the respective proposition is modal-impossible.

However $S4.3$ does not yet capture Diodorus' perspective on time, as it does not include the characteristic axiom needed in order to count the Diodorean logic as temporally discrete. This feature is gained by adding Dum which gives rise to the logic $ModalDIOD$. In this system, as it has been emphasised by [Zeman, 1973, 248], every statement that is true and becomes false has a last instant of being true, as opposed to the dense time of $S4.3$

Among the systems weaker than $ModalDIOD$ or $S4.3.1$, the systems $S4$ and $S4.2$ present some relevant peculiarities. However, some properties of these are quite different with respect to $S4.3$. The property of *branching-time* is characteristic of $S4$ and $S4.2$. This entails that – by interpreting possible worlds as instants of time – the denotation of some propositions which are possible will, nevertheless, never occur on our time-history. What makes the difference for a temporal interpretation of $S4$ and $S4.2$ is the branching property: it is standardly defined in $S4$, while it is *branching-converging* in $S4.2$. Therefore, our philosophical interpretation of $S4.2$ is at least as deterministic as that of $S4.3.1$: indeed, also in $S4.2$ the future is already determined, even if there is at least a counterfactual world before time converges again on the same node of the time-history in the frame.

K_t4P will be defined later.

Let us reconstruct now the genesis of the search for a Diodorean frame. It starts with Prior's studies of an intermediate logical system between Lewis' $S4$ and $S5$.

Initially, Prior conjectured that the Diodorean frame was an analogue of $S4$. This first hypothesis was supported by a previous comparison between the S-logics in [Lewis and Langford, 1932], Diodorus' intuitive account of modalities (*e.g.* the Master Argument and Boeth. *in Int.* 234, 22-26), and the M-calculus in [von Wright, 1951].

M includes the axioms of PC and two axioms:

I $\alpha \rightarrow \Diamond\alpha$

II $\Diamond(\alpha \vee \beta) \leftrightarrow (\Diamond\alpha \vee \Diamond\beta)$

Its rules are the already defined MP, $R\Box$, the *substitution-rule* and, the *modal extensionality rule*, namely

EXT. $\vdash \alpha \leftrightarrow \beta \Rightarrow \vdash \Diamond\alpha \leftrightarrow \Diamond\beta$

Let us suppose that we extend M with the following axiom:

III $\Diamond\Diamond\alpha \rightarrow \Diamond\alpha$

The resulting logic is M^i, *i.e.* an equivalent of $S4$.

Extending M^i by IV:

IV $\Diamond\neg\Diamond\alpha \rightarrow \neg\Diamond\alpha$

we obtain M^{ii}, *i.e.* an equivalent of $S5$.

The search for a Diodorean frame begins in [Prior, 1955a]: in this work the analogies between M-von-Wright, S-Lewis (in particular, $S4$ and $S5$), and Diodorus are investigated in depth for the first time, and Prior concludes that:

> The Diodoran definitions of the modal operators yield a system more like the Lewis system S4 than any other [Prior, 1955a, 209].

In [Prior, 2003b] we find Prior's reasons for endorsing the modal analogy: the outstanding one was the *reflexivity* and the *transitivity* of the accessibility relation in $S4$-frames. Since at the time Parry's $S4.5$ was believed to be the only intermediate system between $S4$ and $S5$ [Parry, 1939], later discovered to be equivalent to $S5$, the Diodorean modal frame should have been $S4$. We recall that the *reflexivity* and *transitivity* of the accessibility relation respectively correspond to the following axioms:

T $\Box p \rightarrow p$

4 $\Box p \rightarrow \Box\Box p$

Furthermore, [Prior, 2003b, 8-28] presents infinite matrices, that are assumed to be adequate to the Diodorean logic. These matrices are also useful for comparing modal and temporal notions, since they depict for every slot a given unit of time. Here is an example, in which of 0 stands for false and 1 for true:

Table 2.1: Diodorean matrix

p	q	$\neg p$	$p \wedge q$	$\Diamond p$	$\Box q$	every $S4$-theorem, *e.g.* $\Diamond p \leftrightarrow \Diamond\Diamond p$
1	0	0	0	1	0	1
1	1	0	1	1	0	1
0	0	1	0	1	0	1
1	0	0	0	1	0	1
0	1	1	0	1	0	1
0	0	1	0	1	0	1
0	1	1	0	1	0	1
1	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	0	0	1	1
0 hereafter	1 hereafter	1	0	0	1	1

A modal formula is verified by the matrix *iff* every series of assignments of values propositional variables returns formulas whose truth value is 1. *E.g.* the

formula $\Diamond p \leftrightarrow \Diamond\Diamond p$ is verified by the matrix – as are, in general, all the theorems of $S4$.

One should also note that:

- This matrix involves an implicit assumption of the discreteness of time;
- Some puzzles may arise in relation to $S4$-branching: we have no information about the branch of the fork at which the truth value assignment is done.

When [Prior, 2003b] was initially published, two years had passed since [Prior, 1955a]. At that time, the intermediate modal systems between $S4$ and $S5$ had not been deeply studied. The only attempt before Prior's studies was by Parry [1939]. However, Parry's $S4.5$ was later proved to be equivalent to $S5$. In relation to Prior's attempt and the examination of Diodorean modalities, a correction with reference to the Diodorean frame as an equivalent of $S4$ as well as a self-criticism to [Prior, 2003b] was provided in [Prior, 1958a]:

> The Diodorean definition being translated into a 'matrix', I state on p. 23 that this matrix is 'characteristic' for $S4$, *i.e.* verifies all those *and only those* formulae which are theorems of $S4$. And this is a mistake. [This] was first pointed out to me by Mr. E.J. Lemmon, whose example of a formula in D but not in $S4$ was $\Box(\Box p \rightarrow \Box q) \vee \Box(\Box q \rightarrow \Box p)$ [Prior, 1958a, 226].

Furthermore, the strongest Lewis' modal $S5$ contains Lemmon's $\Box(\Box p \rightarrow \Box q) \vee \Box(\Box q \rightarrow \Box p)$. Lemmon's formula is not a theorem of $S4$, nor of the weaker Lewis systems. However, on its side, $ModalDIOD$ contains the above characteristic formula, so that Lemmon's formula is valid on the Diodorean frame. Moreover, Lemmon's formula characterises $S4.3$.

Here is the text from Lemmon-Prior correspondence with relevant considerations about some intermediate between $S4$ and $S5$:

> Michael Dummett and I are working on Modal systems between $S4$ and $S5$, and were on the bench of proving that your pseudo-$S4$-matrix is characteristic for the system (we call $S4.2$) which results from $S4$ by adding $\Box(\Box p \to \Box q) \vee \Box(\Box q \to \Box p)$ ($S4.1$ is $S4$ + $\Diamond\Box p \to \Box\Diamond\Box p$ – and we may have a characteristic matrix for that two). I have written a short paper on this, and Michael is beautifying it by reproving the theorems more neatly using lattice theoretic and topological means. The proof that Parry's $S4.5 = S5$ comes out in about one line in lattice theory! The most interesting thing is the connexion between your matrix and Gödel's intuitionist matrix which Dummett has shown characterises IPC + $(p \to q) \vee (q \to p)$.[7] We've shown that α is satisfied by Gödel's matrix if and only if $T(\alpha)$ is satisfied by yours, where T is the Tarski-Mckinsey translation function of their '48 paper. (They show that $\vdash \alpha$ in IPC + $(p \to q) \vee (q \to p)$ if and only if it $\vdash T(\alpha)$ in S, and $\vdash \alpha$ in CPC if and only if $\vdash T(\alpha)$ in $S5$) [Lemmon, 1958].

Hintikka [1958] was on the same par as Lemmon and, in fact, in the review of *Time and Modality* he provides an equivalent formulation of the characteristic formula of Lemmon's $S4.3$. This makes it clearer that $S4$ does not represent the Diodorean frame: the formula is obviously not $S4$ valid. Here is Hintikka's counterexample:

$$(\Diamond p \wedge \Diamond q) \to ((p \wedge q) \vee \Diamond(p \wedge \Diamond q) \vee \Diamond(q \wedge \Diamond p))$$

Therefore, according to Hintikka [1958] (as well as Dummett and Lemmon [1959]), $ModalDIOD$ does not correspond to $S4$. In fact, a Diodorean frame should include Hintikka's modal formula – or some equivalent of it – in order to preserve a *transitive* and *linear* accessibility relation on the frame. By these conditions, $ModalDIOD$ would convey the linearity of *actual* time [Kripke, 1958b]. On the contrary, there are some transitive frames that falsify the previous formula.

[7] IPC is for the Intuitionistic Propositional Calculus.

So, an intermediate modal system including the axiom for linearity emerges. $S4.3 = T + 4$ + the following axiom:

$$\Box(\Box p \rightarrow \Box q) \vee \Box(\Box q \rightarrow \Box p)$$

Nevertheless, Dummett and Lemmon [1959] pointed out that $S4.3$ does not include discreteness, *e.g.* $\Box(\Box(p \rightarrow \Box p) \rightarrow \Box p) \rightarrow (\Diamond\Box p \rightarrow \Box p)$, while we know that, intuitively, an adequate Diodorean system should capture the intuition that time is discrete. Independently and roughly in the same period, a very young Kripke[8] communicates in a letter to Prior that $S4$ does not contain $ModalDIOD$, since neither $(\Diamond p \wedge \Diamond q) \rightarrow (\Diamond(p \wedge \Diamond q) \vee \Diamond(q \wedge \Diamond p))$, namely Hintikka's formula, nor $\Box\Diamond p \vee \Box\Diamond\neg p$, *i.e.*, a sort of modal excluded-middle, are theorems of $S4$ [Kripke, 1958b].[9]

The algebraic approach of [Bull, 1965] clearly indicates which class of frames satisfy the Diodorean properties. Bull's paper proved that $ModalDIOD$ is based on the following properties: *discreteness*, *reflexivity*, *transitivity* and *linearity*. According to Bull, Zeman [1968] identified and confirmed that this logic is $S4.3.1$.

$$S4.3.1 = S4.3 + Dum$$

From a philosophical perspective, we maintain that $ModalDIOD$ necessarily requires the axiom of discreteness in order to characterise Diodorus' intuitions about time. Therefore, we show that in Dum – which connotes $ModalDIOD$ as $S4.3.1$ – we can introduce Priorean tense operators, and endorse discreteness in $DIOD$.

According to [Prior, 1967, 29], Dum is equivalent to:

$$PDum \quad (\Diamond\Box p \wedge \Box(\neg p \rightarrow \Diamond(p \wedge \Diamond\neg p))) \rightarrow p$$

[8] He was only 17!

[9] Kripke proposed to translate $\Box p$ of $S4$ as 'p is true now and will be throughout the *possible* future' rather than 'p is true now and will be throughout the *actual* future'. The $\Diamond$-operator was defined, as usually, as the dual of the box. Thus, [Kripke, 1958b] shows that the modal $S4$ is relevant as a logic of indeterminism.

Prior [1967, 29-31] shows that a tense-modal version of $PDum$ – later referred as $PDum.t$, containing one occurrence of the F-operator – is a theorem in $DIOD$. Thus, Prior reduces to absurd the assumption that $PDum.t$ is not a law of $DIOD$.

$PDum.t \;\; (\Diamond\Box p \wedge \Box(\neg p \rightarrow \Diamond(p \wedge F\neg p))) \rightarrow p$

At first, Prior must to justify the fragment of $PDum.t$ which diverges from $PDum$, namely the step from (i) to (ii):

$(i) \;\; p \wedge \Diamond\neg p$

$(ii) \;\; p \wedge F\neg p$

To achieve this goal, Prior recalls the Diodorean definition of possibility as $\Diamond\alpha \leftrightarrow (\alpha \vee F\alpha)$, and lets $\neg p$ be the instance of α in (i), such that (i) is $p \wedge (\neg p \vee F\neg p)$. We observe that, it is not the case that $\neg p$ is true now because a manifest contradiction would arise in this last conjunction. Therefore p must be true and, consequently, by (i) we infer (ii) by considering the truth of $\neg p$ as postponed in the future, in the sense that there will be an instant in the future at which p changes its truth value. According to Prior, we gained an analogue tense-modal version of Dum, that is $PDum.t$.

[Prior, 1967, 30] confirms that there is no counterexample to $PDum.t$ in $DIOD$, by showing that the following conjunction $1-3$ is inconsistent:

1. $\Diamond\Box p$
2. $\Box(\neg p \rightarrow \Diamond(p \wedge F\neg p))$
3. $\neg p$

So, Dum is confirmed to be a theorem in $ModalDIOD$, and its tense-modal analogue $PDum.t$ is a theorem in $DIOD$. Nevertheless, Prior expresses a doubt about the validity of $PDum.t$, in virtue of his general preference for *density* of time in $DIOD$.

Let us assume 1 and substitute the F-operator for $\Diamond$:

$$\Box p \vee F\Box p$$

Then, it is either the case that 1.1 or that 1.2:

1.1 $\Box p$ is true, then p is true; this supposition is ruled out by 3.

1.2 $\Box p$ is not yet true, but there will be a time at which $\Box p$ is true, and then p will be true thereafter: *i.e.*, there exists a time at which p is false for the last time, and its immediate successor at which p is true.

If the condition 1.2 is valid when p is false for the last time, by 2 we obtain $\Diamond(p \wedge F\neg p)$. But then, it would be absurd to think that when p will be true (and thereafter always true), for some instant of the future, it is at the same time false. Therefore, if the counterexample is rejected, the formula is a theorem, however 1.2 constrains us to take time as a discrete sequence.

If discreteness is a corollary of $DIOD$ or $ModalDIOD$, we may be less worried than Prior. In fact, we are far from considering the Diodorean intuition that time is dense; we actually wish to reject this hypothesis.[10]

Many interpretations of the Diodorean system have been proposed starting from the 80s of the last century. Much work has been done on a semantical level (*e.g.*, [White, 1984], [Trzesicki, 1987], or [Zanardo, 2009]), and the Diodorean system has been interpreted in very different fields, for instance, in the physical Minkowski spacetime account [Goldblatt, 1980].

In spite of the different languages and approaches, it is possible to explain scientifically the temporal meaning of the modal notions: following Jarmużek and Pietruszczak [2009], we will show the power of a pure tense logic linked to the Master Argument. In particular, we will focus on some frames by going over

[10]The historical point of view on the topic has been discussed in section 1.1.

the semantics for K_t4P system. We obtain the system K_t4P from K_t4 tense logic + (P), namely, the axiom:

$$(p \wedge Gp) \rightarrow PGp$$

Furthermore, K_t4 is equal to K_t, plus any one of the axioms among 5-8.

K_t

1. $G(p \rightarrow q) \rightarrow (Gp \rightarrow Gq)$;
2. $H(p \rightarrow q) \rightarrow (Hp \rightarrow Hq)$;
3. $p \rightarrow HFp$
4. $p \rightarrow GPp$

Characteristic axioms for K_t4P

5. $PPp \rightarrow Pp$
6. $Hp \rightarrow HHp$
7. $Pp \rightarrow GPp$
8. $FHp \rightarrow Hp$

After having discussed $DIOD$ and $ModalDIOD$, we will examine a semantics for a pure tense logical system, in the spirit of Boethius' translation from modal to temporal notions. [Jarmużek and Pietruszczak, 2009] analyse the characteristic formula (P) for the tense logic K_t4P. In [Jarmużek and Pietruszczak, 2009] K_t4P is taken as the pure tense logic analogue of $ModalDIOD$, including the fundamental premises of the Master Argument, and likewise for $DIOD$. The characteristic formula (P), namely $(p \wedge Gp) \rightarrow PGp$, is equivalent to what I call $(+d)$ in section 2.2.1, namely the second additional premise of Prior's Master Argument.

Let us first give the acronyms legend, and then recall the definitions from [Jarmużek and Pietruszczak, 2009].

LIP stands for Limited Immediate Precedence. Every LIP-frame is characterised by this condition between two ordered temporal points. An IP-frame is unlimited.

1. All reflexive frames are LIP-frames;
2. All irreflexive LIP-frames are IP-frames and conversely;
3. All IP-frames are left-discrete and cannot have a minimum.

BC stands for Branching Condition. Every BC-frame is characterised by the branching.

1. All reflexive frames are BC-frames;
2. All right-total frames are BC-frames.

Below follows a list of definitions pertaining to the semantics for the logic K_t4P:

- A frame **F** is a (P)-frame *iff* the formula (P) is valid in **F**.
- **F** = $< T, R >$ is defined from the relation '$\prec$' of *immediate-precedence/succession*.
- **F** is a LIP-frame *iff* $\forall x \in T(x \text{ not-}Rx \Rightarrow \exists y \in T, y \prec x)$.
- **F** is a BC-frame *iff* $\forall x, y, z \in T(x \prec y \ \& \ xRz \ \& \ y \neq z \Rightarrow yRz)$.
- LIP-BC-frames are the class of frames satisfying LIP and BC properties.

Jarmużek and Pietruszczak [2009, 98] prove the following theorems:

Theorem 2.1. ***F** is a (P)-frame* iff ***F** is a LIP-BC-frame.*

Theorem 2.2. ***F** is an irreflexive (P)-frame* iff ***F** is an IP-BC-frame.*

From these follows the corollaries:

- Some treelike IP-frames are not BC-frames, therefore they are not (P)-frames
- Some linear and BC-frames are not IP-frames, therefore they are not (P)-frames
- Some irreflexive, transitive, right-total (P)-frames (so also IP-BC-frames) are not treelike frames
- There is a frame **F** = $< T, R >$ such that **F** is a treelike (P)-frame, namely a IP-BC-frame, but it is not right-total, *i.e.*: $\exists x, y, z \in T(zRx \ \& \ zRy \ \& \ x \neq z \ \& \ x\text{not-}Ry \ \& \ y\text{not-}Rx)$.

Thus, it is remarkable that the *branching condition* is weaker than *linearity*, that is, there are some branching but not-linear frames.

Clearly, frames associated to the Diodorean conditions guarantee several interpretations of the Diodorean temporal account.[11] From a logical point of view, this goes beyond the Hellenistic Diodorus' intuition of time and modalities. Following [Copeland, 2006], the above treatment of the topic is possible since Prior's and Meredith's pioneering studies of *possible worlds* [Prior, 1962b] and, mostly, by Kripke's definitive systematisation of the formal notion of semantics [Kripke, 1963a,b].

2.1.2 Some early research on tense: Prior, C. L. Hamblin, and E. J. Lemmon

In Hamblin [1961]'s letter to Prior, the Australian logician voices his opinion about the status of early tense logic:

[11] *E.g.*, White [1984] considers semantical assumptions for discreteness to be of secondary importance: only the assumption of irreflexivity is necessary. On the contrary, Trzesicki [1987] needs a tense-logical semantics satisfying the condition of discreteness. Therefore, the author concludes that even if we introduce irreflexivity, this property is not sufficient to infer the Master Argument's conclusion.

> I feel a little guilty about having left it all so unfinished, but I am glad to see it used.

Actually, if Prior is the pioneer of modern tense logic, Hamblin ought to be considered one of the most active participants in the effort of giving a foundation to the discipline (*see*, appendix B.3).

We can gather some information about the collaboration between the two logicians from [Prior, 1967], which follows [Prior, 2003b, first ed. 1955], and focuses on the official beginning of modern tense logic in the scientific literature; as well as in several papers and in the letters written by Hamblin to Prior from November 30th, 1956 to May 31th, 1966. Moreover, less than a month before Prior's death, the two logicians took part in the conference organized in Oberwolfach, West Germany, by the *International Society for the Study of Time* [Fraser et al., 1972]. From Hamblin [1972] and Prior [1972], we are aware that a fruitful exchange between the two colleagues took place both during that conference and also by letters. Unfortunately, only one side of the Hamblin-Prior correspondence is currently available. The Prior Nachlass at the Bodleian Library – Oxford,[12] collects only letters from Hamblin to Prior.[13]

Hamblin [1958] declares that:

> His first offering is a set of axioms for a simple 'dateless' tense logic, *i.e.* with F and P as monadic operators.

It will be useful to compare the system we call $HAMB$ [Prior, 1967, 48, 177] and the minimal one K_t [Prior, 1967, 51, 176].

[12] All records are available also on internet in [VLP, 2011], some of those are digitised for a critical edition in [NAP, 2014]; we focus on the topic in section 2.3.2.

[13] The Archive contains nine letters from Hamblin to Prior. Their contents are various: tense logics and related implicative schemata, modal language, type theory and some notes on Prior's works. In [Hamblin, 1969a] to the widow Mary Prior, are mentioned 'a number of scribbles' interchanged by Prior and Hamblin during their last Oberwolfach Conference. Further down, Hamblin writes 'I'll bring you the papers or copy of them when I next come to Oxford – probably in about a week'. We do not know if Hamblin [1969a]'s second quotation refers to the scribbles of Oberwolfach or more interestingly to all the letters from Prior. Worse none of those papers from Prior to Hamblin is conserved in the Nachlass, and perhaps they are lost.

An achievement of [Hamblin, 1958] was the diagram which describes the implication relations in $HAMB$. In [Hamblin, 1965] the diagram is augmented with further relations and the primitive strong tense operators G ('It will always be the case that-') for the future and H ('It has always been the case that-') for the past.

Hamblin [1961]'s letter explicitly endorses Prior's choice:

> I like your formulation using all four tense-operators.

The diagram representing the implication relations in [Hamblin, 1965] is explained also in Prior [1967, 46].

We will focus on K_t and $HAMB$, which are usually interpreted as minimal or quasi-minimal tense logical systems. Nevertheless, both K_t and $HAMB$ are based on some peculiar assumptions. Prior [1967, 51] remarks:

> It's odd to begin by insisting on linearity, and it might be better (as Lemmon has suggested) to confine one's 'basic' laws to those which put no special assumptions on the earlier-later relation at all. [...] Lemmon calls this 'minimum' system K_t.[14]

We present K_t axioms and rules.

K_t 1. α, where α is a tautology of the propositional calculus

K_t 2. $G(p \to q) \to (Gp \to Gq)$

K_t 3. $H(p \to q) \to (Hp \to Hq)$

K_t 4. $p \to GPp$

K_t 5. $p \to HFp$

MP If $\vdash p$, and $\vdash p \to q$, then $\vdash q$

[14] A complete explanation of K_t is in [Rescher and Urquhart, 1971, 55-67].

RG If $\vdash p$, then $\vdash Gp$

RH If $\vdash p$, then $\vdash Hp$

On the other hand, the tense logic by Hamblin, namely $HAMB$, is built on a stronger set of axioms and rules:

$HAMB$ 1. α, where α is a tautology of the propositional calculus

$HAMB$ 2. $F(p \vee q) \leftrightarrow (Fp \vee Fq)$

$HAMB$ 3. $\neg F \neg p \rightarrow Fp$

$HAMB$ 4. $FFp \leftrightarrow Fp$

$HAMB$ 5. $FPp \leftrightarrow (p \vee Fp \vee Pp)$

$HAMB$ 6. $\neg F \neg Pp \leftrightarrow (p \vee Pp)$

RG If $\vdash p$, then $\vdash \neg F \neg p$

RF $\vdash p \leftrightarrow q$ iff $\vdash Fp \leftrightarrow Fq$

MI $\vdash F\alpha$ iff $\vdash P\alpha$

Prior [1967] credits E. J. Lemmon with the introduction of K_t. Actually, Lemmon presented to Prior some considerations on K_t in some letters from 1964 till 1966. The correspondence stopped only some months before Lemmon passed away, on July 1966. In particular, [Lemmon, 1966a] contains some observations that are relevant to the semantical viewpoint (*see*, appendix B.3).

K_t 2 and K_t 3 are temporal analogues of schema K in modal logic, $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$, in view of the fact that the strong operators G and H of K_t bears some similarity to the necessity operator in modal systems. In the same vein, K_t 4 and K_t 5 express an analogue of schema B in modal logic. Therefore, K_t in a sense correspond to the modal system KB which is sound and complete with respect to the class of symmetrical frames. One question naturally arises: is the

symmetrical property problematic for Prior's philosophical interpretation of K_t? Before we answer this question, let us pause for some preliminary clarifications.

The exact date at which Lemmon introduced K_t to Prior is not clear. Although it is [Lemmon, 1965a] that lists the K_t axioms, the logician, at the time lecturer at the Claremont Graduate School, previously discussed with Prior about K_t, as witnessed by some notes about K_t [Lemmon, 1965b].[15]

Prior deals with $HAMB$ in several works [Prior, 1962c, 1966, 1967]. The modal analogue of $HAMB$ 2 is $CK9$, namely, $\Diamond(p \vee q) \leftrightarrow \Diamond p \vee \Diamond q$, a standard and relatively uncontroversial axiom for a normal modal logic. On the contrary, the modal analogue of $HAMB$ 3 involves some stronger assumptions. In fact, the analogue of $HAMB$ 3 is a modal equivalent of the schema of *deontic logic*, that is $\Box\alpha \rightarrow \Diamond\alpha$. The modal analogue of $HAMB$ 4 is equivalent to the characteristic axiom of Lewis $S4.3$: $\Diamond\alpha \leftrightarrow \Diamond\Diamond\alpha$. $HAMB$ 5 and $HAMB$ 6 have no direct analogues in any modal logic. The previous comparison makes it clear that K_t is weaker and more minimal than $HAMB$. An even more important question is, which temporal and tense features are represented by K_t and $HAMB$?

Let us start from a syntactical perspective: we prove the K_t axioms in $HAMB$.

Axioms Kt 1 and $HAMB$ 1 express the fact that both systems are extension of the classical propositional calculus.

The following is the proof of K_t 2, in $HAMB$.

1. $F\neg q \vee \neg F\neg q$

   $[\alpha \vee \neg\alpha]$

2. $F\neg p \vee F\neg q \vee \neg F\neg q$

[15] Lemmon formulates K_t axioms and rules using a notation a little bit different from the standard one. He refers to standard strong tense operators G and H by F and P inside *box*. Lemmon uses the two respective operators F and P inside *diamond* denoting the standard weak operators, namely F and P. To avoid any misunderstanding between modal and temporal meaning, we opt to use the standard tense operators G, H, F, P. In any way, Lemmon's formulation may be useful in order to suggest to the reader the analogy between K_t and KB.

[1, $\beta \to \alpha \vee \beta$]

3. $F(\neg p \vee \neg q) \vee \neg F \neg q$

[2, $HAMB$ 2]

4. $F((p \wedge \neg q) \vee \neg p) \vee \neg F \neg q$

[3, **RF**]

5. $F(p \wedge \neg q) \vee F \neg p \vee \neg F \neg q$

[4, $HAMB$ 2]

6. $\neg G \neg (p \wedge \neg q) \vee \neg G p \vee G q$

[5, $Df.\ G \equiv \neg F \neg$, $\neg\neg\alpha$ iff α]

7. $\neg G(\neg p \vee q) \vee \neg G p \vee G q$

[6, $\neg(\alpha \wedge \beta) \leftrightarrow \neg\alpha \vee \neg\beta$]

8. $\neg G(p \to q) \vee (\neg G p \vee G q)$

[7, $(\alpha \to \beta) \leftrightarrow (\neg\alpha \vee \beta)$]

9. $G(p \to q) \to (\neg G p \vee G q)$

[8, $(\alpha \to \beta) \leftrightarrow (\neg\alpha \vee \beta)$]

10. $G(p \to q) \to (G p \to G q)$

[9, $(\alpha \to \beta) \leftrightarrow (\neg\alpha \vee \beta)$]

Then, once K_t 2 is proved in $HAMB$, it is enough to apply **MI** to get K_t 3, namely $H(p \to q) \to (Hp \to Hq)$.

Here we prove K_t 4, that is, $p \to GPp$, from $HAMB$.

1. $p \rightarrow (p \vee Pp)$

[$\alpha \rightarrow \alpha \vee \beta$]

2. $p \rightarrow GPp$

[1, $HAMB$ 5]

Again, by **MI** on K_t 4, we can infer K_t 5, namely $p \rightarrow HFp$. Therefore, it follows that $K_t \subset HAMB$.

Let us now have a look at the meaning of the formulae in K_t and $HAMB$. The uncontroversial axioms K_t 2 and K_t 3 express the properties of the strong future and the strong past operators, respectively, something analogue to the schema K of modal logic. K_t 4 and K_t 5, on the other hand, are reminiscent of the schema B in modal logics, and are respectively equivalent to $FHp \rightarrow p$,[16] and $PGp \rightarrow p$.[17] In particular, $PGp \rightarrow p$ is also the first additional premise of Prior's reconstruction of the *Master Argument* [Prior, 1955a, 1967]. Prior claimed that $PGp \rightarrow p$, and therefore also the equivalent K_t 5, would force determinism. In fact, if we admit K_t 5, we accept that the truth of p was already determined. For any instant in the past it would be true to state that there is a moment in the future in which p will be true and that this instant is now. Later, we will show why this point has significant consequences for the logical semantics, and then we will argue that K_t falls short of the requirements for a minimal tense logic.

Let us turn to $HAMB$. As already noticed, $HAMB$ is based on stronger assumptions than K_t. $HAMB$ 2 simply says that if it will be the case that a disjunction is true in some future, sooner or later at least one of the disjuncts will be true. $HAMB$ 3 says that time has no ending point. Of course, this is a strong assumption about the nature of time. For every instant, there will always be a later one. Basically, time goes on indefinitely. $HAMB$ 4 has a strong consequence too: time is dense.

[16] The first step is $\neg GPp \rightarrow \neg p$, by K_t 4 and contraposition, using the interdefinability of strong and weak tense operators and the replacement principle. Then, $FHp \rightarrow p$ is deduced as equivalent to K_t 4.

[17] It is obtained from K_t 5 by the strategy previously shown in note 5.

Suppose that time is composed by atoms, like a pearl necklace. Then, if we assume that p is true at the immediate successor of the present moment, the result is a counterexample to $HAMB$ 4. In particular, the implication $Fp \rightarrow FFp$ is falsified. From $HAMB$ 5, we derive $(p \wedge Hp \wedge Gp) \rightarrow GHp$, that is *backward-linearity* and its *mirror image*, that is $(p \wedge Gp \wedge Hp) \rightarrow HGp$, namely *forward-linearity*.

Here is the proof:

1. $FPp \leftrightarrow (p \vee Fp \vee Pp)$

[$HAMB$, 5]

2. $FPp \rightarrow (p \vee Fp \vee Pp)$

[1, $(\alpha \leftrightarrow \beta) \rightarrow (\alpha \rightarrow \beta)$]

3. $\neg(p \vee Fp \vee Pp) \rightarrow \neg FPp$

[2, $(\alpha \rightarrow \beta) \leftrightarrow (\neg\beta \rightarrow \neg\alpha)$]

4. $(\neg p \wedge \neg Fp \wedge \neg Pp) \rightarrow \neg FPp$

[3, $\neg(\alpha \vee \beta) \leftrightarrow (\neg\alpha \wedge \neg\beta)$]

5. $(\neg\neg p \wedge \neg F\neg p \wedge \neg P\neg p) \rightarrow \neg FP\neg p$

[4, $p/\neg p$]

6. $(p \wedge Gp \wedge Hp) \rightarrow GHp$

[5, $\neg\neg\alpha \leftrightarrow \alpha$; $Df.\ G \equiv \neg F\neg$; $Df.\ H \equiv \neg P\neg$]

7. $(p \wedge Hp \wedge Gp) \rightarrow HGp$

[6, **MI**]

Linearity exposes an important feature of $HAMB$: time does not branch, neither in the past nor in the future.[18]

Let us turn to the question about discreteness and density, in particular for $HAMB$ 6. Some difficulties arising from $HAMB$ 4 have already been considered. [Prior, 1962c, 136] noticed that $HAMB$ 6 too may be problematic within that system. In fact, the assumption of discreteness may follow exactly from $HAMB$ 6, via its dual $PGp \leftrightarrow (p \wedge Gp)$. Of course, this doesn't mean that Hamblin's postulates for $HAMB$ are not consistent, and again Prior [1962c, 136] credits to T. Smiley a proof that $HAMB$ is consistent if we let $Fp \equiv Pp \equiv p$. Therefore, it seems clear that tense logic needs some readjustment.

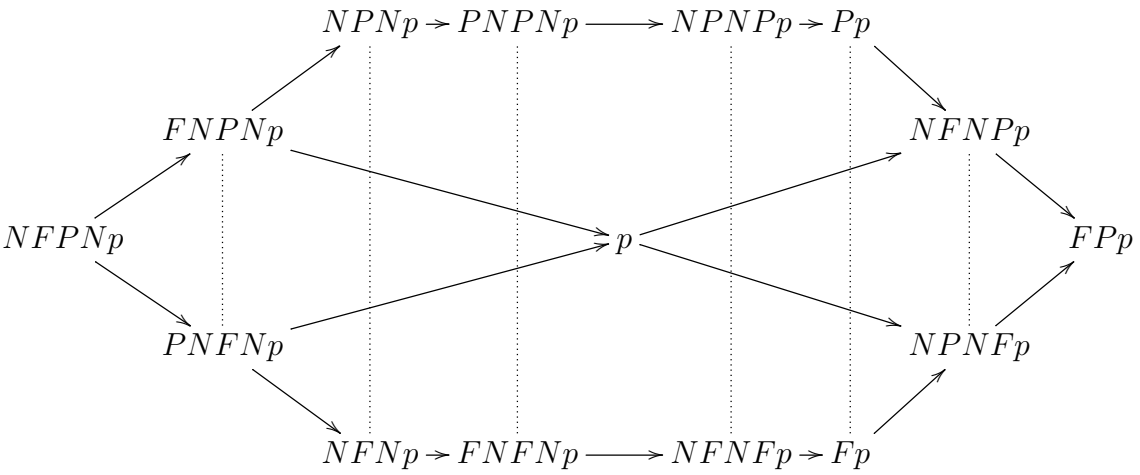
Hamblin's first note to Prior about a structure of implication relations is in [Hamblin, 1958]. Hamblin focuses on a network of relations between 30 distinct tense logical formulae,[19] all of them derivable from the postulates of his $HAMB$ system. Some years later, according to [Prior, 1962c, 134] the importance of Hamblin's heritage for tense logics is attested:

> I think it must be regarded as the first major 'break-through' in this field, namely that if we make certain very natural assumptions, any sequence of the functors P, H, F, G, however long, is equivalent to one or other of a group of 15 (including the null sequence, *i.e.*, the 'present' tense).

Hamblin [1958]'s first tense graphic, enriched by dotted lines, follows on next page.

[18] Except in the case of a *not-total* time structure. *See* [Jarmużek and Pietruszczak, 2009, 99-100].
[19] In fact, Hamblin supposes a parallel table with $\neg p$ for p or *vice versa*.

TABLE 2.2: First version of Hamblin's diagram of tenses with dotted lines for symmetrical relations



Dotted lines highlight one of the features of $HAMB$, namely the symmetrical relations between past and future tenses. Basically, these amount to some consequences of the *mirror image rule*, **MI**. Regarding the tenses we know that:

> The lower 8 are of course the mirror images of the upper ones [...] The right-hand 8 are 'duals' of the left-hand 8, *i.e.* if we have ϕp on the left, we have an equivalent of $\neg\phi\neg$ at the reflecting position on the right Prior [1967, 47].

Later letters, starting from [Hamblin, 1965], simplify this structure, directly introducing strong operators. Hamblin follows Prior's formalism, and anyhow NPN and NFN in the above graphic are respectively equivalent to Hp and Gp.[20] In any case, Hamblin explains that:

> Any tense-modality with more than two tense-operators can be reduced to one with two or less.

References in [Hamblin, 1961] to some missing letters from Prior testify that these presently unavailable documents would be essential to reconstruct

[20] According to Øhrstrøm and Hasle [1995, 177], M. Cresswell has reported that G was inspired by the phrase 'is always *going* to be', while H by '*has* always been'.

their exchange of ideas. Hamblin is accepting the problematic character of the equivalences between FFp and Fp, and GPp and $p \vee Pp$, namely $HAMB$ 4 and $HAMB$ 6. However, we are not able to reconstruct Prior's argument because there are no letters from Prior to Hamblin in the Nachlass, and furthermore, Prior does not publish any paper on tense logics during the period between the letters [Hamblin, 1958] and [Hamblin, 1961]. Moreover, Hamblin does not explicitly give up $HAMB$ 4 and $HAMB$ 6. So, the discussions about some results on tenses goes on, and Hamblin refers to something that Prior suggested in a previous (and missing) letter.

Prior's lectures at the *Polska Akademia Nauk*, Warsaw, 1961, were properly about developments and results in tense logics inspired from the correspondence with Hamblin: the 15 tense theorems and the graphic of implication relation. What Prior introduces in Warsaw in 1961 is closely examined in [Prior, 1962c].

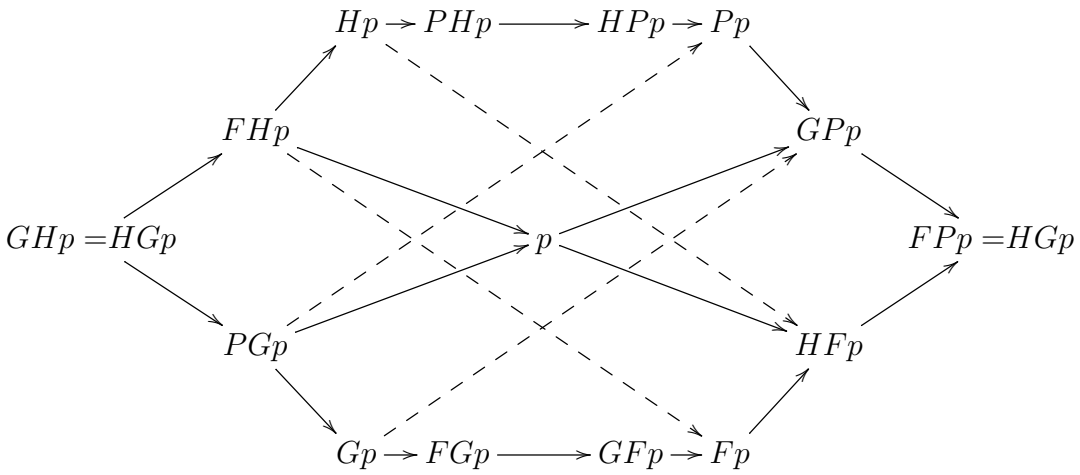
A relevant document in the history of tense logics is [Hamblin, 1965]. Hamblin introduces *a small note on the doctrine of tenses*: 'Shouldn't PGp imply Pp?'

Since $HAMB$ avoids discreteness by virtue of $HAMB$ 4, $PGp \rightarrow Pp$ is in $HAMB$, and so are its 'duals and mirror-images'.

Further, $HAMB$ is shown to be inconsistent with two formulae: $(p \wedge Gp) \rightarrow PGp$, and $(p \wedge Gp \wedge Pp) \rightarrow PGp$. In fact, these are unprovable in Hamblin's logic: by $HAMB$ 4, time is not discrete.

On next page, we show the complete version of Hamblin [1965]'s diagram of tenses, looking 'a bit like a bird's nest'.

TABLE 2.3: The Bird's Nest



The diagram is an attempt to schematise the implication relations between tenses, but temporal logic is a more complicated structure than the *bird's nest*.

[Hamblin, nd] is a relevant unpublished typewritten by Hamblin, titled *The Logic of Tenses*. It is kept in the Prior Nachlass.[21] The section *Iteration of tense operator* in [Hamblin, nd], presents some notes that are deserving of attention, for instance on how to interpret the diagram of the bird's nest. According to [Hamblin, nd, 5], by $HAMB$ we derive 62 formulae:

> Since these 62 formulae can be combined by disjunction at will without repetition, there are 2^{62} non-equivalent things that can be said about the temporal truth of p in terms of them, or about a million million million [...] Many non-equivalent formulae can be generated by the use of truth-operators.

Furthermore, Hamblin credits Prior with strengthening the status of tense logics. Hamblin revises his previous system as follows:

A 1. $G(p \to q) \to (Gp \to Gq)$

[21] The date is not clear, the last references is at [Prior, 1962c]. But in a note at p. 7, Hamblin quotes Prior for a letter dated on July, 1965.

A 2. $Gp \rightarrow GGp$

A 3. $Gp \rightarrow Fp$

A 4. $G(p \rightarrow Gq) \rightarrow (G(Fp \rightarrow q) \rightarrow (G(p \rightarrow q) \rightarrow (Fp \rightarrow Gq)))$

A 5. $PGp \rightarrow p$

A 6. $PGp \rightarrow Gp$

A 7. $p \rightarrow (Hp \rightarrow (Gp \rightarrow HGp))$

A 8. $GGp \rightarrow Gp$

A 9. $p \rightarrow (Gp \rightarrow PGp)$

A 10. $Gp \rightarrow p$

A 11. $Gp \rightarrow Hp$

By the above list, the so-defined T systems and the modal $S5$ may be compared [Hamblin, nd, 8].

T 1. A 1 - 7 (weak system)

T 2. A 1 - 8 (dense time)

T 3. A 1 - 7, 9 (discrete time)

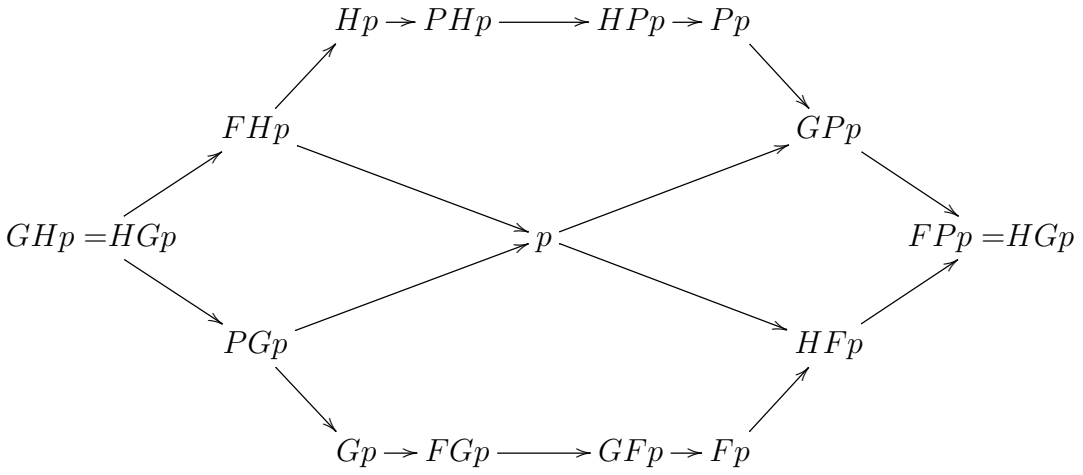
T 4. A 1, 2, 4, 6, 7, 10 (strong system)
(A 3, 5, 8, 9 deducible)

$S5$. A 1, 6, 10, 11
(A 2 - 5, 7 - 9 deducible).

T 1 is a basic tense logic. By adding $A8$, one obtains density; T 2 leads to a dense-time-scale. $T3$ works for discrete time and it is neither weaker nor stronger than $T2$.

$T3$ makes the diagram of tenses look a bit different. In fact it loses $PGp \rightarrow Pp$, its dual and their *mirror image*:

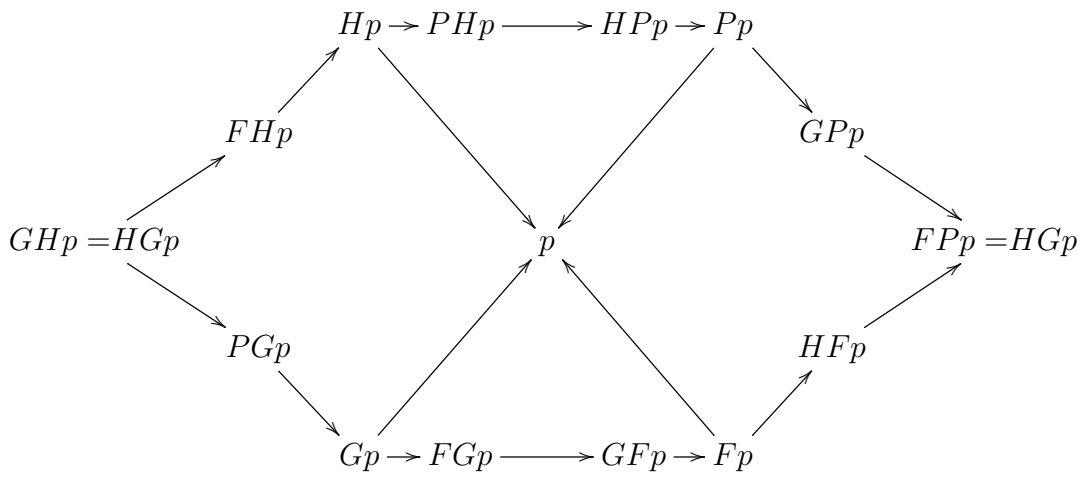
Table 2.4: T 3 diagram of implications



Then, Hamblin points out that adding A 10 to T 2 or T 3 results T 4, in which A 8 and A 9 are redundant, in the same way as A 3, and A 5. Moreover, $T4$ changes the meaning of the G operator, to 'It both is and always will be the case that-' rather than 'It will be the case that-'. Of course, the other tense operators change their meaning as well if we include the present instant in their classical definition.

The resulting T 4 diagram follows on next page.

TABLE 2.5: T 4 diagram of implications



Finally, by adding A 11 to T 4, the logic collapses into the modal system $S5$, in which all tenses to the left of the centre of the diagram become equal to the modal operator for necessity, that is $\Box$ or L in Prior's and Hamblin's notation; while the tenses on the right, become identical to the modal operator for possibility, that is $\Diamond$ or M [Hamblin, nd, 7].

Among the previously mentioned systems, only T 1 may be considered minimal. In fact, only $T1$ does not introduce assumptions of density or discreteness for the time series. However, in the same way as K_t or $HAMB$, T 1 entails the symmetry between the past and the future, and one may wonder if that property might be problematic for 'minimal' interpretations.[22]

The next argument concerns the symmetry between the past and the future, in relation to the question of *minimality* in tense logics.

[22] An engaging paper on Hamblin's 15 tense theorem, and recent advance on the topic is in Kudlek [2010]. In order to be brief, we notice only one of the results of Kudlek's: extending $HAMB$ to a logic for branching time. Kudlek [2010, 72-73] introduces models on $\mathbb{R}$, $\mathbb{Q}$, and $\mathbb{Z}$, for tense logics in $HAMB$ style, which include branching. The proposed models have the following structures:

(1) $\mathbb{R} \times \mathbb{R}$ with $(x_1, y_1) \prec (x_2, y_2) \iff ((x_1 \leq x_2 \wedge y_1 < y_2) \vee (x_1 < x_2 \wedge y_1 \leq y_2))$

(2) $\mathbb{Q} \times \mathbb{Q}$ with $(x_1, y_1) \prec (x_2, y_2) \iff ((x_1 \leq x_2 \wedge y_1 < y_2) \vee (x_1 < x_2 \wedge y_1 \leq y_2))$

(3) $\{(x, y) | x \in \mathbb{Z} \wedge y \in \mathbb{Q}\} \cup \{(x, y) | x \in \mathbb{Q} \wedge y \in \mathbb{Z}\}$
with $(x_1, y_1) \prec (x_2, y_2) \iff ((x_1 < x_2 \wedge \lfloor y_1 \rfloor \leq \lfloor y_2 \rfloor) \vee (\lfloor x_1 \rfloor \leq \lfloor x_2 \rfloor \wedge y_1 < y_2))$.

We will introduce the topic by means of a short passage from Prior [1996a]'s *Some free thing about time* (later included in the NAP [2014] too):

> One of the big differences between the past and the future is that once something has become past, it is, as it were, out of our reach – once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make [...] We can lay it down as a law that whatever *now* is the case *will always have been* the case; but we can't interchange past and future here and lay it down that whatever *now* is the case *has always been going to be* the case – I don't think that's a logical law at all.

A tense logic is minimal when it does not involve any physical assumption. It can be argued that the symmetry between past and future, as encoded in the *mirror image* principle, violates this minimality requirement.

Secondly, we outline several relevant philosophical considerations about K_t. Since we already noticed that $K_t \subset HAMB$, we should extend our remarks to $HAMB$ and to the other systems obtained from $HAMB$. Again, by admitting determinism we contradict what it has been said about *minimality* of tense logic: can we define K_t, $HAMB$ and the derived systems as minimal tense logics?

Lemmon [1966a] presents the following argument.

Let us consider a model $\mathcal{M} = \langle U, R_1, R_2, \rangle$ where:

- U is the set of moments of time;
- R_1 is the forward relation;
- R_2 is the backward relation.

And let t, t' be instants in U. What K_t assumes is that:

$$tR_1t' \text{ iff } t'R_2t.$$

According to Lemmon, this implies that R_1 is the converse of R_2, *i.e.*:

$$R_1 = \check{R}_2.$$

But, from the last equality, a further result follows. Namely, the *weaker* assumption that:

$$R_1 \subseteq \check{R}_2.$$

That is:

$$\text{if } tR_1t' \text{ then } t'R_2t,$$

with the consequence that $FHp \rightarrow p$ (an equivalent formulation of K_t4) is preserved, while $PGp \rightarrow p$ (an equivalent formulation of K_t5) is not.

According to Lemmon, there are further reasons to conclude that K_t is not minimal. In fact, Lemmon [1966a] offers a suggestion to the opponent of determinism, reaffirming the strategy of weakening K_t:

> This is what an anti-determinist worried by K_t 5 should assume, and no more: namely that if t' is later than t, then t is earlier than t', but *not the converse*.

Lemmon [1966a] concludes with several interesting observations and a remark:

> Maybe it's only an ordering language prejudice that 'earlier' is the strict converse of 'later'!

We have seen that K_t, $HAMB$, and other logics extending K_t 4, K_t 5, or with symmetrical relations between past and future, disregard the principle of 'minimality' of tense logic. Furthermore, if logic is about what is real, there are noteworthy consequences for our language, and therefore, both for our interpretation of physics – in particular about its temporal structure – and for our life and actions. In particular, if we accept both K_t 4 and K_t 5, we have to admit that what is true now, has already been determined. But it boggles the mind to consider such a deterministic logic as minimal.

2.2 The *Master Argument* of Prior

In order to give a formal analysis of the aims, the reconstruction, and the strategies, behind the Master Argument, we preliminarily give an essential bibliographical overview.[23] Thus, here we would like to present the thread that led Prior to the Master Argument, what the effects of the formalisation were and, more generally, we will link the Master Argument to the invention of tense logics.

Beyond of formal considerations on the Diodorean frame, Prior's interest on the Master Argument grew as a consequence of his interest in avoiding determinism. During his initial academic training, Prior has embraced the theological views of Methodism in the Calvinist context. Therefore, Prior's belief in predestination had as a consequence the certainty of determinism as shown *e.g.* by his *logic of Calvinism* [Prior, 1940]. Within a few years, Prior began to experience a crisis of belief (*see*, [Prior, 1942][24]) which brought with it an aversion to predestination. This desire to escape predestination lead him to develop strategies to elude determinism [Hasle, 2012]. In [Prior, 1951] modalities, indeterminism, and free will, appear to be conciliated for the first time. And, since Prior was developing his interest in ancient and medieval logic [Uckelman, 2012], without

[23] [Prior, 2003a, 311-328] provides the reader with the complete list of pertinent bibliographical references.

[24] The style of this work reminds to a magnificent dialogue by Petrus Abelardus [Thomas, 1970].

underestimating the importance of modern technicalities (*e.g.*, [Prior, 1954]), the study of indeterminism was applied to a three-valued logic, as in the way of Łukasiewicz [1920]'s logic. Starting from those researches, Prior wrote his first contribution on what is considered a modern version of the *κυριεύων λόγος*.

From the mid 50's till the last works in 1969, the Master Argument was a recurring topic and one of Prior's more trendy and prolific themes, where several logical and philosophical results intertwined. In order to substantiate this claim, we will briefly overview the following works:

- [Prior, 1955b, 194]
- [Prior, 1955a, 209-213]
- [Prior, 2003b, 1, 12-13, 86-93, 96]
- [Prior, 1958a]
- [Prior, nda]
- [Prior, 1962a, 120]
- [Prior, 1962c, 137-139]
- [Prior, 1966]
- [Prior, 1967, 17, 32-58, 59, 116, 121-122, 148-149]
- [Prior, 1969b]

[Prior, 1955b, 194]'s section 'The Logic of Modality' focuses on Diodorus, modalities, the ancient debate on implication, a comparison with the Stoic Chrysippus, and the modern discussion by C.S. Peirce and C.I. Lewis on the theme. The Master Argument is not directly quoted, but Prior is preparing the ground for future contributions on it.

[Prior, 1955a, 209-213] is the first paper in which, after searching for a Diodorean modal-analogue system, Prior proposes the Master Argument formalisation in detail, proves its conclusion, and gathers that the argument is correct but unsound. Prior's replay to Diodorus is based on Łukasiewicz three-valued logic and a background assumption of indeterminism.

[Prior, 2003b, 1, 12-13, 86-93, 96] is the first book about tense logic, and many sections of it are dedicated to Diodorean topics and the Master Argument. The debate on implication between Diodorus and Philo, and some repercussions in the logic of Middle Ages are the *incipit* of section 'Basic Modal Logic and the Ł-Modal System'. 'Tense-Logic and an analogue of $S4$' deals with the Diodorean modalities. In particular, on p. 12, Prior proves that $DIOD$ is at least as strong as the modal *Lewis* $S4$, that is $S4 \subseteq DIOD$. The Master Argument and Prior's proof are analysed in relation to free will and the three-valued logic in 'Intentional Logic and Indeterminism'; on pp. 86-92 the discussion on the Master Argument is resumed and, in the same way as in [Prior, 1955a], it is linked to an indeterministic three-valued solution in Łukasiewicz's style. In those pages, Prior takes the first premise, namely $Pp \rightarrow \Box Pp$, to be mistakenly assumed as a law in the Master Argument. Then, Prior gives a Philonian counterexample against determinism. In a footnote to 'Indeterminism without neuter propositions' Prior challenges the correctness of the formula $p \rightarrow \neg P \neg Fp$, equivalent to one of Prior's premises of the Master Argument.

[Prior, 1958a] returns to the Diodorean modal logic, and makes some relevant correction on $DIOD$.

[Prior, nda] is an unpublished manuscript. It is a lecture with an undated postscript, in which Prior gives a philosophical presentation of the aims of tense logic. What is very relevant is at the bottom of page, where there is a version of Prior's reconstruction of the Master Argument which seems to be an analogue of the one in [Prior, 1962c, 138].

[Prior, 1962a, 120] gives the reader a wider framework about divine foreknowledge, time, and future contingencies, in which the author quotes many

historical accounts, *e.g.*, *Ockhamism*, Peter de Rivo's and Thomas Aquinas' doctrines, Cic. *Fat.* Prior also comments upon a premise of the Master Argument, *i.e.* $p \rightarrow HFp$.

In [Prior, 1962c, 137-139], section 2 of the paper is dedicated to 'The Continuity of Time and the Diodorean Master-Argument'. Prior recalls the third premise of the Master Argument, represented by the formula $(p \wedge Gp) \rightarrow PGp$. However, Prior maintains that this formula is troublesome dealing with the conception of time. In fact, it requires the assumption of discreteness, namely the atomistic view of time. But we can add that the formula is not as problematic as it would seem, provided the Master Argument is consistent with the atomistic account of Diodorus Cronus. Afterwards, Prior challenges the conclusion (on grounds of unsoundness) of the Master Argument in order to refute the Aristotelian view that 'while it is now beyond the power of men or gods to affect the past, there are alternative futures between which choice is possible.' Then, he sets forth the contrasting view against Diodorus' definition of possibility. In fact, this last is formalised by $\Diamond p \leftrightarrow (p \vee Fp)$, namely the possible is defined as what either is or will be true. All the Diodorean premises are examined by Prior, and he gives a formalisation and a proof in order to obtain the conclusion of the argument. The Master Argument is connected to the search for $DIOD$, and particular attention is given to Dummett and Lemmon [1959]'s formula $\Box(\Box(p \rightarrow \Box p) \rightarrow \Box p) \rightarrow (\Diamond\Box p \rightarrow \Box p)$, and some other variation for it. Moreover, the Dummett-Lemmon formula is been proved to be consistent with the atomistic view point. Finally, the Master Argument is also linked to $HAMB$.

[Prior, 1966] does not mention the Master Argument, but in this paper Prior gives the OCK and PEI formal systems. From these, we are able to build a reply to the Master Argument. For this purpose, we need to further assume the postulate $PFp \rightarrow (p \vee Pp \vee Fp)$. For instance, by the OCK solution we easily rule out the determinism of the Master Argument, in spite of its conclusion and the definition of the possible as $\Diamond p \leftrightarrow (p \vee Fp)$. In fact, we can admit some path or history of our frame, in which p is false at every node/time provided that there is at least one node in another path of the same frame at which p is true such

that we can truly affirm now that Fp.[25] On the contrary, in Prior's works we find several applications of OCK and PEI to tense logical theses, and also for some tenets pertaining to the Master Argument, for instance in [Prior, 1967, 113-136]'s section titled 'Time and determinism'.

Regarding this last work, Prior [1967, 17, 32-58, 59, 116, 121-122, 148-149] dedicates many pages to the Master Argument in what that may be considered the best of Prior's contributions to tense logic: in section 'Precursor of tense logic' Prior introduces the argument by Diodorus, modal and tense definitions; in section 3, Prior writes several pages about the Master Argument: *The topology of time* is the first paragraph and gives, in detail an 'Analysis of the Master-argument of Diodorus' (pp. 32-34). Also in connection to the Master Argument, Prior deals with a *logic of futurity*, by adding tense operators and axioms to the propositional calculus. Then, Prior searches and verifies some formulas in order to find the axioms of $DIOD$.[26] For instance, the formula $p \rightarrow HFp$ is considered as derivable by Oc. *Praed.* q. 1.[27] On p. 49 the author discusses the strength of the conclusion of the Master Argument, *i.e.*, $(\neg p \wedge \neg Fp) \rightarrow \neg\Diamond p$; whereas on pp. 121-122, he reviews some ancient solutions to avoid the determinism of Diodorus' argument: the first, denies that past-tense truths are always necessary (Cleanthes); the second, turns against the thesis that the impossible cannot follow from the possible (Chrysippus). In section 'Time and existence', on pp. 148-149, the Master Argument is again discussed in relation to both Chrysippus' reply and the law $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$, and compared with the Aristotelian perspective and tenets involved in the first two premises. In general, the *PF-calculus*[28] was one of the most important aims of [Prior, 1967] and the basis for

[25] *E.g.* Braüner and Øhstrøm [2000, 193] consider a similar strategy, even if we can safely say that Diodorus would deny the branching time. Furthermore, [Braüner and Øhstrøm, 2000, 194-195] confirms that 'the conceptual price for involving branching time [...] is that the notion of the future is conflated with the notion of possibility' and 'Prior was not ready to pay such a price'.

[26] The treatment continues by focusing on tense logical systems (*e.g.*, $HAMB$), and some axioms and features (*e.g.*, discreteness).

[27] Si haec propositio sit modo vera: Haec res est, quacumque re demonstrata, semper postea erit haec vera: Haec res fuit.

[28] The name given by Prior to the logic of P and F operators, and concerning a tensed approach.

further developments of tense systems.[29]

[Prior, 1969b] is a short, detailed, and disapproving review of Stahl [1963]'s interpretation of the Master Argument, that is useful for the reader who wishes to recapitulate the right formalisation of Diodorus' argument. In particular, in this work Prior discusses the first and the second premises of the Master Argument, focusing on some preventable misinterpretations.

2.2.1 Prior's formalisation of the Master Argument

Prior takes an interest in the Master Argument after reading the first edition of Mate's influential treatise on *Stoic logic* (*see*, appendix B.2). Here is Mates [1973, 38]'s recapitulation of *Epict.* 2, 19, 1:

> Diodorus argued that the following three propositions could not all be true.[30]
>
> 1. Every proposition true about the past is necessary.
> 2. An impossible proposition may not follow from a possible one.
> 3. There is a proposition which is possible, but which neither is true nor will be true.
>
> Since, according to Epictetus, the first two propositions seemed to Diodorus to be more plausible than the third, he dropped the third, and this accounts for his definition of the possible as 'that which either is true or will be true.'

[29] [Prior, 1967] goes through several tense logics, *e.g.*, by Hamblin, Cocchiarella, Scott, Lemmon, one after the other, and step by step by adding characteristic axioms.

[30] In [Prior, 1967, 32] the three propositions are:

1. Every true proposition concerning the past is necessary
2. The impossible does not follow from the possible
3. Something that neither is nor will be is possible.

According to [Prior, 1955a, 210], Diodorus' argument is valid but unsound. Other ancient logicians – Cleanthes and Chrysippus among them – judged in the same way, opting to reject at least one of its premises. The problem of modern logicians is to reconstruct the Master Argument as a valid argument. Prior's challenge, however, is more general: he aims at explaining the consistency of the propositions of the Master Argument with Diodorus' modal definitions. Moreover, he wants to find the logical matrix that provides a counterexample to the Diodorean conclusion, in accordance with his own indeterministic perspective.

In order to provide a clear formalisation of the argument, we need some preliminary clarifications.

- Ordinary propositional variables as p, q, r, etc., have to be considered as Hellenistic and Diodorean sentences,[31] such that they may change their truth value.[32]
- As a corollary: p, q, r, etc., are tensed propositions.
- Propositions about the past have to be assumed as prefixed by the operator 'It has been the case that-'. It works analogously for propositions about the future, which have to respect the form 'It will be the case that-'.[33]
- The formal calculus within which the formalisation is carried out is the Diodorean system ($DIOD$).

Let us follow Prior [1955a, 209-213]'s formalisation, uncovering his view on the Master Argument and Diodorus' tenets. Here is Prior's interpretation:

(a) When anything has been the case, it cannot not have been the case.

$Pp \rightarrow \neg\Diamond\neg Pp$

[31] The approach to Diodorean propositions is formally different in [Mates, 1973, 33-36], which presents these as propositional functions. Prior [1955a, 205] opts to use tense and modal operators, by prefixing propositional variables as their arguments.

[32] *Cf.* D.L. 7, 65.

[33] *E.g.*, 'Prior was born in 1914' cannot be considered an atomic tensed proposition. We should translate the sentence 'Prior was born in 1914' as 'It has been the case that, both Prior is born and it is the year 1914'. That is, the translation should be into a tensed language allowing us to express the proposition $P(p \wedge q)$.

(b) If anything is impossible, then anything that necessarily implies it is impossible.

$\neg\Diamond q \rightarrow (\Box(p \rightarrow q) \rightarrow \neg\Diamond p)$

(+c) When anything is the case, it has always been the case that it will be the case.

$p \rightarrow HFp$ or directly $\Box(p \rightarrow HFp)$[34]

(+d) When anything neither is nor will be the case, it has been the case that it will not be the case.

$(\neg p \wedge \neg Fp) \rightarrow P\neg Fp$

(z) What neither is nor will be true, is not possible.

$(\neg p \wedge \neg Fp) \rightarrow \neg\Diamond p$

The argument is a modern formulation of the ancient *κυριεύων λόγος*; Prior's Philonian example of *a shell at the bottom of the sea* shows the pertinence of his formalisation for both the Hellenistic way of thinking, and the context of Diodorus' debate. Notice that Prior's example is not neutral. It already indicates that he is taking a stand against the Master Argument, to the extent that the example follows an anti-deterministic tradition. In fact, in order to explain better the way the proof proceeds, Prior refers to Philo as Diodorus' opponent with respect to claim (z), and the notions of possibility and impossibility:

> By (+d), if the shell neither is nor will be seen, it has been the case that it will not be seen. Hence, by (a), it cannot (now) not have been the case that it has been the case that it will not be seen. That is, the proposition that it has not been the case that it will not be seen, i.e. that it has always been the case that it will be seen, is impossible. But

[34] Prior took the string $\Box(p \rightarrow HFp)$ as (+c) in [Prior, 1967]; while in [Prior, 1955a, 211] (+c) is not prefixed by the box ($\Box$), although the previous formulation is deduced at a later stage. We note the passage at line 12 of the proof. Furthermore, a necessary implication, as it is in $\Box(p \rightarrow HFp)$, is equivalent to what the same Prior [1955a, 206] defines as *Diodorean implication*. We refer to S.E. *M.* 8, 115, for the ancient debate, in particular, the *truth criterion for conditional sentences* by Diodorus, whilst [Hurst, 1935] is one of the best modern contributions on the topic.

> by (+c), the proposition that the shell is now being seen entails this impossible proposition that it has always been the case that the shell will be seen. Hence, by (b), the proposition that the shell is being seen is itself impossible [Prior, 1955a, 210-201].[35]

If (a) and (b) are modern translations of Diodorus' first and second premises, (+c) and (+d) are premises added by Prior.[36] Prior has two kinds of reasons to add (+c) and (+d), to the originals (a) and (b) of the Master Argument:

- *Validity of the proof.* Because it has to be ruled out that (z) is inferred by adding to (a) and (b), as assumptions, the Diodorean modal notions, namely $\Diamond p \leftrightarrow (p \vee Fp)$, and $\Box p \leftrightarrow (p \wedge Gp)$ (Boeth. *in Int.* 234, 22-26) instead of (+c) and (+d). This would be circular, since the conclusion (z) is already implicit in the definition of possibility. In fact, it is sufficient to apply to $\Diamond p \leftrightarrow (p \vee Fp)$ the *negation law* and *De Morgan* to obtain (z).[37]
- *Philosophical pertinence with Diodorus' doctrines.* (+c) and (+d) are not foreign to Diodorus. They can be taken as (implicit) consequences of his tenets.

In particular, (+c) is connected to the position expressed *e.g.* in Cic. *Fat.* 12, 27: if p is true now, then at any instant in the past it was the case that p will be true. For, seen from the past, the actual now was a time in the future, therefore, in virtue of his anti-deterministic convictions, Prior is suspicious about (+c), because it would deny free will. However, even if it is true – as a result of logical semantics – that the actual now is seen from the past this does not mean that 'seen' refers to some alleged fatalistic prediction. In fact, we already showed that Diodorus posited a kind of minimal, but at the same time essential, gap between logic and ontology, such that from his deterministic perspective, the

[35] *See*, [Prior, 2003b, 87] also.
[36] This is why we prefixed them with $+$.
[37] This is confirmed, *e.g.*, by Alex.Aphr. *in APr.* 184, 4-6: the Master Argument is deployed in order to assert that the possible is only what either is or, in any event, will be, that is, we can infer (z) only from this.

Megaric dialectician could keep contingent propositions and reject counterfactual possibilities. Furthermore, in section 2.1.1, we showed that (+d) alone does not entail determinism. In fact, we are able to obtain an IP-BC-frame which is branching but not-linear. Of course, we cannot say that an IP-BC-frame was what Diodorus had in mind, and surely (+d) is necessary to infer the Master Argument's deterministic conclusion. In any case, we treat time as a discrete sequence, in order to respect a historically faithful Diodorean interpretation.[38]

Here is Prior's strategy to prove the conclusion (z):

1. $(p \to q) \to ((q \to r) \to (p \to r))$

[*Instance of the law of transitivity*]

2. $(p \to (q \to r)) \to (q \to (p \to r))$

[*Instance of the law of exchange*]

3. $P\neg Fp \to \Diamond\neg P\neg Fp$

   (a) $p/\neg Fp$

[*Substitution in (a)*]

4. $P\neg Fp \to \neg\Diamond HFp$

[*From 3 by definition of H*]

5. $((\neg p \wedge \neg Fp) \to P\neg Fp) \to P\neg Fp) \to (((P\neg Fp \to \neg\Diamond HFp) \to ((\neg p \wedge \neg Fp) \to ((\neg p \wedge \neg Fp) \to \neg\Diamond HFp))$

   (1) $p/\neg p \wedge \neg Fp$; $q/P\neg Fp$; $r/\neg\Diamond HFp$

[*Substitution instance of (1)*]

[38] *See* section 1.1 and also [Zeman, 1968] for a contemporary system, namely $S4.3.1$, as the adequate atomistic outline for Diodorus' account.

6. $(P\neg Fp \rightarrow \neg\Diamond HFp) \rightarrow ((\neg p \wedge \neg Fp) \rightarrow \neg\Diamond HFp)$

[*MP (5, (+d))*]

7. $(\neg p \wedge \neg Fp) \rightarrow \neg\Diamond HFp$

[*MP (6, 4)*]

8. $((\neg p \wedge \neg Fp) \rightarrow \neg\Diamond HFp) \rightarrow ((\neg\Diamond HFp \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)) \rightarrow ((\neg p \wedge \neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)))$

   (1) $p/\neg p \wedge \neg Fp$; $q/\neg\Diamond HFp$; $r/\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p$

[*Substitutions in (1)*]

9. $\neg\Diamond HFp \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)$

   (b) q/HFp

[*Substitution in (b)*]

10. $((\neg\Diamond HFp \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)) \rightarrow ((\neg p \wedge \neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)))$

[*MP (8, 7)*]

11. $(\neg p \wedge \neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)$

[*MP (10, 9)*]

12. $\Box(p \rightarrow HFp)$

[$R\Box$ *on (+c)*]

13. $(\neg p \wedge \neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)) \rightarrow (\Box(p \rightarrow HFp) \rightarrow ((\neg p \wedge \neg Fp) \rightarrow \neg\Diamond p))$

   (2) $p/\neg p \wedge \neg Fp$; $q/\Box(p \rightarrow HFp)$; $r/\neg\Diamond p$

[*Substitutions in (2)*]

14. $\Box(p \to HFp) \to ((\neg p \wedge \neg Fp) \to \neg\Diamond p)$

[*MP (13, 11)*]

(z). $(\neg p \wedge \neg Fp) \to \neg\Diamond p$

[*MP (14, 12)*]

Prior's proof of (z) uses (a) and (b) of the ancient Master Argument, and, by adding (+c) and (+d), reaches Diodorus' conclusion. Nevertheless, although Prior accepts the validity of Diodorus' argument, he objects to its soundness. In fact, and once again, Prior is afraid of validating determinism. The crucial step of the first part in the examination of Prior's reconstruction is criticising the truth of (+d).

The second part of Prior's analysis reveals his best move. Prior refers to Łukasiewicz three-valued logic. Prior takes $\frac{1}{2}$ as the truth value of future propositions that are not true from now on, while 1 and 0 respectively denote *true* and *false* in the standard fashion. The ∇-operator will be used as an operator for contingency, such that $\nabla p \equiv \Diamond p \wedge \neg\Box p$.

> The three-valued logician is not in fact free to accept even the negative formula $(\neg p \wedge \neg Fp) \to \neg\Diamond p$ as expressing a logical law [Prior, 1955a, 212].

We discover the background of Prior's quote in Łukasiewicz [1920], implemented with [Prior, 1953], and according to which we propose the three-valued system.

Let us start from the following matrices for $\neg$ and $\to$. The tables consider the truth-values of the primitive logical connectives.

TABLE 2.6: Łukasiewicz's three-valued matrices for $\neg$ and $\rightarrow$

$\neg$	
1	0
$\frac{1}{2}$	$\frac{1}{2}$
0	1

$\rightarrow$	1	$\frac{1}{2}$	0
1	1	$\frac{1}{2}$	0
$\frac{1}{2}$	1	1	$\frac{1}{2}$
0	1	1	1

The other connectives are defined in the usual manner.

Furthermore, as in [Prior, 1953, 321], modal operators are definable in the following way. A further Priorean condition for the three-valued logic, concerning possibility and implication, is:

$$\Diamond p \equiv \neg p \rightarrow p$$

The other modal notions are definable from this.[39]

The following matrices show the value combinations for modal operators. According to [Prior, 1953], the operator Q,[40] which appears, in our notation, as $\triangledown$, can be understood as 'It is contingent that-'.

TABLE 2.7: Prior's three-valued matrices for modal $\Box$, $\Diamond$, and $\triangledown$

$\Box$	
1	1
$\frac{1}{2}$	0
0	0

$\Diamond$	
1	1
$\frac{1}{2}$	1
0	0

$\triangledown$	
1	0
$\frac{1}{2}$	1
0	0

Let us observe that these tables do not include any matrix for the F-operator. The reason is as follows.

Suppose we consider the following three-valued interpretation of F:

[39] It means that either (i) $\neg p$ is no closer to truth than p is; or (ii) both p and $\neg p$ are equivalent to $\frac{1}{2}$.

[40] *See* also [Prior, 2003b, 41-54] for Q many-valued system and future contingents.

- truly futurable propositions (1) return propositions that are uncertainly true in the future ($\frac{1}{2}$);
- uncertainly futurable propositions ($\frac{1}{2}$) return propositions that are uncertainly true in the future ($\frac{1}{2}$);
- falsely futurable propositions (0) return propositions that are false in the future (0)

This truth-function for the F-operator is nonregular.[41] Thus, we refer to Bergmann [2008, 94]'s result 5.34, namely:

> No nonregular truth-function is definable by Łukasiewicz's matrices.

As a matter of fact, Prior [1955a, 212]'s counterexample is aimed at the implication in (z), by interpreting as indeterminate (or $\frac{1}{2}$) its truth value. So, Prior's focuses on the conjunction $\neg p \wedge \neg Fp$, by obtaining the truth value of the antecedent, and on the formula $\neg\Diamond p$ for the truth value of the consequent. [Bergmann, 2008] 5.34 leads us to interpret the formula Fp as an atom. We assume the formula $\neg p \wedge \neg\alpha$, in which α stands for a generic propositional variable, *e.g.*, instantiated by Fp. In this case, by the evaluation on $\neg p \wedge \neg\alpha$, we infer the following result: $\frac{1}{2} \wedge \frac{1}{2}$.

Nevertheless, the three-valued approach does not do justice to Diodorus' modal intuitions.

The table with the counterexamples appears on next page.[42] We assume the $\Diamond p$-values, by comparing those with the $p \vee Fp$-values. Then, we show that in some cases the value of $\Diamond p$ does not coincide with the value of $p \vee Fp$. We will give particular attention to $p = \frac{1}{2}$, that is, the indicted case by [Prior, 1955a].

[41] Following [Bergmann, 2008, 92], nonregular truth-functions in Łukasiewicz's three values logic are all those that do not produce classical truth-values when (but not necessarily only when) applied exclusively to classical truth-values. *See*, also the result 5.33

[42] They are given by the following couples for p and Fp: $(\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(0, 1)$, $(0, \frac{1}{2})$.

TABLE 2.8: Counterexample matrix for three-valued $\Diamond$ in $DIOD$

p	Fp	$\Diamond p$	$p \vee Fp$
1	1	1	1
1	$\frac{1}{2}$	1	1
1	0	1	1
$\frac{1}{2}$	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
$\frac{1}{2}$	0	1	$\frac{1}{2}$
0	1	0	1
0	$\frac{1}{2}$	0	$\frac{1}{2}$
0	0	0	0

After this critical digression against an indeterministic three-valued approach to $DIOD$, we recapitulate Prior's work. As a result of these matrices, the Master Argument wavers. The first threated principle is (+d). Namely, the tenet that *if anything neither is nor will be the case, then it has been the case that it will not be the case*. In fact, if we consider some future contingency, such that it is sufficient for p to be indeterminate or $\frac{1}{2}$, then it returns that both the atomic propositions of the conjunction in the antecedent are $\frac{1}{2}$, and:

$$\frac{1}{2} \wedge \frac{1}{2} = \frac{1}{2}.$$

Consequently, $P\neg Fp$ turns out to be false, that is 0. Under the same assignment, (+d) is indeterminate and gets the value $\frac{1}{2}$ [Copeland, 1996, 18-19].

$$\frac{1}{2} \rightarrow 0 = \frac{1}{2}.$$

And so, analogously, the same result is obtained with the Master Argument's conclusion, namely (z). In fact, Prior [1955a]'s first philosophical aim is to prove that, even if we deal with a valid argument, and despite some doubts regarding its soundness, by the previous assignment, it is certain that (z) is not a logical law in $DIOD$. Therefore, there exists at least a solution in order to

defeat the deterministic doctrine of Diodorus. Before focusing on (z), and in order to obtain our claim, that is a counterexample to (z) interpreted as a $DIOD$-tautology, we show the matrices for $\wedge$, $\vee$, and find out the one for $\neg\Diamond$.

TABLE 2.9: Prior's three-valued matrices for $\wedge$, $\vee$, and $\neg\Diamond$

$\wedge$	1	$\frac{1}{2}$	0
1	1	$\frac{1}{2}$	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
0	0	0	0

$\vee$	1	$\frac{1}{2}$	0
1	1	1	1
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
0	1	$\frac{1}{2}$	0

$\neg\Diamond$	
1	0
$\frac{1}{2}$	0
0	1

For the reader's convenience, we recall the formula (z):

$$(\neg p \wedge \neg Fp) \rightarrow \neg\Diamond p.$$

If the conjunction in the antecedent is contingently true or indeterminate ($\frac{1}{2}$), as was already stated; and if the proposition $\neg\Diamond p$ is false (0), because by definition of $\Diamond p \leftrightarrow (p \vee Fp)$, we obtain $0 \vee \frac{1}{2} = \frac{1}{2}$, and $\neg\Diamond$ on $\frac{1}{2}$ returns the value 0, then, we are able to identify at least one case in which the conclusion of the Master Argument, namely (z), is not always true, but only indeterminate, *i.e.* $\frac{1}{2}$:

$$\frac{1}{2} \rightarrow 0 = \frac{1}{2}.$$

And so, the claim that the premises of the Master Argument truly lead to the conclusion that 'it is impossible that the shell will be seen' – on the supposition of the Łukasiewicz indeterministic or three-valued logic – according to Prior 'is definitely false' [Prior, 1955a, 212].

Furthermore, (z) produces two counterexamples, just as in the following matrix:[43]

[43] They are given by the following couples for p and Fp: $(\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, 0)$.

TABLE 2.10: Counterexamples showing that (z) is not a three-valued tautology

p	Fp	$\neg p$	$\wedge$	$\neg Fp$	$\rightarrow$	$\neg\Diamond p$
1	1	0	0	0	1	0
1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	0
1	0	0	0	1	1	0
$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	1	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0
0	1	1	0	0	1	1
0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1
0	0	1	1	1	1	1

Therefore, Prior shows the conclusion of the argument to be correctly inferred from the premises, but he proves that we are also able to get our assignment and reach that (z) has the value of $\frac{1}{2}$. Moreover, also Prior [2003b, 88] alludes to the strength of a three valued or indeterministic approach, by means of which we can deduce that the Master Argument fails to reach its deterministic target.

However, there is no doubt that Diodorus supported determinism. So, he would never admit a third or indeterminate value between 1 and 0. Moreover, it is possible to obtain (+d) from the fourth axiom of Hamblin's system, namely $(p \vee Pp) \leftrightarrow \neg F\neg Pp$.[44] This is relevant if we consider the Diodorean authorship of the Master Argument, because, although Hamblin claimed that time was dense, Prior noted that the previous axiom supports the discreteness of time, and Diodorus account was atomistic [Denyer, 1981b, 49].
Furthermore, if we contrapose (+d), we get the formula $HFp \rightarrow (p \vee Fp)$, that seems to enclose both discreteness and determinism.

[44] $HAMB$ 6. $(p \vee Pp) \leftrightarrow \neg F\neg Pp$
$\neg F\neg Pp \rightarrow (p \vee Pp)$
$\neg F\neg Pp \rightarrow \neg(\neg p \wedge \neg Pp)$ [*by De Morgan*]
$(\neg p \wedge \neg Pp) \rightarrow F\neg Pp$ [*by contrapositon*]
i.e. (+d): $(\neg p \wedge \neg Fp) \rightarrow P\neg Fp$ [*by mirror image*]
Prior gives an analysis of Hamblin's system in [Prior, 1967, 45-50]; Hamblin deals with the theme in the correspondence conserved in the *Prior Collection*.

Certainly, Prior thought that (z) is not a threat in a modern context for an indeterministic approach. From Prior's works on OCK and PEI, *e.g.*, [Prior, 1966], [Prior, 1967, 122-136], we gain new elements for the debate. These are already disclosed in advance by some remark in Prior-Kripke correspondence, in relation to the themes of branching time, indeterminism and the status of tense logics (*see*, [Ploug and Øhrstrøm, 2012]). In sum, by interpreting the structure of time as branching rather than linear, and opting for an OCK semantics, the formula $(\neg p \wedge \neg Fp) \rightarrow \neg\Diamond p$ does not rule out that, in some given future path, p might be true.

Nevertheless, Diodorus' doctrine seems scarcely compatible with the clause about the branching of time. This last is also contrary to a genuine determinism, as Prior himself admits.

> Genuine determinism would be the belief that there is only one possible future, and to express this you really do need to go beyond K_t and add a postulate for nonbranching of the future, *e.g.* $PFp \rightarrow p \vee Pp \vee Fp$ ('Whatever has been "on the cards" either is the case or has been the case or is "on the cards" still') [Prior, 1969a, 329].

Altogether, as a result of the focus on the Master Argument and on Prior's analyses of it, since the argument is valid, but (z) is not a three-valued tautology, we acknowledge the following alternatives:

- Łukasiewicz three-valued logic is not adequate to $DIOD$.
- This logic is adequate to $DIOD$, but at least one of the premises of the Master Argument is questionable.[45]

[45] Against [Akama et al., 2011], the fact that the Master Argument cannot be justified by a three-valued logic does not entail that determinism is false. By the first horn of thesis, the three-valued approach does not do justice to Diodorus' intuitions about $\Diamond$ and $\Box$. By the second horn, we would assume a not-Diodorean perspective in $DIOD$, thus betraying the original intent of the Master Argument and dealing with a different logical puzzle.

2.3 The reception of Prior's account

We will look at the reception of Prior's account on two coordinates: the *Danish Master Argument* (2.3.1), and the search concentrated on Prior's unpublished manuscripts and the genesis of his archive (2.3.2).

2.3.1 P. Øhrstrøm and P. Hasle: a Danish Master Argument

A fascinating reconstruction of the Master Argument is given in [Øhrstrøm and Hasle, 1995, 23-28]. This reconstruction which we call *Danish Master Argument*, does not add additional premises to Diodorus' argument. Nevertheless, the *Danish Master Argument* does require some background assumptions:

(a) Time is discrete;[46]

(b) The relation **T**(t, p) means 'p is true at t'. Further, the verb *akolouthein* in the second premise refers to Diodorean implication, defined by $(p \Rightarrow q)$ iff $(\forall t)(\mathbf{T}(t,p) \rightarrow \mathbf{T}(t,q))$;

(i) The Master Argument refers to statements which correspond to propositional functions.

These assumptions should be taken alongside the following definitions of possibility and necessity of Boeth. *Int.* 9, 234: $\Diamond p \leftrightarrow (p \vee Fp)$, $\Box p \leftrightarrow (p \wedge Gp)$.
The first premise of the Master Argument is as in Prior: $Pp \rightarrow \Box Pp$.
The second premise uses the concept of Diodorean implication, which is formalised as: $((p \Rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$.
Finally, the third premise is: $\neg q \wedge \neg Fq \wedge \Diamond q$.

Øhrstrøm and Hasle use semantical methods to show the contradiction between the third premise and the previous two. As a first step, they pose a

[46] In the case (a) is brought into question – but we believe it is not the Diodorean case – Øhrstrøm and Hasle [1995] suggest to substitute (a) by (A): no proposition has a first instant of truth. If a proposition is true, it has already been true for some time (Arist. *Ph.* 236a, 12-14): it is true over intervals with last but without first instant of time.

hypothesis about the meaning of q, allowing that $\neg q \wedge \neg Fq \wedge \Diamond q$, *i.e.* the excluded juncture by Diodorus. Suppose 'Dion is here' is q. Let w be the statement 'The prophet says: Dion never will be here', that is supposed to be true only in the atomic instant immediately before the present instant. Hence, Pw is false at any past time, and it is true from now on.

From the first premise $Pp \rightarrow \Box Pp$, we are able to get, for the present time, the formula $Pw \rightarrow \Box Pw$, whose consequent we write as $\neg\Diamond\neg Pw$. Then, we get the following matrix, by (a), where t_0 stands for the present time, t with positive n for the future, and t with negative n for the past.

Table 2.11: Danish matrix for the Master Argument

t_{-3}	t_{-2}	t_{-1}	t_0(now)	t_1	t_2	t_3
$?q$	$?q$	$?q$	$\neg q$	$\neg q$	$\neg q$	$\neg q$
$\neg w$	$\neg w$	w	$\neg w$	$\neg w$	$\neg w$	$\neg w$
$\neg Pw$	$\neg Pw$	$\neg Pw$	Pw	Pw	Pw	Pw

We deduce the Diodorean implication between q and $\neg Pw$, that is $q \Rightarrow \neg Pw$. In fact, it is evident that $(\forall t)\neg(\mathbf{T}(t,q) \wedge \mathbf{T}(t,Pw))$, therefore $(\forall t)(\mathbf{T}(t,q) \rightarrow \mathbf{F}(t,Pw))$. From the Master Argument, and assumption (i), we get the second premise $((p \Rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$, by the substitutions p/q, $q/\neg Pw$. Therefore we obtain $((q \Rightarrow \neg Pw) \wedge \Diamond q) \rightarrow \Diamond\neg Pw$. But by the substitutions in the first premise, we already get $\neg\Diamond\neg Pw$, a contradiction between the last sentence and the consequent $\Diamond\neg Pw$. Moreover, we also obtain the negation of the second sentence of the Master Argument, *i.e.* the impossible does not follow from the possible, therefore Øhrstrøm and Hasle rule out the third proposition $\neg q \wedge \neg Fq \wedge \Diamond q$.

We should notice that Prior's Master Argument includes the formalisation of the original *κυριεύων λόγος* and from four premises deduces the conclusion. On the other hand, the Danish Master Argument formalises the Hellenistic argument, but does not propose any decisive strategy to infer the conclusion. In fact, Øhrstrøm and Hasle assume the third premise, which contradicts the first two.

Moreover, Prior has used four premises, two of which are Diodorean, while the other two are supposed to be consistent with Diodorus' doctrine.

Øhrstrøm and Hasle's Master Argument main achievement consists in avoiding the use of new premises. However, to reach their goal they require the assumptions (a), (b), (i).[47]

Let us see how the premises are involved in these different accounts.

- The Danish first premise is (a) in Prior. It suffices to define the box ($\Box$) by the diamond ($\Diamond$) and *vice versa*.
- The second premise of the Master Argumet is different in Prior's argument and in the Danish one. In Prior it is $\neg\Diamond q \rightarrow (\Box(p \rightarrow q) \rightarrow \neg\Diamond p)$, in Øhrstrøm and Hasle's version is $((p \Rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$. The second premise of the Danish Master Argument requires an interpretation for the double arrow, that is, a semantics for the Diodorean connective $\Rightarrow$: $(p \Rightarrow q)$ iff $(\forall t)(\mathbf{T}(t,p) \rightarrow \mathbf{T}(t,q))$. And yet, it is provable that Prior's second premise is equivalent to the Danish Master Argument second premise.[48] In any case the formalisation of Øhrstrøm and Hasle extends the system from propositional logic to first order logic.
- If (A) is, in some way, also valid in Prior's account, (i) is properly present. If we substitute an actual sentence to a variable in a law we still have a tautology.

[47](i) guarantees the opportunity to substitute sentences or constants to the variables in the premise.

[48]We prove the equivalence from Prior's formula to the Øhrstrøm and Hasle's one:

1. $\neg\Diamond q \rightarrow (\Box(p \rightarrow q) \rightarrow \neg\Diamond p)$
2. $\neg(\Box(p \rightarrow q) \rightarrow \neg\Diamond p) \rightarrow \neg\neg\Diamond q$

[by *contraposition*]

3. $\neg(\Box(p \rightarrow q) \rightarrow \neg\Diamond p) \rightarrow \Diamond q$

[by eliminating the *double negation*]

4. $(\Box(p \rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$

[inferred by *Chrysippus C11*]

Since $(p \Rightarrow q)$ iff $(\forall t)(\mathbf{T}(t,p) \rightarrow \mathbf{T}(t,q))$, we get $\Box(p \rightarrow q) \equiv \neg\Diamond(p \wedge \neg q) \equiv (p \Rightarrow q)$. Therefore, 1 (Prior's second premise) is equal to 4 (Øhrstrøm and Hasle's second premise).

- Contrarily to Øhrstrøm and Hasle, even if Prior uses some additional premises, he refuses the assumption of the modal definitions. In fact, the same Alex.Aphr. *in APr.* 1, 184, 5 mentioned that the ancient Master Argument was proposed by Diodorus to obtain the modal definitions, in particular for the possible.

In general, Prior's strategy has the advantage of having a clear syntactic side, via a Hilbert style proof. On the other hand, the Danish Master Argument seems more perspicuous on the semantics side, by exemplifying or considering explicit counterexamples.

In both proofs, we are trying to define time and modality, the metaphysical topics of Diodorus Cronus (*cf.*, [Denyer, 1999]), using the tools of modern tense logic.

2.3.2 Prior's Nachlass

An essential source for the researcher who deals with Priorean studies and the birth of temporal logics is *the Nachlass of A. N. Prior*. In this section we would present:

- the *Prior Collection*, at the Bodleian (Oxford) and on the internet [NAP, 2014]
- structure and contents of Prior's Nachlass
- the genesis of an archive for Prior
- an invite to the researcher on Priorean fields [VLP, 2011]
- cooperative research and web-multimedia

The great part of *Prior Collection* is held in the Bodleian Library, Oxford, in 22 boxes; while, 7 further boxes are kept by the Philosophy Department Library,

10 Merton Street, OX1 4JJ; the whole Prior's Nachlass consists of a total of 29 boxes, originals materials are made available for researches at the Weston, *Special Collections Reading Room*, Broad Street, OX1 3BG. The core is represented by boxes 1-11, including materials by Prior. Boxes 12-29 mainly hold miscellaneous documents, by friends and colleagues who cooperated with Arthur.

The Nachlass counts various documents: letters, offprints from published papers, originals for published (typewritten and handwritten), unpublished (typewritten and handwritten), other kinds of material (*e.g.* lecture notes). The researcher in Priorean fields may avail of 297 papers and drafts (200 of these are by Prior), and 1102 letters (247 from Prior, 829 to Prior, 26 between others persons and that Prior received as notes or attachments).

The history of Prior's Nachlass in a sense is traced both in some posthumous contributes [Prior, 2003a] edited by Øhrstrøm and Hasle, in [Copeland, 1996] and [Øhrstrøm and Hasle, 1995]. Furthermore, a *virtual path* to retrace the history of the Nachlass is on the internet. Since 1999, a first web-source was launched on the web by Øhrstrøm and Hasle. The current version of that web-project is *Foundations of Temporal Logic – The WWW-site for Prior-studies* [wPS, 2014] (`http://www.priorstudies.org/`).

In 2007, Mary Prior authorised Øhrstrøm and Hasle to publish the scientifically relevant material from *Prior Collection*. But only in 2011, the Danish group obtained the full permit from the Bodleian Libreries to photograph the documents in the Nachlass and to insert these in an interactive web-database. The database was opened to Priorean researchers of all the world, authenticated by a web-login. The main purpose is a first critical edition of relevant Prior's unpublished. Jørgen Albretsen created *The Virtual Lab for Prior Studies* database [VLP, 2011], and a Priorean communities in all the world has begun to work and cooperate on the material of Prior Nachlass.

By recapitulating on the archive, we underline its dual characterisation. A common user may go to the Weston at Oxford and ask for consulting the *Prior Collection*, otherwise he can benefit from the web. [wPS, 2014] introduces us

to the Prior's Nachlass, and shows its catalogue. On the wPS [2014] we find a detailed outline and some tools for the web-user of [VLP, 2011]. Here is the list which shows structure and contents of Prior's Nachlass. Both Prior's letters in boxes 1-3 and materials of boxes 12-21 follow the alphabetical order of the addressed or involved persons:

Box1 Correspondence (with Anderson-Bennett)

Box2 Correspondence (with Gabbay-Montgomery)

Box3 Correspondence (with Nerlich-von Wright)

Box4 Correspondence (with various persons), and other items

Box5 Mainly offprints of Prior's papers

Box6 Mainly papers/drafts

Box7 Mainly papers/drafts and theological papers

Box8 Logical drafts, lectur notes, etc.

Box9 Lectur notes on logic and ethics, Medieval handhouts and translations

Box10 Miscellaneous of courses materials

Box11 Notes and scrapbooks

Box12 Various from (Aaqvist-Anderson)

Box13 Various from (Baier-Clifford)

Box14 Various from (Cocchiarella-Frege)

Box15 Various from (Gale-Kenny)

Box16 Various from (Laraiche-Meredith)

Box17 Various from (Montague-Prucnal)

Box18 Various from (Reid-Rosser)

Box19 Various from (Routley-Ryan)

Box20 Various from (Scott-Sobel)

Box21 Various from (Stenius-Ziedins), and printed materials

Box22 Full version of Prior's unpublished *The Craft of Formal Logic* donated by Mackie

Boxes23-29 Offprints from the collection of Prior donated by Mary Prior

The NAP [2014] project as an internet open-source is only minimally developed. However, many researchers are working at the VLP [2011] to increase available documents for a critical edition of Prior's unpublished papers and correspondences. First results have been announced by Øhrstrøm in the final session of the *A. N. Prior Centenary Conference (August 20–22, 2014) Balliol College, Oxford*, but much more remains to be done.[49] Here is the NAP [2014] in its current configuration on the internet, at the web-page `http://nachlass.prior.aau.dk/`:

- 40 papers
- 5 letters

Papers concern several topics in many disciplines (logic, theology, philosophy of language, ontology, and various from diary notes), reproducing the eclecticism of Prior's interests. Letters refer to two important correspondences, namely Prior-Kripke (4 items) and Prior-Mates (1 item).

Our survey on the genesis of Prior's Nachlass, starts with an important testimony, a piece of the interview given by Mary Prior, widow of Arthur, to Hasle and Øhrstrøm in 1997, later transcribed in the expanded edition of [Prior, 2003a, 293-310] – original video is available through the YouTube channel [yPS, 2014].

[49] A Special Issue of *Synthese*, Vol. 193, 11, November 2016, is dedicated to *the Logic and Philosophy of A. N. Prior*, and collects 26 contributes presented during the Conference at the Balliol [Albretsen et al., 2016b].

> After the memorial service in Oxford which took place about three weeks later people came back in the house for drinks, and Bill Kneale took me aside and suggested I ask Peter Geach to help me cope with Arthur's papers, as he had proved himself capable in such matters – he was one of Wittgenstein's executors. Peter was there and I asked him. The only window of opportunity for some time was that week-end, and so we went down to Arthur's room at Balliol and did a vast but rapid preliminary sorting of papers and correspondence. These letters were of course letters to Arthur. Collecting letters from Arthur was more difficult. I wrote around to most likely recipients, but of course few people keep letters to the extent Arthur did. They had for long been his lifeline and were never destroyed. However, some were saved, though the only massive collection is from Alan Ross Anderson. Tony Kenny and Peter Geach put into publishable form one uncompleted book manuscript by Arthur, which appeared as *Objects of Thought* [Prior et al., 1971]. They edited a collection of Arthur's papers posthumously [Prior, 1976b]. [...] Kit Fine edited and supplemented the material for a book of which Arthur had left only one completed chapter [Prior and Fine, 1977]. [...] The archive has existed now for over 25 years. Only recently has it been much used, but it is a rich source not only of Arthur's work, but also because it contains so many letters from logicians and philosophers about their own work – letters written in the days when letters were the common method of communication over a distance.

The idea of a Nachlass of Prior is by W.C. Kneale. It is meaningful that less than a month from Prior's death, the author of one of the best handbooks in history of logic of the last century [Kneale and Kneale, 1962] judges as worthwhile to realise an archive which preserves Prior's manuscript and unpublished papers.

Mary Prior embraced Kneale's proposal, and Peter Geach – who worked together Prior during the years in Oxford, and has already curated with G.H.

von Wright the archive of Wittgenstein in Cambridge – seemed to be the best candidate to start the task. Today, the higher ambition increased also from those efforts is in improving the NAP [2014], by dedication of many scholars at the VLP [2011]. As suggested by Kneale, then, Mary Prior asked and obtained by Geach his joining in the challenge for Prior's Nachlass.

Therefore, as a first step, a selection of documents keept in Prior's room at the Ballion formed the nucleus of further investigation: in particular, drafts by Prior and correspondences. However, as Mary Prior declares in the interview, many letters signed by colleagues of Prior were found in his room, but no letter by Prior. Nevertheless, since 1969 a lot of Prior's letters have been recovered, while many others are yet missing and should be retraced. In fact, Prior's letters are very relevant under both personal and technical aspects. At the time, after Arthur's death, Mary Prior wrote to all the husband's colleagues, fellow logicians, and friends: she asked them to send her copies of the letters from Prior, so they could be deposited in the Bodleian, that is, the Nachlass location. However, also till nowadays, there are different gaps hiding many correspondences.

Conciliating between personal researches and presentations of unpublished, many friends and colleagues of Prior cooperated in order to reconstruct his thought by manuscripts and letters. Mary Prior encouraged some of them well-knit with Prior – as it is for A. Kenny, P. Geach, and K. Fine – to complete and shed light on some shared but yet unpublished works. Thus, during the 1970s, four important books were edited, starting from some researches and studies on unpublished drafts.

Geach and Kenny edited a posthumous typewritten dated 1964 – at the time possessed only by some Prior's friends. Prior's work was further expanded by completing notes and remarks [Prior et al., 1971].

At second, Geach and Kenny arranged a new collection of papers, with some published and unpublished contributes, in relation to the topics of logic and ethics [Prior, 1976b].

Geach and Kenny edited also [Prior, 1976a], that is a first draft, as a survey in history of logic. Prior had revised the original version by more formal and less historical aspects in [Prior, 1955b], as a consequence of editorial choices.

At last, Fine, who expanded the project of a book to be titled as *Worlds, Times and Selves*. In fact, only the first chapter of this book has been completed by Prior before he died [Prior and Fine, 1977].

Let us recapitulate two points of the last piece of Mary interview, namely, (i) the birth of Prior's Nachlass and its fruition, and (ii) further reasons because the Nachlass is of utmost importance.

By referring to (i), the history of Prior's Nachlass develops over time, and new recovering of Prior's drafts and letters increase step by step the number of items and the value of the Nachlass. Thus, even if in a sense the archive exists already from the beginning of the 1970s, a richer fruition of documents is guaranteed only with the second part of 1980s and the 90s. David and Stephanie Lewis played a leading role, year after year, in order to deposite and organise papers in the Bodleian. R. Gallie, O. Flo, Mary Cresswell, J. Copeland, P. Øhrstrøm, T. Braüner, T. Kleif, P. Hasle and T. Müller have been habitual users of the Nachlass in Oxford [Copeland, 1996, 312].

In relation to (ii), the Nachlass is not only the home of Prior's documents; more in general, it conveys a pioneering way to intend temporal logics and philosophy of logic. Many contributions and papers by different scholars are kept in the Nachlass of Prior (*e.g.*, by N. Belnap, R. Bull, N. Cocchiarella, M. Cresswell, D. Gabbay, S. Kripke, J. Łukasiewicz, I. Thomas). They were involved into the debate with Prior, and their drafts and letters to Prior mark the route for new trends in the history of logic of the last century.

We underlined that the wPS [2014] was a first step to pursue a more important aim on the internet, in order to increase knowledge on Prior's life and works. Furthermore, the underway building of a web-archive, that is [NAP, 2014], is related to the dedication of a community of researchers from different countries

of all the world. According to [Albretsen et al., 2016a], people participating in the *Virtual Lab* are a close-collaborative-community. Accesses of everyone are allowed after a web-login, in order to join a crowdsourcing work on documents of the Nachlass. We can define the *virtual web potentiality* in this sense: users have the chance to make better every contribute in the database by digitising unpublished, starting or improving comments, attaching remarks to the original and – lastly but not least – theirselves grow skills and knowledge by each other. Moreover, these efforts on the *Prior Collection* guarantee to researchers original works, and produce new and qualified criticism in relation to Prior's unpublished documents. This is the importance of [VLP, 2011] as the internet *locus* in which the NAP [2014] is under development. The VLP [2011] final purpose is to make open-sources the digital edition of transcribed and proofread documents, by recovering Prior's heritage.

Every researcher in temporal logics and the birth of a logic of tenses, philosophy of time, and all those with deal with Priorean fields, are invited to join the VLP [2011] in order to contribute at the NAP [2014]. To pursue this aim, qualified approach and intuition on unpublished drafts and correspondences are required to scholars working at the *Virtual Lab*, in order to recover Prior's heritage. The editors of the project wrote a letter of invitation by claiming that all significant papers should be transcribed and the texts should be enriched with relevant comments [Hasle and Øhrstrøm, 2013]. For all digitised and commented papers follows a standard peer review process, which defines the quality of singulars efforts.

Let us now elucidate the VLP [2011] functioning, funded by the Danish Council for Independent Research as part of the project *The Primacy of Tense: A.N. Prior Now and Then*, within digital humanities perspective. According to [Albretsen et al., 2016a], the status as of September 2016 counts 48 researchers from around the world that have been granted a login to the *Virtual Lab* web-site.

Once we have logged by username and password, a first interface makes reference both to the contents of the *Virtual Lab*, and to the transcribed and proofread letters and papers already located in the web *Nachlass*. By opting for the *Virtual Lab*, the main menu is divided in:

(a) Papers

(b) Letters

We can link to 4 groups of documents:

$(a1)$ Papers – entire database

$(b1)$ Letters – entire database

$(b2)$ Letters ordered 'From Prior to X'

$(b3)$ Letters ordered 'From X to Prior'

In the case of $(a1)$ and $(b1)$ a new interface invites us to enter search criteria in one or more fields: *entry number, author, title, content, date, comment*. While, just click on one among the previous link $(b2)$ and $(b3)$, the corresponding list of item is catalogued in alphabetical order, plus the additional info of the number of documents contained for every entry. Further, in every entry obtained from $(b2)$ and $(b3)$, the list of documents is displayed on the screen by chronological order. Finally, for every document $(a1 - b3)$ the user finds two kind of description:

- A legend denoting typologies of record by colours:
  - *Red*: not yet digitised,
  - *Yellow*: transcribed but not yet proofread,
  - *Green*: transcribed, proofread, and already catalogued in [NAP, 2014].
- Full name, affiliation, and email address of who has edited a given document (transcribed or partially transcribed and commented).

We show why the Nachlass online and efforts for the *Virtual Lab* are to be understood within a cooperative research and as a web-multimedia project.

[VLP, 2011] is a web platform including the database of photographed contents from *Prior Collection*. This database gives to users the opportunity to be actively involved in the digital transcription and the critical process by producing a collective work. The main aim is to produce the edition of Prior's relevant unpublished works. The approach in [VLP, 2011], concerns collective participation, and lends surplus effects on the research: scholars from many universities, by different scientific areas, academic experiences, and perspectives make available their active contribution. Prior was an eclectic logician and philosopher, interested in several fields, *e.g.* theology or physics. Multidisciplinary approaches to Prior's papers and correspondence should lead to understand better his works, both historically and also by referring to new trends, *e.g.* in hybrid logics and their applications.

According to [Albretsen et al., 2016a], and also by our experience joining in the VLP [2011], we explain the usual practise: after the registration as a member of the *Lab*, the user that likes to give a contribution may ask to the web-site administrator one or more texts to transcribe. In any case, all users are able to click and read every document uploaded in the database, or also to add notes and remarks. However, the user need to require to sign up in order to get credit for transcribing. Once letters or papers have been commented and transcribed by users, and proofread by the editorial board, then the approved document is placed at the NAP [2014] web-page. To access these first results no login is required to visitors on the internet, but they have only to respect the rule to mention the NAP [2014] for every quotation from the archive.

In conclusion, we would remark how web-multimediality is pursued also by the proposal of a YouTube channel, within a project concerning both Arthur Prior's logic and philosophy and digital humanities perspectives. In [yPS, 2014] there are fifteen videos. A very precious document is the first part of *Mary Prior interview* (1997). Mary tells about Prior as his life-partner, by following him with

her love also in Arthur's studies and investigation. Max Cresswell is protagonist of another biographical interview, in relation to *his professional career and his work with Arthur Prior* (2014), starting from Prior supervision of his Ph.D at the University of Manchester. In [yPS, 2014] we can find other two documents by Cresswell. The first is the Aalborg lecture dedicated to *The History of Temporal and Modal Logic in the 1950s* (2008). The second is included among a series of talks given during the *Arthur Prior Centenary Conference - at Balliol, Oxford (2014)*; the other uploaded videos from that Conference are by Kit Fine, Jack Copeland, Hans Kamp, and Anthony Kenny, as well as two further videos regarding the *Opening* and the *Closing session*. Five videos of [yPS, 2014] are from the conference titled *Birthday Party*, Roskilde, December 4th, 2014, by Peter Øhrstrøm, Per Hasle, David Jakobsen, Torben Braüner and Jack Copeland.

Conclusions

The focus of this thesis are modal and temporal notions, both in the first Hellenistic Age and in particular in Diodorus Cronus' thought (1) and in contemporary philosophy, especially in the logico-philosophical work of Arthur Norman Prior (2).

Results

The main results of this thesis are as follows:

A reconstruction of Diodorus Cronus' thought. Our historical analysis concerned three aspects of Diodorus' thought: physics, language, logic. On physics, we argue that Diodorus' view is a species of ancient atomism, linked with arguments against motion (1.1), thus firmly placing Diodorus inside the tradition represented by Democritus and Epicurus. On language, we show that Diodorus favours conventionalism. Furthermore, we unravel and explicate his tense doctrine (1.2).

A detailed analysis of the Hellenistic *κυριεύων λόγος*. The starting point of our investigation are the various accounts of capacities and modalities in ancient thought. We provide a comparative analysis of some of the relevant tenets and we propose a formal interpretation of these notions, as properties and operators. The historical adequacy of this formal analysis is ensured by paying attention to the debate initiated by Arist. *Metaph.* 9. Drawing on [Makin, 1996, 2006], we propose a novel account of Diodorus' doctrine of capacity (1.3.1.1).

We show that this (conjectural) reconstruction is coherent with Diodorus' treatment of modalities. With respect to these last notions, we provide a detailed study by considering Boeth. *in Int.* 234-235, and other historical sources 1.3.1.2 (cf., next appendix A). In section 1.3.1.3, we distinguish fatalism and Diodorus' determinism. In section 1.3.1.4 we continue our investigation with an analysis of the definitions of *possible* and *capable* as reported by Simpl. *in Cat.* 196, 4-6; this strengthens the case for our hypothesis that the Diodorean view of capacity is consistent with the established definition of modality. In section 1.3.2, we provide an interpretation of the *κυριεύων λόγος*, by starting from the original structure and key-words of the argument, as reconstituted starting from the Greek text of *Epict.* 2, 19; in this section we introduced a tense formalism for a modern version of the Master Argument.

A reconstruction of the birth of tense logic. In section 2.1.1, we reconstruct the historical debate on Diodorean systems, in order to compare three such systems, namely $DIOD$, $ModalDIOD$ e K_t4P. In section 2.1.2 we study the Hamblin-Prior and Lemmon-Prior correspondence: we analyse K_t and $HAMB$ systems, with a particular attention to the semantics, and some important properties such as linearity, symmetry, discreteness, density, and minimality.

An in-depth study of Prior's version of the Master Argument. In section 2.2, we trace the motivation driving the New Zealander logician to deal with the Master Argument. In section 2.2.1, we provide a step by step analysis of Prior's formalisation, proof and strategies against the deterministic conclusion (z). Following [Prior, 1955a], we take the four premises of Prior's Master Argument as theorems, but alternatively we can interpret the inferential chain as a syntactic derivation of (z). We argue that the trivalent logic approach is not consistent with given Diodorus' modal intuitions. Finally, in section 2.3.2, we compare Prior's version of the Master Argument with the *Danish* version.

Further lines of investigation

Our present research is open to further developments. In particular, the following topics provide fruitful directions for further inquiry:

Re-thinking the ancient truth criteria for conditional sentences. We would like to focus on stoic logic, in relation to our study of the Master Argument, modal and temporal notions. One of Diodorus Cronus' doctrines concerns the truth criterion for conditional statements. According to S.E. *M.* 8, 112-117, as well as *P.H.* 2, 110-112, there are four such criteria at play in the epoch: Philo's, Diodorus', that based on *συνάρτησις*, and the one based on *ἔμφασις*. The Diodorean criterion can be formulated as follows: a conditional is true if, it neither was nor is it the case that its antecedent is true and the consequent is false. This was a contested and much discussed topic starting from the first part of the 20th century, e.g., [Nelson, 1930], [Hurst, 1935], [Mates, 1949], [Mates, 1973, 42-51]. Later, e.g. [Mignucci, 1966] attempted to link this criterion of truth with the Master Argument, and [Bobzien, 1998, 97 and ff.] with modality. Starting from the present thesis it is envisageable that one can (i) provide a novel interpretation of the 4 criteria; (ii) gain a better insight of the connection between the Diodorean criterion, modality and the second premise of the Master Argument.

From Prior's heritage and hybrid logics. Many a scholar has pointed out that Prior's work is directly connected with modern hybrid logics, e.g. [Blackburn, 2006], [Øhstrøm and Hasle, 2006], [Areces, 2007], [Blackburn and Jørgensen, 2016a,b], [Pleitz, 2016], [Tulenheimo, 2016]. There is scope for a in-depth historical investigation of the roots of this connection starting from the first claims [Prior, 1967, 187-203], developments and problems [Prior, 2003a, 117-158, 171-256] and further lines [Prior and Fine, 1977]. Likewise, we plan to explore the idea that hybridisation characterises a universal modal theory concerning *modal model theories* and, as a consequence, to be able to apply this perspective in a cross-disciplinary and pluralistic approach to one or more fields, for instance cognitive science, linguistics, artificial intelligence, computer science.

To continue the VLP [2011] project. Our investigation has been enriched by the study of some of Prior's unpublished manuscripts. We have also pointed out to the urgent need to recover the missing parts of the correspondence between Prior and Hamblin and, repectively, Prior and Lemmon. Prior Collection still holds a many documents which, although irrelevant to the present investigation, are, nevertheless, worthwhile being studied. Such a research would help us gain a better understanding of Prior's work, it being known that, given his wide and eclectic range of interests, Prior made significant contributions to the birth of various logics or better logical traditions. We are especially interested in studying the problem of 'entailment', with its relation to *strict implication* and *relevant logics*. The correspondence between Prior and Ross Anderson (1955-1969) and Belnap (1960-1962) may hold some interesting suprises.

Appendix A

Formal semantics for ancient doctrines: Diodorean ontologies

In this appendix, we develop a formal model for times and modalities. Our starting point is, again, Diodorus, whose modal notions can be perfectly well included into a modern system of modal logic.[1]

Furthermore, Diodorus' definition of the possibility as what is or will be, and the necessity as what is and will always be true (Boeth. *in Int.* 234, 22-26), makes it clear that modalities and temporal notions are interdefinable. I would will the reader the basic technicalities of a modal/temporal semantics for a Diodorean logic and not just that.

Let us consider a given formalism and, following [Denyer, 1999, 244-246], a formal language mixing tense with modalities. In fact, the semantics for that language concerns both modal and temporal notions. According to Diodorus, what is possible or necessary, and what has been or will be at one time, respectively depends on what is true in other worlds and at other times. This is the relevant difference between modal/temporal notions and capacities (*see*, section 1.3.1.4). Valuations on capacities concern the truth of individual properties at

[1] *See*, [Prior, 1955a]; as well as above, section 2.1.1, in order to obtain a historical reconstruction.

a single world, while valuations on modal/temporal notions are in relation to different worlds.

In order to make this clear, we will focus on six frames, which depict some ancient ontologies of Diodorean modal and temporal notions.

Possibility (as well as its dual, *i.e.* necessity) being a modal notion, presupposes a binary relation between worlds that also grounds temporal notions. In the second graphic we will also represent the exceptional case in which the frame only consists of one point. Actually, only from the third graphic onwards we will be able to fully express the view of Diodorus Cronus, because it describes his deterministic ontology, in addition to his temporal and modal perspective.

Let us consider W, that is the set of worlds/times, represented by the variables: $j, r, i, z, q, s, w, x, y, m, ...$

Consider also a relation $R \subseteq W^2$.

More formally, suppose that $\mathcal{L}$ is a propositional language containing, besides the usual connectives, the additional unary operators: $\Diamond, \Box, F$.

A model for $\mathcal{L}$ is a triple $\mathcal{M} : \langle W, R, v \rangle$, such that:

W and R are as above; v is $FOR(\mathcal{L}) \rightarrow \mathcal{P}(W)$ that respects the following conditions:

- $v(p) \subseteq W$ for every variable p;
- $j \varepsilon v(\neg\alpha)$ iff $j \notin v(\alpha)$;
- $j \varepsilon v(\alpha \wedge \beta)$ iff $j \varepsilon v(\alpha)$ and $j \varepsilon v(\beta)$;
- $j \varepsilon v(F\alpha)$ iff there exists $i \varepsilon W$ s.t. jRi and $i \varepsilon v(\alpha)$;
- $j \varepsilon v(\Box\alpha)$ iff for every $i \varepsilon W$ s.t., if jRi then both $j \varepsilon v(\alpha)$ and $i \varepsilon v(\alpha)$;
- $j \varepsilon v(\Diamond\alpha)$ iff there exists $i \varepsilon W$ s.t., jRi and either $j \varepsilon v(\alpha)$ or $i \varepsilon v(\alpha)$.

Then,

1. α is true in $\mathcal{M} : \langle W, R, v\rangle$ iff for every $j\varepsilon W$, $j\varepsilon v(\alpha)$;

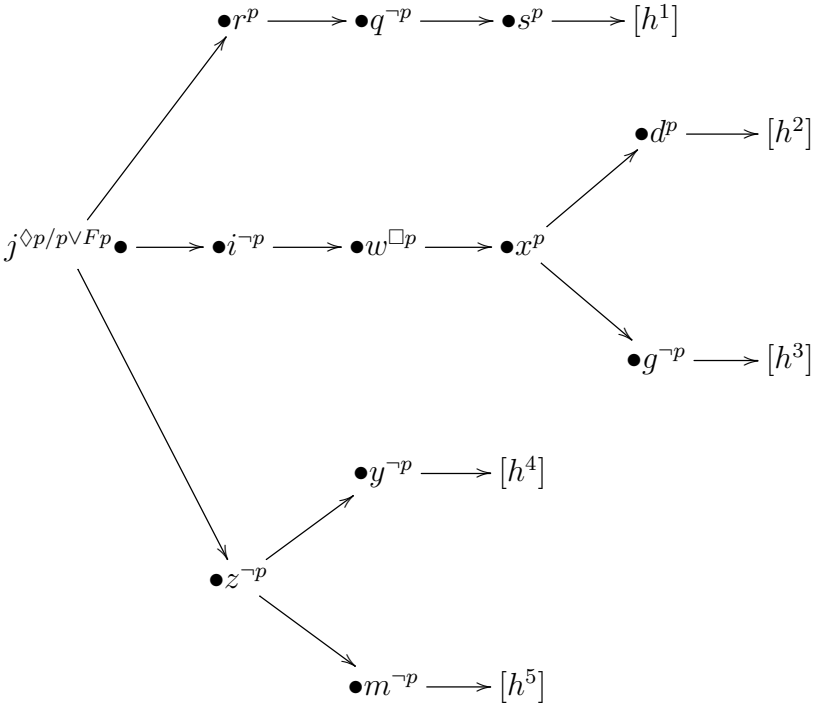
2. α is true in the frame $\langle W, R\rangle$ iff for every valuation v, α is true in $\mathcal{M}$.

Here is briefly how we should read notation and diagrams. Sometimes, instead of $j\varepsilon v(\alpha)$, we write $j \vDash \alpha$; while, the easily understood jRi, stands for $(j, i)\varepsilon R$. If jRi, in our graphics we use an arrow pointing from j to i:

$$\bullet j \longrightarrow \bullet i$$

Furthermore, every variable on the frame-nodes, represents the proposition forced at that world/time in virtue of an indexical reference. For instance, the index p of the proposition r, namely r^p, means that as a result of $v(p)$ at r, $r \vDash p$. Finally, we use h or the word *history* for every given path on the frame such that, h is the set of nodes that are considered for the valuation v on the frame. Let us begin with the first frame.

Table A.1: Counterfactual modalities and times



We can opt for one of two different interpretations of the frame. These are usually defined as *Ockhamist* and *Peircean*, and in relation to them, we respectively resort to OCK and PEI.[2]

OCK There exists only one real history, namely, only a given h, and there are several counterfactual worlds/times alongside the other paths h^1-h^5;

PEI Simultaneous histories (as well, h^1,h^2,h^3,h^4, and h^5) are real, as in the case of living in parallel universes.

OCK depicts the view of someone who gives importance to the present moment and generally prefers a tensed approach to temporal notions. Therefore, it makes sense to opt for a given path, that is, the only real history. Let us consider modal and temporal sentences on the world/time j as the actual one, and the history h^2 as our path, namely $\langle j, i, w, x, d, ...\rangle$. There exists at least one node in h^2, at which $(j)\varepsilon v(\alpha)$, that is for j, w, x, namely, $j\varepsilon W$ and $j \vDash p$. While, by PEI, $j\varepsilon W$ s.t. jRi, only provided that $j \vDash p$. This is the only chance, because by looking at the nodes subsequent to j on h^4 and h^5, it follows that $z, y, m \nvDash p$, and we need that, for every h, there is at least one world/time point forcing p on W.

To be thorough about W of table A.1 we refer also to modal necessity ($\Box p$) and its Diodorean temporal interdefinition ($p \wedge Gp$). Let the actual world be the node w. The approach presented in OCK ensures us about h^2; in fact, $w, x, d \vDash p$. Things are different if we opt for PEI: there is at least one node, as it happens, on h^3, namely g, s.t. $g \nvDash p$.

Let us recapitulate our discussion in relation to Hellenistic dialecticians. Even if the tree in table A.1 has been built to represent Diodorus' notions, it is not authentically Diodorean. None of the ancient reports on Diodorus allow us

[2]Both approaches have been introduced by Prior [1966, 157-161] and Prior [1967, 113-136]. Prior himself adopted PEI as covering his own philosophical tenets [Øhrstrøm and Hasle, 1995, 195], at least in a first period, but we can feel also his *Ockhamism*, for instance as a more appropriate view to avoid the determinism. OCK has been axiomatised in Burgess [1979] and Thomason [1984], studied by *e.g.* Brown and Goranko [1999],Øhrstrøm [2009], and Malpass and Wawer [2012], dealing with the so-called *thin red line*. On the other hand, PEI has been axiomatised by Zanardo [1990], and studied *e.g.* by Carmo and Sernadas [1989], and Øhrstrøm [1996].

to hypothesise that counterfactual worlds/times were a part of the Megaric's ontology. The tree above allows us to consider propositions referring to counterfactual possibilities, which are avoided in Diodorus' theory. For instance, branching of a frame may be interpreted as a result of counterfactuals: starting from a given node, our history may fork. An example is provided by the node x, and what follows it, on each of h^2 and h^3. *E.g.*, we suppose p to be the proposition 'This piece of wood is burning'. Then, on the path h^2, it is the case at d that, as a result of a blaze, $d \vDash p$. However, we should also consider g on the path h^3, and maybe that piece of wood dropped to the bottom of the sea: as driftwood, it does not burn, $g \nvDash p$. We can rightly hypothesise that, the described branching represents the point of origin for counterfactual worlds/times, such that the proposition p is true at d on h^2; and, simultaneously, the same proposition p is false at g on h^3. This is why the tree in table A.1 is certainly appropriate for Philo's definition of possibility. The Philonian view of possibility as *ἐπιτηδειότης* allows us to maintain that it is possible for a given piece of wood to burn, even in the driftwood case. In fact, if the disposition of that wood in x is to burn, then in virtue of its *ἐπιτηδειότης* that piece of wood can burn, and our proposition p may be true; but in every path, as on h^3, that piece keeps its disposition, even though on h^3 it does not burn, so that $g \nvDash p$.

Only two interpretations locate one point in order to describe modality on a temporal frame: the time conceived as totality (according to Parmenides's doctrine) and, the last moment of time (as a borderline case for Diodorus' perspective). In those two cases, there are no simultaneous paths or histories, while modality is expressed by necessity on a given *dead-end* of the frame.

Table A.2: The dead-end case

With only one history, entirely situated on one point, it does not make sense to distinguish OCK from PEI. The dead-end case is the frame representing the fact that everything is eternally given in one world/time: past, present and future are the same ($Pp \equiv p \equiv Fp$), and every modality is collapsed into necessity. In virtue of the frame in table A.2, we can assume the best Parmenidean claim for the reality of the world, the *One Being*, that is the proposition p: 'being is'. On the other hand, interpreting the Diodorean dead-end case demands that we are aware of Diodorus' determinism. The dead-end point, as the last moment of time, stands for the last actualization of a contingent proposition. If something is possible, then it will happen sooner or later. Let us give a romantic example. Bob is a very bashful and timid guy. If p is the contingent proposition 'Bob kisses Viola', and supposing that $\Diamond p$, we have no reason to be worried since $j \vDash p$. The grand finale of Bob and Viola's love story is safe, even if, at that moment, time will end, and everything collapses in the Parmenidean *One Being*. In logical terms, every proposition is necessary in a dead-end since jRj.

We have already examined the borderline case represented by the last moment of time in Diodorus' perspective in table A.2. The Diodorean complete doctrine can be described by logical frames in virtue of the following four tables. Diodorus' determinism is conveyed by the chain-frame and the singular history, *h*. No counterfactual world/time is given in Diodorus' view, and they do not allow for counterexamples, not even momentary ones to tensed Diodorean propositions.[3] These last points may bring some clarity about Diodorus' determinism; this claim is confirmed by some ancient texts, as Cic. *Fam.* 9, 4 and Simpl. *in Cat.* 196, 4-6. Furthermore, since it will be possible to formulate contingent propositions, we state that Diodorus' theory is not a fatalism (*see*, section 1.3.1.3). Therefore, I propose four tables, in which are represented Diodorus' modal and temporal notions on the supposed worlds/times (j, r, i, z) of the W frame.

[3] In relation to the topic, *see* the diatribe between Denyer [1981b, 52-53] and Bobzien [1998, 102-108].

Table A.3: No counterfactual and determinism. Time and possibility

$$j^{\Diamond p/p \vee Fp} \bullet \longrightarrow \bullet r^{p} \longrightarrow \bullet i^{\neg p} \longrightarrow \bullet z^{\neg p} \longrightarrow [h]$$

The Diodorean possible is interdefinable with the future tense operator on p, namely F. $j \vDash \Diamond p/p \vee Fp$ on W. In fact, there is at least one node, namely r, such that jRi and $r \vDash p$. On the other hand, $j \nvDash \Box p/p \wedge Gp$, since there exists at least one node, *e.g.* i, such that $i \nvDash p$.

Table A.4: No counterfactual and determinism. Time and necessity

$$j^{\Box p/p \wedge Gp} \bullet \longrightarrow \bullet r^{p} \longrightarrow \bullet i^{p} \longrightarrow \bullet z^{p} \longrightarrow [h]$$

The Diodorean necessity is interdefinable with the strong future tense operator on p, namely G. $j \vDash \Box p/p \wedge Gp$ on the frame. In fact, for every node on h, namely j, r, i, z, $(j) \vDash p$.

Table A.5: No counterfactual and determinism. Time and impossibility

$$j^{\neg \Diamond p/\neg p \wedge G\neg p} \bullet \longrightarrow \bullet r^{\neg p} \longrightarrow \bullet i^{\neg p} \longrightarrow \bullet z^{\neg p} \longrightarrow [h]$$

The Diodorean notion of impossibility is the contradictory of the notion of possibility. $j \vDash \neg\Diamond p/\neg p \wedge G\neg p$. In fact, there is no node j, r, i, z, such that $(j) \vDash p$, then $(j) \vDash \neg p$.

Table A.6: No counterfactual and determinism. Time and non-necessity

$$j^{\neg \Box p/\neg p \vee F\neg p} \bullet \longrightarrow \bullet r^{p} \longrightarrow \bullet i^{\neg p} \longrightarrow \bullet z^{p} \longrightarrow [h]$$

Finally, the Diodorean notion of non-necessity is the contradictory of the notion of necessity. We state that $j \vDash \neg\Box p/\neg p \vee F\neg p$. In fact, there is at least one node on W, *e.g.* i, such that $i \nvDash p$, that is $i \vDash \neg p$.

Appendix B

Unpublished MS from the *Prior Collection*

This appendix presents some documents from *Prior Collection*. Original materials from [VLP, 2011] are provided with transcripts, editorial notes, and comments. These remarks are presented in the form of footnotes to transcriptions of original manuscripts (MS). If an MS is composed of two or more pages, the numbers into brackets denote the (explicit or implicit) page number of the MS in transcript. For the reader's convenience, every transcription in this appendix is followed by photographic reproductions of the original MS from the VLP [2011]. All the contents of this appendix are available also on the web, being this author's contribution to the NAP [2014].

We opt for a selection of texts which illustrate different types of unpublished MS kept in the archive. Thus, we propose six documents gathered into the following three thematic sections:

1. Time and logic
2. Prior's letter concerning Mates, *Stoic Logic*
3. Tense logics and the missing letters

The first section contains one paper, namely [Prior, ndb]. This is a draft by Prior, and it focuses on a study of logical features of temporal determinations. The paper has been studied during the *Workshop on the Nachlass of Prior* (Aalborg, Aug. 11-14, 2014). The second section presents a letter addressed to Mates, [Prior, 1954], dealing with some remarks about [Mates, 1973]. In the third part of this appendix, we collect three letters to Prior ([Hamblin, 1961, 1965], and [Lemmon, 1966a]), and a letter from Hamblin [1969a] to Mary Prior. The topic of this last section is not only related to the birth of tense logics, but it concerns the problem of the missing letters as well.

B.1 Time and logic

The Place of Time in Logic by Arthur Prior[1]

My first purpose in this paper is to discuss precisely where, within the whole body of logical theory, we ought to locate the study of the logical features of temporal determinations. Putting it that way, however, suggests that the structure of logic as a whole is something given and fixed, into which we must fit the logical study of temporal determinations as best we can. In fact, however, the situation is not quite as straightforward as that, and it may turn out that the logical study of temporal determinations suggests a radical re-structuring of logic as a whole.[2] So my second purpose is to take a good look at this possibility.[3]

[1] Edited by P. Øhrstrøm and the present author. The original is kept in the *Prior Collection* at Bodleian Library, Oxford, box 6. *Virtual Lab* n. 147, transcribed in [NAP, 2014]. The first title for the manuscript was *Time and logic*, later modified. The MS is composed of two pages (front and paginated 2), three other pages (paginated 14-15-16), and one note page, numbered 68.

[2] Following Prior's thought, logic doesn't concern static values. Logic embraces dynamical conceptions.

[3] Prior proposed a general rethinking of logic; in particular, a fitting formalism is required in order to build paradigms of temporal logics by tensed languages. Some references are gathered by Prior-Kripke correspondence in order to underline why, according to Prior, a tenseless logic does not work with temporal descriptions, and in particular with indeterminism [NAP, 2014]. Others references, by *re-structuring of logic as a whole*, may be found in Leśniewski's calculus, quoted in [Prior, 2003b, 76-83]: furthermore, we can trace an analogy concerning Leśniewski's 'a is a b' (in particular its *weak* interpretation) and Prior and Kamp's *Since* and *Until* logic, e.g. in [Kamp, 1968].

There is what might be called a standard modern view as to the structure of logic as a whole. According to this standard modern view, the basis on which the remainder of logic rests – the ground floor of the subject, as it were – is the propositional calculus, conceived as a study of truth-functions of {2} propositions, roughly the study of the logical features of words like 'if', 'and', 'or' and 'not'. In this branch of the subject, or maybe root of the subject, we have such laws as "if p-and-q then p", "if if p then q then if not q then not p", "if if p then q and if q then r then if p then r", and so on. On top of that we build the first-order predicate calculus, roughly the study of the logical features of the words "Everything" and "Something". In this branch of the subject we have such laws as "If everything that is F is G, then if something is F, something is G". In representing such laws, we generally employ not the vernacular "Everything" and "Something" but the quantifiers "For all x" and "For some x", so that the law just mentioned is given the form "If for all x if fx then gx, then if for some x, fx, then for some x, gx."[4]

{14}[5] "I remember reading a paper" and "That ship is a long way off" are contractions of "I remember myself reading a paper" and "That ship is a long way off from there". The speaker, we would like to say, is a constituent of the fact remembered or spoken of, even if it is unnecessary (given on speech conventions) to say so. It could be argued, however, that it is not like this at all, and that what is the case is always what is the case from a certain point of view, just as it is always what is the case at a particular time.[6]

Let us briefly develop this parallel. What is the case, most of it, hasn't always been the case and will not always be the case; but from what is the case, if it's given fully enough, we can infer, what has been the case and what will be the case. Tomorrow, what is the case today will have ceased to be the case, but it will also come to have been the case the day before, or as we will then put it,

[4]Pages 3-13 are missing in the MS.

[5]The following has been crossed out: "but it could be argued that in fact it is not so, and that in fact everything that we say or think, even everything that is the case, has built into it a certain point of view."

[6]Temporal relations are compared with spatial relations. It is significant that Prior was searching for a *topology of time* for his tense logics [Prior, 1967, 32-58].

to have been the case yesterday. Today {15}[7] – these things are commonplace – will be tomorrow's yesterday, and tomorrow will be tomorrow's today. If you want to know what day really is today, will of course today – today's today, if you want to put it like that – really is today; but if you want to know what day is timelessly today, there's no such day as that.[8] Similarly, it might be said, that a ship really is a long way off; that is what is the case; but if you want to know, not what just is the case, but what is the case on the ship, then what is the case on the ship isn't that the ship is a long way off but that the ship is right here. What just is the case, is of course – according to this story – what is the case from my point {16} of view, and from that, if it will be given in enough detail, we can work out what is the case from the ship's point of view, or from any point of view; but if you want to know what is the case without being the case from my point of view, it could be said that this is senseless; it is like asking what is the case without being the case now.[9]

I don't know whether I've expressed this third reaction to special relativity very well. I probably haven't, because it isn't in fact my own reaction, which I am

[7] The following has been crossed out: "say, will be tomorrow's yesterday, and tomorrow will be tomorrow's today. Similarly, what is the case, in the sense of what is the case from my point of view, e.g. that the ship is a long way off, isn't the case from the ship's point of view, what is the case from the ship's point of view is that the ship is right here."

[8] Prior's logic is tensed. Logic is about the real world. Every point of a temporal structure has a relative actual present with its temporal determinations.

[9] By this MS-passage at pp. 15-16, Prior's philosophical perspective about time fits for introducing us from an *earlier-and-later* to a *pure temporal logic*, as an anticipation of today's hybrid logics. So, Prior states: 'not only are *instants* not genuine individuals, but there are no genuine individuals, only certain propositions that can be formally treated as if they were individuals' [Prior, 2003a, 219]. Therefore, we can refer more in detail to three extensions: *Nominals*, *Satisfaction operator on nominals*, and the binder $\forall$. Nominals recur as $a, b, c, ...$, like propositional symbols denoting a sum of formalisable occurrences in relation to a given moment. Satisfaction operators are in the form of $@_a, @_b, @_c, ...$, such that $@_a\varphi$ is true if and only if φ is true at the moment a refers to. And $\forall a\varphi$ is true if and only if φ is true in whatever moment a refers to. Blackburn [2006] presents to the reader an analysis about Prior's heritage for hybrid logics. In relation to formalism, we specify that Prior writes $Ta\varphi$ rather than $@_a\varphi$.

At p. 16, Prior gives further reasons for preferring his tense logical approach – in which we would add a semantical level through an accessibility relation between world-states or hybrid accounts, rather than *e.g.* standard Kripkean tree-structures in which every world is already fixed and timeless. In fact, it is pointless both to ask what is the case without being the case now, and also to consider as indifferent the singular point of view from which every individual can describe time and its features.

afraid is the old and crude one, the first. But now I want to add something which isn't so much about the nature of the present as about the nature of physics.[10]

Note page (68)[11]

$$\forall(m)P(n)P(m)p \to P(n)P(l-n)p$$
$$\forall(m)P(n)P(m)p \to P(n+l-n)p$$
$$*\forall(m)P(n)P(m)p \to P(l)p$$
$$[l \geq n]$$[12]

$$\forall(m)P(n)F(m)p \to P(n)F(n-l)p$$
$$\forall(m)P(n)F(m)p \to P(n-n+l)p$$
$$*\forall(m)P(n)F(m)p \to P(l)p$$
$$[n \geq l]$$

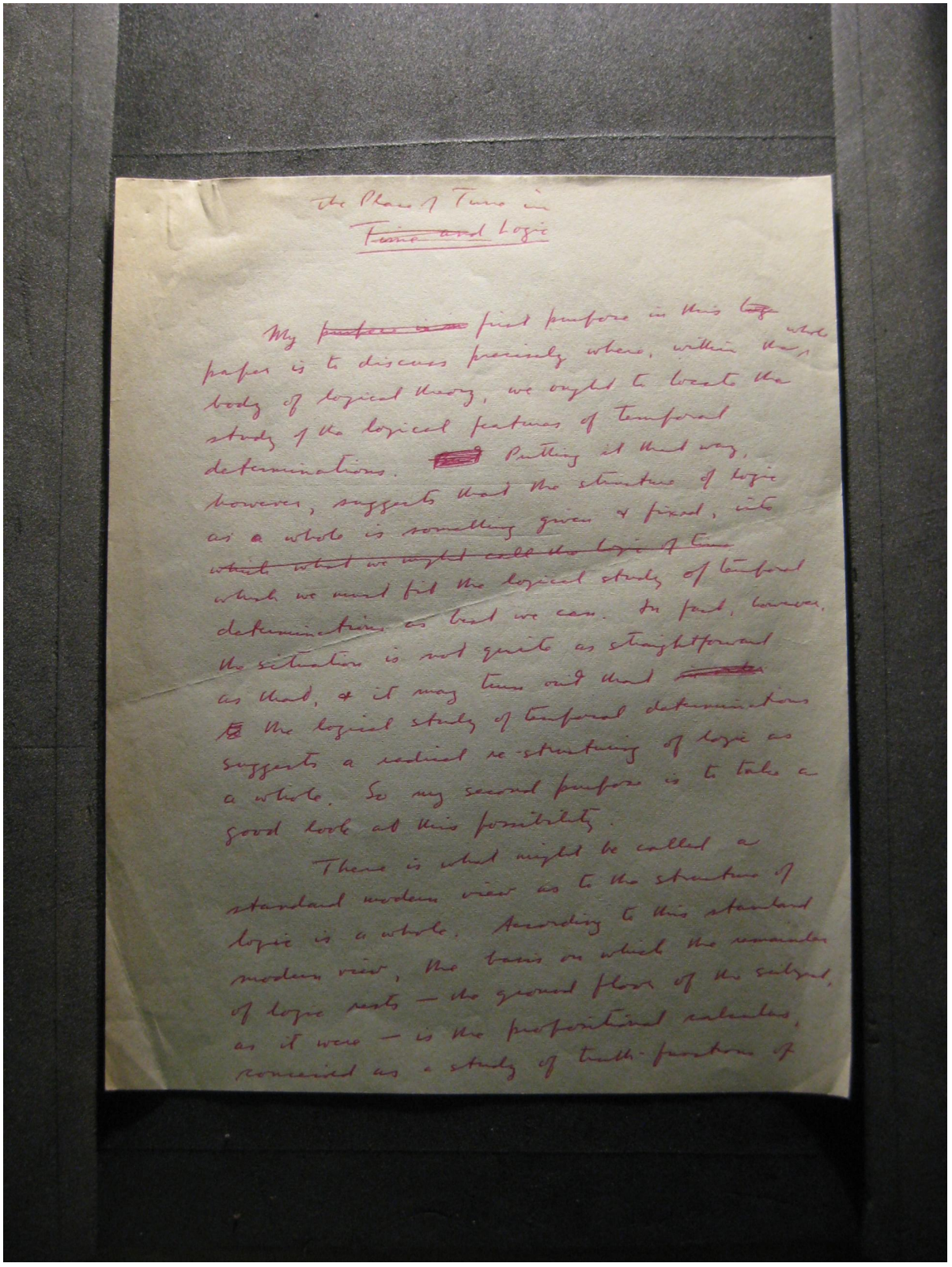
$$\forall(m)P(n)F(m)p \to P(n)F(n+l)p$$
$$\to \forall(m)P(n)F(m)p \to F(l)p$$

OK

[10] The following has been crossed out: "I have set up to sorts of metaphysical pictures ... there is a difference between what will appear to be the case ...".

[11] We don't know the page to which this note makes reference. It concerns a derivation in a metric tense logic, *cf.* [Prior, 1967, 95-112]. The derivation is in Polish notation in the original MS, we transcribe it in standard notation.

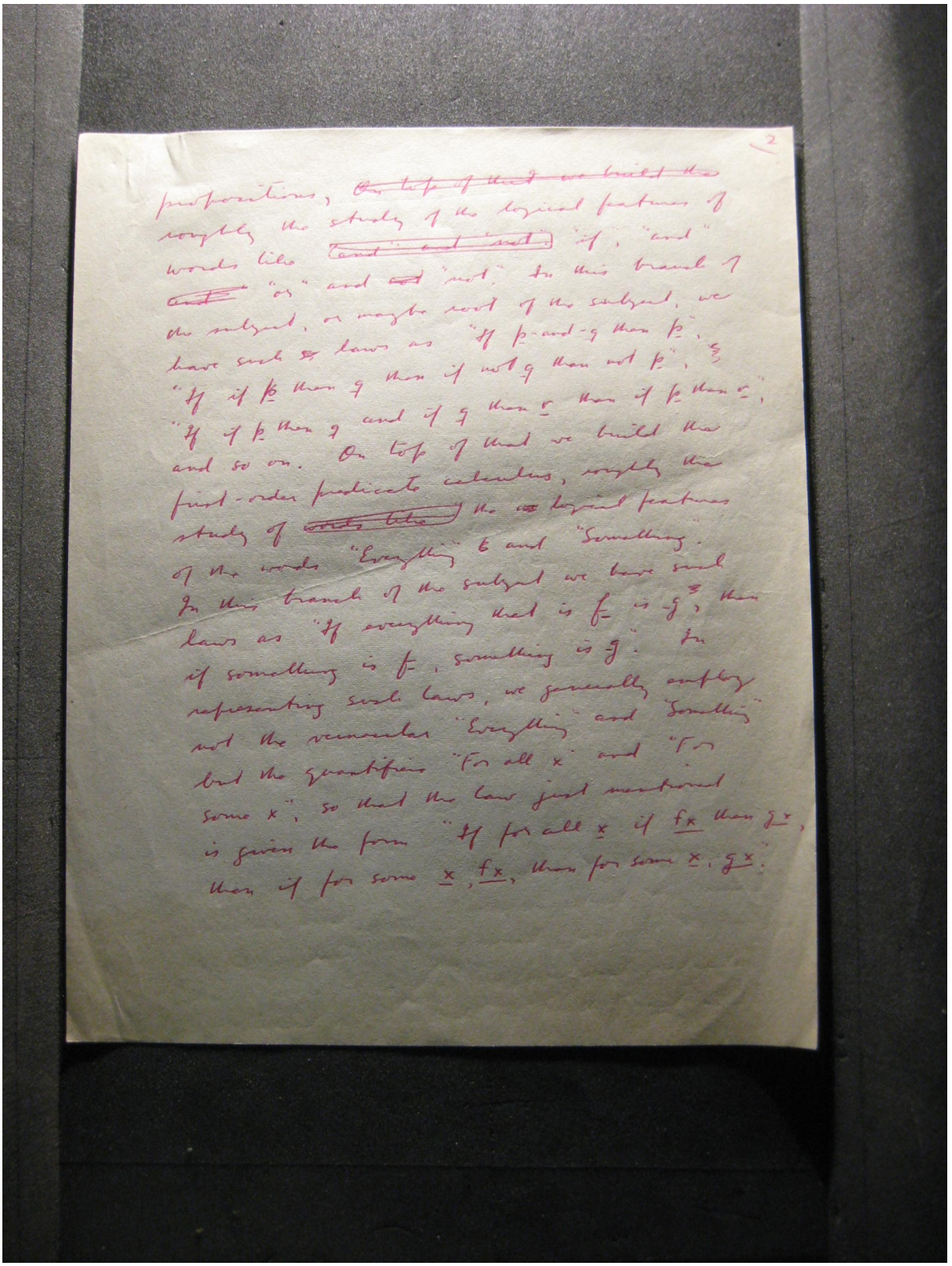
[12] The following lines have been crossed out:
$\forall(m)P(n)F(m)p \to P(n)P(l-n)p$
$\forall(m)P(n)F(m)p \to P(n+l-n)p$
$\forall(m)P(n)F(m)p \to P(l)p$
$[n \geq l]$

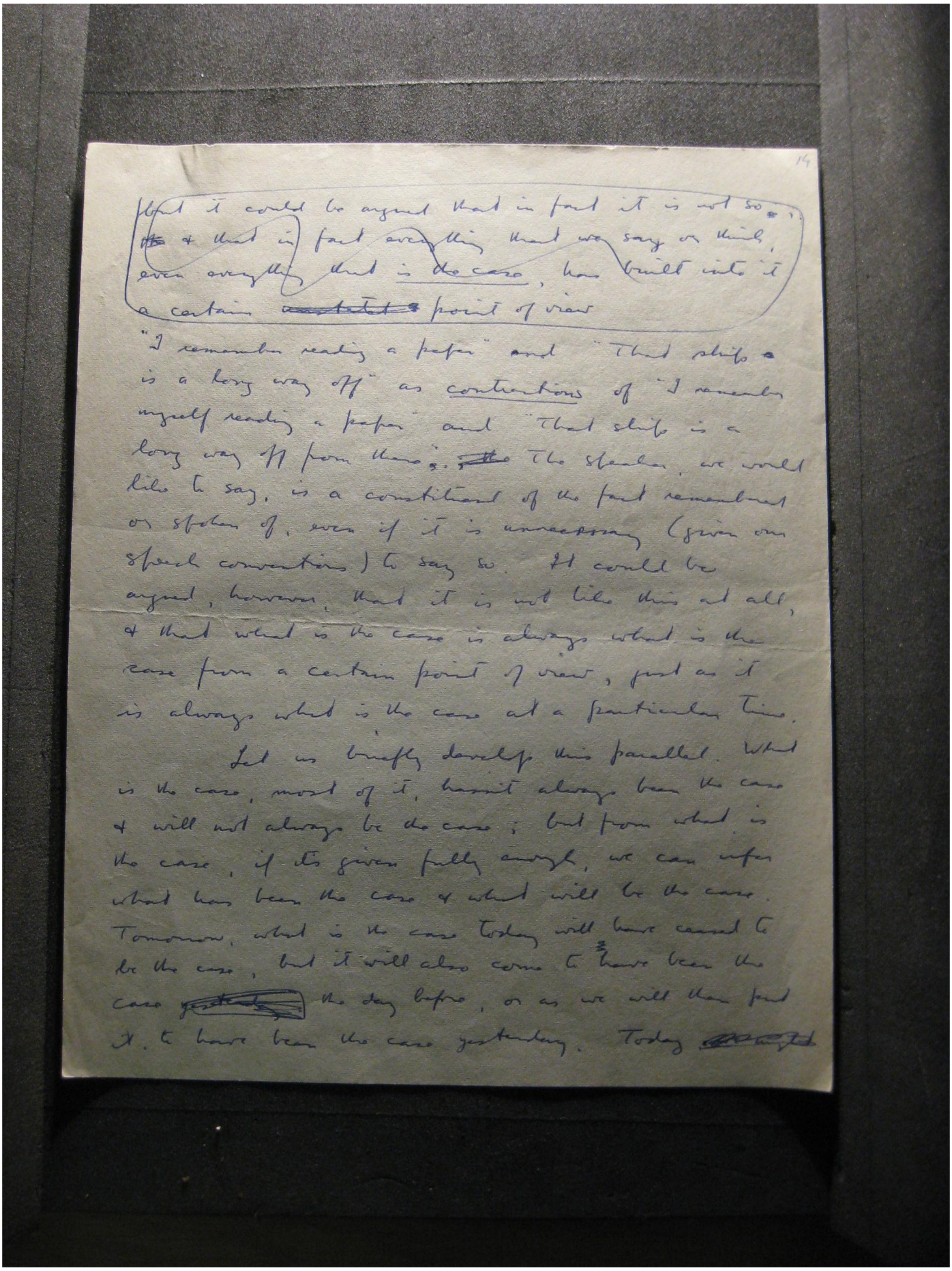


The Place of Time in
~~Tense and~~ Logic

My ~~purpose~~ first purpose in this ~~lecture~~ paper is to discuss precisely where, within the body of logical theory, we ought to locate the study of the logical features of temporal determinations. Putting it that way, however, suggests that the structure of logic as a whole is something given & fixed, into ~~which~~ what ~~we might call the logic of time~~ which we must fit the logical study of temporal determinations as best we can. In fact, however, the situation is not quite as straightforward as that, & it may turn out that ~~the~~ the logical study of temporal determinations suggests a radical re-structuring of logic as a whole. So my second purpose is to take a good look at this possibility.

There is what might be called a standard modern view as to the structure of logic is a whole. According to this standard modern view, the basis on which the remainder of logic rests — the ground floor of the edifice, as it were — is the propositional calculus, conceived as a study of truth-functions of





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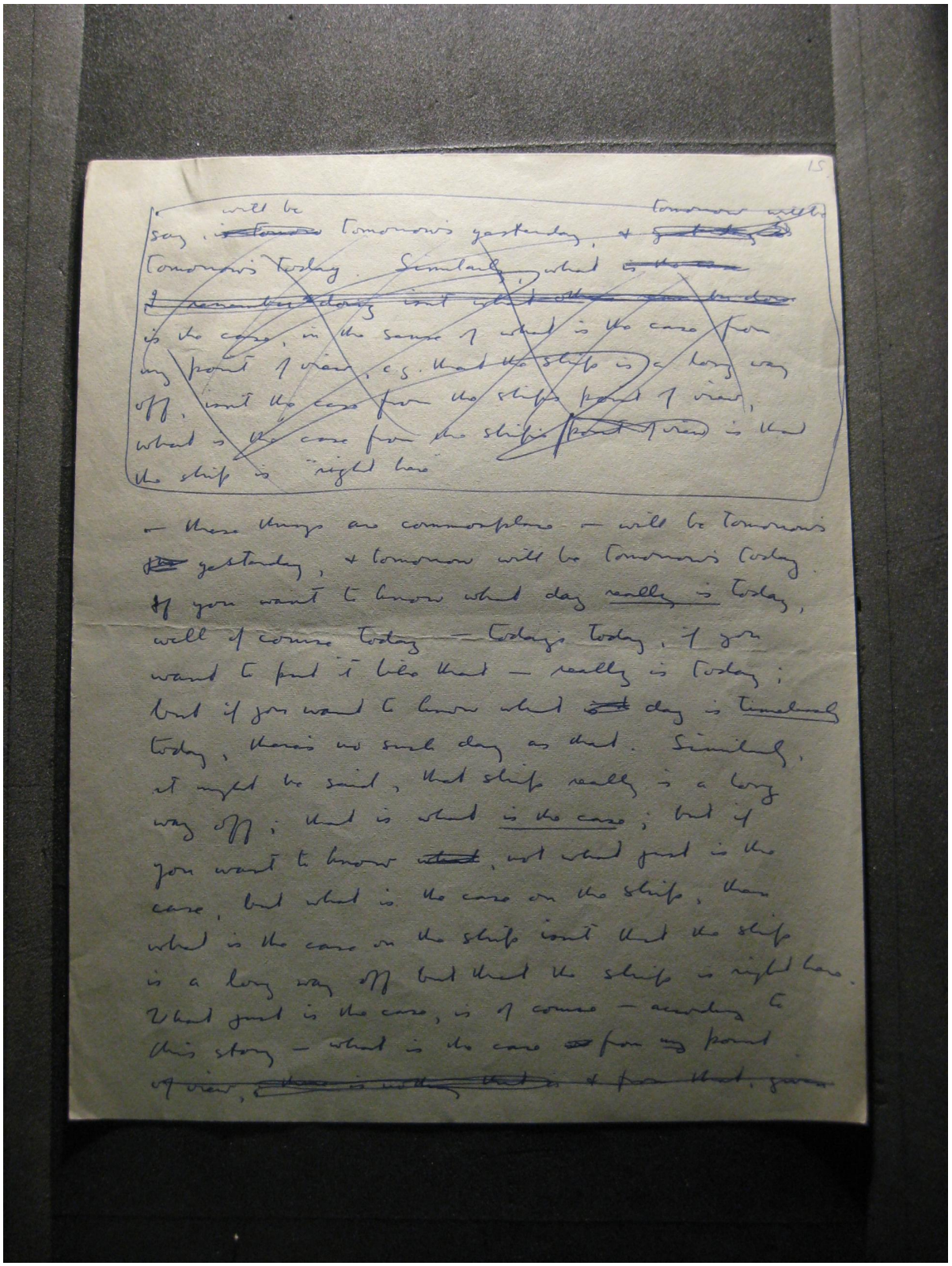
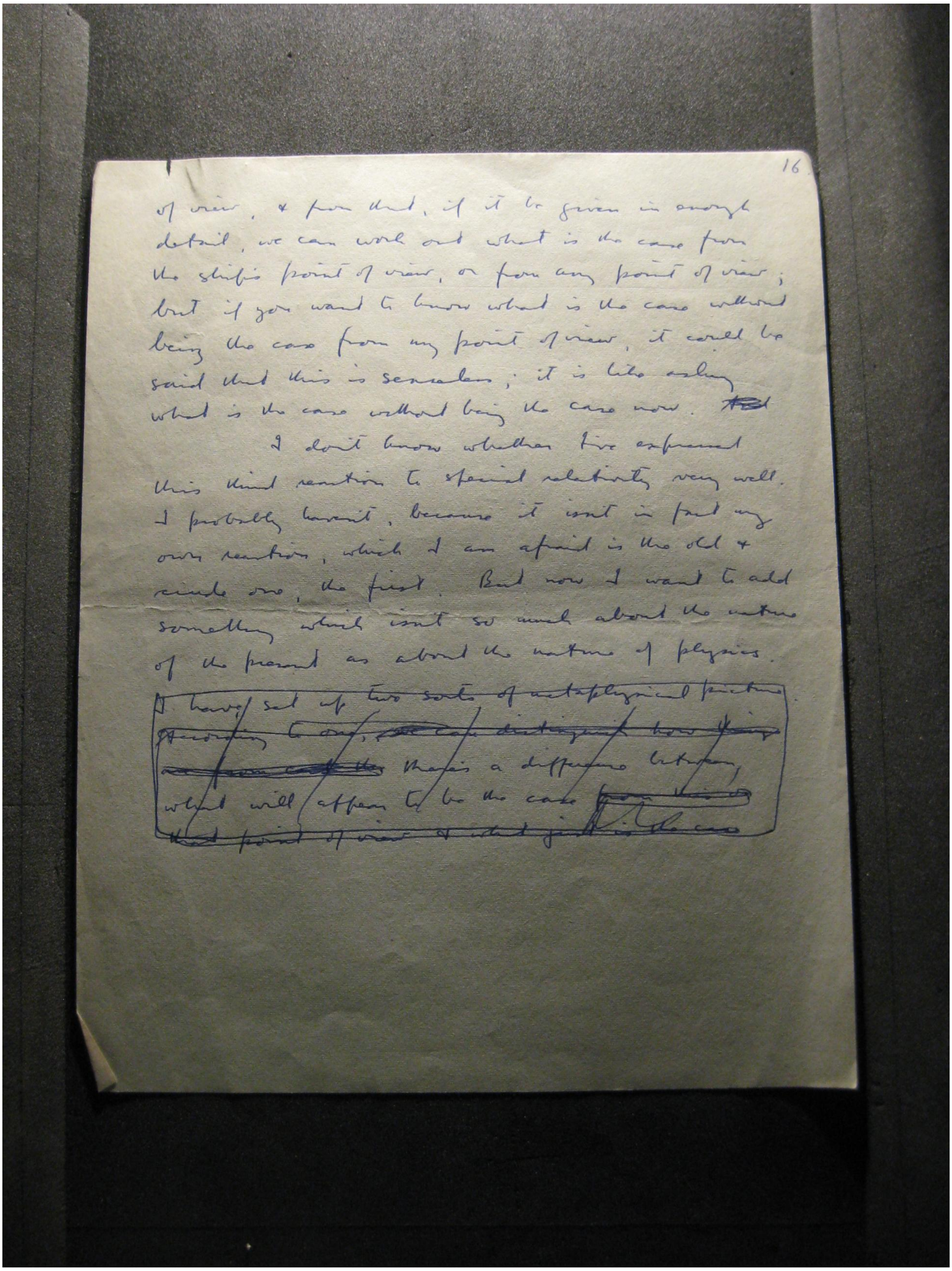
But it could be argued that in fact it is not so; & that in fact everything that we say or think, even everything that is the case, has built into it a certain point of view

"I remember reading a paper" and "That ship is a long way off" as contractions of "I remember myself reading a paper" and "That ship is a long way off from there". The speaker, we would like to say, is a constituent of the fact remembered or spoken of, even if it is unnecessary (given our speech conventions) to say so. It could be argued, however, that it is not like this at all, & that what is the case is always what is the case from a certain point of view, just as it is always what is the case at a particular time.

Let us briefly develop this parallel. What is the case, most of it, hasn't always been the case & will not always be the case; but from what is the case, if it's given fully enough, we can infer what has been the case & what will be the case. Tomorrow, what is the case today will have ceased to be the case, but it will also come to have been the case the day before, or as we will then put it, to have been the case yesterday. Today





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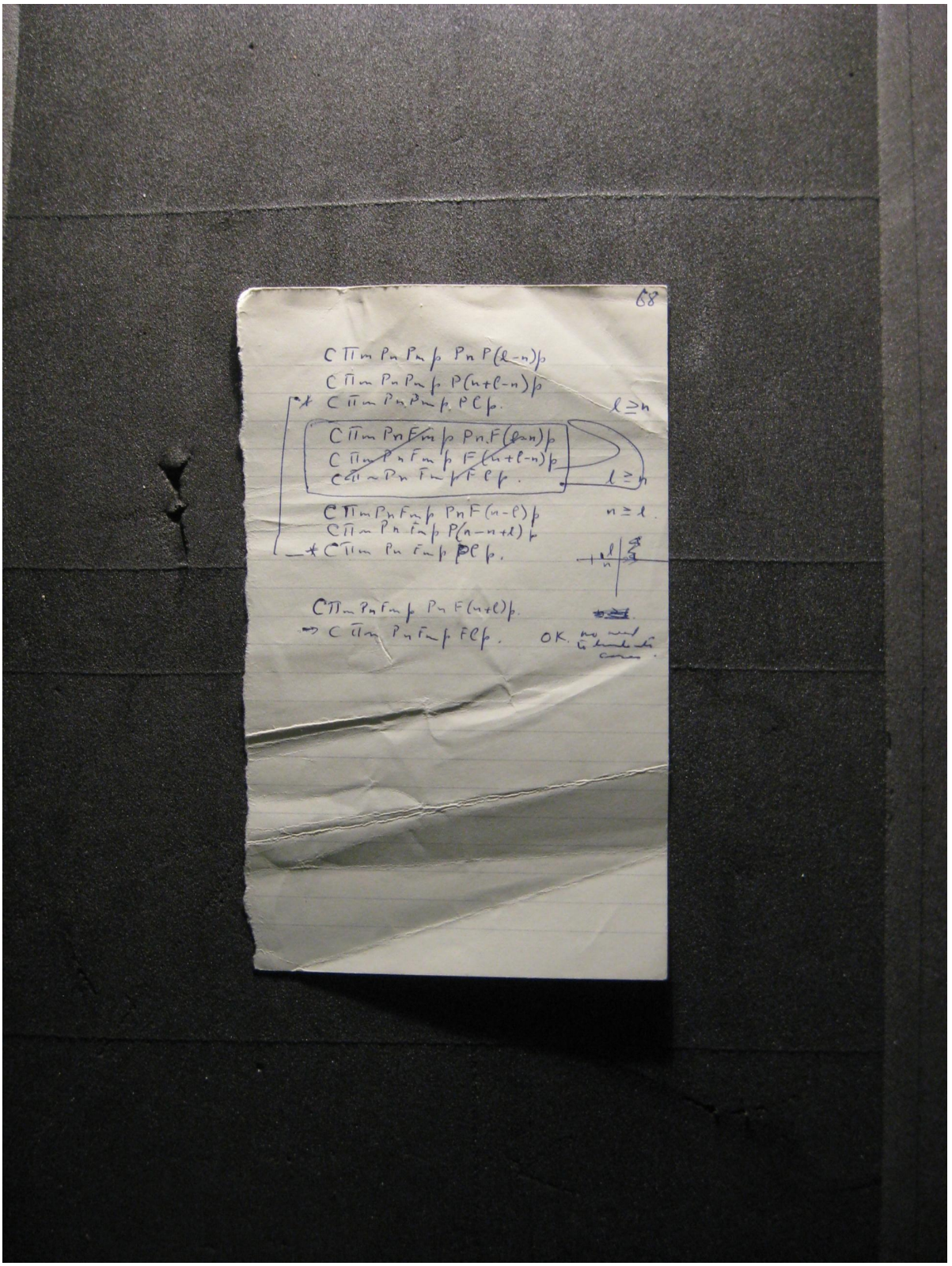
of view, & from that, if it be given in enough detail, we can work out what is the case from the ship's point of view, or from any point of view; but if you want to know what is the case without being the case from any point of view, it could be said that this is senseless; it is like asking what is the case without being the case now. ~~And~~

I don't know whether I've expressed this third reaction to special relativity very well. I probably haven't, because it isn't in fact my own reaction, which I am afraid is the old & crude one, the first. But now I want to add something which isn't so much about the nature of the present as about the nature of physics.

~~I have set up two sorts of metaphysical picture. According to one, we can distinguish how things are from [illegible] there is a difference between what will appear to be the case from that point of view & what is in the case~~



B.2 Prior's letter on the *Stoic Logic* by Mates

Letter from Prior to Mates, dated Aug. 6, 1954[13]

Dept. of Philosophy
CANTERBURY UNIVERSITY COLLEGE
CHRISTCHURCH, N.Z.
6/8/54

PS. It goes without saying that I've enjoyed & profited by your book immensely. A.P.

Dear Professor Mates,

I enclose a small paper provoked by your book on Stoic Logic (I've submitted it to a periodical for publication, but haven't yet heard about it).

A couple of other points arising out of the same work:

(1) On p. 81 you say of the schema

$p \to p, \neg p \to p, p \vee \neg p \Rightarrow p$[14]

that 'no clue is offered on how this sort of argument would be analysed into the five basic arguments'.[15] But isn't just such a clue given in the paragraph from Sextus that you have at the top of p. 106? This paragraph suggests that they had some such principle of composition as this:- If one of the members of a

[13]Edited by the present author. The original is kept in the *Prior Collection* at Bodleian Library, Oxford, box 4. *Virtual Lab* n. 39, transcribed in [NAP, 2014]. The correspondence contains two other letters: one of these is signed by Prior, and dated Mar. 5, 1968, VL n. 40; the other letter is from Mates to Mary Prior, dated Jan. 31, 1970, VL n. 575, and talks about recovering letters n. 39 and n. 40.

[14]The original MS refers to this schema in Polish notation: $Cpp, CNpp, ApNp \to p$.

The Stoic schema is reported by S.E. *M.* 8, 292, and *P.H.* 2, 242, 243. Arguments for this schema are in S.E. *M.* 8, 281, 466; *P.H.* 2, 186.

[15]The above schema is linked to the second and the fifth of five basic types of undemostrated arguments, *e.g.*, respectively in S.E. *M.* 8, 227, Boeth. *in Int.* 351; and in Gal. *Inst.Log.* 16, Cic. *Top.* 56. As reported by S.E. *M.* 8, 223-224, the undemostrated arguments are so defined because we do not need any proof for them, in fact they are evident *per se*.

true disjunction yields a certain conclusion when combined with one premiss, and the other member yields the same conclusion when combined with another premiss, then the two premisses yield the conclusion.

By this principle the argument in question is 'reduced' to two (disjunctively combined) applications of ponendo ponens.[16]

{2}

- It's of some interest that the post-Renaissance logicians (e.g. the Port Royalists & Watts) classified the dilemma along with the sorites as a species of 'compound argument'.[17] – Possibly the thing I've suggested was another of the 4 principles of reduction.

(2) Regarding the fourth opinion about implication recorded in the extract from Sextus at top of p. 48 - 'Those who judge by "suggestion" '. It seems to me that there's something to be said for the view that there were Peripatetics.[18] The word that you translate 'in effect' ('its consequent is in effect included in its antecedent') is δυνάμει & it is noteworthy that in An. Pr. 67b 3,5 Aristotle uses the complementary term ἐνεργεῖν in describing the difference between a man who has the premisses but hasn't drawn the conclusion & the man who has drawn it.[19]

[16]The locution *modus ponendo ponens* is the expanded for the *modus ponens*, namely MP. $\frac{p \to q, p}{q}$.

[17]*Compound arguments* are characterised either by complex subjects or complex predicates, for instance, propositions of the form "either α or β is A", or "β is A and B." Compound arguments may refers to conditionals as statements of inference. Other cases of compound arguments involve complex subjects or predicates. We can distinguish *exponibles* and *not-exponibles*, *e.g.* the sorites is an example of the latter kind.

[18]Sextus' general aim in *P.H.* 2, 110-112, and *M.* 8, 112-117, is to establish, against the dialecticians, that there is no agreement about the truth criterion – in those specific cases in relation to conditional sentences or implication – and, if that, consequently no agreement about truth would be possible. Sextus compares the criteria by Philo, Diodorus, by people who decide by ἔμφασις, and by those who decide by δυνάμει.

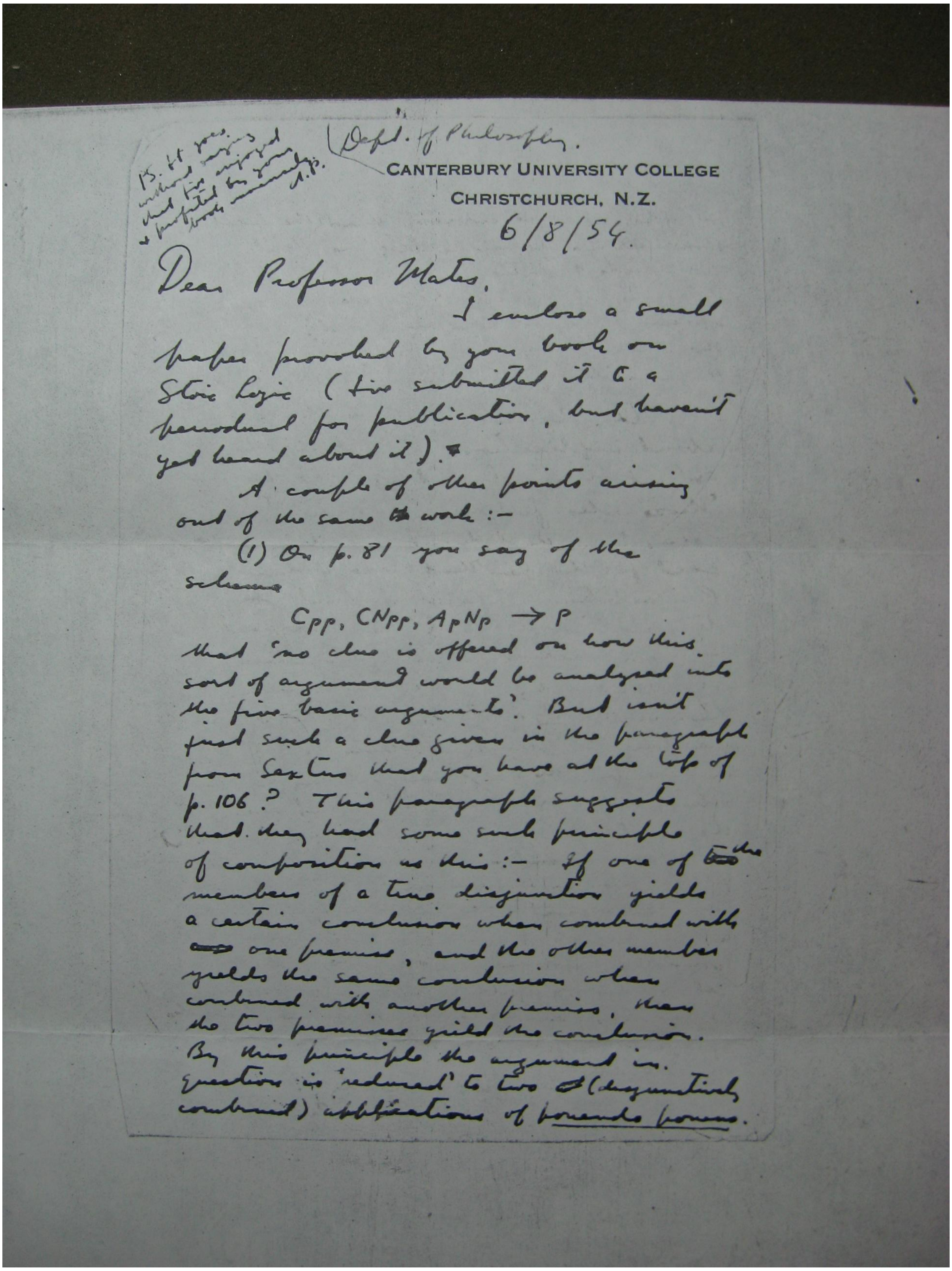
[19]The following has been crossed out: "~~('The consequent is potentially~~ 'The antecedent is potentially the consequent & the consequent is the antecedent brought to actuality."

And the quotation you give from Alexander on pp. 125-6 about 'useless' syllogismus reinforces the conjecture that the proponents of Sextus 4th opinion were Peripatetics.[20]

- Yours sincerely

Arthur N. Prior

[20] According to [Barnes and Bonelli, 2012, 443, n.1], we cannot take for granted that the fourth criterion for conditional sentences in S.E. *P.H.* 2, 112, is ascribable to the Peripatetics. Certainly, Prior makes a good conjecture, and the reference to Arist. *APr.* 2, 67b 3-5, seems to support his claim.



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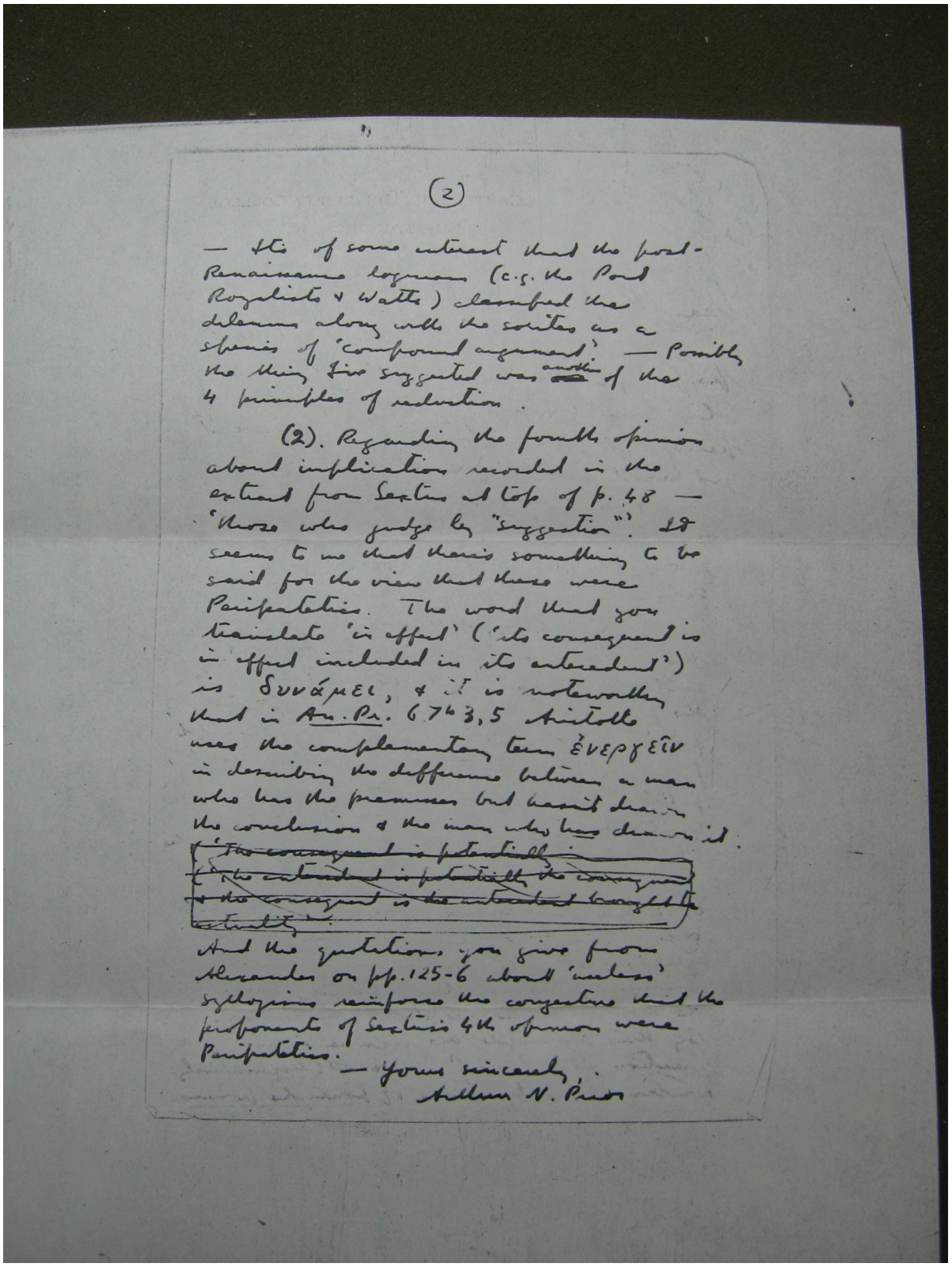
I enclose a small paper provoked by your book on Stoic Logic (I've submitted it to a periodical for publication, but haven't yet heard about it).

A couple of other points arising out of the same work:—

(1) On p. 81 you say of the schema

Cpp, CNpp, ApNp → p

that 'no clue is offered on how this sort of argument would be analysed into the five basic arguments'. But isn't just such a clue given in the paragraph from Sextus that you have at the top of p. 106? This paragraph suggests that they had some such principle of composition as this:— If one of the members of a true disjunction yields a certain conclusion when combined with one premiss, and the other member yields the same conclusion when combined with another premiss, then the two premisses yield the conclusion. By this principle the argument in question is 'reduced' to two (disjunctively combined) applications of ponendo ponens.



(2)

— this of some interest that the post-Renaissance logicians (c.g. the Port Royalists + Watts) classified the dilemma along with the sorites as a species of 'compound argument' — Possibly the thing first suggested was ~~one~~ another of the 4 principles of reduction.

(2). Regarding the fourth opinion about implication recorded in the extract from Sextus at top of p. 48 — 'those who judge by "suggestion"': it seems to me that there is something to be said for the view that these were Peripatetics. The word that you translate 'in effect' ('its consequent is in effect included in its antecedent') is δυνάμει, & it is noteworthy that in An. Pr. 67b3,5 Aristotle uses the complementary term ἐνεργείᾳ in describing the difference between a man who has the premisses but doesn't draw the conclusion & the man who has drawn it. ~~('its consequent is potentially in its antecedent, its antecedent is potentially the consequent, the consequent is the antecedent brought to actuality')~~

And the quotations you give from Alexander on pp. 125-6 about 'useless' syllogisms reinforce the conjecture that the proponents of Sextus's 4th opinion were Peripatetics.

— Yours sincerely,
Arthur N. Prior

B.3 Correspondence on tenses, and the problem of missing letters

This section focuses on the birth of tense logic by illustrating some developments in the history of logic of the last fifty years. We would illustrate the development of tense logics by way of two specific epistolary exchanges involving Arthur Prior. Furthermore, we also discuss the problem of missing letters, or one-sided correspondences, in relation to Hamblin-Prior and Lemmon-Prior exchanges.

The correspondence between Hamblin and Prior contains ten letters, from November 30, 1957 to November 3, 1969. All the records are signed by Hamblin, and very strangely no letter from Prior has been found. One letter by Hamblin is addressed to Mary Prior, and it will be later presented in this section.

Recovering Prior's missing letters to Hamblin would be a very significant achievement. These are precious pieces of historical evidence which have not been studied yet. In section 2.1.2, we emphasised that their importance can be guessed from several declarations, proofs, and suppositions as well as from the surviving part of the correspondence (Hamblin's letters). Prior's missing letters would be useful in order to clarify some questions about the formal account of tense logic and, in particular, about the outcomes of the correspondence, *e.g.* in relation to *Bird's nest* diagram, the conception of time, and its minimal elements. In the current situation, *Prior Collection* preserves nine letters from Hamblin to Prior, a letter to Mary Prior, and no letter from Prior to Hamblin.[21]

[21] An attempt to trace Prior's letter is underway. At the moment, we know that neither the University of New South Wales (UNSW) Archive, nor Hamblin family, nor P. Staines – one of Hamblin's best students and collaborator – have Prior's letters. Starting from the second part of 2013, our investigation led us to contact *e.g.* Lloyd Humberstone ('I did see Hamblin in action once at a conference, but I don't think I met him. The work I did on interval based semantics was inspired by his publications and grew out of my Oxford BPhil thesis before I ever came to Australia. So I don't have any first hand information about his correspondence with Prior.' [Aug. 26, 2014]), and Julie Hamblin ('I can see how valuable it would be for you to be able to see Arthur Prior's letters to my father, but unfortunately, I have to report that we have been unable to find them and believe that my father did not keep them. One of my father's former students and colleagues, Philip Staines, holds the boxes of my father's papers and has recently searched through them again in response to your request. He has confirmed that there are no letters there

A similar problem concerns Prior-Lemmon correspondence. In front of fortynine letters from Lemmon, no letter from Prior is kept in the *Prior Collection*. The correspondence covers the interval between Aug. 16, 1956 and Mar. 16, 1966. Some interesting topics of this correspondence are: linearity conditions, the $U-$calculus, K and K_t logics, symmetry of time and determinism, definition of modal necessity in Cocchiarella's and Hintikka's systems.

Prior starts many of his works about minimal tense logics from Lemmon's K_t; presumably, the letters from Prior to Lemmon are highly relevant for this. Nevertheless, some questions remain pending and might be guidelines for our research: *e.g.* what philosophical remarks would emerge from Prior's letters to Lemmon? What did Prior write to Lemmon? What is Prior's philosophical judgment about K_t? Further, who did Mary Prior contact in 1969, since Lemmon died in 1966?[22]

from Arthur Prior. This is consistent with my mother's recollection, and also her belief that it is most unlikely that my father would have retained correspondence of this kind.' [Oct. 10, 2013]). On December 2014, from the report of an anonymous referee of *Synthese*, a strange case of academic thievery seemed to come to light. He gave the following clues by email: 'The majority of Hamblin's papers were stolen from his widow by two unknown American academics. Perhaps Prior's letters to Hamblin were part of the loot.' Furthermore, here is what has been declared by the reviewer when we asked for further details: 'I'm afraid this is a bit of a dead end. I have tried to find out who the two American academics were but I have never been able to, else I would have tracked them down and insisted they return the papers to Australia. Sorry I can't be more help.' And so, he proposed us 'A couple of ideas about Hamblin studies: (1) to email Vaughan Pratt and ask if he knows the whereabouts of any of Hamblin's unpublished papers; (2) I wrote to the Stanford some years ago and suggested that I write an entry on Hamblin. Zalta wrote back saying something like who was Hamblin, no one has heard of him, no thanks. Perhaps if Fabio makes the suggestion that he write an entry, they will start to get the idea that there is gap in their coverage' [Dec. 3, 2014]. Unfortunately, our contacts with Pratt did not revealed any relevant information. Pratt told us 'The theft of Hamblin's papers is news to me, and extremely unfortunate if true' and 'The referee should be able to supply such a name – just explain the situation to the *Synthese* editor. I'm very sorry I don't have anything to offer that would be of assistance to you' [Feb. 13, 2015].

[22] Analogously to Prior-Hamblin missing letters, we have begun a search for Prior's letters to Lemmon. We have sent emails to several scholars, *e.g.* Dana Scott, in order to obtain some further information. Unfortunately, we have not got any relevant result yet. Dana Scott emailed us the following response: 'I of course knew John Lemmon very well, and I met Arthur Prior many times. I am afraid, after so many years, I have no memories of connections between Lemmon and Prior that could help you out' [Aug. 17, 2014].

Letter from Hamblin to Prior, dated Oct. 19, 1961[23]

School of Philosophy
Univ. of NSW
Box 1, Kensington NSW
Australia.

19th October 1961

Dear Prior,

Thank you for your letter.[24] I had a go at it last night, but since I have hardly touched tense-logic for a long time I don't feel absolutely confident that anything I say is accurate. But you would like an early reply, so this is it.[25]

It all looks fine to me: I like your formulation using all four tense-operators: there isn't a single negation sign in your whole letter except where the operator are interdefined, and that is obviously a gain in setting the issues. I agree that it was quite "inconsistent" of me (in some sense) to suggest both $FFp = Fp$ and $GPp = p \vee Pp$[26].[27] Your resolution of it, though I haven't checked all the incidental derivations, has an air of intuitive correctness. (Though I wonder if you agree with me that the axiom-system still awaits some major simplification...?

[23] *Virtual Lab* n. 18, transcribed by the present author.

[24] This is clear evidence that Prior wrote some letter to Hamblin: the remark underlines the missing letters problem.

[25] Hamblin and Prior have already started a fruitful collaboration in order to axiomatise tense logics and investigate their properties. For instance, Hamblin [1958] proposed for the first time 'a set of axioms for a simple "dateless" tense logic, i.e. with F and P as monadic operators' and a system involving thirty distinct tenses that can be interpreted on 'a two-way infinite continuous time-scale.' In that letter was presented a first diagram of tenses, which characterises the relations of implication in Hamblin's logic. [Hamblin, nd] – not dated but most likely from 1962 – is an interesting document titled *the Logic of Tenses*, while among Prior's published works on the topic we mention *e.g.* [Prior, 1962c, 1966, 1967].

[26] Polish notation in the original typewritten: $GPp = ApPp$.

[27] The above formulas are respectively $HAMB$ 4 and $HAMB$ 6, see section 2.1.2. The modal analogue of $HAMB$ 4 is equivalent to the characteristic axiom of Lewis $S4.3$: $\Diamond\alpha \leftrightarrow \Diamond\Diamond\alpha$. Hamblin and Prior were supposing a dense account for time. But the assumption of discreteness may be derived both by $HAMB$ 6, via its dual $PGp \leftrightarrow p \wedge Gp$, and more directly by $HAMB$ 4 if we consider the implication from right to left.

Particularly my $FPp \leftrightarrow p \vee Pp \vee Fp$ looks now to me terribly ad hoc.[28] Perhaps this isn't important any way but aesthetically).

Regarding $FGp \rightarrow GFp$:[29] I have searched through any former scribbles on the subject, and sure enough I can't find a proof. It looks, in fact, as if it ought to be provable as independent using an interpretation such that time "stops" somewhere in the future – I think you suggested something similar in a previous letter –[30] but I do not for the moment see how this would go in detail.

One small detail, more a slip of the pen that anything: in justifying $Gp \rightarrow GPGp$ for the case of discrete time you write "If time is discrete, $Gp = GGp = GGp\&Gp = G(Gp\&p) = GPGp$ (using your (5) at the end). But $Gp = GGp$ ~~in (3), and~~ is "inconsistent" with (5), and presumes continuous time. You have, of course, $Gp \rightarrow GGp$ and hence, $Gp \rightarrow GGp\&Gp = ...etc.$ and all is well since this is all you need.[31]

By all means publish the lot. I feel a little guilty about having left it all so unfinished, but I am glad to see it used.

By the way, I shall be in Oxford on study-leave for a year from about Jan. 10th. I hope I may meet you sometime.

Yours sincerely,

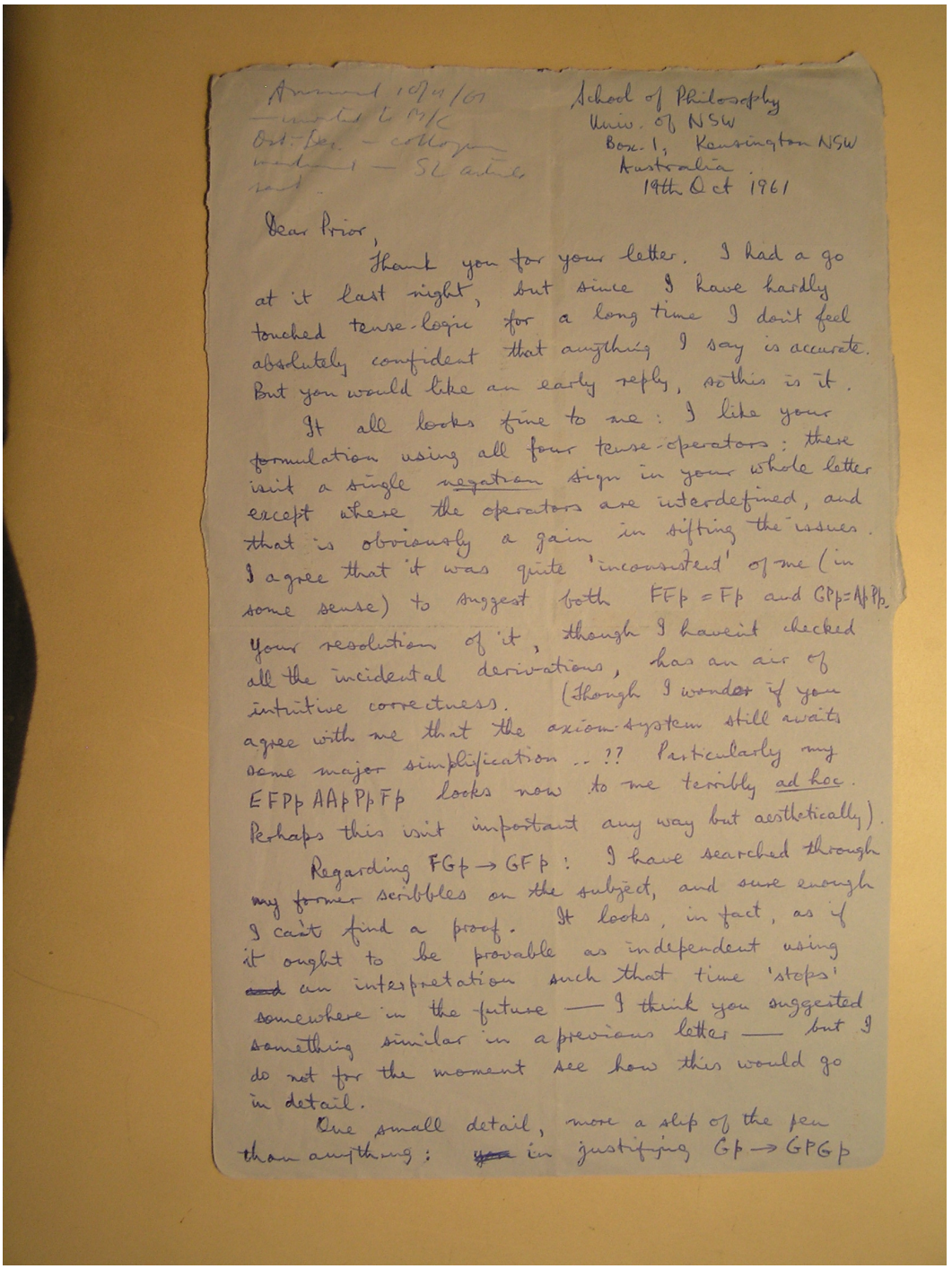
Charles Hamblin

[28] In the original, the Polish notation is used: $EFPpAApPpFp$. This formula is $HAMB$ 5, from which we can derive $p \wedge Hp \wedge Gp \rightarrow GHp$ for *backward-linearity*. Again, from its *mirror image*, namely $p \wedge Gp \wedge Hp \rightarrow HGp$, we can infer *forward-linearity*.

[29] The analogue $\Diamond\Box p \rightarrow \Box\Diamond p$ is a theorem of $S4$, while in a quantified modal logic we get $\Diamond\forall xp \rightarrow \forall x\Diamond p$.

[30] Again a reference to some Prior's letter.

[31] Any tense-modality with more than two tense-operators can be reduced to one with two or less (*cf.* [Prior, 1967, 47], [Øhrstrøm and Hasle, 1995, 177]). Prior [1962c] considers several results from this correspondence.



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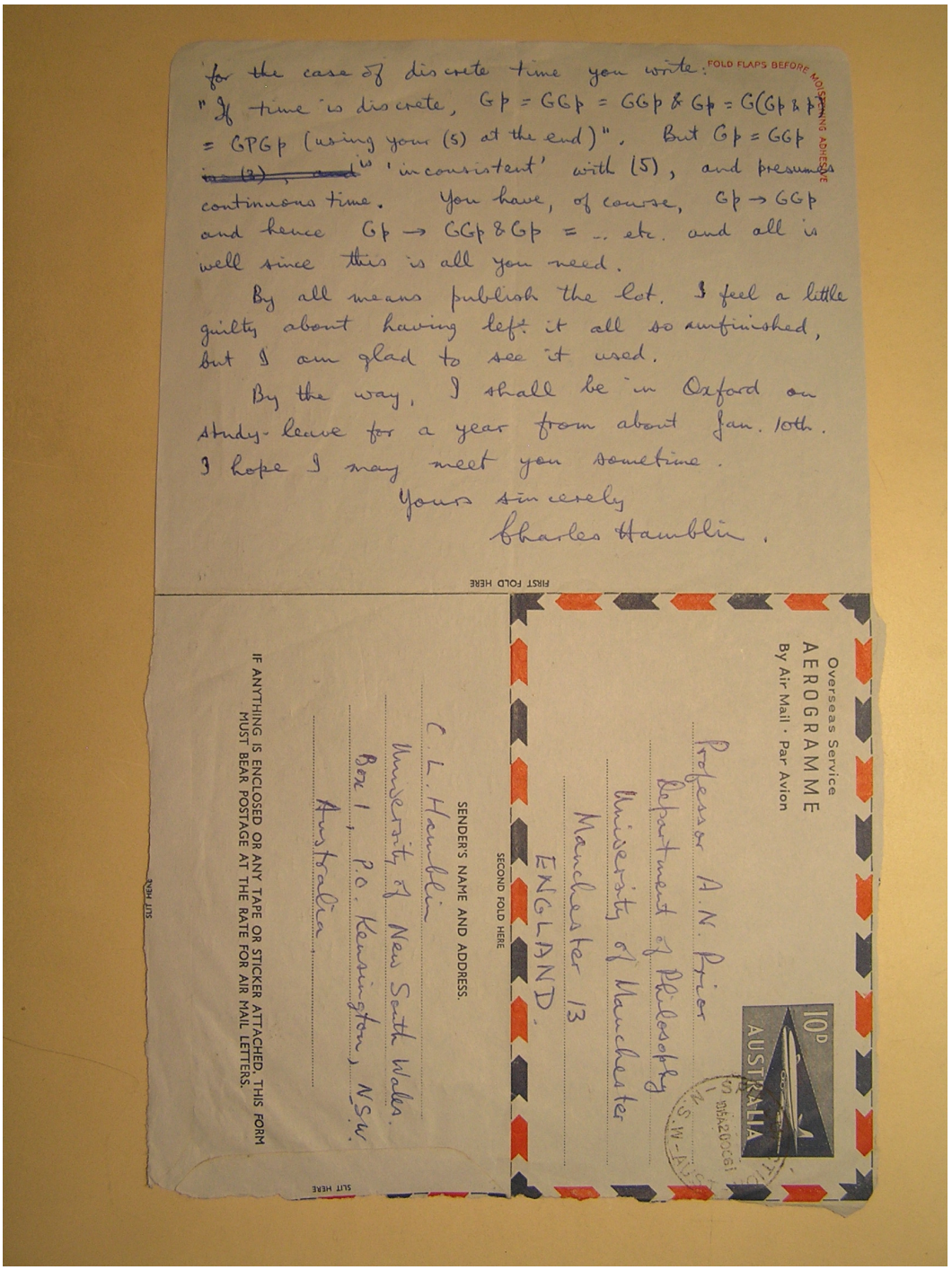

for the case of discrete time you write:
"If time is discrete, $G\phi = GG\phi = GG\phi \,\&\, G\phi = G(G\phi \,\&\, \phi)$ = $GPG\phi$ (using your (5) at the end)". But $G\phi = GG\phi$ ~~is (5), and~~ is 'inconsistent' with (5), and presumes continuous time. You have, of course, $G\phi \to GG\phi$ and hence $G\phi \to GG\phi \,\&\, G\phi$ = .. etc. and all is well since this is all you need.

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Yours sincerely
Charles Hamblin.

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10D AUSTRALIA

Professor A.N. Prior
Department of Philosophy
University of Manchester
Manchester 13
ENGLAND.

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SENDER'S NAME AND ADDRESS.

C.L. Hamblin
University of New South Wales.
Box 1, P.O. Kensington, N.S.W.
Australia.

SLIT HERE

IF ANYTHING IS ENCLOSED OR ANY TAPE OR STICKER ATTACHED, THIS FORM MUST BEAR POSTAGE AT THE RATE FOR AIR MAIL LETTERS.

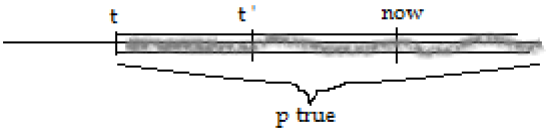
Letter from Hamblin to Prior, dated Jul. 6, 1965[32]

THE UNIVERSITY OF NEW SOUTH WALES
BOX 1, POST OFFICE, KENSINGTON, N.S.W.
Telegraphic Address: UNITECH, SYDNEY

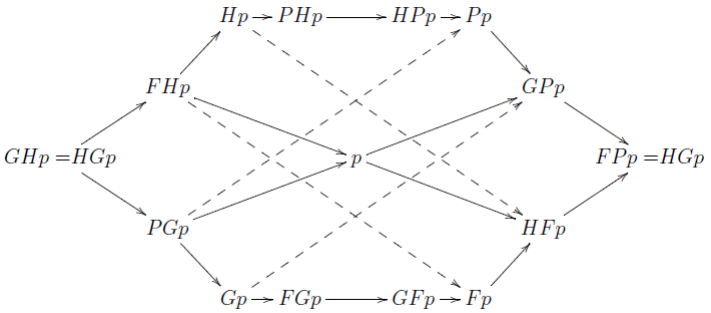
Tuesday 6th July 1965.

Dear Arthur,

A small note on the doctrine of tenses: shouldn't PGp imply Pp? If at some time t in the past it was the case that p would be the case for all times future to t, there must have been some other time t^i, between t and now, at which p was the case



Similarly for duals and mirror-images. This makes the diagram of tenses a bit like a bird's nest.[33]



[32] *Virtual Lab* n. 10. This letter is strictly linked to other *Virtual Lab* items (7, 18, 21).

[33] Prior refers to this diagram in several works, *e.g.* [Prior, 1966, 1967]. *Cf.* section 2.1.2. The diagram showed in this letter is the definitive version concerning the implication relations. We consider it as an implementation of a previous diagram in [Hamblin, 1958], later explained also in [Hamblin, nd].

I can't see how the additions could be derived from any of the axioms that have been proposed so far:[34] an extra one is necessary. Moreover, these are rationally inconsistent with the discarded $(p \wedge Gp) \rightarrow PGp$.[35] But $(p \wedge Gp) \rightarrow (Pp \wedge PGp)$ doesn't work work either.[36] Pity!

I didn't get round to talking about the Meredith thing you sent.[37] I was intrigued with the definition of negation in all contexts: i.e. it sounds like confusing, in the jungle-example, "There is no lion here" with "There is a non-lion here".[38] And can you ask the negation of a question Q by asking the join[39] of such other questions as do not contain Q?

Should I send these papers back to Manchester marked "Not to be forwarded"?

I expect Rita will meet Ann on Saturday morning. Best wishes to Mary and yourself.

Yours

Charles Hamblin

[34] *Added to the text:*
$CPGpp$
$\rightarrow CPPGpPp$
$\rightarrow CPGpPp$.

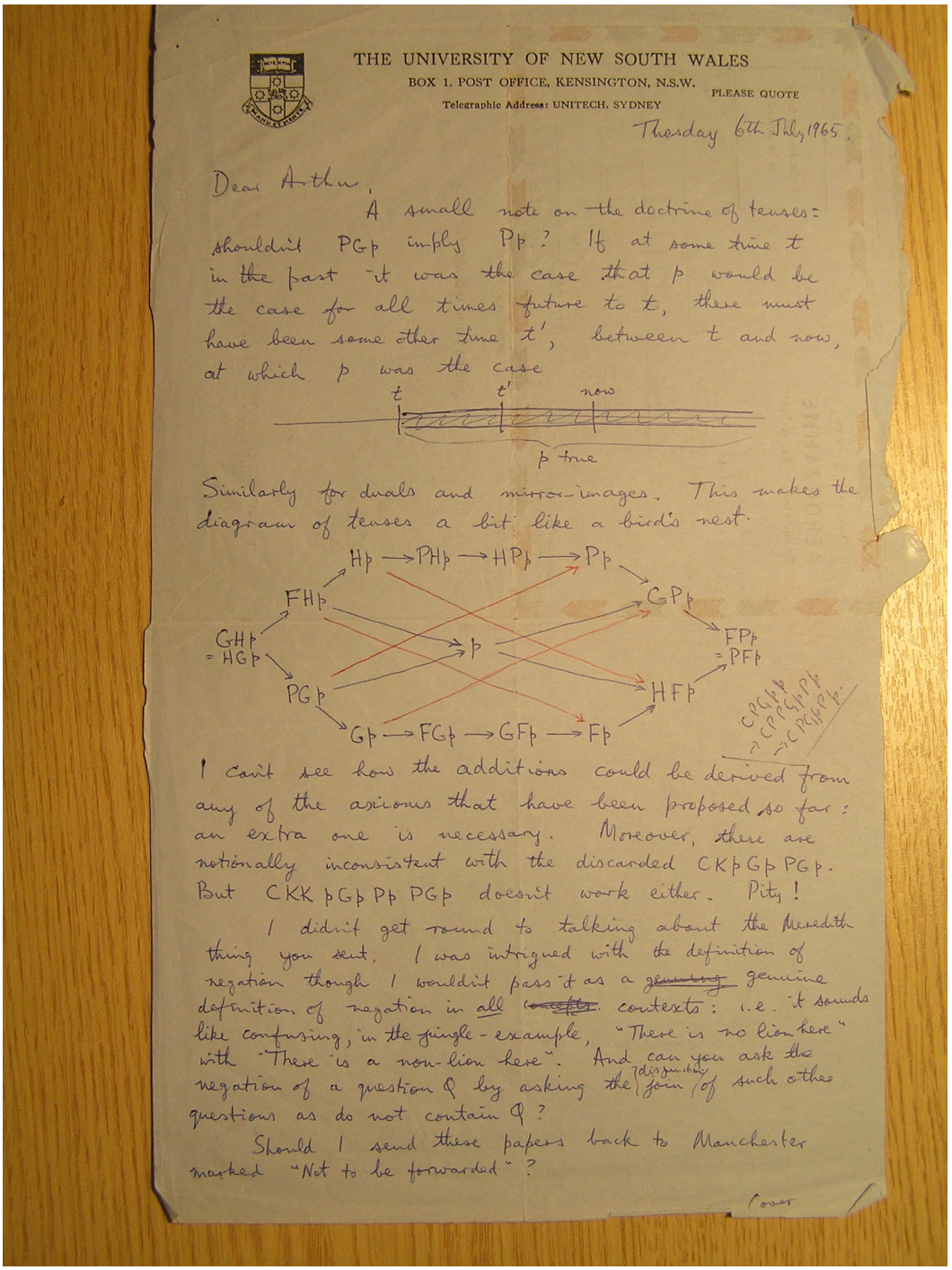
[35] The Polish notation is used in the typewritten manuscript: $CKpGpPGp$. The formula occurs as $(+d)$ in the Prior's version of the Master Argument (*Cf.* section 2.2.1). Furthermore, it is also the third axiom in $DIOD$. $DIOD$ 3 avoids density and, in fact, its relation of implication is ruled out in the *Bird's nest* diagram (*Cf.* section 2.1.1).

[36] Polish notation in the typewritten: $CKKpGpPpPGp$.

[37] The reference is to C. A. Meredith. We do not know if Hamblin is alluding to a paper, draft or oral reports or discussions, since we do not have Prior's letters. However, Prior and Meredith were in touch, and we can mention *e.g.* [Meredith and Prior, 1965] to emphasise their fruitful cooperation (*see*, also [Copeland, 2006]). Many records of Prior-Meredith works are also in [VLP, 2011] and they cover the years 1952-1968, both as letters (24 items) and notes found among correspondence (22 items).

[38] Classic topic of *obversion*. *Cf.* [Prior, 1955b, 126-134], concerning categorical forms with negative terms.

[39] *Added to the text:* disjunction.



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PLEASE QUOTE

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Similarly for duals and mirror-images. This makes the diagram of tenses a bit like a bird's nest.

I can't see how the additions could be derived from any of the axioms that have been proposed so far: an extra one is necessary. Moreover, there are notionally inconsistent with the discarded $CKpGp\,PGp$. But $CKKpGp\,Pp\,PGp$ doesn't work either. Pity!

I didn't get round to talking about the Meredith thing you sent. I was intrigued with the definition of negation though I wouldn't pass it as a ~~genuine~~ genuine definition of negation in all ~~contexts~~ contexts: i.e. it sounds like confusing, in the jingle-example, "There is no lion here" with "There is a non-lion here". And can you ask the negation of a question Q by asking the disjunction of such other questions as do not contain Q?

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Best wishes to Mary and yourself.
Yours
Charles Hamblin
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c/o Philosophy Department
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Letter from Lemmon to Prior, dated Jan. 2, 1966[40].

January 2, 1966.
CLAREMONT GRADUATE SCHOOL
AND UNIVERSITY CENTER
CLAREMONT, CALIFORNIA

Dear Arthur:

Writing up the Scott completeness results for tense logic this morning.[41] I suddenly realised they had the consequence that $Hp \to HHp$[42] is redundant in the presence of $Gp \to GGp$.[43] (If time is transitive forward, it has to be transitive backward, so this is not a hit surprising).[44] Another nice exercise for your class. Here's how the direct proof goes: I use my own symbols: We rely of course on:

$TF: \langle P\rangle[F]A \to A$

$TP: \langle F\rangle[P]A \to A.$

Assume $\vdash [F]A \to [F][F]A$. Then $\vdash \langle P\rangle[F]A \to \langle P\rangle[F][F]A$, so that $\vdash \langle P\rangle[F]A \to [F]A$ by TF. Now since $\vdash A \to [F]\langle P\rangle A$ by TP, we have $\vdash \langle P\rangle A \to \langle P\rangle[F]\langle P\rangle A$. By the result just proved, taking A as $\langle P\rangle A$, we have

$\vdash \langle P\rangle A \to [F]\langle P\rangle A$. Hence

[40] *Virtual Lab* n. 1106, transcribed by the present author. In this letter Lemmon deals with some semantics features of K_t, namely the logic including K + TF + TP. We notice that, according to section 2.1.2, TF is equivalent to K_t 5, and TP to K_t 4. Therefore, by contraposition we can deduce also the modal analogue of symmetry, namely $p \to \Box\Diamond p$. In order to make clear Lemmon's notation, let us consider the operators $\langle F\rangle$, $[F]$, $\langle P\rangle$, $[P]$, respectively as the standard tense operators F, G, P, H. In order to follow Lemmon's explanation of his temporal modalities, we suggest [Lemmon, 1965b] to the reader. Some other letters in which K_t is quoted are [Lemmon, 1965a, 1966a,b]

[41] Scott proved the completeness and decidability of axiomatic tense logics. However, his results are largely in unpublished correspondence with Lemmon, some are reported in places such as *e.g.* [Scott, 1965]. Further pertinent materials are in [Scott, 1970] and [Lemmon and Scott, 1977].

[42] Polish notation in the typewritten manuscript: $CHpHHp$.

[43] Polish notation in the typewritten manuscript: $CGpGGp$.

[44] Note that $Hp \to HHp$ and $Gp \to GGp$ are temporal analogues (both for the past and the future) of $\Box p \to \Box\Box p$, namely 4 or modal *transitivity*.

$\vdash \langle P\rangle\langle P\rangle A \rightarrow \langle P\rangle[F]\langle P\rangle A$, whence $\vdash \langle P\rangle\langle P\rangle A \rightarrow \langle P\rangle A$ by TF.

This of course gives $\vdash [P]A \rightarrow [P][P]A$. Actually, it's clear from the above proof that the schemata

$$[F]A \rightarrow [F][F]A$$
$$[P]A \rightarrow [P][P]A$$
$$\langle P\rangle[F]A \rightarrow [F]A$$
$$\langle F\rangle[P]A \rightarrow [P]A$$

are all deductively equivalent in the presence of the basic system K_t.[45]

I've not got a better semantical viewpoint on the white business. Our time models have the structure $\langle U, R_1, R_2\rangle$, where U is the set of moments of time and R_1, R_2 relations in U – R_1 is the forward direction of time, R_2 the backward direction. What we lay down
Semantically is that

$$[F]A$$

is time at $t\varepsilon U$ iff for all t^i such that tR_1t^i A is time at t^i (at all moments later that t):

$$[P]A$$

is time at $t\varepsilon U$ iff for all t^i such that tR_2t^i A is time at t^i (at all moments earlier that t). Now what the system K_t assumes is that

$$tR_1t^i \text{ iff } t^iR_2t$$

ie. that $R_1 = \check{R}_2$. This is what makes TF and TP come out valued.

That is all that K_t assumes as its completeness with respect to those models show. But if we make the weaker assumption that $R_1 \leq \check{R}_2$, ie. Only that

[45] This means that, by adding one among these formulas to K_t, we are able to deduce each one of the others.

$$\text{if } tR_1t^i \text{ then } t^iR_2t$$

TP is preserved and TF lost. This is what an anti determinist worried by TF should assume, and no more: namely that if t^i is later than t, then t is earlier than t^i, but not the converse.[46] Completeness results are forthcoming accordingly for systems with TP but not TF. This gives the proper semantics for tense logics which do not insists in the symmetry of time. The moment we do this, of course, the above interdeniability result broke down – time may be forward transitive but not backward transitive, and vice versa.[47]

Don't you think this helps to see what's at stake here? Maybe it's only an ordering language prejudice that 'earlier' is the strict converse of 'later'![48]

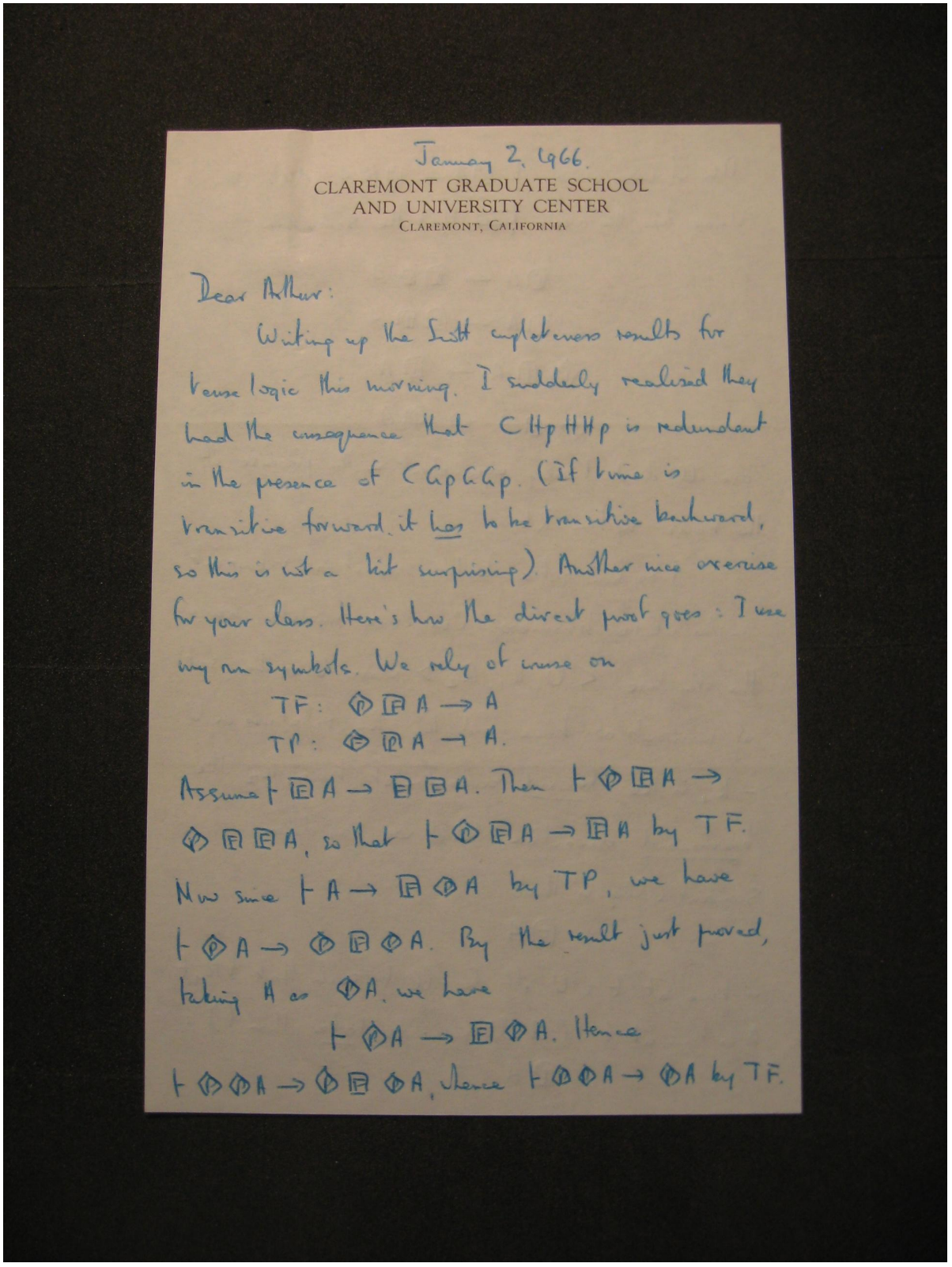
Best wishes for the New Year to you all.

John

[46] Of course, this is a clear assumption (*ad hoc*) on the semantics.

[47] To assume K_t might be a tricky endeavour for an indeterminist like Prior. In fact, since K_t includes both TF and TP, this means that the past is already determined and, *symmetrically* also the future. Therefore, we are in front of the following alternative, but we are not free to opt for the first horn: K_t is a minimal tense logic, or K_t is deterministic and then not-minimal.

[48] Lemmon's final remark gets to the core of the matter, so much so that [Prior, 1996a, 48] gives it a philosophical justification: 'Nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make for ourselves. And this is a distinction which a tenseless logic is unable to express. In my own logic with tenses I would express it this way: We can lay it down as a law that whatever *now* is the case *will always have been* the case [*i.e.* TP (K_t 4)]; but we can't interchange past and future here and lay it down that whatever *now* is the case *has always been going to be* the case [*i.e.* TF (K_t 5)] – I don't think that's a logical law at all.'



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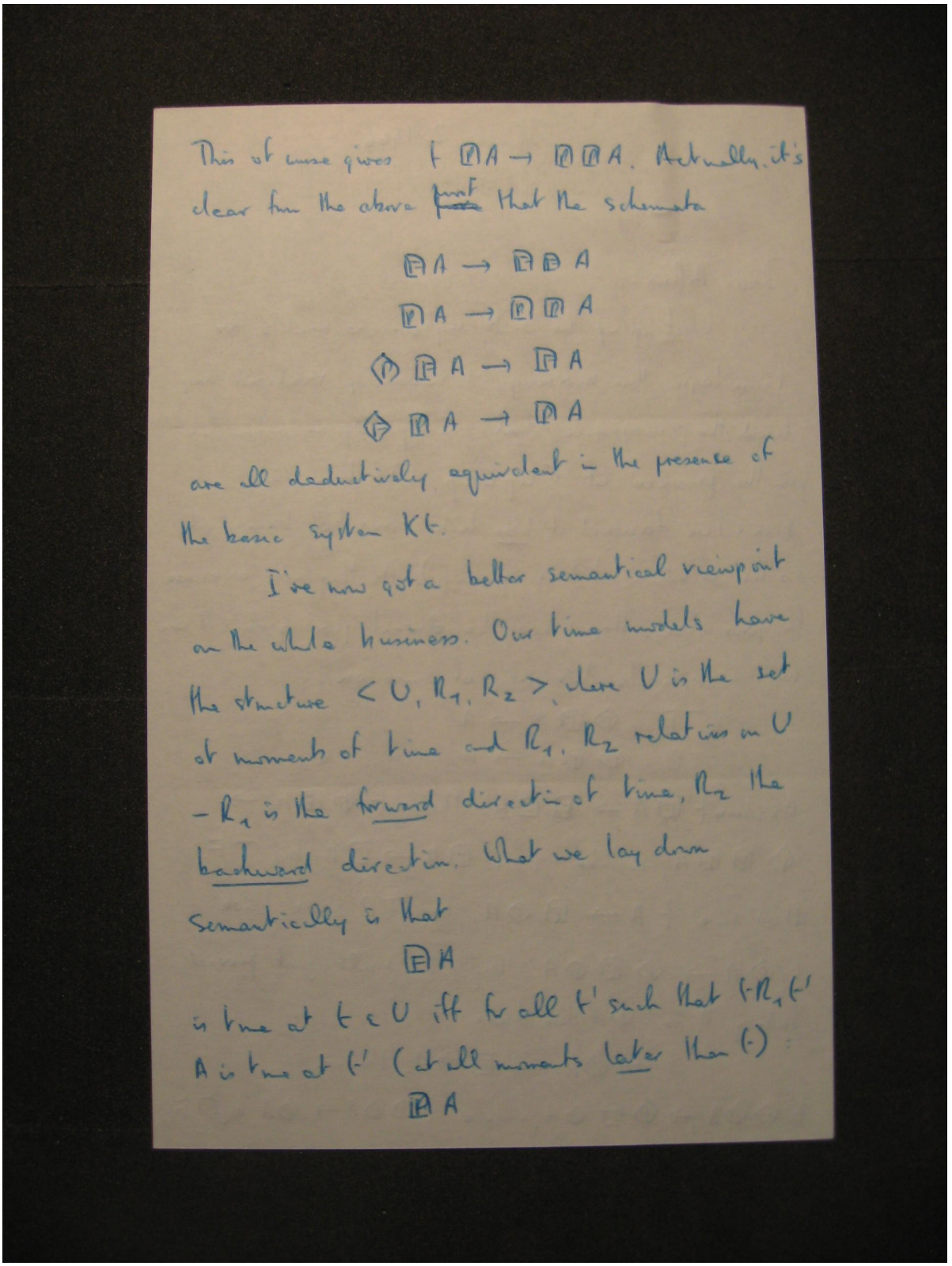
TF: $\Diamond \Box A \to A$

TP: $\Box \Diamond A \to A$.

Assume $\vdash \Box A \to \Box\Box A$. Then $\vdash \Diamond \Box A \to \Diamond \Box\Box A$, so that $\vdash \Diamond \Box A \to \Box A$ by TF. Now since $\vdash A \to \Box \Diamond A$ by TP, we have $\vdash \Diamond A \to \Diamond \Box \Diamond A$. By the result just proved, taking A as $\Diamond A$, we have

$\vdash \Diamond A \to \Box \Diamond A$. Hence

$\vdash \Diamond\Diamond A \to \Diamond \Box \Diamond A$, hence $\vdash \Diamond\Diamond A \to \Diamond A$ by TF.



This of course gives $\vdash \Box A \to \Box\Box A$. Actually it's clear from the above ~~fact~~ that the schemata

$$\Box A \to \Box\Box A$$

$$\Box A \to \Box\Box A$$

$$\Diamond \Box A \to \Box A$$

$$\Diamond \Box A \to \Box A$$

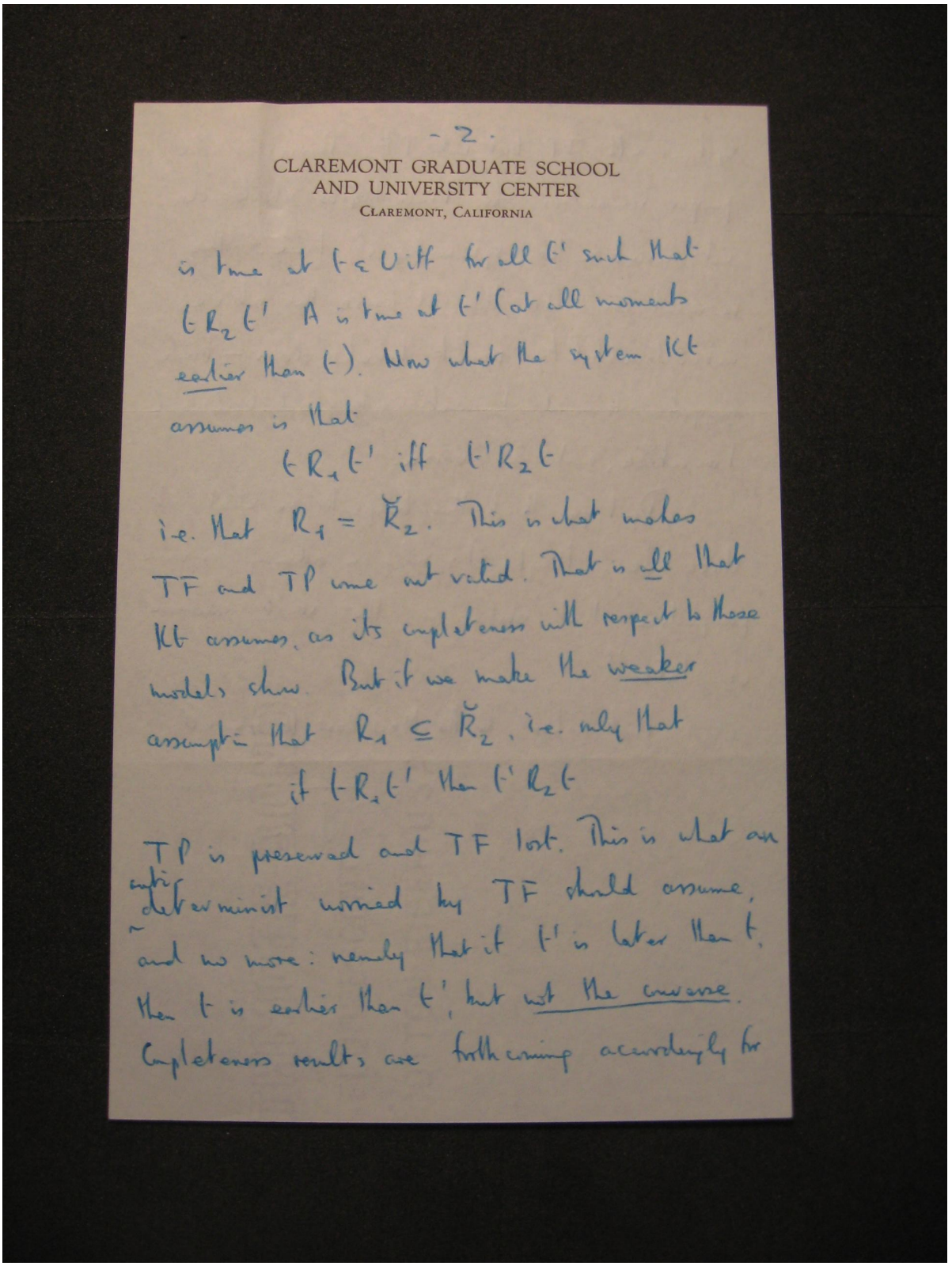
are all deductively equivalent in the presence of the basic system K4.

I've now got a better semantical viewpoint on the whole business. Our time models have the structure $\langle U, R_1, R_2 \rangle$, where U is the set of moments of time and R_1, R_2 relations on U - R_1 is the forward direction of time, R_2 the backward direction. What we lay down semantically is that

$\Box A$

is true at $t \in U$ iff for all t' such that $t R_1 t'$ A is true at t' (at all moments later than t):

$\Box A$



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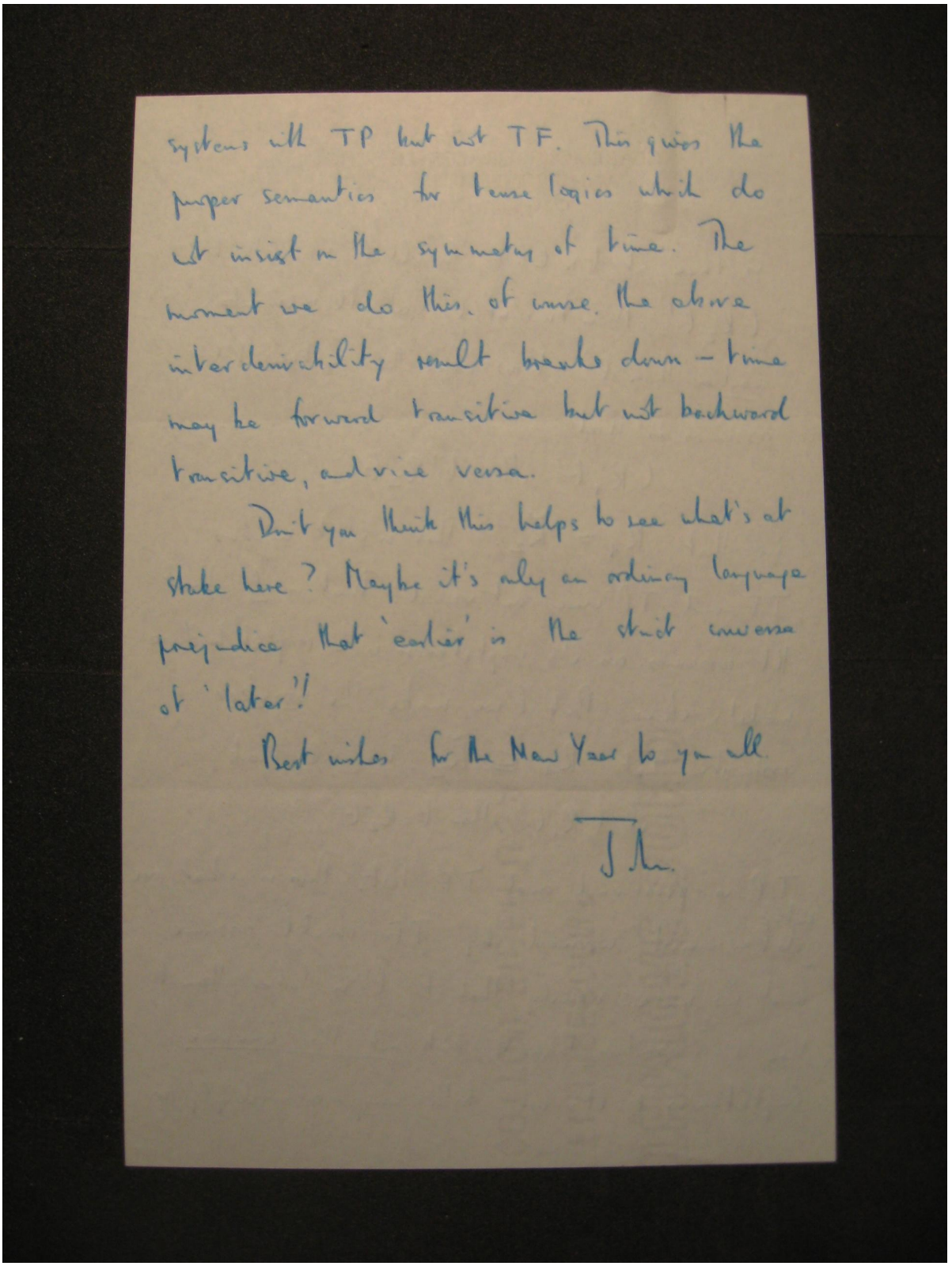
is true at $t \in U$ iff for all t' such that $t R_2 t'$ A is true at t' (at all moments earlier than t). Now what the system KK assumes is that

$$t R_1 t' \text{ iff } t' R_2 t$$

i.e. that $R_1 = \breve{R}_2$. This is what makes TF and TP come out valid. That is all that KK assumes, as its completeness with respect to those models shows. But if we make the weaker assumption that $R_1 \subseteq \breve{R}_2$, i.e. only that

$$\text{if } t R_1 t' \text{ then } t' R_2 t$$

TP is preserved and TF lost. This is what an anti-deterministic variant of TF should assume, and no more: namely that if t' is later than t, then t is earlier than t', but not the converse. Completeness results are forthcoming accordingly for



systems with TP but not TF. This gives the proper semantics for tense logics which do not insist on the symmetry of time. The moment we do this, of course, the above interdefinability result breaks down – time may be forward transitive but not backward transitive, and vice versa.

Don't you think this helps to see what's at stake here? Maybe it's only an ordinary language prejudice that 'earlier' is the strict converse of 'later'!

Best wishes for the New Year to you all.

JA.

Letter from Hamblin to Mary Prior, dated Nov. 3, 1969[49]

31 Parliament Hill London NW3 3rd November 1969.

Dear Mary,

Thank you for your nice letter: we've been away for a few days & I'd have answered sooner.

"Anti-fuzz" refers to what Arthur and I were talking about in Oberwolfach:[50] I have a number of scribbles he and I interchanged. A proposition is "fuzzy" in a given interval of time if it is sometimes true and sometimes false in the interval but there is no extended subinterval in which it is true & in which it is false:[51] Example: "Present clock time is expressible as a terminating decimal". He was worried about axioms, & lack of them, in a system of mine aimed at excluding temporal fuzzes.[52] I'll bring you the papers or copies of them when I next come to Oxford – probably in about a week. I shall, at least, footnote them in the printed version of my Oberwolfach paper.[53]

[49] *Virtual Lab* n. 1284.

[50] This city in West Germany, Black Forest, was the location of a conference organized by the *International Society for the Study of Time*, from August 31 to September 6, 1969 [Fraser et al., 1972]. During this conference Prior and Hamblin met each other for last time before Prior's death, on October 6, 1969.

[51] In occasion of the conference both Prior and Hamblin gave a talk about what they thought as basic elements of time: according to Prior, the notion of present [Prior, 1972], according to Hamblin, the notion of interval [Hamblin, 1972], gained by a comparison between instants and intervals. Prior highlighted the tight relation between the concepts of what is real, present, true. A correct idea of his view can be gleaned by an exploration of its presentism, *i.e.* there exist only present and actual events. This theory influenced many Prior's linguistic and ontological papers, *e.g.* [Prior, 1968, 1972]. In the first paper, Prior pointed out a 'genuine difficulty', but he did not renounced to the problematic claim that 'the present is an instant'. Hamblin's talk focused on a philosophically no less problematic solution. Hamblin [1972] suggested that 'instants can have no content' and especially that 'if time divides into instants we can give no account of temporal relations'. Hamblin decided to refer to intervals of time rather than to instants, giving a formal system.

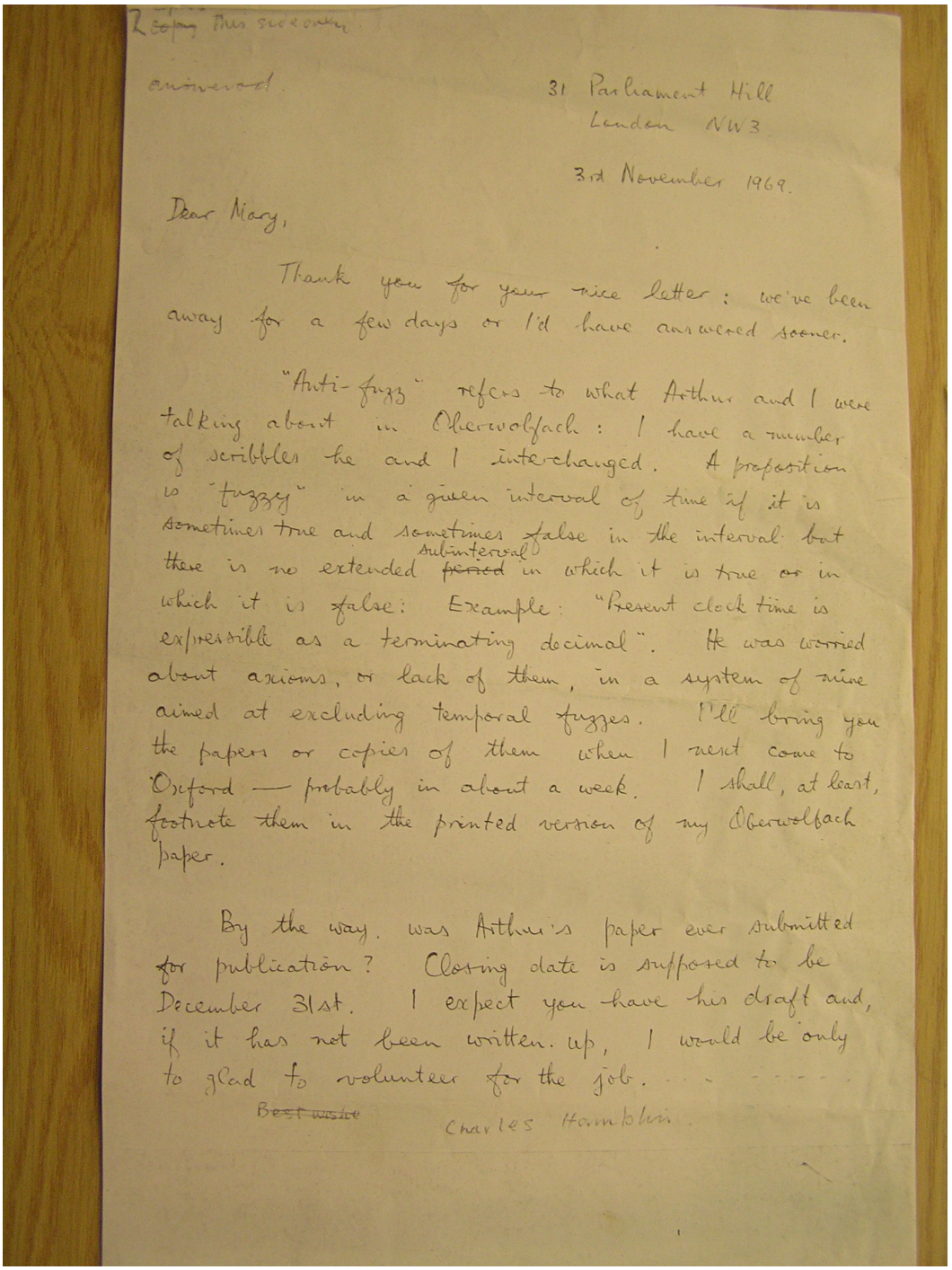
[52] A logical syntax of intervals has already been proposed by [Prior, 1967, 95-112] in the way of a metric tense logic; while [Hamblin, 1969b] is aimed to a study of topological properties of intervals.

[53] In the *Proceedings* of the Oberwolfach Conference, Hamblin alluded to the fruitful exchange with Prior, both during the conference and also through letters [Hamblin, 1972, 328]. Unfortunately, in spite of Hamblin's promise to Mary Prior, not a trace of Prior's documents to Hamblin is kept in the *Prior Collection*, and where the scribbles of Prior ended up is unknown.

By the way, was Arthur's paper ever submitted for publication?Closing date is supposed to be December 31st. I expect you have his draft and, if it has not been written up, I would be only to glad to volunteer for the job. .....

~~Best wishe~~

Charles Hamblin



Keeping this screwed

answered.

31 Parliament Hill
London NW3.

3rd November 1969.

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~~Best wishe~~

Charles Hamblin.

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