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Optimization Methodologies in Complex Water Supply Systems for Energy Saving and a Correct Management under Uncertainty

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ABSTRACT

Nowadays, the management of complex water supply systems needs to pay a close attention to economic aspects concerning high costs due to energetic management. Among them, the optimization of water pumping plants activation schedules is a significant issue when managing emergency and costly water transfers under drought risk. These problems are affected by a high uncertainty level, which is difficult to be faced. In this class of problems, uncertainty lies in water availability, demand behavior, electric prices and so on. Therefore, in order to provide a reliable solution, this research wants to develop some approaches of *optimization under uncertainty*, dealing with *water resources management problems* concerning multi-users and multi-reservoirs systems, especially referring to the definition of optimal activation rules for emergency pumping stations in drought conditions. In scarcity situations, the evaluation of different solutions is intimately related to the future water resource availability and the opportunity to provide water through the activation of emergency and costly water transfers. Hence, the water system optimization problem needs to deal with uncertainties particularly in treating the effectiveness of emergency measures activation to face droughts.

The research analysis wants to assure simultaneously an energy saving and a correct management in complex water supply system under uncertainty conditions. The formulation of this problem highlights a complicated decision procedure, considering the requirements duality: to guarantee a complete water demands fulfilment respecting an energy saving policy. The obtained results should allow the water system's authority to get a *robust decision policy*, minimizing the risk of wrong future decisions. A *cost-risk balancing approach* has been here developed to manage this

problem, in order to balance the damages due to shortages of water and the energy-cost requirements of pumping plants. In a first step, the problem has been solved using a traditional *Scenario Analysis Approach* with a two stages stochastic programming. The obtained results using *Scenario Analysis Approach* were appreciable considering a limited number of historical scenarios characterized by a short time horizon. Nevertheless, in a second phase, when increasing the number of considered scenarios by generation of a new synthetic database in order to take into account the effect of climate and hydrological changes, some computational problems related to the dimensions of the model arose. Therefore, to solve these computational difficulties, it is been necessary to apply a specialized approach for optimization under uncertainty. Hence, a simulation model has been coupled with an optimization module using the *Stochastic Gradient Methods*.

Testing the effectiveness of this proposal, an application of the modelling approach has been developed in a water-shortage prone area in South-Sardinia (Italy), characterized by Mediterranean climate and high annual variability in hydrological inputs to reservoirs. By applying the combined simulation and optimization procedure a robust decision strategy in pumping activation was obtained considering also the synthetic database.

RIASSUNTO

Oggigiorno, la gestione dei sistemi di approvvigionamento idrico complessi presta una particolare attenzione rispetto gli aspetti economici concernenti gli elevati costi derivanti dalla gestione energetica. Tra questi, la problematica relativa all'ottimizzazione delle soglie di attivazione degli impianti di sollevamento idrico è particolarmente sentita nella gestione di sistemi in cui sono presenti dei trasferimenti idrici onerosi d'emergenza e, al contempo, fenomeni di carenza idrica. Tali problemi sono affetti da un alto livello d'incertezza, che è difficile da fronteggiare. In tale classe di problemi, l'incertezza affligge la disponibilità idrica, il comportamento delle domande, i prezzi dell'elettricità etc. Pertanto, al fine di pervenire ad una soluzione affidabile, l'attività di ricerca si pone come obiettivo lo sviluppo di alcuni approcci di *ottimizzazione sotto incertezza* con riferimento all'ambito della *gestione delle risorse idriche*, analizzando sistemi d'approvvigionamento multi-utenza e multi-risorsa, e, nello specifico, mirando alla definizione di regole ottimizzate per la gestione degli impianti di sollevamento idrico di emergenza, atti a far fronte ad eventuali eventi siccitosi.

In condizioni carenza idrica, l'individuazione di valide soluzioni sostenibili è strettamente connessa alla futura disponibilità di risorsa e l'opportunità di garantire un approvvigionamento idrico attraverso dei trasferimenti onerosi di emergenza. Pertanto, il problema di ottimizzazione del sistema di risorse idriche sarà affetto da un elevato livello d'incertezza, in particolare, analizzando possibili soluzioni finalizzate alla mitigazione dei fenomeni di carenza idrica. L'analisi sviluppata nel progetto di ricerca mira ad assicurare contestualmente un risparmio energetico e una corretta gestione dei sistemi idrici complessi in condizioni d'incertezza. La formulazione del problema evidenzia una complessa procedura decisionale, dovuta alla dualità

nell'obiettivo perseguito: garantire un completo soddisfacimento delle domande provenienti dalle utenze del sistema osservando al contempo una politica di risparmio energetico. I risultati ottenuti garantiranno all'autorità del sistema idrico in esame di attuare delle *robuste politiche decisionali*, in grado di minimizzare il rischio di incorrere in decisioni errate in riferimento alla gestione futura. La modellazione è stata formulata attraverso un *bilanciamento costi-rischi*, affinché sia assicurato un bilanciamento tra i danni conseguenti da fenomeni di carenza di risorsa e gli oneri energetici dovuti all'azionamento degli impianti di sollevamento idrico. In prima istanza, il problema è stato risolto adoperando l'approccio tradizionale di *Analisi di Scenario* attraverso un algoritmo di programmazione stocastica sviluppato su due fasi.

I risultati ottenuti sono stati soddisfacenti considerando un numero limitato di scenari idrologici storici, caratterizzati da un breve orizzonte temporale di riferimento. In una seconda fase è stato incrementato il numero di scenari di riferimento, generando un nuovo database sintetico, in modo da prendere in esame gli effetti dovuti alla variabilità idrologica e climatica. In tale configurazione emergono alcune problematiche computazionali derivanti dalle dimensioni del modello analizzato. Pertanto, al fine di superarle è stato opportuno adoperare un approccio specialistico di ottimizzazione sotto incertezza. Quindi, è stato implementato un modello di ottimizzazione e simulazione ricorsiva mediante il *Metodo dei Gradienti Stocastici*.

Al fine di testarne l'efficacia, il modello è stato applicato ad un caso studio reale, relativo al sistema di approvvigionamento multi-settoriale della Sardegna meridionale (Italia). Tale sistema è caratterizzato da un clima Mediterraneo e da un'ampia variabilità degli input idrologici. Applicando le procedure di simulazione ed ottimizzazione ricorsiva è stato possibile individuare una strategia decisionale robusta considerando entrambi i database idrologici.

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Notation – List of Symbols

This section describes the major symbols and notations used in each chapter of this thesis. To the greatest extent possible, I have attempted to keep unique meaning for each item. In those cases where an item has additional uses, they should be clear from the context. Some notations may be used in more than one section, therefore their description have been reported just one time exactly in concomitance of the chapter where they appear at the first time.

▪ Chapter 2

- *G: directed graph;*
- *N: nodes of the directed graph;*
- *L: arcs of the directed graph;*
- *T: considered time-horizon;*
- *t: time step (basic period);*
- *S. sea node of water network;*
- *R: reservoirs in the water system;*
- *x: vector of decision variables of the water system (i.e. water flows along arcs):* $x \in X \subseteq \mathbb{R}_n$;
- *X: set of feasible solutions;*
- *c: cost vector;*
- *b: RHS (Right Hand Side);*
- *u: upper bounds vector;*
- *l: lower bounds vector;*
- *A: coefficient matrix of the constraints system (with* a_{ij}*);*
- B_τ*: bundles at each branching-time;*
- Γ_τ*: groups of scenarios to include in each bundle;*
- *g: single scenario,* $\forall g \in G$ *;*
- *p: weight assigned to each scenario;*

- x^*: *set of vectors submitted to non-anticipative constraints (congruity constraints);*
- *t: time period representing a branching time;*
- y^t: *reservoir's water stored configuration at the time step t;*
- *w: random parameters,* $\omega \in \mathbb{R}_k$;
- $\mathbb{E}_\omega$: *expected value of random parameters w;*
- π_X: *projector operator;*
- ρ_s: *step size of Stochastic Gradient Methods procedure;*
- ξ^s: *stochastic gradient of the objective function* $\mathbb{E}_\omega f_0(x,\omega)$;
- *z: arbitrary value of decision variables x:* $z \in \mathbb{R}^n$;
- $\mathbb{B}_s$: *sigma field defined by the history of the process;*
- $F_{0x}(x^s)$: *gradient of the function* $F_0(x) = \mathbb{E}_\omega f_0(x,\omega)$;
- a^s: *it represents the bias of the process;*
- x^b: *vector of barycentric values of decision variables;*
- $\hat{x}_g$: *observed value of decision variable* x_g *in the specific g-th scenario;*
- w_g: *costs related to the risk occurrences in a specific scenario g;*
- λ: *weight factor, it regulates the relationship between cost and risk elements.*

▪ Chapter 3

- *R: vector of reservoirs located in the water system;*
- *D: vector of water demands located in the system;*
- *T: vector of junction nodes located in the water system;*
- *P: vector of pump stations located in the water system.*

▪ Chapter 4

- $\bar{x}$: *average volumes referred to historical series [*10^6 m^3*];*
- $\bar{x}_f$: *average flow rates referred to historical series [*m^3/s*];*
- $\bar{x}_i$: *average water inflows referred to historical series [mm];*
- m_x: *median value;*
- σ_x^2: *variance value;*

- σ_x: *standard deviation value;*
- g_x: *skewness value;*
- cv_x: *coefficient of variation value;*
- μ_y: *water inputs value in logarithmic scale;*
- σ_y^2: *variance value in logarithmic scale;*
- σ_y: *standard deviation value in logarithmic scale;*
- g_y: *skewness value in logarithmic scale;*
- σ_{yR}: *standard deviation value calculated with the residuals;*
- ε: *stochastic components for synthetic series generation;*
- μ_x: *water average volumes of synthetically series [10^6 m^3];*
- c_{ij}: *monthly breakdown coefficients (i and j refer respectively to the observed year and month);*
- Q_{ij}: *water volumes referring to the j-th month and i-th year [10^6 m^3];*
- Q_i: *average annual volumes [10^6 m^3].*

Chapter 5

- I_t: *hydrological inflows at the period t [10^6 m^3];*
- D_t: *water demand from users and activities at the period t [10^6 m^3];*
- *S: sea node of the water system;*
- $i \in P\{1, \ldots, n_P\}$: *pumping stations in the water system;*
- $j \in R\{1, \ldots, n_R\}$: *reservoirs in the water system;*
- x_d: *amount of water flowing along the dummy arc connected to the d-th demand center [10^6 m^3];*
- x_i: *amount of water flowing along the arc connected to the i-th pump station [10^6 m^3];*
- x_{nt}: *amount of water flowing along the n-th arc of the system during the period t-th [$10^6 m^3$];*
- xv_t: *amount of storage volume transferred by the inter-period connection in the t-th period [$10^6 m^3$];*
- xs_{jt}: *amount of water spilled by the j-th reservoir to the sea node in the t-th period [$10^6 m^3$];*
- S^b: *barycentric activation value for i-th pumping station [10^6 m^3];*
- $\hat{S}_i^g$: *activation storage value for i-th pumping station in g-th scenario [10^6 m^3];*

- $h_i \in \{0,1\}$*: binary variable of on/off condition referred to the i-th pump station;*
- *CM_i: activation penalization coefficient;*
- *BM: Big M, large scalar;*
- *K_j: reservoirs' capacity [10^6 m^3];*
- *c_i: pumping cost referred to the i-th pump station [€/10^6 m^3];*
- *c_d: deficit cost referred to the d-th demand node [€/10^6 m^3];*
- *Z_{TOT}: average value of annual costs [10^6 €/year];*
- *Z_{pump}: average value of pumping costs [10^6 €/year];*
- *Z_{def}: average value of the deficit costs [10^6 €/year].*

▪ Chapter 6

- *q: set of parameters that describe the pumping rules activation;*
- *Q: feasible set for parameters q;*
- *v^t: water volumes stored in reservoirs [10^6 m^3];*
- *d^t: water volumes transferred to the water demand d at the period t [10^6 m^3];*
- *r^t: water inflows arrived at the period t [10^6 m^3];*
- *x^t: water flows along the arcs of network at the period t [10^6 m^3];*
- *C^t: average steady state costs supported at the period t [10^6 €];*
- *δ: it is a small positive value;*
- *e_k: it is a vector of zeros with one value in the a generic k-th position;*
- $\prod_Q(\cdot)$*: projection operator on feasible set q,*
- *α_t: averaging parameter which usually assumes the 0.3 value;*
- $\bar{C}_0^t$*: it is an average cost referred to all periods of the considered until the period t [10^6 €];*
- C_0^t*: they are the costs evaluated at the time-step t;*
- Θ_m^t*: configuration of the network state during the generic period* $m \in M$ *and at the time-step t;*
- s*: they are random vectors with known distributions;*
- τ*: length of the moving window horizon* $(\tau > 1)$;
- η_s*: discontinuing coefficients;*
- *U: vector of transshipments nodes of the system;*
- *P: set of pumping links of the system;*
- $\bar{P}$*: it is an arbitrary operating subset of the set P;*

- ς_i^t: *it is a parameter that represents the fraction of water, which is evaporated from the reservoir* $i \in R$ *during the period t;*
- K_i^-: *it represent the set of parent nodes of node i;*
- K_i^+: *it represent the set of children nodes of node i;*
- g_{ij}: *it represents the capacity of the link (i,j) ∈ L;*
- V_i^{max}: *it is the maximum capacity for the reservoir* $i \in R$ *[10^6 m^3];*
- u_{pdi}^t: *they are the planned deficit for the user i at the period t [10^6 €];*
- u_{ndi}^t: *they are the unplanned deficit for the user i at the period t [10^6 €];*
- c_{pd}^i: *costs related to planned deficit occurrences [10^6 €];*
- c_{nd}^i: *costs related to unplanned deficit occurrences [10^6 €];*
- β_i: *maximal fraction of planned deficit allowed from demand value at each node;*
- c_w^i: *costs related to spilled water [10^6 €];*
- x_{iS}: *water volumes transferred to the sea node S from the generic node i [10^6 m^3];*
- h_{ip}: *coefficient that summarizes the functional dependences between volumes in reservoir i and p;*

1. Introduction

The management optimization of complex multi-sources and multi-demand water resource systems, aimed to the energy saving, is an interesting and actual research topic (*Gaivoronski et al., 2012b; D'Ambrosio et al., 2015; Nault and Papa, 2015; Napolitano et. al., 2014; 2016*). Problems pertaining to management policies and concerning the effectiveness of emergency and costly water transfers, mainly to alleviate droughts, are faced with different methodological approaches *(Menke et al., 2016, Lerma et al., 2015; Pasha and Lansey, 2014)*, resulting frequently hard to solve. They need some *robust approaches* to deal with many uncertainties modeling the system to achieve optimal decision rules. In general terms, the last decades have seen many studies concerning the development of operating rules for water resources systems under scarcity conditions (*Asefa et al., 2014; Hanel et al., 2017; Mateus et al., 2017*). These problems are generally treated in order to provide efficient solutions to the water resource system's Authorities.

The original "simple optimization" problem was referred to a single reservoir regarding: "*How much water to deliver and how much to withhold for immediately benefit and retention in reservoir for possible future use*" (*Bower et al., 1962*; *Maas et al., 1962*). This problem has been gradually complicated considering multi-reservoir operating rules, multi-user demands and, moreover, including pumping transfers, relaxing the water blending standards, activating some emergency transfers, conjunctive use of conventional and non-conventional resources, etc. Some general reviews of reservoirs operating policies can be found in *Lund and Guzman* (*1996; 1999*), *Loucks and Sigvaldson* (*1982*), *Loucks and Van Beek* (*2005*), *Sulis and Sechi* (*2009; 2013*). Particularly, *Draper and Lund* (*2004*) demonstrated that the optimal hedging policy for water supply reservoir operations depends on a balance between beneficial release and carryover storage values. Moreover, *Draper* (*2001*) estimated parameters for quadratic carryover storage economic value functions considering the reservoirs supply system in California.

Undoubtedly, problems related to multi-sources water supply system management under drought risk are an important field of research, documented by several applications of specifically derived mathematical optimization models defining decision rules. The optimization of the operation of large-scale multi-sources water supply systems has been also considered in order to provide water managers of a useful decision support systems such as: AQUATOOL (Universidad Politecnica de Valencia, *Andreu et al., 1996*), MODSIM *(Colorado State University, Labadie et al., 2000)*, ResSim (*U.S. Army Corps of Engineers, Hydrologic Engineering Center, USACE, 2003*) WARGI-SIM *(University of Cagliari, Sechi and Zuddas, 2000)*, WEAP *(Stockholm Environmental Software and Services, SEI, 2005)* and RAISON *(National Water Research Institute Environment of Canada, Young et al., 2000*). Moreover, in *Vieira et al.* (*2011*) have been defined a prioritization of preferences for the start of contingencies measures in a conjunctive-use decision model.

In water resource field, problems affected by *uncertainty* have been widely treated implementing several computational solutions, especially with application of stochastic dynamic programming to multi-reservoir systems, see *Labadie (2004)* for survey of different relevant optimization techniques.

In scarcity conditions, the system reliability evaluation is intimately related to a quantitative evaluation of future water availability and the opportunity to provide water through the activation of emergency and costly water transfers. Hence, the water system optimization problem needs to deal with uncertainties particularly in treating the effectiveness of emergency measures activation to face droughts.

On the occasion of drought occurrences, water managers must be able to manage this criticality, alleviating the effect of shortages on water users. A useful solution could be to develop some emergency policies, supplying additional water to users. Modern water supply systems frequently count on the presence of emergency sources with enough capacity for these critical occurrences. Frequently, the activation of these emergency transfers requires additional costs, such as for activation of pumping schedules. Optimal decision on this costly transfer activation is a hard decision problem: it is conditioned by uncertainties on future demands and inflows that are normally characterized by high variability in time and quantity. Moreover, in water resources management problems, uncertainty could affect other system components, as the assurance of proper water quality, elements malfunction, etc.

According some authors (*Pallottino et al., 2004*; *Cunha and Sousa, 2010; Vieira and Cunha, 2017*; *Napolitano et al., 2016***)**, the uncertainty could be described as different scenarios, which occurrence follows a probability distribution. In case of previous pessimistic forecasts, uncertainty and variability can generate water excess or subsequent spilling from reservoir, causing some losses and therefore resulting "*regrets costs*" (*Kang and Lansey, 2014*). On the other hand, the definition of emergency policies in reservoirs management could consider early warning measures, taking advantages of lower energy prices in some time-periods and achieving economic savings. Moreover, the management of multi-source water supply systems can be conditioned by difficulties in definition of water deficit penalties and system management costs, becoming extremely complex to search for economic efficiency.

When higher demand conditions or lower than expected hydrologic inputs actually occur, this policy may result in a significant system failure and intolerable system adaptation costs. *Hobbs and Hepenstal (1989)* demonstrated this effect by showing that an optimal solution for a specific condition will be always biased in an uncertain future.

In *Pallottino et al.* (*2004*), *Gaivoronski et al.* (*2012a*) and *Napolitano et al. (2016)*, the development of a cost/risk balanced management of scarce resource has been done by formulating a multistage scenario optimization model, in order to define a reduced target values in supplying water as a drought mitigation measures effectively linked with reservoir-storage triggers.

The hereafter described research activities aim to develop an *optimization under uncertainty modeling approach*, in order to deal with water resources management problems especially referring to the definition of *optimal activation rules* for emergency activation of pumping stations in drought conditions. Therefore, this study wants to define a cost-risk trade-off considering the minimization of water shortage damages and the pumping operative costs, under different hydrological scenarios occurrence possibilities. The results should be able to provide the water system's authority with a strategic information, defining *optimal rules, and specifically optimal activation triggers for water pumping stations*.

Pursuing the objectives it is necessary to assure simultaneously energy saving policies and correct water management of the supply system. The formulation of the related modelling approach highlights this duality: to guarantee water demands fulfillment respecting an energy saving policy. Results should assure optimal rules in the supply network, in order to minimize the emergency transfer's costs.

Following described analysis has been developed with some promising technics for optimization under uncertainty (*Birge and Louveaux, 2001*). In particular, a *Scenario Analysis Approach* and a *Stochastic Gradient Method*. A typical *Cost-Risk Balancing Approach,* have been set up evaluating management strategies. Results could provide an efficient *Decision Support System* able to lead the *Water System's Authority* in taking the best management rules.

To test the effectiveness of the proposed modeling approaches, they have been applied to a real case concerning a multi-reservoir and multi-user water supply system in a drought-prone area, located in the South-Sardinia (Italy) region, characterized by South-Mediterranean climate.

2. Methodologies

2.1. Mathematical Formulation

Treating the water management problems, the *graph theory* (*Ahuja et al., 1993; Diestel, 2005*) is considered as an efficient support for the mathematical modeling. According to this theory, a graph is a pair $G = (N, L)$ of sets satisfying $L \subseteq [N]^2$, where the elements of N are the nodes of the graph G, while the elements of L are its arcs. In the common hydraulic notation, nodes could represent groundwater, sources, reservoirs, demands, etc. Arcs represent the connections between nodes, where flows water.

This approach allows schematizing a complex water system problem through a simple flow network on a graph. Therefore, each possible scenario corresponds with a *dynamic multi-period graph* (*Pallottino et al., 2004; Sechi and Zuddas, 2008*), following a particular sequence of decisions. For example, in the water resource management, it is possible to consider a hydrological series or a sequence of management decisions related to a reservoir.

This kind of analysis could be extended to a wide time-horizon T, assuming a time step (period) t. The number of considered time steps must be adequate to reach a significant representation of the variability of the hydrological inflows and water demands in the system.

Each dynamic multi-period graph represents the possible complete realization of the system in an examined scenario. In the single period, we can represent the physical system and the static situation by a direct network called *basic graph*, as reported in Figure 1.

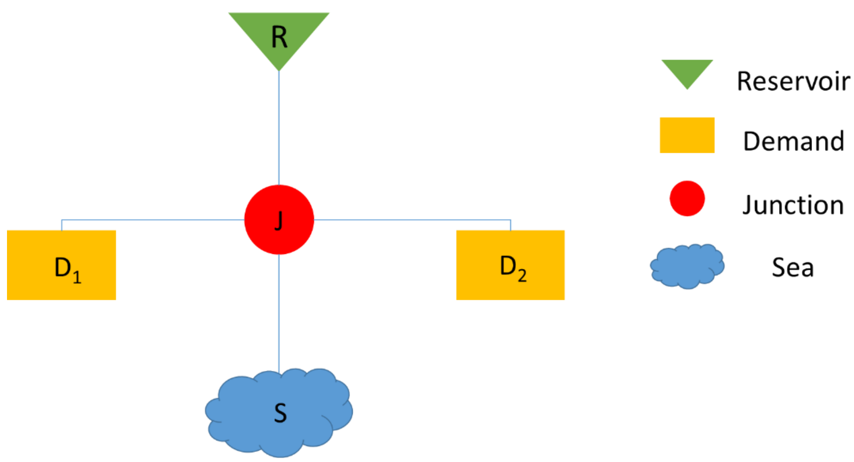


Figure 1: Basic Graph

The dynamic multi-period network is generated by replicating the basic graph for each period of the time horizon. Moreover, it possible to add some dummy nodes and arcs to represent not only physical components but also events that may occur in the system. The dummy nodes represent a possible external system acting as a supposed source or demand flow. The dummy arcs allow carrying water stored at the end of each period, for this reason they are called inter-period arcs. The reservoir nodes could be considered as a memory of the system, storing the unused resource in the period *t* and transferring it to *t+1*.

In order to guarantee a correct balance of the system, we insert a dummy node *S*. It is called sea node, and it represents an external demand center or source with an unlimited resource availability. Therefore, each arc *(i, S)* represents a spillway from a generic reservoir-nodes *i*, each arc *(S, i)* represents a supposed additional flow in case of shortage in order to meet request in the demand nodes *i* and prevent solution which are not feasible. Flow on arcs *(S, i)* highlights possible water system's deficits and the necessity to make recourse to external water resources. Moreover, at each time period *t*, a water input I^t fulfils each reservoir *R*.

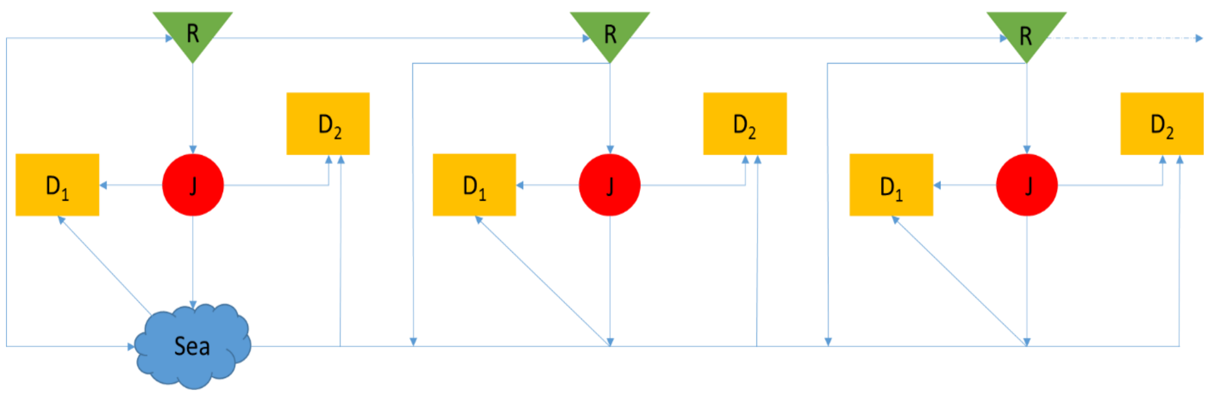


Figure 2: Dynamic Multi-Period Graph

The sketch reported in Figure 2. shows a multi-period graph in a simple problem of water resource management. All the examined optimization approaches could be written through a mathematical formulation of the problem.

A mathematical model is usually used describing the main features of the optimal solution in an optimization problem. With this formulation, we are able to describe even a complex problem (using suitable variables and constraints) and to build the basis for an application of standard optimization algorithms, in order to achieve the optimal solution.

In this way, dealing with a Linear Optimization (LP), the multi-period dynamic approach could represent an efficient support; indeed, a LP formulation allows achieving easily the optimal solution of the multi-period model (1a - 1c).

$$\underset{x}{Minimize}\ c^T x \tag{1a}$$

subject to

$$Ax = b \tag{1b}$$

$$l \leq x \leq u \tag{1c}$$

Few parameters describe this generic deterministic model:

- c: it is a *cost vector*, it appears in the objective function, whose components c_j can represent costs, benefits, penalizations or specific weights assigned by the system's manager to the variables x_j;
- b: it is the RHS (Right Hand Side), this vector appears in the constraints system. The components b_i can represent a supply or a demand associated to a node i;
- u and l: they represent respectively the lower and upper bound vectors, whose components l_j and u_j represent lower and upper limits (possible zero and infinity values are admitted, respectively) imposed on the variable x_j by physical, technological, environmental and/or political requirements;
- A: it is the coefficient matrix of the constraints system, with elements a_{ij}.

Using deterministic optimization models, we assume that the manager has a complete knowledge of the system and the variability of its main features along the examined time horizon. These models are characterized by several decision variables, which are frequently divided into *operation* and *project variables*.

In the water resource management, the operation variables can refer to different types of water transfers such as water transfer in space along physical arcs in the system or connecting similar nodes at different time. Project variables refer to the dimension of the physical elements of the system (e.g. pipe dimensions, reservoir capacity and so on).

The constraints represent the relations among flow variables and project works, technological limitations on decision variables, mass balance equations and demands for the centers of water consumption. Here, the vector x represents the decision and project variables, while the equations (1b-1c) summarize the constraints.

Nevertheless, the deterministic linear programming (LP) models are not adequate to describe efficaciously the variability and uncertainty of some crucial variables and parameters. Modeling real-world problems, difficulties arise from a classical LP-deterministic implementation. As a matter of fact, a huge uncertainty and several aleatory characterize many considered entities and it requires looking for non-deterministic approaches, as will be discussed in the following.

2.2. Scenario Analysis Approach

Scenario analysis approach arises from considering that the future events can evolve from a set of different and statistically independent scenarios (*Rockafellar and Wets, 1991; Dembo, 1995; Pallottino et al., 2004*). A single scenario describes a possible realization of some sets of uncertain data in the examined time horizon. Considering the inner structure of scenarios temporal evolution, it is possible to obtain a *robust decision policy*, minimizing the risk of wrong future decisions.

In the first step of the analysis, it is possible to work using a configuration with parallel scenarios, followed by a simulation aimed at confirming the reliability of each solution. In the second phase, a model that contextually examines the set of different scenarios can be elaborated, proving a barycentric solution to the multi-scenario problem. This modelling approach can be represented as a *tree-graph*, according to appropriate aggregation rules (*Rockafellar and Wets, 1991; Pallottino et al., 2004*). Two scenarios sharing a common initial portion of data can be considered together and partially aggregated with the same decision variables for the aggregated part, taking into account the two possible evolutions in the subsequence diverse parts.

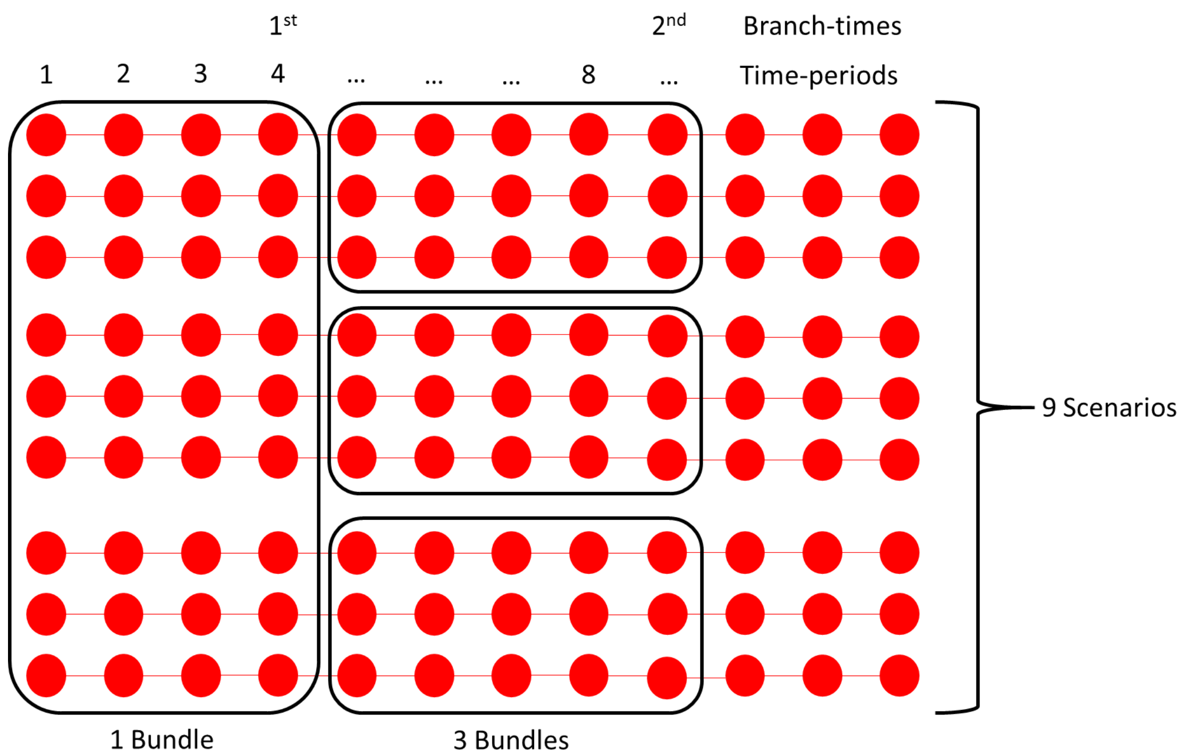


Figure 3: Scenario-Tree with Parallel Scenarios

The scenario aggregation rules could be independent of the extension of the examined time horizon, number of periods and adopted optimization techniques. Figure 3 shows a set G of nine parallel scenarios before the aggregation step. Herein, each single scenario corresponds to a dynamically generated multi-period network, associated to a particular sequence of inputs. In this draft, the dynamic multi-period network is represented by a sequence of dots, where each dot represents the system in a time period t.

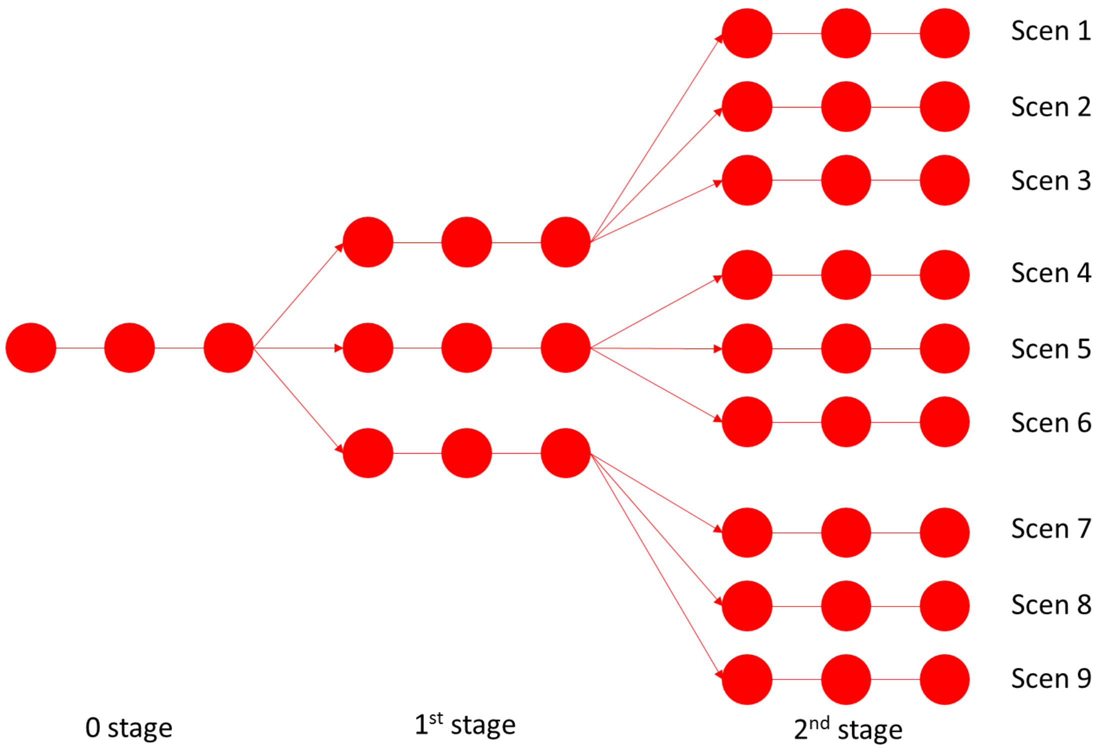


Figure 4: Scenario-Tree Aggregation

In order to perform the scenario aggregation, in the second step are defined *stages* and *branching-times*. A branching-time t identifies the time-period in which some scenarios, that are identical up to that period, begin to differ. A stage corresponds to a sequence of time periods between two consecutive branching-times. In particular, the stage 0 identifies the initial characterization of the system. Therefore, it represents the *root* of the *scenario-tree*. In stage 1 a number β_1 (3 in the Figure 4) of different possible different configurations can occur, in stage 2 a number $\beta_1 x \beta_2$ (9 in the Figure 4) can occur and so on.

The Figure 4 shows a scenario-tree characterized by 3 stages and 9 scenarios, which has been built through a set of parallel scenarios aggregated in order to obtain a tree-structure.

Two branches characterize this tree-structure: the first branching time appears after three periods, while, the second one is after the six periods. All along the time periods that precede the first branching time, all scenarios are gathered in a single bundle, whereas three bundle characterize the second branch. This implies that the zero bundle includes in a unique group all scenarios; in the first stage, 3 bundles are generated identifying the groups Γ_τ: {(scen1, scen2, scen3); (scen4, scen5, scen6); (scen7, scen8, scen9)} to include in each bundle.

Summarizing, the main rules adopted to organize the set of scenarios in the scenario-tree are:

- *Branching*: to identify the branching times t as time periods at which to bundle parallel sequences, while identifying the stages at which to divide the scenario horizon;
- *Bundling*: to identify the number B_τ of bundles at each branching time;
- *Grouping*: to identify groups Γ_τ, of scenarios to include in each bundle.

The root of the scenario-tree corresponds to the time at which decisions (common to all scenarios) have been taken and the leaves of the scenario-tree represent the performance of the system in the last stage. Each path from the root to a leaf identifies a possible sequence of occurrence along the entire time horizon.

The optimization problem can be described by a mathematical model that includes all single-scenario-g deterministic models, taking into account all scenarios $\forall g \in G$.

$$\underset{x^g}{Minimize} \sum_{g \in G} p^g c^g x^g \tag{2a}$$

subject to

$$A^g x^g = b^g \qquad \forall g \in G \tag{2b}$$

$$l^g \leq x^g \leq u^g \qquad \forall g \in G \tag{2c}$$

$$x^* \in S \tag{2d}$$

All decision variables in the model (2a-2d) and referred data are scenario-dependent. The vector of decision variables x^g is then identified by index g. The objective function is defined as the average of the cost objectives of all scenarios weighed by their probability p^g. Each scalar p^g represents a weight that could be assigned by the manager to each scenario to characterize its relative importance. Hence, it could represent the probability of occurrence of the scenario, if this probability can be estimated by stochastic techniques or statistical test based on historical data. The resulting aggregated multi-stage stochastic programming model can be expressed as the collection of single deterministic models for each $g \in G$, plus a set of variables corresponding to the indistinguishable parts in each scenario be equal among themselves (*Pallottino et al., 2004*).

The additional set of non-anticipative constraints $x^* \in S$ represents the congruity constraints derived from the scenario aggregation conditions. Then, equations (2d) are collected from all scenarios and placed into the aggregated model. Specifically, x^* represents the vector of variables submitted to congruity constraints.

Congruity constraints allow that the decision variables in those scenarios that are indistinguishable up to branching-time τ are the same up to τ. Dealing with reservoirs, this means that, in these scenarios, the amount of resource stored in the reservoir at the end of the time τ to transfer in the period $\tau+1$, must be the same. Introducing these constraints, the model will be redundant and the components of the model (variables and constraints), that are associated to overlapped scenarios, can be reported just once.

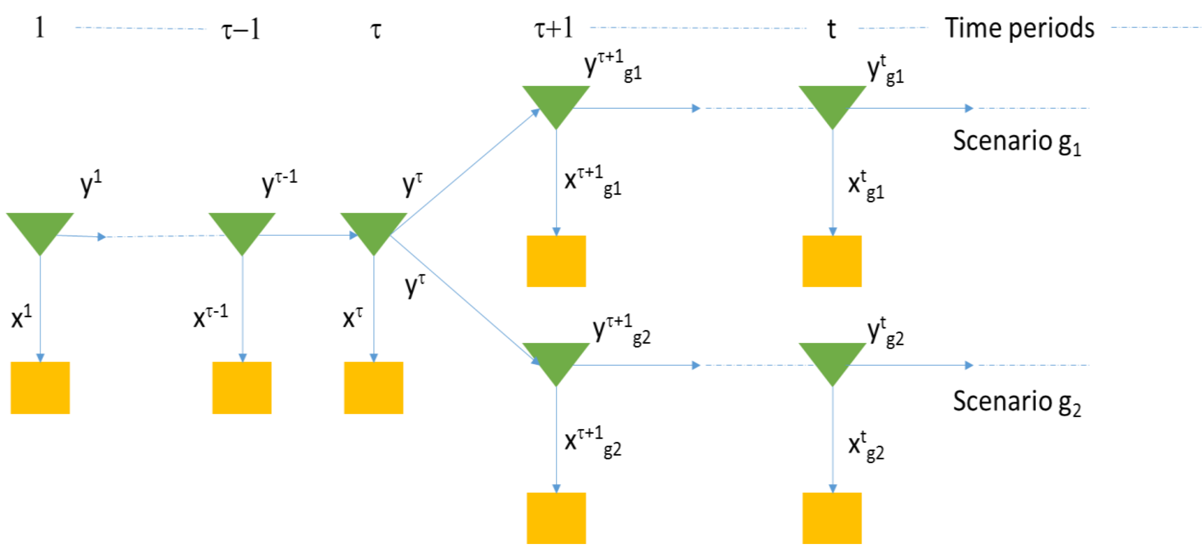


Figure 5: Scenario-Tree Generation

The Figure 5 shows the scenario-tree evolution for a simple one reservoir-one demand scheme. It highlights a branch going to two scenarios *g1* and *g2*, identical up to the branching-time τ. Until the period τ, the decisions taken must be the same for both scenarios, hence, they will share a common past from the initial period up to the branching-time τ.

2.3. Stochastic Gradient Methods

The *stochastic gradient approaches* belong to a family of methods (*Ermoliev and Wets, 1988; Birge and Louveaux, 2001; Gaivoronski, 2005; Conejo, 2010*) specifically designed having in mind continuous distributions of random parameters and nonlinear optimization problems. These methods are suited for optimization of simulation models where specific analytical relations between the objective function and random parameters are difficult to trace. Therefore, they are frequently characterized by several decision variables and a long time horizon.

An application area of such methods consists of multi-period dynamic stochastic model with parametrized decision rules. Supply chain management, energy generation, financial applications, water resource management are among the sources of such kind of problems.

The stochastic gradient is a statistical estimate of the gradient of the objective function, which provides an optimal direction for iterative updating of the current approximation to the optimization problem solution. These methods allow employing clear structures of the models through a gradient estimation schemes specific to the application domain. Otherwise, without these schemes, a general estimation procedure can be applied, such as the average evaluation of finite difference approximation.

The stochastic gradient has its roots in the stochastic approximation and in the mathematical programming algorithms with gradient technics.

They try to solve the problems (3).

$$\underset{x \in X}{Minimize}\ \mathbb{E}_{\omega}\, f_0(x, \omega) \tag{3}$$

Where $\mathbb{E}_{\omega}$ is an expected value considering $\omega \in \mathbb{R}_k$ as a vector of random parameters and $x \in X \subseteq \mathbb{R}_\mathrm{n}$ as a vector of decision variables. X represents the set of feasible solutions and usually it is a convex set characterized by a simple structure. The problem statement describes a large set of dynamic and static stochastic optimization problems. In detail, these methods could be divided in two categories: *deterministic equivalents* and *iterative sampling algorithms*.

The *deterministic equivalents* start by approximating the (3) with a problem where the original probability distribution of the random parameters ω is substituted by discrete distribution concentrated in a finite number of points ω_i, which describe different scenarios. Therefore, the original problem could be substituted by a deterministic optimization problem with a particular structure, where the expectations in (3) are replaced by sums.

Iterative sampling algorithms have their stochasticity in the statistical estimates of functions $F_i(x) = \mathbb{E}_{\omega} f_i(x, \omega)$, their gradients and Hessians. These estimates are obtained through the generation of the different scenarios ω_i. The generation process deals with the solution process; herein the estimates are adopted as substitution of the exact values in the iterative algorithms, which come from linear and nonlinear programming. *Stochastic Quasi-Gradient methods* (SQG) belongs to this class of problems.

In general, SQG problems start from an initial point x^0, moving forward to the current approximation of the optimal solution x^s and trying to achieve the best solution of the original problem (3).

$$\mathrm{x}^{s+1} = \pi_X(x^s - \rho_s \xi^s) \tag{4}$$

The ρ_s represents a step-size, that occurs in direction opposite to the current estimate ξ^s of the gradient of the objective function $F_0(x) = \mathbb{E}_\omega f_0(x, \omega)$, evaluated solving the problem (3), at the point x^s and projecting the resulting point onto the set X. The projection operator π_X deals with X and it transforms an arbitrary $z \in \mathbb{R}^n$ into the point $(z) \in X$ such that:

$$\|z - \pi(z)\| = \min_{x \in X} \|z - x\| \tag{5}$$

The structure of the set X allows a fast solution of the problem (5). Indeed, this problem should be solved several times during the optimization process, but, the set X is defined by linear constraints and, therefore, it guarantees to achieve an efficient solution.

The crucial part of the SQG implementation consists in the evaluation of statistical estimates ξ^s of the gradient of the objective function $F_0(x)$. Its estimation is characterized by a considerable flexibility, following the equation (6).

$$\mathbb{E}(\xi^s | \mathbb{B}_s) = F_{0x}(x^s) + a^s \tag{6}$$

Here, $\mathbb{B}_s$ is the σ-field defined by the history of the process (composed as a sequence $x^0, x^1, \dots, x^s$), while $F_{0x}(x^s)$ represents the gradient of the function $F_0(x) = \mathbb{E}_\omega f_0(x, \omega)$ at the current approximation x^s. Term a^s is the bias that should decrease as the number of iterations increase, in order to vanish asymptotically. When the vector ξ^s satisfies the property (6) is defined as stochastic gradient.

A generic evaluation of the stochastic gradient ξ^s is given solving the following equation:

$$\xi^s = f_{0x}(\omega, x^s) \tag{7}$$

Where ω^s is a single observation of a random vector ω. When the classical gradient does not exist, such as with convex but nonsmooth functions, ξ^s could be estimated through a generalization of the gradient that is called *stochastic quasi-gradient*.

The step-size definition is another fundamental component for the SQG algorithm. Indeed, the correct convergence to the optimal solution depends on the choice of the correct step-size ρ_s. Convergence theory suggests that any series with the properties (8) can be used as a sequence of step-sizes.

$$\rho_s > 0; \qquad \sum_{s=1}^{\infty} \rho_s = \infty; \qquad \sum_{s=1}^{\infty} {\rho_s}^2 = \infty \tag{8}$$

ρ_s selection depends on the precision of the stochastic gradient. If the variance of ξ_s is bounded from above and from below away from zero then the step-size should tend to zero, but not excessively fast. Therefore, ρ_s tends to zero faster if the variance of stochastic gradient grows with the number of iterations. Usually the best strategy seems to be to choose the step-size using an interactive method.

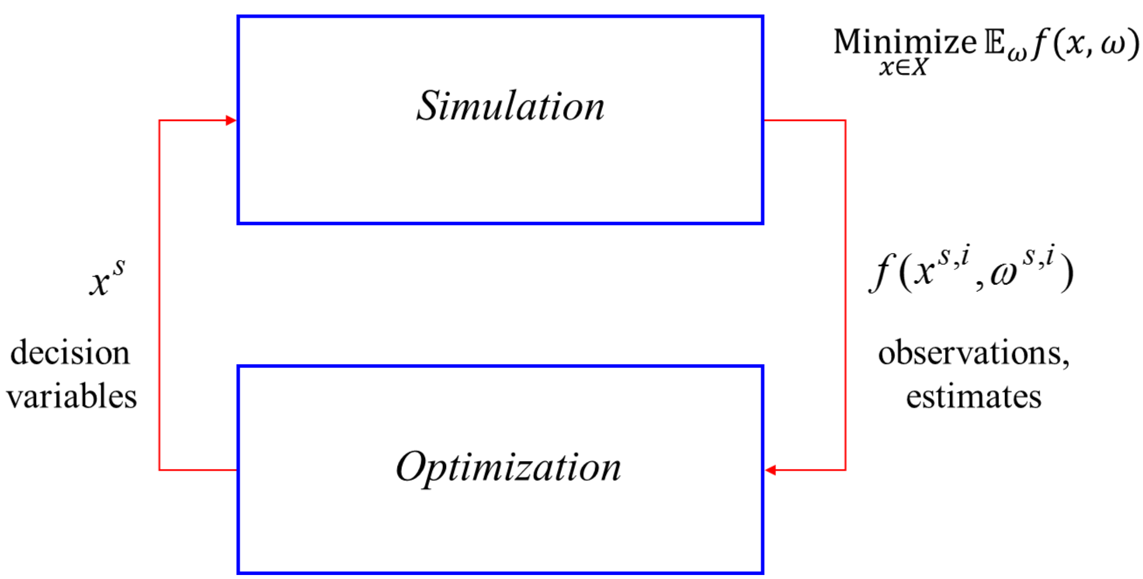


Figure 6: Recursive Simulation and Optimization Algorithm

A recursive interaction between a simulation module and the optimization process allows achieving efficaciously the best solution of this evaluation problem, evolving gradually by the SQG algorithm, as shown in the Figure 6.

This methodology is particularly suited for optimization problems characterized by a large number of decision variables, describing by a simulation model each single problem.

The main novelty of this research refers to the SQG development in order to afford real problems in water resource system management.

2.4. Cost-Risk Balancing Approach

Dealing with uncertainty, in order to find a balanced water system management criterion between alternatives, and to improve the decision-making process robustness, we propose a Cost-Risk Balancing optimization approach.

The *Cost-Risk balancing approach* tries to attain to a *robust* decision policy, which can minimize the risk to take wrong and harmful decisions for the future management. Therefore, in a problem of water system management, the water system's authority should be able to define a reliable target-value of water, assuring to deliver it to the centers of consumption, minimizing the system costs and reducing the deficit hardships for users. The problem mainly arises in the condition of scarcity of available resource and, for this reason, the demand could not be satisfied in considered future scenarios. In such scarcity situations, managers should develop an emergency policy, alleviating the effect of shortages. Hence, each stakeholder should know in advance and possibly should make agreements on decisions regarding expected reduced level of satisfaction in water demand. The reduced target level in demand satisfaction that the system's manager is willing to deliver to him, considering possible scenarios, should be accurately evaluated. According this approach, the target value has to result *barycentric* relating to demand requirements and future scenarios occurrences. In these situations, the deficit levels in the supply systems could be reduced introducing some management emergency measures, such as decreasing the allocated water to users or activating costly transfers. As result of cost-risk balancing process, the user will obtain the barycentric value x^b, which will be used like a parameter to be optimized in a new dynamic deterministic model called *re-optimization*.

The aim of re-optimization phase is to evaluate the sensitivity of the examined system compared to the stochastic process. The re-optimization face the optimization problem through a deterministic process. Once evaluated the main variables of the model during the stochastic process, they are assigned to the re-optimization phase as parameters in order to try out the reliability of the water system and to obtain the optimal network's flows configuration. Through this phase is possible to reach a robust solution and planning a part of the risk of incorrect decisions caused by wrong assumptions about adopted parameters.

Applying this methodology is possible to take into account the temporal evolutions of some crucial data, providing a barycentric solution to the multi-period decision problem. A weight can be assigned to each scenario characterizing its relative importance. Weights could represent the probability of occurrence of each scenario, if this probability can be estimated by stochastic techniques or statistical tests based on historical data. The objective function (9a) tries to minimize the weighted distance of the flow values $\hat{x}_g$ related to the barycentric value x^b, namely for each scenarios g and period t. The cost-risk balancing problem can be formulated modifying according the following model (9a-9d) in a form containing both the risk and cost terms, as in (*Gaivoronski et al., 2012a*; *Gaivoronski et al., 2012b; Napolitano et al., 2016*):

$$\underset{x_g,\, x^b}{Minimize}\;\; (1-\lambda)\sum_g p_g c_g(x_g) + \lambda \sum_g p_g [w_g \|\hat{x}_g - x^b\|^2] \qquad (9a)$$

$subject\ to$

$$A_g x_g = b_g \qquad \forall g \in G \qquad (9b)$$

$$l_g \leq x_g \leq u_g \qquad \forall g \in G \qquad (9c)$$

$$x^* \in S \qquad (9d)$$

All decision variables and data are scenario dependent, hence the index g.

Therefore, x_g represents the vector of decision variables in scenario g; the vector c_g describes the unit cost of different activities like delivery cost, opportunity cost related to unsatisfied demand, opportunity cost of spilled water, and so on. The objective function is defined as the average of the cost objectives of all scenarios weighted with their probabilities p_g. The set of standardized equality constraints describes the relationships between storage, usage, spill, and exchange of water at different nodes and in subsequent time periods. The Right Hand Sides (RHS) b_g are formed from scenario data of inflows and demands. The lower and upper bounds l_g and u_g are defined by structural and policy constraints on the functioning of the system. All constraints (9b-9d) in equation are collected from all scenarios and put in the aggregated model. The additional set (9d) of non-anticipative constraints $x^* \in S$ represents the congruity constraints derived by aggregation rules.

Usually this new target level x^b will be less than the user's demand due to inherent scarcity of the resource. The difference between original demands and reduced target x^b will represent the *planned shortages*, which the single user is asked to accept under drought conditions. Besides the planned shortages, in very critical scenarios could also occur some *unplanned shortages* due to severe lack of resources under these scenarios. In these situations the available supply is even less than reduced target. The decision variables related to each scenario $g \in G$ should be highlighted. Consequently, in the model formulation the vector x_g will represents resource delivery in the multi-period graph under scenario g; moreover, under scarcity scenarios we have to calculate the new demand target x^b, which has to be barycentric referring to scenarios occurrences and related delivery $\hat{x}_g$.

The weighted difference $\left(\hat{x}_g - x^b\right)^2$ is the Euclidean measure of distance between the target delivery and actual delivery to demands and w_g is the cost of related to the risk occurrences of scenarios requiring accepted planned deficits.

In (9a), c_g is the cost associated to flow values x_g . Particularly, these costs refer to flows in pumping stations under scenario $\forall g \in G$. In more general terms the first term of the objective function (9a) can be defined as a *cost function* and it tries to look for the system flows configuration that allows minimizing the costs supported by water system's management. The second term can be defined as a *risk function*, it is quadratic and it can be considered like a non-linear *social function*, in order to guarantee users' major priority fulfillments, referring to future scenarios. In this way, giving a weight to both terms of the objective function, we can find a solution of the cost-risk balancing problem.

The relationship between cost function and risk function is regulated by the parameter λ called *weight factor*. This parameter can vary between *0* and *1*, where $\lambda = 0$ corresponds to the pure cost minimization problem, while for $\lambda = 1$ the problem becomes one of risk minimization. Intermediate values of λ provide different tradeoffs between costs and risks. An efficient frontier in the space of risk-cost could be built by solving our problem for different values of λ.

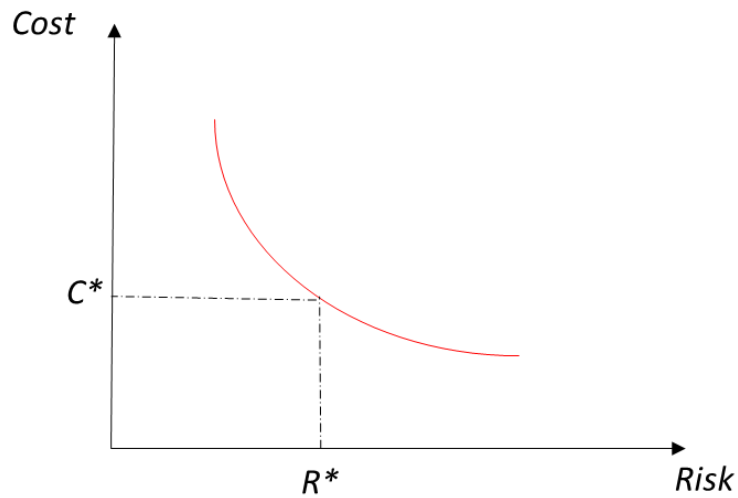


Figure 7: Economic Relationship between Cost and Risk Elements

The Figure 7 shows an economical inversely proportional behavior between cost and risk. In general terms, minimizing users' risk involves some investments on preventative measures and, consequently, a higher system management cost. While, minimizing costs allows saving money in preventive measure investments but an increment in terms of risk of damages associated to water shortages.

3. Case of study

3.1. Tirso - Flumendosa - Campidano system

The stochastic methodologies illustrated have been applied to a real case of study, which refers to the south Sardinia water supply system. It is also known as *Tirso - Flumendosa - Campidano* water system.

During the last decades, this area has been deeply analyzed, it has a strategical importance in the Sardinia water resource plan and, for this reason, a large number of data and information are available. In this area, the Water System Authority needs to build an *integrated water system*, interconnecting among them different areas, located in the region and characterized by different water resource availability. Moreover, balancing this water network, the Authority will be able to move resource from areas with a higher water resource availability to prone zones affected by frequent deficit occurrences. These connections are mainly organized allocating some emergency and costly water transfer by pump stations, whose need information regarding the activation threshold levels. Therefore, for this reasons this water system has been chosen as case study testing the effectiveness of the optimization approaches previously described.

In *Sechi et al.* (2012a) it has been recognized that in each year the energy average costs supported for the wholesale water-system management is around 4 million of euros per year. Therefore, considering the high incidence of these energy costs, the Sardinian water system's authority needs an optimization tool in order to minimize the energy consumptions, particularly relating to pumping station activation rules, and, at the same time, to provide water demands fulfilment minimizing water deficit occurrences.

Table 1: Main Rivers of Sardinian Region

River	Length [km]	Basin [km^2]
Tirso	153.6	3365.78
Flumendosa	147.82	1841.77
Fluminimannu	95.77	1779.4
Cedrino	77.18	1075.9
Taloro	67.71	495.02
Coghinas	64.4	2551.61
Liscia	51.83	570.74
Temo	47.71	839.51

Sardinian island is located in the middle of the Mediterranean Sea and is characterized by an area of 24.000 km^2 and a population of 1.658.000 citizens (*ISTAT*, 2016). The hydrological regime of its rivers is typical of the Mediterranean basins of limited dimensions and low potentiality aquifers. Even the main rivers have not a perennial stream and are reported in the Table 1.

There are also smaller rivers, which have mainly a torrential behavior.

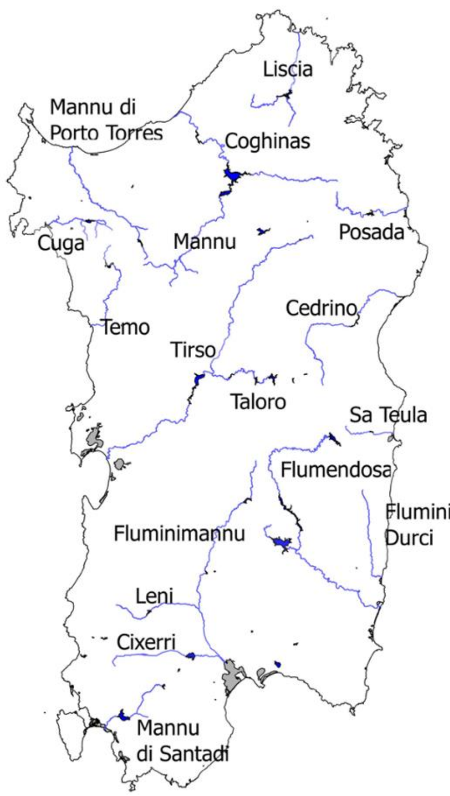


Figure 8: Main Rivers of Sardinia

By climate regime, in late autumn heavy precipitation can occur, plentiful floods and water overflow phenomena characterize flows. Shortage periods are frequent in the summer seasons and early autumn. The drought events refer almost to the total absence of rainfall that could affect for many consecutive months. In order to avoid the drought risk, the water resource authority manages the whole system through a stream-flow regulation by reservoirs, safeguarding water supply to the users considering related priority.

The main lakes, reported in the Table 2, are artificial and have a strategic role in the water resource management.

Table 2: Main Lakes of Sardinia

Lake	Volumes [10^6 m^3]	Area [km^2]
Omodeo	793	29.4
Mulargia	347	12.4
Flumendosa	316	8.7
Coghinas a Muzzone	297	20.1
Cedrino	117	7.8
Liscia	108	5.7

The Sardinian water supply system is then characterized by a strictly interconnected and multi-sectorial conveyance system by storing surface water in artificial reservoirs. Only local-needs and demands are satisfied through groundwater sources.

Therefore, reservoirs may have several aims, such as:

- Water outflows regulation to satisfy demands;
- Flood events lamination;
- Hydroelectric energy production.

Considering current planning activity by Regional Sardinian Authority (RAS), the region is divided in seven hydrographical zones as shown in the Figure 9.

In compliance with the Regional Law n. 19 of the 2006 (*RAS, 2006c*), the multi-purpose system management has been entrusted to *Ente Acque della Sardegna* (ENAS), which supplies the wholesale region. In planning acts, three principal macro demands are considered: civil, irrigational and industrial; moreover, the same restrictions are made to water retention by dams considering environmental flow requirements.

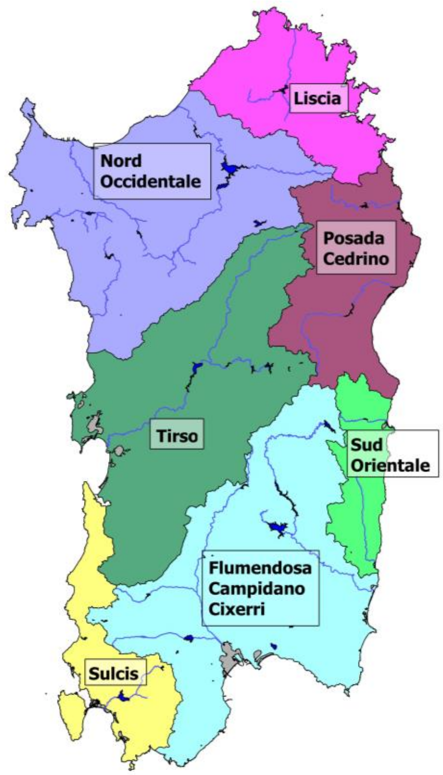


Figure 9: Sardinian Hydrographical Zones

The hydrographical zones are reported in Figure 9 and in Table 3 are given main information.

Table 3: Sardinian Hydrological Zones

System	Name	Basin [km²]
1	Sulcis	1646
2	Tirso	5372
3	Nord Occidentale	5402
4	Liscia	2253
5	Posada - Cedrino	2423

System	Name	Basin [km²]
6	Sud Orientale	1035
7	Flumendosa - Campidano -Cixerri	5960

3.1.1. Multi-purpose water supply system

The multi-purpose water supply system (*SIMR*) has been adopted after the enactment of the Regional Law n.19 of 6.12.2006 (*RAS, 2006c*). According this law, the SIMR represents "*a set of works for water supply that, individually or as parts of a complex system, have the possibility to feed, directly or indirectly, more territorial areas or more different categories of users, contributing to an equalization of quantities and costs of supply*". It deals with the management and the wholesale water transfer to the multi-user system: civil, irrigational, industrial and hydroelectrically production.

As previously set, the multi-purpose supply network is managed by ENAS; specific network are then managed by operators in different sector of uses. These main operators are reported in the Table 4.

Table 4: Water Service Operators

Operator	User
ENAS	Multi-users and (partially) irrigation and hydroelectric
ABBANOA S.p.A.	Civil sector
9 Consorzi di Bonifica	Irrigation sector
8 CIP	Industrial sector
ENEL S.p.A.	Hydroelectric energy production

In recent years, thanks to the SIMR definition, by the Regional Law n.19 of 6.12.2006, a rationalization in the water resource planning and management has been allowed. Concerning the amount of resource delivered to downstream users, the Table 5 reports the volumes supplied form SIMR for the year 2009.

Table 5: SIRM Delivered Volumes in 2009

River	Water Demand [$10^6 m^3$]	Percentage [%]
Civil	220	38%
Irrigational	326	57%
Industrial	31	5%
Total	577	100%

According to Regional Law n. 29/1997 (*RAS, 2006b*), Sardinia Region Authority establishes also an accurate management of the *civil sector water supply*, in fulfilling the Galli Law, National Law n. 36/1994 (*Law, 1994*). In the region has been defined a unique ATO (*Ambito Territoriale Ottimale - Optimal Territorial Field*) which management, starting from 2006, was given to Abbanoa S.p.A. enterprise. Therefore, downstream the SIMR, the regional ATO has been organized in eight districts, managing the drinking distribution system in the residential areas.

The *industrial sector* aims to manage, usually downline of the SIMR, the water supply to the industrial centers. Industrial centers are organized by CIP (*Consorzio Industriale Provinciale – Provincial Industrial Consortium*), which manage also the service of wastewater collection and treatment. Industrial demand partially do not use the resource supplied by SIMR because of local sources, as wells and water springs, or treatment plants for water reuse and in some cases small desalination plants.

In Sardinian island, the *irrigational demand* represents the main consumption of water resource. Irrigation areas are managed from nine land reclamation syndicates (Consorzi di Bonifica). The syndicates, as well as guarantee water supply to associated users, aim to manage and to maintain of irrigational water distribution system. The SIMR network represents the main source of water supplied for irrigation areas; only in few cases waters are retrieved by local sources.

3.1.2. The South Sardinia water supply system

The so-called *Tirso – Flumendosa – Campidano* system is the principal multi-purpose water systems in Sardina. This system consists of a union of four different multi-user water systems interlinked among them:

- Tirso
- South Oriental
- Sulcis
- Flumendosa – Campidano – Cixerri

interconnected among them through natural or artificial links (watercourses, diversions, pipes, drains, canals, etc.).

The principal resources of the system are represented by stored water in the Flumendosa and Mulargia river reservoirs. Moreover, a relevant contribution could be transferred from Tirso river, stored in lake Omodeo, which is the biggest in Sardinia in terms of dimensions. These resources are regulated by several infrastructures and, consecutively, delivered to demand centers through a net of pipelines, tunnels, canals and pump stations.

3.2. System's analysis

The described water supply system has been schematized using the graph theory (*Ahuja et al., 1993; Diestel*, 2005), building a complex directed connected graph composed by nodes and arcs as shown in the pattern adopted by SIMR and drafted in the following Figure 10.

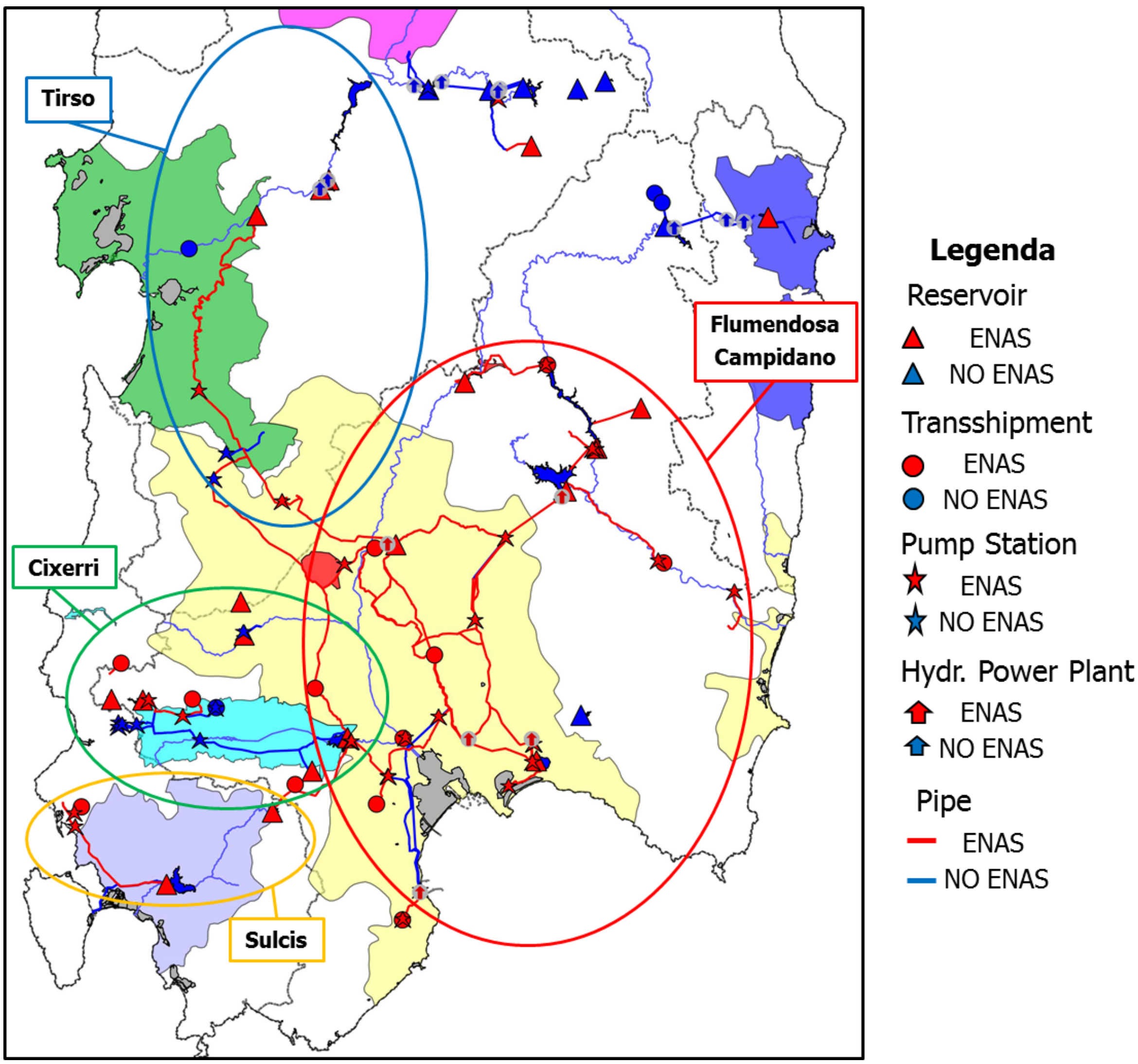


Figure 10: ***Tirso – Flumendosa - Campidano*** **water supply system - SIMR**

This network structure is very complex and describes all the nodes and connections among them, taking into account each different feature of the water system. Analyzing this particular optimization problem, the network structure is excessively detailed and, therefore, some of its features can be aggregated in order to build an easier configuration. The sketch reported in Figure 11 simplifies the real configuration complexity of the system SIMR in Figure 10, but it can be used for analyzing flow configurations in the multi-period network.

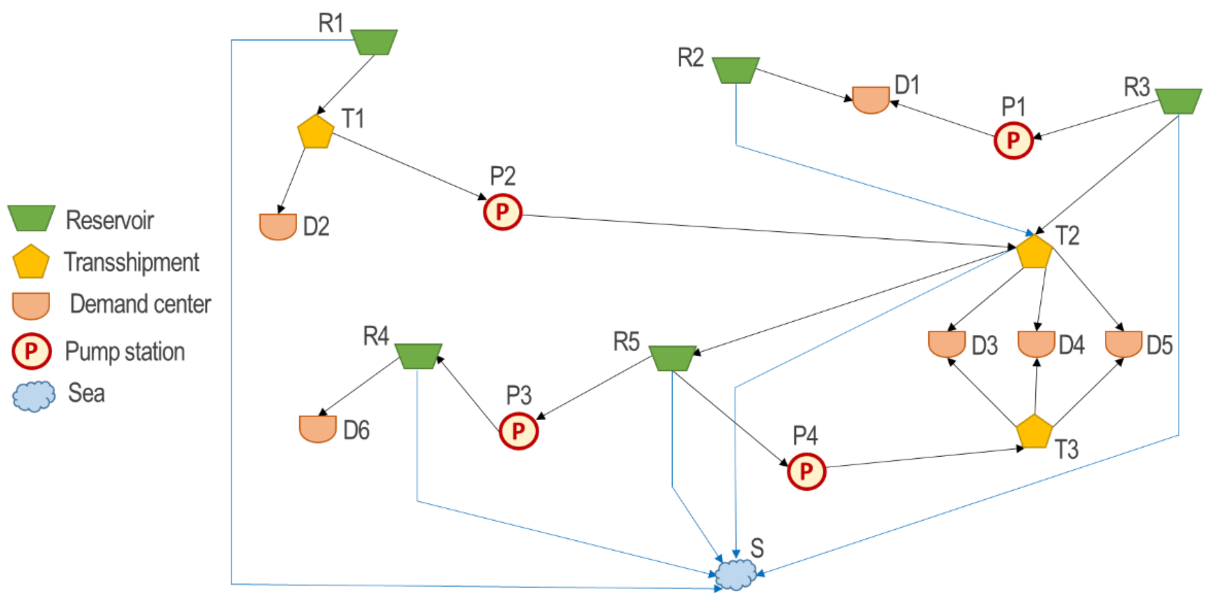


Figure 11: Southern Sardinia Water Supply System

In the next paragraphs will be given a brief qualitatively and quantitatively description of the main system's elements, in order to facilitate a comprehension of its operation.

3.2.1. Reservoirs

Water sources are mainly given by five artificial reservoirs. Their main features are reported in Table 6.

Table 6: Reservoirs Features

Code	Name	Capacity [10^6 m^3]
R1	Diga Cantioniera	450
R2	Is Barrocus	12.24
R3	Flumendosa + Mulargia	623
R4	Bau Pressiu	8.25
R5	Cixerri (Genna is Abis)	24

These reservoirs are represented in the sketch by system's storage nodes. The evaluation of potentiality in the water supply arises from historically hydrological inflows evaluated from 1922 to 1992 considering the values reported in the Sardinia Region Water Plan (*RAS, 2006a*).

3.2.2. Demand centers

Water demands have been grouped into six centers, according to three different users: civil, irrigation and industrial. An annual volume of demand and a deficit cost are associated to each demand center. Deficit costs quantify the damages supported by the user in the case of shortages occurrences. These values are given in Table 7.

Table 7: Demand Centres Requirements and Deficit Costs

Code	Demand center	Demand [10^6 m^3/year]	Planned Deficit Costs [€/10^6 m^3]	Unplanned Deficit Costs [€/10^6 m^3]
D1	Civil Sarcidano	11	25	250
D2	Irrigation Oristano	118	6	60

Code	Demand center	Demand [10^6 m^3/year]	Planned Deficit Costs [€/10^6 m^3]	Unplanned Deficit Costs [€/10^6 m^3]
D3	Industrial Campidano	17	256	2560
D4	Irrigation Campidano	81	6	60
D5	Civil Campidano	90	25	250
D6	Civil Bau Pressiu	9	25	250

Deficit costs have been evaluated starting from the water annual rates for unit of volume applied by each stakeholder (*Sechi et al., 2012b*).

The monthly distribution of the irrigation demand will be done using evaluated monthly coefficients, assuming equal to one the total annual requirement.

Table 8: Monthly Breakdown Coefficients – Water Demands

Monthly Distribution		
Month	Irrigation Demand	Civil and Industrial Demand
October	0.041	0.083
November	0.021	0.083
December	0.015	0.083
January	0.016	0.083
February	0.016	0.083
March	0.029	0.083
April	0.062	0.083
May	0.109	0.083
June	0.186	0.083

Month	Irrigation Demand	Civil and Industrial Demand
July	0.248	0.083
August	0.183	0.083
September	0.075	0.083

3.2.3. Transshipment limits

Conveyance arcs of the system are subject to upper bound limits in the capacity of water that monthly can be transferred; upper bounds refer to conveyance works characteristics.

Table 9: Water Transportation Limits

Connected Nodes	Name	Capacity [10^6 m^3]
R3 -T2	Galleria Uvini - Sarais	80
R1 - T1	Canale Sinistra - Tirso	30
T2 - R5	Canale Est - Ovest	30

3.2.4. Junction nodes

Junction nodes allow the resource to pass through without any consumption. They cannot store resource but only transfer it to the downstream nodes and demand centers. In the system sketch are located three nodes with these features and they are named T1, T2, T3.

3.2.5. Pump stations

Pump stations allow demand centers to be supplied with an increased economic burden, namely incurring pumping costs in addition to the ordinary management costs. Capacities and pumping costs referred to each pump station are given in Table 10.

Table 10: Pump Stations Features

Code	Name	Capacity [10^6 m^3/month]	Pumping Cost [€/m^3]
P1	Sarcidano	0.7	0.193
P2	Tirso - Campidano	5.2	0.2056
P3	Cixerri - Bau Pressiu	2.1	0.218
P4	Cixerri - Campidano	10.4	0.078

The pattern in the Figure 11 simplifies the real configuration of the system, indeed, main pump stations of the SIMR system have been inserted.

3.2.6. Pumping schedules activation thresholds

Modelling the system, threshold levels for pumps activation refer to the stored volume in reservoirs that supply the downstream demand nodes. Each pump threshold could be related, in some cases, to a single reservoir (such as P1 and P3) and, in the other cases, to two reservoirs (P2 and P4); in these cases the threshold value has been evaluated as the sum of the stored water in both reservoirs. These functional dependencies are reported in the Table 11, where 1 means dependence, while 0 independence.

Table 11: Pump Stations' Activation Dependences

Reservoir → / Pump station ↓	R1	R2	R3	R4	R5
P1	0	1	0	0	0
P2	0	1	1	0	0
P3	0	0	0	1	0
P4	0	1	1	0	0

For example, the second row shows that the activation of the pump station P2 depends on the sum of stored volumes in reservoirs R2 and R3.

4. Hydrological Inflows

4.1. Historical inflows and reference scenarios

As explained in the previous chapters, in a problem of water resource management the main uncertainties are closely related to future water inflows, due to hydrologic variability and to water demand behavior, connected to users' requirements. In this kind of problems, for planning requirements, we need to consider a large time horizon, normally model extensions of several decades are considered, but time steps could be settled by considering one-month extension.

In order to provide an efficient database, the hydrological inputs have been extracted from the Regional Water Plans. Particularly, the *PSURI* Plan (*Piano Stralcio Direttore per l'Utilizzazione delle Risorse Idriche - RAS*, 2006a) has been considered. This planning document provides the reference series of water inflows to reservoirs for 53 hydrological years (from October to September). These series were based on observed data in the main hydrographical stations of Sardinia. Streamflow values were evaluated considering the historical values, originally reported in the SISS Plan (*Studio Idrologia Superficiale della Sardegna - SISS*, 1998; *RAS, 2006a*) and upgrading them considering expected reductions in order to take into account the recent climatic trends. These climatic trends were evaluated on the basis of the dramatic drought periods and water-shortage occurrences recorded in the island, particularly in the period from 1986 to 2002.

The main statistical indices of considered reference-series of water inflows to reservoir, as reported in PSURI, are reported in the Table 12.

Table 12: Statistical Indices of Historical Series

Statistical Indices		Reservoirs				
		R1	R2	R3	R4	R5
Average -Volumes [10^6 m^3]	$\bar{x}$	148.3	12.3	187.1	3.0	32.9
Average - Flow rates [m^3/s]	$\bar{x}_f$	4.7	0.4	5.9	0.1	1.0
Average - Inflows [mm]	$\bar{x}_i$	70.8	129.6	201.2	102.7	66.0
Median	m_x	117.1	10.9	162.2	2.2	28.5
Variance	σ^2_x	9079.7	64.3	9141.6	6.1	441.6
Standard Deviation	σ_x	95.29	8.02	95.61	2.47	21.02
Coefficient of Variation	cv_x	0.64	0.65	0.51	0.83	0.64
Skewness	g_x	1.14	0.70	0.76	1.51	1.06

4.1.1. Scenario tree definition

In a first modelling step, the 53 years' PSURI reference-series of hydrological inputs have been used defining a suitable database to be used inside the scenario analysis approach. The optimization problem has been solved using a two stage stochastic programming model.

Therefore, the 53 years' series have been organized considering 4 hydrological scenarios, composed with a common root of 10 years and the following data diversified in 10 year scenarios. Hence, each scenario has been characterized by 20 years, 240 monthly periods, with the branching time located at the 120-th period.

- *First scenario (g1)*: this scenario is composed with a common root of 10 years and the second series decade;
- *Second scenario (g2)*: this scenario is composed with a common root of 10 years and the third series decade;
- *Third scenario (g3)*: this scenario is composed with a common root of 10 years and the fourth series decade;
- *Fourth scenario (g4)*: this scenario is composed with a common root of 10 years and the fifth series decade.

The root of the scenario tree could be considered as deterministic part of the model and has been composed by the first decade of the reference-series and it will be kept the same for all considered scenarios. Each scenario has its hydrological peculiarity and criticalities, which will influence the results and consequently the global future system management.

The structure of the scenario-tree is reported in the Figure 12.

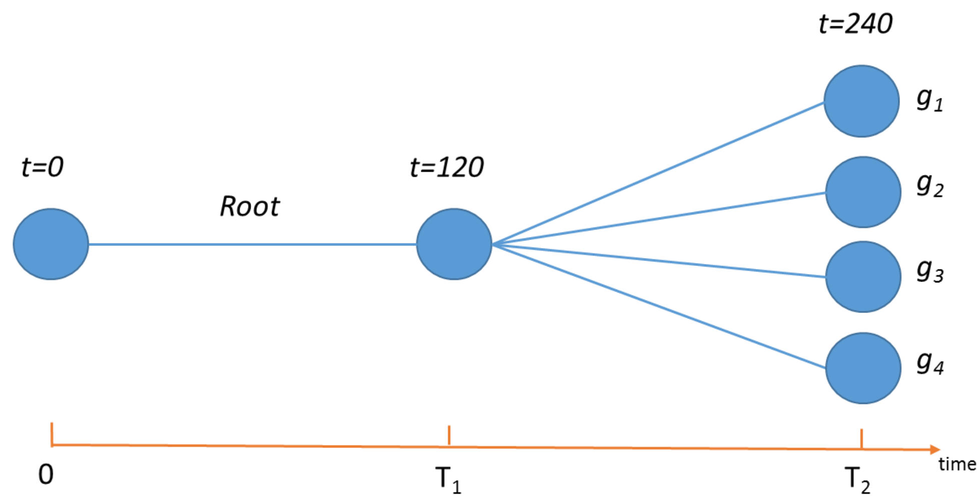


Figure 12: Scenario-Tree using Historical Scenarios

Therefore, the monthly hydrological inflows, which characterize the five reservoirs in the water system, have been organized following the previously defined aggregation rules, in order to build correctly the scenario-tree.

In the following Tables are reported the monthly hydrological inflow values for the reservoir R1 along the scenarios; this same procedure has been developed considering all the other reservoirs in the system.

Herein, the total and the average annual values have been also evaluated. Furthermore, the hydrological inputs in each scenario, considering each system's reservoir, are reported in Figure 13-17. These Figures highlight the value differences in hydrologic scenarios: in the tree-structure, all scenarios share a common root until the branching-time located at 120-th period. Up to this time-step a branching phase has been done, developing differences from all four scenarios independently.

Table 13: Hydrological Inflows to R1 – Historical Scenario g_1

HYDROLOGICAL INFLOWS R1 - DIGA CANTONIERA SCENARIO (g1) - [10^6 m^3]													
	OCT	*NOV*	*DEC*	*JAN*	*FEB*	*MAR*	*APR*	*MAY*	*JUN*	*JUL*	*AUG*	*SEP*	*TOT*
22-23	0.11	3.55	6.05	16.5	47.39	17.92	24.0	4.51	0.87	0.08	0.04	2.73	123.7
23-24	3.06	10.1	74.02	37.21	86.83	50.74	18.1	4.82	0.81	1.33	0.39	0.69	288.0
24-25	5.32	1.18	23.69	3.23	7.79	17.14	16.4	7.69	1.84	0.96	0.8	0.94	87
25-26	2.65	13.1	19.44	22.46	10.42	4.69	7.0	8.29	1.07	1.5	1.06	0.86	92.52
26-27	1.14	2.41	7.24	37.74	20.23	7.28	3.2	0.63	0.42	0.01	0.03	0.42	80.75
27-28	3.92	3.6	63.04	34.53	14.34	25.41	16.6	6.59	0.39	0.11	0.03	0.32	168.9
28-29	3.22	5.6	17.84	35.64	21.66	8.11	2.86	1.61	1.25	0.03	0.01	0.56	98.39
29-30	0.68	32.4	10.3	45.96	75.17	29.97	57.0	11.09	4.16	0.88	0.44	0.33	268.3
30-31	0.6	0.76	10.24	11.21	21.43	20.12	5.95	4.34	1.15	0.32	0.12	0.04	76.28
31-32	0.32	6.27	10.33	6.95	16.58	12.63	3.1	1.83	0.73	0.11	0.04	0.22	59.11
32-33	3.15	43.6	64.78	35.72	60.6	56.2	74.3	10.81	1.27	1.08	0.73	1.23	353.5
33-34	1.39	2.05	8.02	6.18	12.16	7.88	1.59	0.52	0.21	0.09	1.58	0.33	42
34-35	0.89	8.7	55.39	44.9	22.35	92.2	5.55	7.85	2.61	2.29	1.12	0.64	244.5
35-36	2.3	9.35	38.13	23.97	26.35	47.01	19.2	22.9	6.66	1.39	1.25	1.37	199.9
36-37	1.88	2.41	3.32	3.83	21.94	35.96	9.39	4.83	2.22	1.67	0.81	1.87	90.13
37-38	1.85	4.84	20.33	12.72	14.19	3.2	3.03	4.42	1.6	0.94	0.22	0.8	68.14
38-39	1.27	3.47	15.89	25.22	17.19	24.28	12.42	10	2.12	0.1	1.16	3.93	117.05
39-40	4.61	3.48	23.15	77.04	34.23	4.38	2.52	4.75	1.04	0.45	0.01	0.01	155.67
40-41	17.95	41.88	48.15	89.86	146.27	33.34	29.79	7.96	3.82	1.84	0.61	0.37	421.84
41-42	4.21	1.95	9.99	17.89	69.22	26.76	9.18	6.6	2.5	0.12	1.26	1.56	151.24
ANNUAL AVERAGE													***159.35***

Table 14: Hydrological Inflows to R1 – Historical Scenario g2

HYDROLOGICAL INFLOWS R1 - DIGA CANTONIERA SCENARIO (g2) - [10^6 m^3]													
	OCT	*NOV*	*DEC*	*JAN*	*FEB*	*MAR*	*APR*	*MAY*	*JUN*	*JUL*	*AUG*	*SEP*	*TOT*
22-23	0.11	3.55	6.05	16.5	47.39	17.92	23.98	4.51	0.87	0.08	0.04	2.73	123.73
23-24	3.06	10.06	74.02	37.21	86.83	50.74	18.07	4.82	0.81	1.33	0.39	0.69	288.03
24-25	5.32	1.18	23.69	3.23	7.79	17.14	16.42	7.69	1.84	0.96	0.8	0.94	87
25-26	2.65	13.08	19.44	22.46	10.42	4.69	7	8.29	1.07	1.5	1.06	0.86	92.52
26-27	1.14	2.41	7.24	37.74	20.23	7.28	3.2	0.63	0.42	0.01	0.03	0.42	80.75
27-28	3.92	3.6	63.04	34.53	14.34	25.41	16.58	6.59	0.39	0.11	0.03	0.32	168.86
28-29	3.22	5.6	17.84	35.64	21.66	8.11	2.86	1.61	1.25	0.03	0.01	0.56	98.39
29-30	0.68	32.38	10.3	45.96	75.17	29.97	56.97	11.09	4.16	0.88	0.44	0.33	268.33
30-31	0.6	0.76	10.24	11.21	21.43	20.12	5.95	4.34	1.15	0.32	0.12	0.04	76.28
31-32	0.32	6.27	10.33	6.95	16.58	12.63	3.1	1.83	0.73	0.11	0.04	0.22	59.11
42-43	0.49	1.14	1.52	19.02	2.92	15.87	2.54	1.21	0.97	0.02	0.1	0.36	46.16
43-44	3.15	10.12	29.23	6.51	15.1	27.4	4.7	2.9	1.02	0.33	1.15	0.74	102.35
44-45	2	1.84	4.57	38.4	20.09	5.23	2.18	0.6	0.06	0.02	0.06	0.04	75.09
45-46	0.4	0.44	3.93	3.86	2.57	3.98	1.72	2.15	0.39	0.04	0.01	0.02	19.51
46-47	14.07	56.6	165.43	47.87	66.85	30.39	10.79	3.9	2.14	3.62	1.93	2.0	405.59
47-48	2.82	2.71	15.06	75.05	18.84	14.55	5.52	6.95	0.88	1.94	0.21	1.4	145.93
48-49	1.63	2.76	4.7	9.02	5.51	4.02	0.72	5.75	0.52	0.28	0	0.06	34.97
49-50	0.15	13.44	22.63	9.04	7.48	10.01	25.32	5.28	1.51	0.39	0.38	1.62	97.25
50-51	1.91	2.66	18.07	17.69	26.66	45.06	5.11	14.5	1.53	0.48	0.03	0.35	134.05
51-52	52.05	23.94	11.97	27.12	19.27	3.47	4.54	3.19	1.74	0.86	0.01	1.42	149.58
ANNUAL AVERAGE													*127.67*

Table 15: Hydrological Inflows to R1 – Historical Scenario g_3

HYDROLOGICAL INFLOWS R1 - DIGA CANTONIERA SCENARIO (g3) - [10^6 m^3]													
	OCT	*NOV*	*DEC*	*JAN*	*FEB*	*MAR*	*APR*	*MAY*	*JUN*	*JUL*	*AUG*	*SEP*	*TOT*
22-23	0.11	3.55	6.05	16.5	47.39	17.92	23.98	4.51	0.87	0.08	0.04	2.73	123.73
23-24	3.06	10.06	74.02	37.21	86.83	50.74	18.07	4.82	0.81	1.33	0.39	0.69	288.03
24-25	5.32	1.18	23.69	3.23	7.79	17.14	16.42	7.69	1.84	0.96	0.8	0.94	87
25-26	2.65	13.08	19.44	22.46	10.42	4.69	7.0	8.29	1.07	1.5	1.06	0.86	92.52
26-27	1.14	2.41	7.24	37.74	20.23	7.28	3.2	0.63	0.42	0.01	0.03	0.42	80.75
27-28	3.92	3.6	63.04	34.53	14.34	25.41	16.58	6.59	0.39	0.11	0.03	0.32	168.86
28-29	3.22	5.6	17.84	35.64	21.66	8.11	2.86	1.61	1.25	0.03	0.01	0.56	98.39
29-30	0.68	32.38	10.3	45.96	75.17	29.97	56.97	11.09	4.16	0.88	0.44	0.33	268.33
30-31	0.6	0.76	10.24	11.21	21.43	20.12	5.95	4.34	1.15	0.32	0.12	0.04	76.28
31-32	0.32	6.27	10.33	6.95	16.58	12.63	3.1	1.83	0.73	0.11	0.04	0.22	59.11
52-53	3.39	9.13	50.25	37.65	46.2	20.68	7.4	8.57	16.99	3	0.76	0.54	204.56
53-54	3.35	3.73	4.59	5.12	12.84	6.46	4.55	3.68	0.37	0.33	0.14	0.01	45.17
54-55	0.01	0.2	1.17	6.91	29.58	22.92	5.25	1.18	0.84	0.03	0.05	0.16	68.3
55-56	0.79	3.2	14.03	8.13	32.72	18.94	17.2	11.81	4.23	0.55	0.22	0.19	112.01
56-57	0.15	2.51	2.52	8.6	5.16	2.64	2.85	3.9	1.58	0.29	0.01	0.01	30.22
57-58	0.84	8.68	48.96	52.94	12.68	30.08	49.85	9.58	2.23	0.14	0.16	0.17	216.31
58-59	3.07	22.08	61.21	27.8	18.78	14.31	13.63	28.84	8.25	0.68	1.14	3.62	203.41
59-60	24.41	29.57	76.12	52.31	28.46	58.1	37.65	13.57	3.95	1.47	0.38	0.17	326.16
60-61	1.18	9.28	49.79	66.78	22.85	3.02	16.84	7.34	1.67	0.71	0	0.41	179.87
61-62	5.62	22.64	15.97	4.08	7.63	20.01	7.18	1.7	0.15	1.09	0.18	0.58	86.83
ANNUAL AVERAGE													***140.79***

Table 16: Hydrological Inflows to R1 – Historical Scenario g_4

HYDROLOGICAL INFLOWS R1 - DIGA CANTONIERA SCENARIO (g4) - [10^6 m^3]													
	OCT	*NOV*	*DEC*	*JAN*	*FEB*	*MAR*	*APR*	*MAY*	*JUN*	*JUL*	*AUG*	*SEP*	*TOT*
22-23	0.11	3.55	6.05	16.5	47.39	17.92	23.98	4.51	0.87	0.08	0.04	2.73	123.73
23-24	3.06	10.06	74.02	37.21	86.83	50.74	18.07	4.82	0.81	1.33	0.39	0.69	288.03
24-25	5.32	1.18	23.69	3.23	7.79	17.14	16.42	7.69	1.84	0.96	0.8	0.94	87.0
25-26	2.65	13.08	19.44	22.46	10.42	4.69	7	8.29	1.07	1.5	1.06	0.86	92.52
26-27	1.14	2.41	7.24	37.74	20.23	7.28	3.2	0.63	0.42	0.01	0.03	0.42	80.75
27-28	3.92	3.6	63.04	34.53	14.34	25.41	16.58	6.59	0.39	0.11	0.03	0.32	168.86
28-29	3.22	5.6	17.84	35.64	21.66	8.11	2.86	1.61	1.25	0.03	0.01	0.56	98.39
29-30	0.68	32.38	10.3	45.96	75.17	29.97	56.97	11.09	4.16	0.88	0.44	0.33	268.33
30-31	0.6	0.76	10.24	11.21	21.43	20.12	5.95	4.34	1.15	0.32	0.12	0.04	76.28
31-32	0.32	6.27	10.33	6.95	16.58	12.63	3.1	1.83	0.73	0.11	0.04	0.22	59.11
62-63	2.53	23.86	25.7	62.79	134.61	42.04	19.19	10.01	1.3	3.19	1.68	3.77	330.67
63-64	0.11	1.95	34.74	13.28	18.94	20.42	10.58	2.68	2.32	1.47	1.24	0.85	108.58
64-65	9.78	28.21	59.16	52.64	26.23	36.71	16.53	3.23	2.31	2.11	1.93	1.78	240.62
65-66	1.48	5.64	4.91	5.52	27.27	10.57	5.74	3.08	1.18	0.06	0.02	1.86	67.33
66-67	26.56	32.48	59	23.82	20.37	3.81	10.29	1.31	1.38	0.26	0.5	0.07	179.85
67-68	0.06	1.78	28.06	34.14	25.45	10.27	1.33	1	0.51	0.08	0.71	0.01	103.4
68-69	2.18	6.7	66.41	22	37.7	55.43	12.99	1.23	0.57	2	0.02	2.58	209.81
69-70	1.39	4.2	48.98	47.62	24.43	24.85	5.2	1.2	0.02	0.36	0.4	0.01	158.66
70-71	4.54	5.87	12.01	21.62	22.3	31.66	16.07	6.53	0.48	0.77	0	0.93	122.78
71-72	0.08	11.73	16.47	31.75	75.19	26.51	18.99	12.92	2.04	2.39	0.24	0.9	199.21
ANNUAL AVERAGE													*153.20*

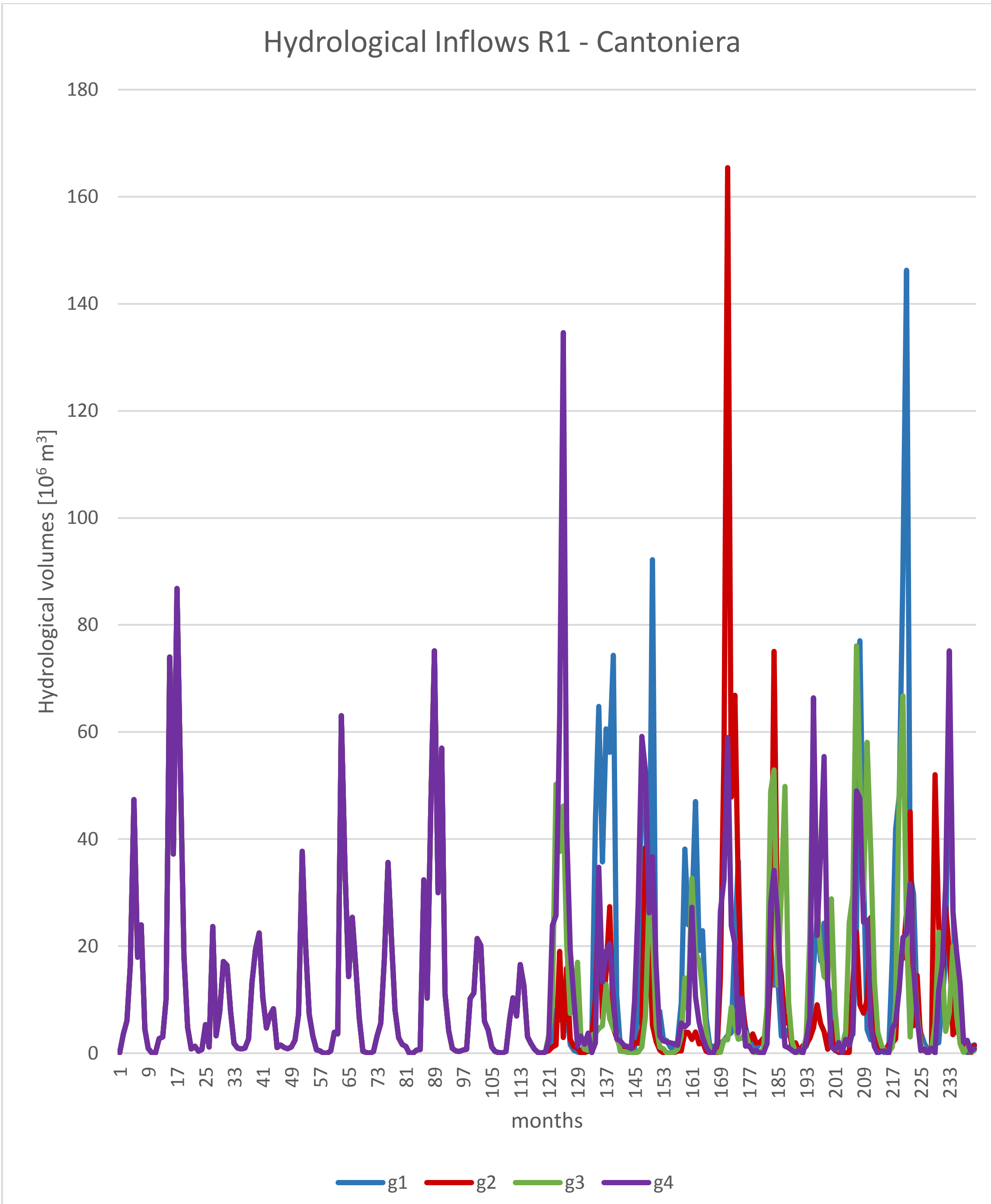


Figure 13: Hydrological Inflows to Reservoir R1 - Cantoniera

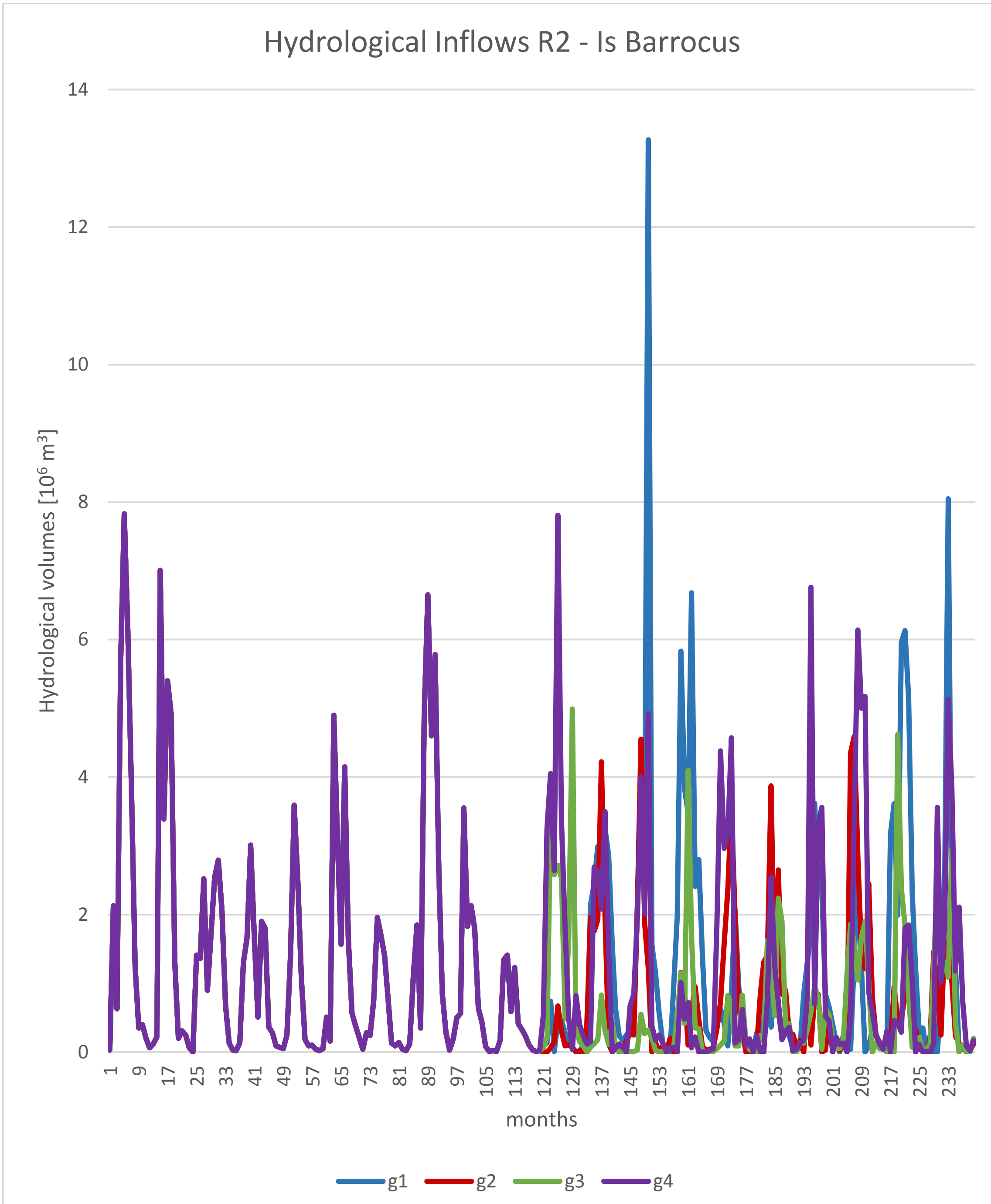


Figure 14: Hydrological Inflows to Reservoir R2 - Is Barrocus

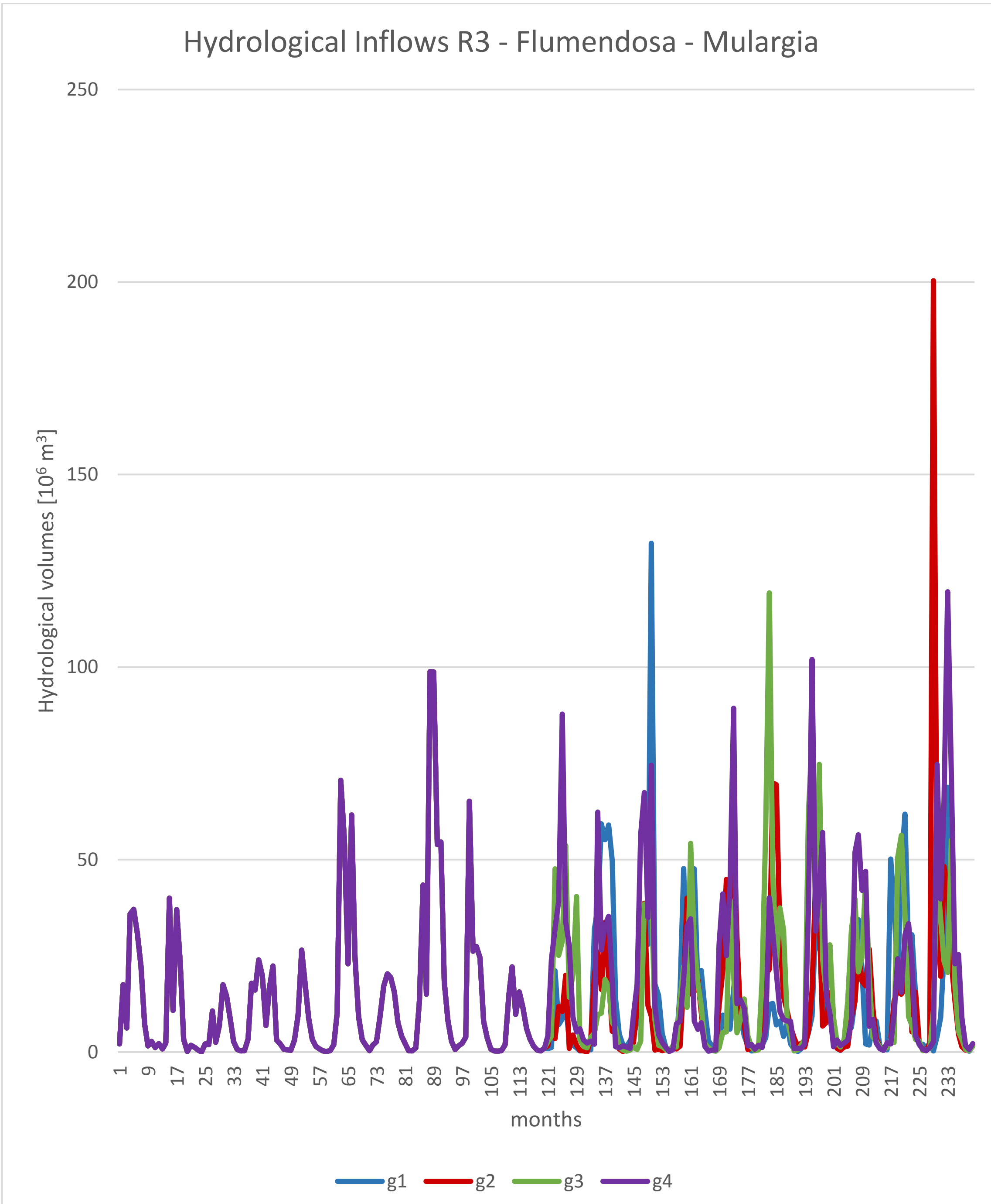


Figure 15: Hydrological Inflows to Reservoir R3 - Flumendosa + Mulargia

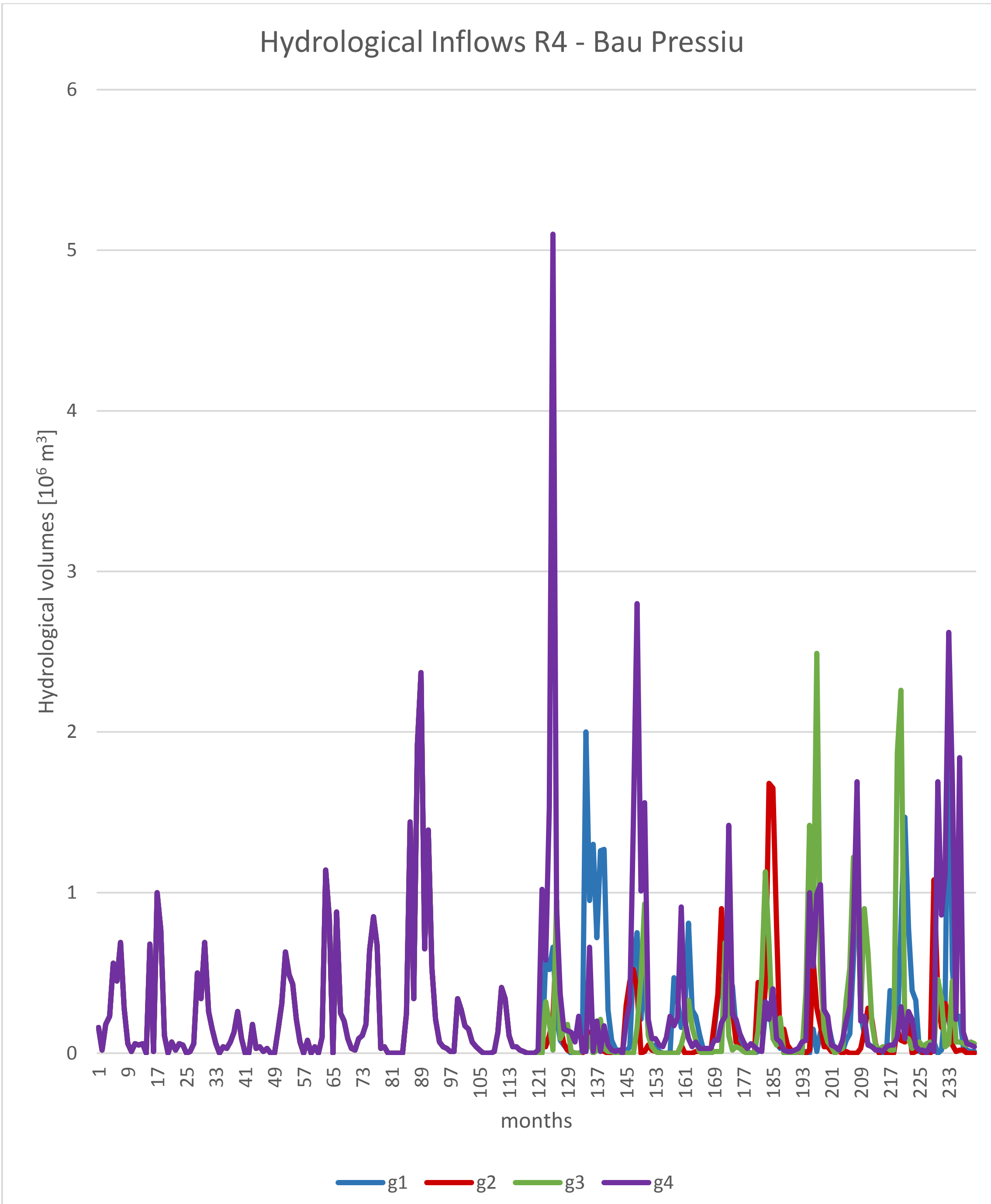


Figure 16: Hydrological Inflows to Reservoir R4 - Cixerri "Genna is Abis"

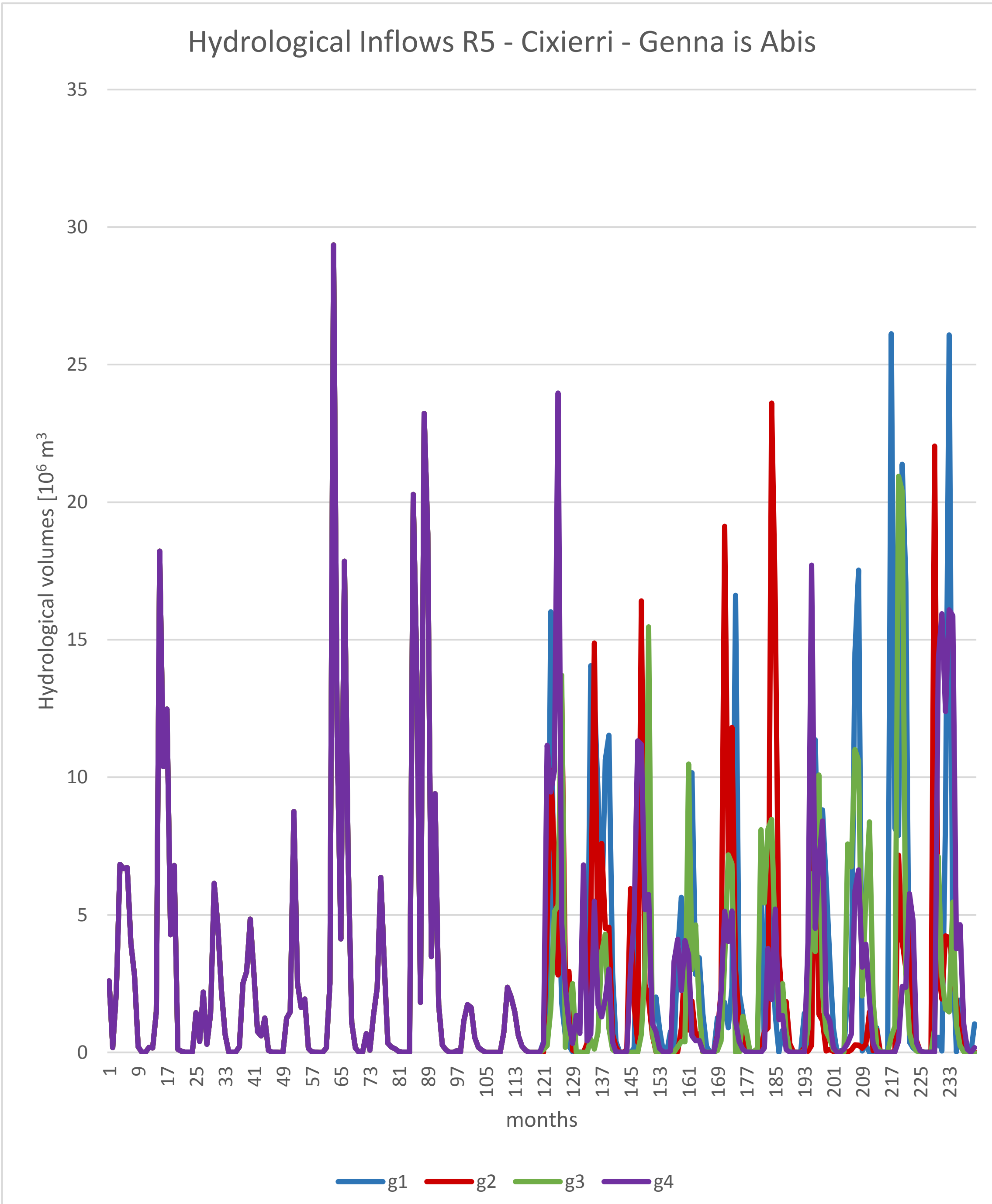


Figure 17: Hydrological Inflows to Reservoir R5 - Bau Pressiu

4.2. Synthetic hydrologic scenarios

Taking into account climate variation, the optimization processes should be extended to a more significant number of synthetic scenarios extending the only four branches tree that has been considered using the historically based reference scenarios in previous paragraph. Following the procedure considered in the PSURI regional plan (*RAS, 2006a*), lognormal probability distributions were used in order to fit the annual inputs to reservoirs. Subsequently, a Monte Carlo generation approach (*Box et al., 1994*) was developed in order to generate inputs to reservoirs. On the basis of a preliminary cross-correlation analysis, reservoir R1 (Cantoniera Tirso) was assumed as the reference station in the generation process and the usual goodness of fit, through a χ^2-test, has been done comparing the expected and observed frequencies. The average (μ_y), variance (σ_y^2), standard deviation (σ_y) and skewness (g_y) of annual logarithmic values considered in the generation process for reservoir R1 are given in Table 17.

Table 17: Statistical Indices in Logarithmic Scale - R1

Reservoirs R1 - Water Inputs - Logarithmic scale (y = ln x)		
Average	μ_y	4.79
Variance	σ^2_y	0.45
Standard Deviation	σ_y	0.67
Skewness	g_y	-0.33

The generation of synthetic inputs to reservoirs was extended to 10 series of the same length of 53 years of the reference database. Even if the number of synthetic series could potentially more extended, for the here considered modelling approach application this number has been considered sufficient. As usual, the generation model has been characterized by considering a deterministic and a stochastic component. The statistical indices of average, variance, standard deviation and skewness of the 10 synthetically generated series for reservoir R1 are given in Table 18.

Table 18: Statistical Indices of Generated Series - R1

Statistical Indices	Series 1	Series 2	Series 3	Series 4	Series 5	Series 6	Series 7	Series 8	Series 9	Series 10
μ_y	4,925	4,753	4,735	4,767	4,756	4,831	4,789	4,701	4,715	4,872
σ^2_y	0,478	0,362	0,487	0,343	0,398	0,402	0,543	0,335	0,402	0,317
σ_y	0,691	0,601	0,698	0,586	0,631	0,634	0,737	0,579	0,634	0,563
g_y	-0,310	0,092	0,243	-0,145	0,513	-0,31	0,557	0,212	-0,611	0,370

The *deterministic components* for other reservoirs R2, R3, R4, R5 were evaluated through a standard cross-correlation analysis by considering R1 as the reference station.

The *stochastic components* ε were generated through a random extraction of white noise following a normal distribution with an average equal to zero and a standard deviation calculated with the residuals.

$$\varepsilon \sim \mathrm{N}[0, \sigma_{yR}] \qquad (10)$$

Hence, the synthetic hydrology was settled by composing the new set of 50 scenarios equal in the length to the reference ones.

The mean and the variance of the synthetic hydrological annual series were tested using a classical Student's t-test for the mean and the Fisher test for the variance (*Box* *et al., 1994*). Table 19 highlights all statistical indices of the synthetic series of hydrological inflows to reservoirs.

Table 19: Statistical Indices of Synthetically Series

Statistical Indexes		Reservoirs				
		R_1	R_2	R_3	R_4	R_5
Average -Volumes [10^6 m^3]	μ_x	147.37	12.35	187.15	3.07	32.81
Median	m_x	118.22	11.13	178.99	2.58	29.42
Variance	σ^2_x	11345.34	61.53	10006.25	4.88	437.35
Standard Deviation	σ_x	106.55	7.85	100.01	2.21	20.91
Coefficient of Variation	cv_x	0.72	0.64	0.53	0.72	0.64

Finally, evaluating the monthly distribution of hydrological volumes in each reservoir, monthly breakdown coefficients c_{ij} were generated using observed historical values, as shown in following equation:

$$c_{ij} = \frac{Q_{ij}}{Q_i} \tag{11}$$

where:

i refers to the observed year;

j refers to the observed month;

Q_{ij} are the water volumes referring to the j-th month in the i-th year;

Q_i are the average annual volumes.

In each reservoir, the independence between the annual volumes Q_i and the coefficients c_{ij} has been verified. According to this, the monthly value generation procedure was randomly extracted by the observed values.

In the following will be described as the optimization approach has been applied by considering the synthetically generated hydrologic scenarios.

4.2.1. Building synthetically scenarios tree

The synthetically generated series are used implementing a new scenario optimization process. In order to do it, an extended scenario-tree has been built using 54 scenarios with a common root: 4 historical scenarios plus 50 synthetic scenarios.

As shown in the Figure 18, the problem has been developed with a two stages model, with a single branching time located in the 120-th period.

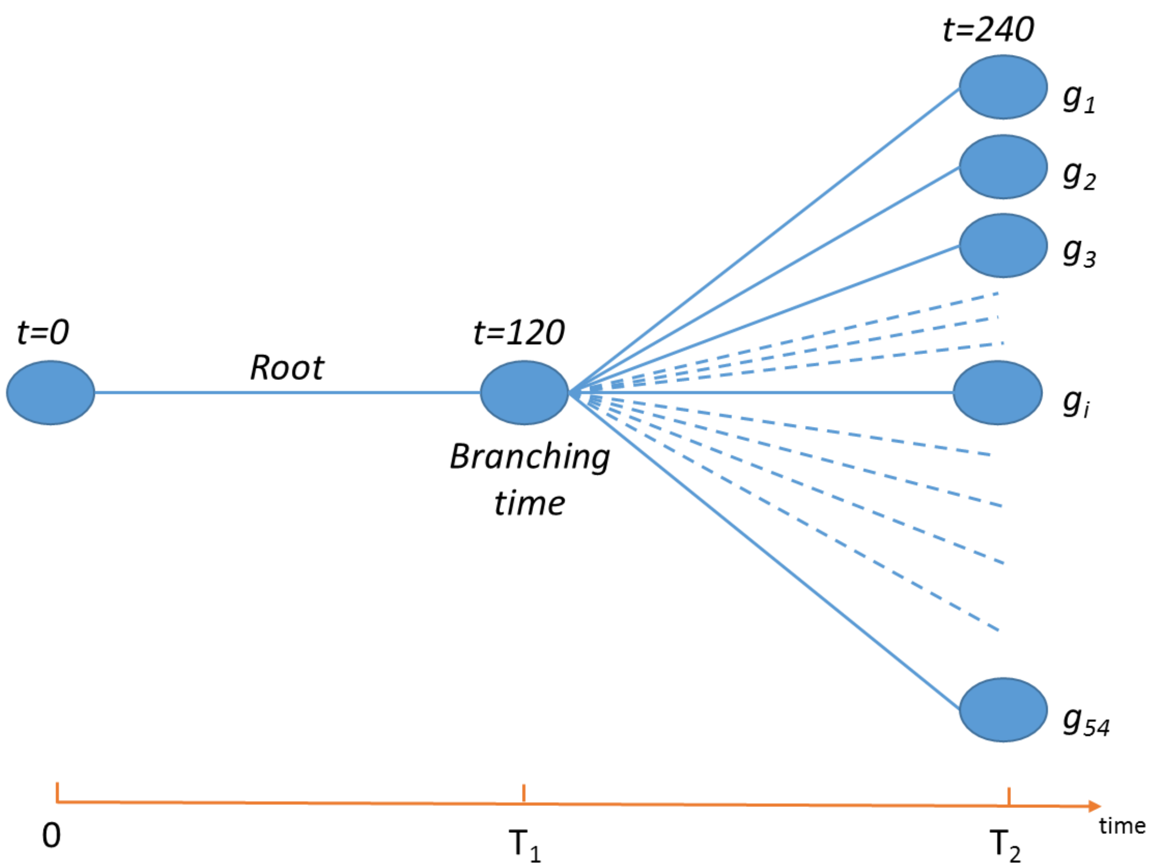


Figure 18: Scenario-Tree using Synthetic Scenarios

The same root of 10 years plus a different decade characterizes each path in the scenario tree. The total amount of periods along a single scenario is equal to 240, so there are twenty years of analysis with a monthly temporal step. This new scenario tree will be used as reference input for an extended analysis using the Scenario Analysis Approach.

4.2.2. Hydrological inputs for SQG methods

SQG methods are able to afford optimization problems characterized by a long time horizon, huge number of variables and, consequently, high level of uncertainty. Their processes solutions develop through a nonlinear variables parametrization, solving easier linear programming (LP) problems.

The suitable database for this SQG implementation is characterized by a long database as schematized in the Figure 19.

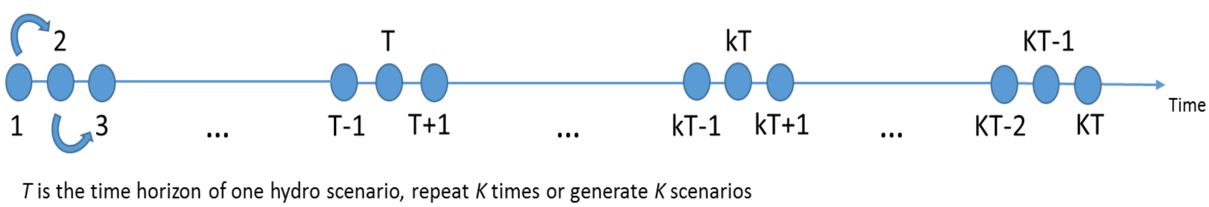


Figure 19: SQG Synthetic Inflows

The monthly inputs are organized along a unique database, arranging the synthetic generated scenarios consecutively. Therefore, the data structure will be completely different from the previously described scenario analysis. This SQG data organization allows dealing efficiently with parallel processes that develop simultaneously interactions between simulation and optimization modules.

The goodness of the final solution is strictly related to the length of this constructed unique scenario and the number of adopted iterations in the optimization. Therefore, after each iteration the set of parametrized solution are improved.

Considering the whole set of available hydrologic database, the final dimensions of the problem are summarized in the Table 20.

Table 20: Database dimension for SQG Methods

SQG Database dimensions	
Months	12
Number of Hydrological Years	530
Hydrological Periods	6360
Number of Iterations	1000
Total Monthly Periods	6360000

5. Scenario Analysis Approach for the Water Pumping Schedules Thresholds Optimization

5.1. Scenario Analysis Approach in a water resource management problem

As previously described, these methodologies have been applied identifying the optimal *threshold activation schedules* for the pump stations located in the examined system (*Napolitano et al., 2014; 2016*).

In this chapter, the optimization problem has been solved implementing a classical scenario analysis approach. At first, this methodology has been developed assigning as hydrological inputs the historical scenarios reported in the Sardinian Region Water Plan (*RAS, 2006a*), while, in a second phase, the same analysis has been extended to a significant number of generated scenarios, in order to evaluate hydrologic and climate variation influence on proposed *pumping schedules activation rules*.

The scenario analysis approach needs a complex mathematical formulation in terms of model dimensions and number of variables and constraints considered. In order to afford efficaciously this problem, guaranteeing a reliable solution with a reasonable computational time, the pumping schedules optimization has been done using a multi-period scenario analysis implemented by the software GAMS (GAMS, 2009).

5.2. Implementation using the software GAMS

GAMS is an acronym of General Algebraic Modeling System. This software is completely dedicated to mathematical programming and, in particular, it allows solving linear or non-linear complex models using several solvers and considering continuous or integer variables. It provides a high-level language for the compact representation of large and complex models, permitting model descriptions that are independent of solution algorithms through a *declaration* and *definition* phases.

The design of GAMS has incorporated ideas drawn from relational database theory and mathematical programming and has attempted to merge these ideas to suit the needs of strategic modelers. Hence, relational database theory provides as structured framework for developing general data organization and transformation capabilities.

While mathematical programming provides a way of describing a problem and a variety of methods for solving it.

Among all the available solvers, GAMS/Cplex allows users to combine the high level modeling capabilities of GAMS with the power of Cplex optimizers. Cplex optimizers are able to solve large, difficult problems quickly. In particular, the solver Cplex provides solution to algorithms for liner (LP), quadratically constrained (QCP) and mixed integer programming (MIP) problems.

- *Linear programming* (*LP*): its solvers work in *continuous optimization* field; these are able to solve problems where are not considered any non-linear or binary variables. The majority of LP problems solve best using Cplex's state of art dual simplex algorithm. Certain types of problems benefit from using the primal simplex algorithm, the network optimizer, the barrier algorithm or the sifting algorithm. Solving linear programming problems is memory intensive. Even through Cplex manages memory very efficiently, insufficient physical memory is one of the most common problems when running large LP.
- *Quadratically Constrained Programming* (*QCP*): Cplex can solve models with quadratic constraints. These are formulated in GAMS as models of type QCP. QCP models are solved with the Cplex Barrier method. QP models are a special case that can be reformulated to have a quadratic objective function and only linear constraints. Those are automatically reformulated from GAMS QCP models and can be solved with any of the Cplex QP methods (Barrier, Primal Simplex or Dual Simplex).
- *Mixed-Integer Programming* (*MIP*): these methods work in the *discrete optimization* field and they are used to solve pure integer and mixed integer

programming problems require dramatically more mathematical computation than those for similarly sized pure linear programs. Many relatively small integer-programming models take enormous amounts of time to solve. For problems with integer variables, Cplex uses a *Branch and Cut algorithm* which solves a series of LP subproblems. Because a single mixed integer problem generates many subproblems, even small mixed integer problems can be very compute intensive and require significant amounts of physical memory. Cplex can also solve problems of GAMS model type MIQCP (*Mixed Integer Quadratically Constraints Programming*). As in the continuous case, if the base model is a QP the Simplex methods can be used and duals will be available at the solution.

GAMS combines its high efficacy in writing the optimization models with a flexibility of data management, variable and constraints definition. Indeed, GAMS language allows a single user to formulate mathematical models in a way that is very similar to their mathematical statement. Herein, GAMS has be interfaced with other software designed for input/output data management (e.g. Excel).

5.3. Mathematical Model Statement

Pursuing the optimal solution and in order to guarantee a correct description of all management rules along the examined time horizon, the mathematical model has been designed by a *multi-period dynamic optimization*. As discussed in the Chapter 2, in a first step the *basic graph* of the system should be drawn in order to describe the static situation. The multi-period dynamic configuration will describe period by period the temporal evolution of all the water system's components; it arises by replicating and interconnecting the static configurations for all periods of the considered time horizon.

At the beginning period, the stored initial volume in each reservoir has been kept equal to the maximum capacity; this boundary condition will not influence the system behavior for extended time horizons. Hydrological inflows I_t will provide a monthly water input to reservoirs. The sea node S allows a water system balance: indeed, it catches spilled water from reservoirs and provides resource to demand nodes in drought occurrences.

The water system balance is regulated by the following equation:

$$\sum_{t=1}^{t}(I_t - D_t) = S \qquad (12)$$

Where D_t represents a water demand from users and activities at the period t.

In order to achieve an easier and faster solution, the optimization process has been developed neglecting the physical characteristics represented by the evaporation components. Each hydrological scenario is characterized by a 20 years' time horizon, where the basic step is the monthly period and the total length is 240 times step with a root of scenario three composed by the initial 120 periods.

Once defined the *static draft* of the system through a *basic graph* definition, it should be repeated as much as are the monthly periods. The network states are connected among them by dummy arcs, therefore: the water availability during the *k+1* period is strictly dependent by the situation at the *k*-th period.

The scenario analysis approach is characterized by a formulation that allows building the scenario-tree model through a set of congruity constraints. These impose that the subsets of decision variables, corresponding to the indistinguishable part of each scenario, must be equal among themselves (13a - 13i).

$$x_{1t}^{g1} = x_{1t}^{g2} = x_{1t}^{g3} = x_{1t}^{g4} \qquad t=1,\dots,120 \tag{13a}$$

$$x_{2t}^{g1} = x_{2t}^{g2} = x_{2t}^{g3} = x_{2t}^{g4} \qquad t=1,\dots,120 \tag{13b}$$

[…] […]

$$x_{17t}^{g1} = x_{17t}^{g2} = x_{17t}^{g3} = x_{17t}^{g4} \qquad t=1,\dots,120 \tag{13c}$$

$$xv_{1t}^{g1} = xv_{1t}^{g2} = xv_{1t}^{g3} = xv_{1t}^{g4} \qquad t=1,\dots,120 \tag{13d}$$

[…] […]

$$xv_{5t}^{g1} = xv_{5t}^{g2} = xv_{5t}^{g3} = xv_{5t}^{g4} \qquad t=1,\dots,120 \tag{13e}$$

$$xs_{1t}^{g1} = xs_{1t}^{g2} = xs_{1t}^{g3} = xs_{1t}^{g4} \qquad t=1,\dots,120 \tag{13f}$$

[…] […]

$$xs_{5t}^{g1} = xs_{5t}^{g2} = xs_{5t}^{g3} = xs_{5t}^{g4} \qquad t=1,\dots,120 \tag{13g}$$

$$x_{d1t}^{g1} = x_{d1t}^{g2} = x_{d1t}^{g3} = x_{d1t}^{g4} \qquad t=1,\dots,120 \tag{13h}$$

[…] […]

$$x_{d6t}^{g1} = x_{d6t}^{g2} = x_{d6t}^{g3} = x_{d6t}^{g4} \qquad t=1,\dots,120 \tag{13i}$$

Where:

x_{nt}: represents the water flowing along the *n*-th arc of the system during the period *t*-th [10^6 m^3];

xv_t: storage volume transferred by the inter-period connection in the *t*-th period [10^6 m^3];

xs_{jt}: spilled water by the *j*-th reservoir to the sea node in the *t*-th period [10^6 m^3].

In this way, these variables will assume the same values in each single scenario until the branching-time and, after that, they will develop independently.

5.4. Optimization of the pumping schedules thresholds

As stated in the Chapter 2, in the analyzed systems the water resource availability is scarce: therefore, the water demands cannot be satisfied in many scenarios. In such drought conditions an emergency policy to alleviate the effect of shortages should be developed. using emergency and costly transfer by activating water pumping stations.

The aim of the optimization process is to identify the optimal *barycentric* pump activation thresholds S^b, mainly using reservoirs' storage volumes as trigger values in the decisions of managers. These activation values should be defined with a cost-risk balancing process that considers the minimization of energy required by pumping stations and related management costs (*cost element*) and damages caused by water deficits for users (*risk elements*). Optimizing these activation rules, a critical stored volume in reservoirs that may supply the downstream demand centers by gravity should be defined.

The mathematical model (14) will be established in two parts: an *objective function formulation* (14a) and a *constraints definition* (14b - 14d). The optimization process aims to fulfil the user's water demands, minimizing the operative and energy costs supported. The activation of pumping stations is supposed to be dependent on the stored volume levels in reservoirs that supply the downstream demand nodes by gravity. Therefore, to model pump activation, assuring a correct operation along the considered time horizon, a binary variable h_i^g to each *i*-th pump station should be assigned. This variable represents the *on/off condition* for a single pump station as it can assume *one* or *zero* values. In the optimization model h_i^g is dependent on the sum of the stored levels xv_i^g in the *j*-th reservoirs supplying water by gravity, according the activation dependences shown in Table 11.

The mathematical model of the scenario optimization problem, considering the activation of pumps, consequently assumes the following form:

$$\underset{x^g, \hat{S}_i^g, S_i^b}{Minimize}(1-\lambda)\sum_{g\in G} p^g c^g x^g + \lambda \sum_{g\in G}\sum_{i\in P} p^g[w^g(\hat{S}_i^g - S_i^b)^2] + \sum_{g\in G}\sum_{i\in P} CM_i(1 * h_i^g) \quad (14a)$$

$subject\ to$

$$A^g x^g = b^g \quad (14b)$$

$$l^g \leq x^g \leq u^g \quad (14c)$$

$$x^* \in S \quad (14d)$$

This formulation arises from the equations (9a-9d) with the introduction of a set of variables represented by the activation thresholds values S_i in the second term. Moreover, it is necessary to introduce a new cost term in the objective function (9a) by multiplying h_i^g by an activation penalization coefficient CM_i that can be recognized as the fixed-cost management activation of the station. Nevertheless, the correctly balanced evaluation of these CM_i penalization coefficients is a complex task: this process has been developed during a preliminary parametric analysis, considering only few sample scenarios. This calibration has a relevant importance, indeed, an inappropriate *CM* value could generate some errors: i.e. pumping an additional amount of water after an overestimation will generate some *regret costs* (*Kang and Lansey, 2014*) or some high deficit phenomena after an underestimation.

During this analysis, the penalization coefficient values (one for each station) are retrieved by model optimization as they are considered like variables to be optimized.

In order to guarantee a correct operation of pumping stations, the model (14) should be completed introducing a new set of constraints (15 – 18).

$$\sum_{j=1}^{R_i} xv_j^g - S_i^b \leq h_i^g BM \qquad \forall i \in P, \forall g \in G \tag{15}$$

$$x_i^g = (1 - h_i^g) P_i \qquad \forall i \in P, \forall g \in G \tag{16}$$

$$\hat{S}_i^g = \sum_{j=1}^{R_i} K_j \qquad \forall i \in P, \forall g \in G \tag{17}$$

$$S_i^b = \sum_{j=1}^{R_i} K_j \qquad \forall i \in P \tag{18}$$

Where:

$i \in P\{1, \dots, n_P\}$ Pumping stations in the system;

$j \in R\{1, \dots, n_R\}$ Reservoirs in the system;

$h_i^g \in \{0,1\}$ Binary variable.

$\hat{S}_i^g$ and S_i^b are respectively the activation storage value for *i*-th pumping station in *g*-th scenario and the barycentric activation storage value for *i*-th pumping station, considering R_i as a set of reservoirs connected to *i*-th pumping station node.

Constraint (15) allows the i-th pump station activation if the sum of the stored volume in reservoir j is under the threshold value S_i^b. In this constraint, the parameter BM is a large scalar.

Constraint (16) guarantees that, in the case of activation of the i-th pump station, the flow along the pumping arc starting from the i-th station will be equal to its capacity P.

Constraints (17 - 18) impose an upper bound on the activation storage levels of the i-th pumping station equal to the sum of the reference reservoir's capacity Kj.

To complete the multi-period model with these constraints, it is necessary to guarantee the correct working of pump stations.

According the scenario analysis approach the optimization process will be developed in the following main steps:

1. Parametric analysis of a single scenario in order to define CM_i $i=1,\ldots,P$;
2. Single scenario optimization to define starting values of reservoir threshold levels S_i;
3. Parametric multi-scenario optimization to check starting values of reservoir threshold levels S_i defined in the previous step;
4. Multi-scenario optimization and barycentric value evaluation of threshold levels S^b;
5. Re-optimization process to validate the results on each single scenario $g \in G$.

5.4.1. Reference scenarios results

In a first step, this process has been developed using historical data (*RAS, 2006*). The model has been organized considering 4 hydrological scenarios, composed with a common root of 10 years and the following data diversified in 20-year scenarios (the resulting scenario tree was discussed in *Paragraph 4.1.*).

The total model dimensions have been reported in Table 21.

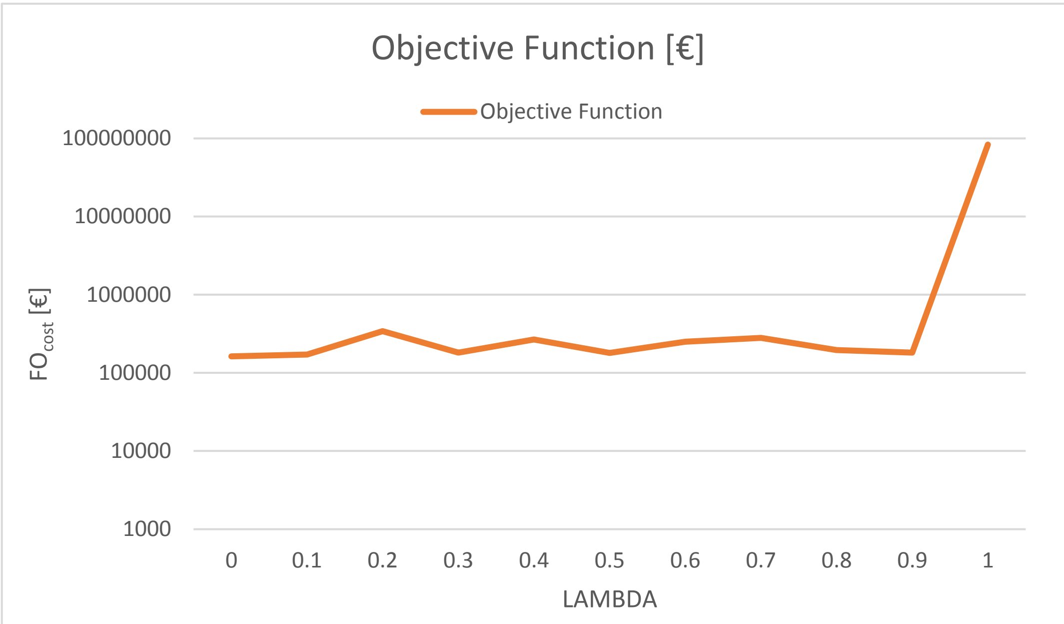
Table 21: Model Dimensions - Historical Scenarios

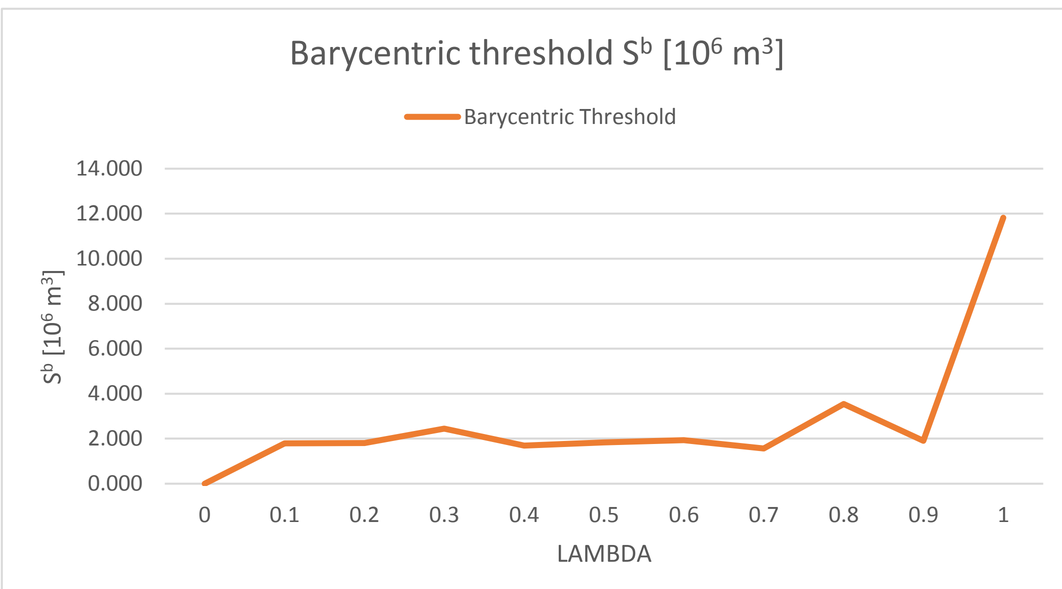
Constraints	Variables	
	Binary	Continuous
568	16	164

Moreover, Figures 20Figures 20a-b show that a specific analysis has been conducted on the λ value, in order to evaluate the influence of its variation on activation threshold and objective function values. This sensitivity analysis particularly refers to a part of the whole network, in particular the Figures 20a-b analyze λ behavior considering the water system connected with the activation threshold S_1^b.

Varying λ between zero and one it is possible to observe decreasing value of activation threshold when the λ value is close to zero; while, for λ near to one this balancing value requires to set the activation threshold towards the reservoir capacity.

This behavior arises from the relationship between cost and risk elements: indeed, for low values of λ the cost term prevails on risk element, leading the water system's authority into take an energy saving management policy, activating the pumps station as later as possible. Otherwise, when λ value is closer to the unit, risk element prevails and a careful management policy should be taken assuring a water fulfilment for users and avoiding drought occurrences. In this case, pumps stations will be switched on as soon as possible with a high-energy costs influence.





Figures 20a-b: Parameter LAMBDA Analysis

Nevertheless, it is evident that a wide variation of λ maintains about constant the pumping activation value. Therefore, the barycentric values for activation thresholds were obtained setting the λ value equal to 0.5, thus considering an equal balance between the cost and risk elements in the objective function.

Barycentric values of activation thresholds, considering stored water in reservoirs, were evaluated by solving the model (14); the obtained results are reported in Table 22.

Table 22: Barycentric Activation Thresholds

Barycentric Activation Thresholds [10^6 m^3]	
S_1^b	1.624
S_2^b	35.451
S_3^b	5.070
S_4^b	37.558

According to scenario analysis, these values are barycentric among all hydrological scenarios and they will guarantee a compromise among different water resource availabilities.

These barycentric values will be adopted during the *re-optimization phase*, where the whole process has been developed assigning these thresholds as fixed parameters.

The re-optimization phase highlights a complete fulfilment for all users along the considered time horizon, arising only some small-unplanned deficit.

As shown in Figure 21, the irrigational demand node D_4 is affected in a single period of the second scenario by a low water deficit equal to 0.035 10^6 m^3.

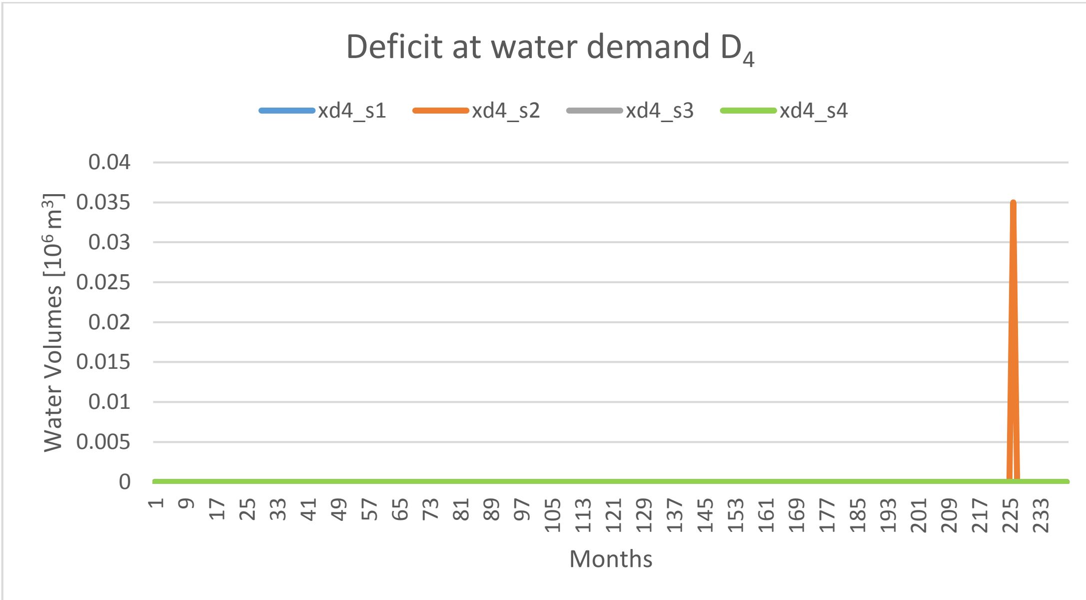


Figure 21: Water Deficit at Demand Node D_4 [10^6 m^3]

The average annual pumping costs were evaluated by an *economic post-processor* taking into account *unplanned deficit* and *pumping costs*. This post-processor has been built downline of the re-optimization phase. Only the real costs of the system, related to energy consumption and drought occurrences, have been considered during this analysis.

The average value of annual total costs (Z_{TOT}) was split into two contributions: pumping costs (Z_{pump}) and unplanned deficit costs (Z_{def}).

$$Z_{TOT} = \sum_{g \in G} (Z^g_{pump} + Z^g_{def}) \tag{19}$$

Z^g_{pump} was evaluated considering the amount of water yearly pumped by each *i*-th station in scenario *g*, assuming unitary pumping cost equal to c_i (Table 10):

$$Z^g_{pump} = \sum_{i \in P} x^g_i \, c_i \tag{20}$$

Z^g_{def} was evalutated considering the amount of water deficit at each d-th demand center in scenario g, assuming deficit cost equal to c_d (Table 7).

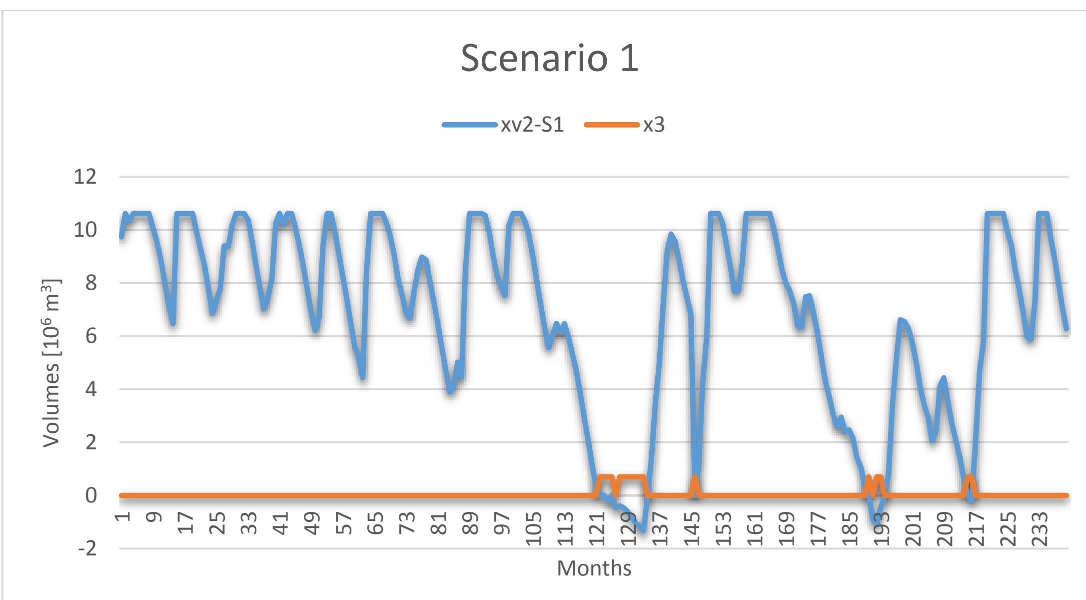
$$Z^g_{def} = \sum_{d \in D} x^g_d \, c_d \tag{21}$$

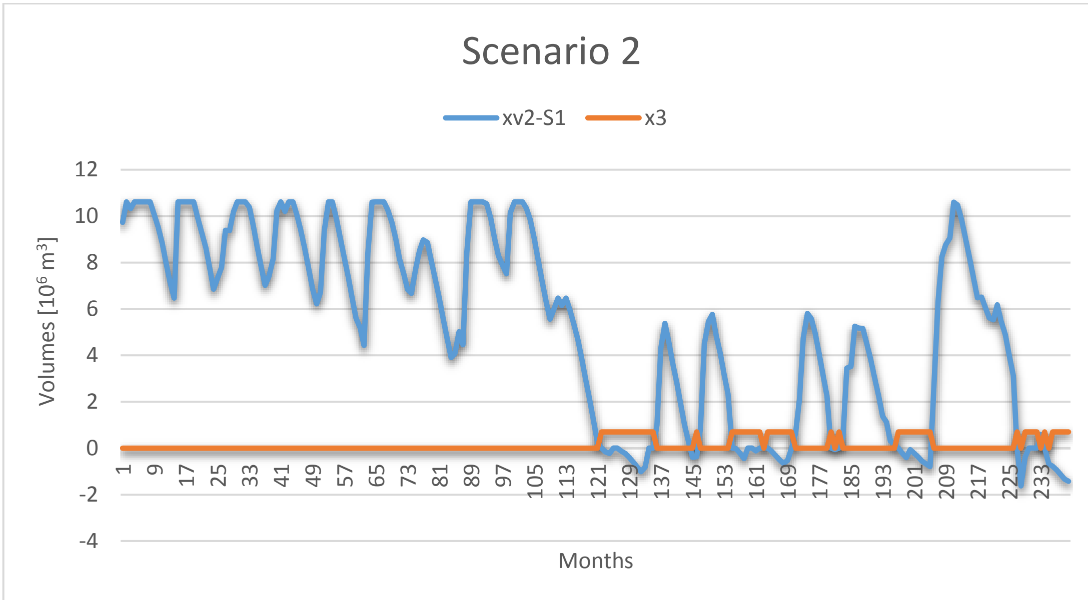
The results are shown in Table 23. It is confirmed that the optimization process decreases the value of unplanned deficit about to zero.

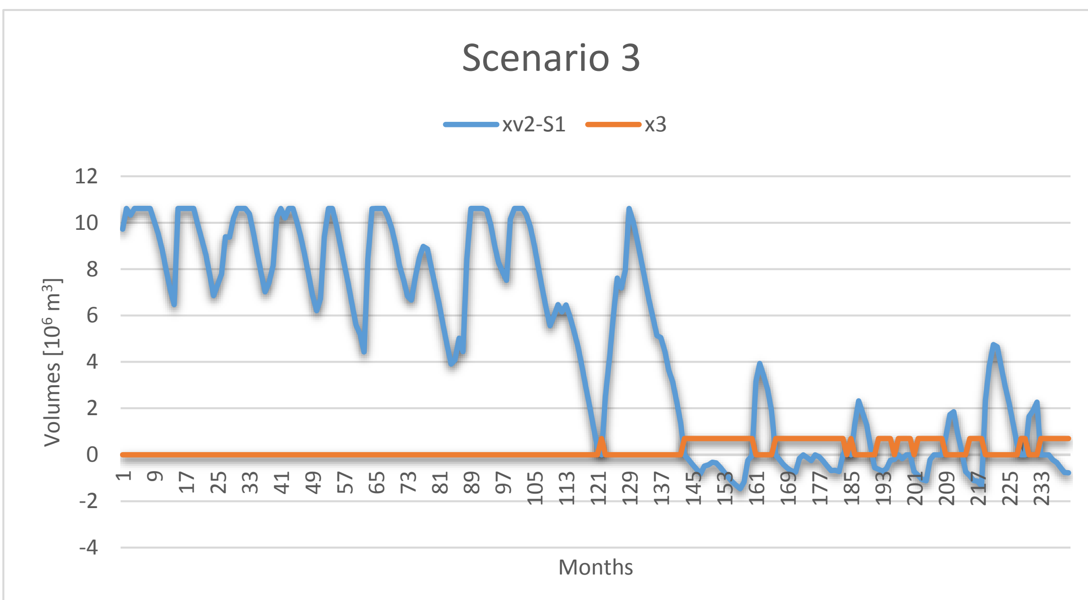
Table 23: Post-Processor Cost Function Results

Annual Average Costs	$C_{deficit}$	$C_{pumping}$	C_{TOT}
[10^6 €/year]	0	0.191	0.191

Optimization results are also given in Figure 22 representing the correct operation of P_1 pumping station in four historical scenarios: the pumped and storage volumes are given by subtracting the threshold activation value *S1* in the reservoir.




Scenario 2
xv2-S1
x3
Volumes [10⁶ m³]
Months


Scenario 3
xv2-S1
x3
Volumes [10⁶ m³]
Months

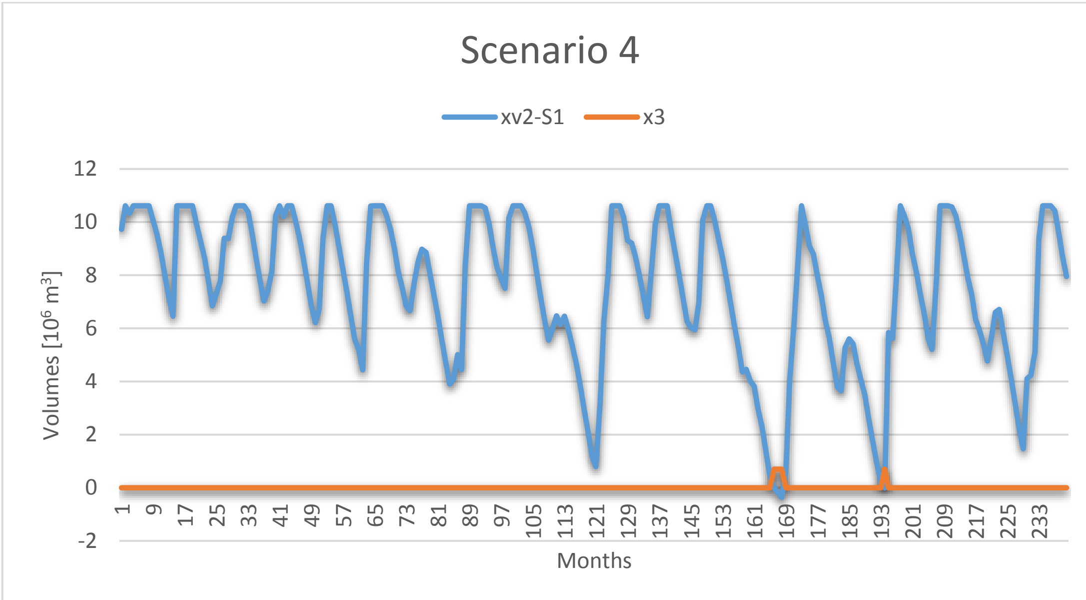


Figure 22: Historical Scenarios: Pumped Volumes by P1 and Storage Volumes [10^6 m^3] (minus threshold value S1)

As shown in the Figure 22, an additional amount water flows along the emergency pumping arcs just during some drought periods along the time horizon.

This activation rule is strictly dependent by the trigger threshold value. Each pump station will be switched on just in the periods when the stored volumes in reservoir will be lower than the optimized activation threshold value. Moreover, if the pump station is working, the amount of pumped water will be equal to the pumping maximum capacity.

Table 24 highlights the amount of each pump station on the total cost.

Table 24: Post-Processor Single Pump Station Costs

Pumping costs	P_1	P_2	P_3	P_4	Σ
[10^6 €/year]	0.245	0.182	1.403	0.083	1.913

5.4.2. Synthetic scenarios results

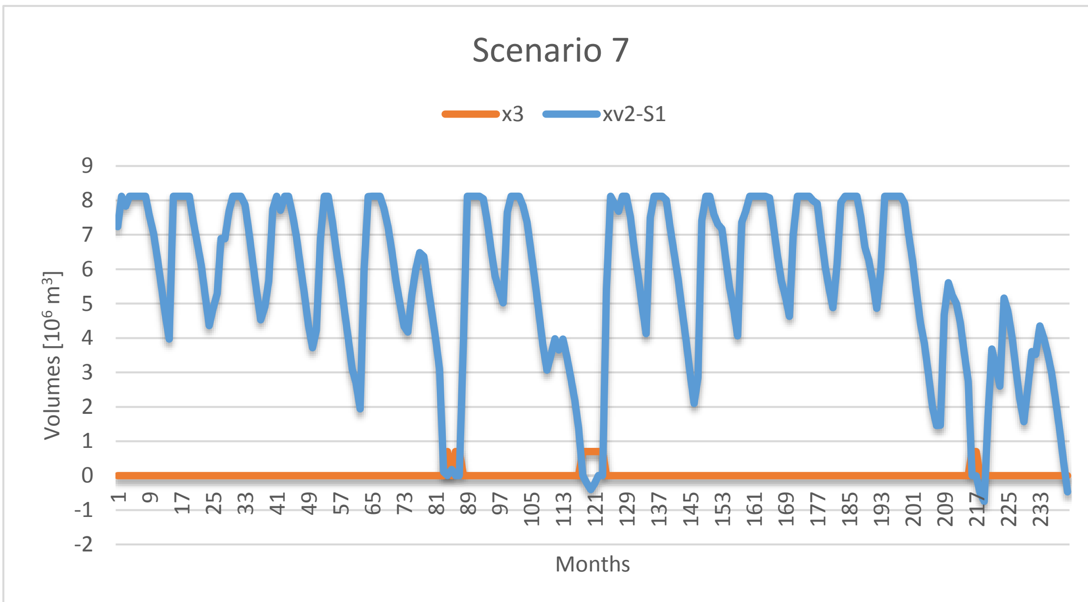
In the second phase of this analysis, scenario optimization was extended to a significant number of synthetic scenarios, as previously described, in order to evaluate the influence of climate variations on the proposed pumping activation rules. As explained in the *Paragraph 4.2.1.*, a new scenario tree was built using the new set of synthetic scenarios with a common historical root: therefore, the tree contains 4 historical scenarios plus 50 synthetic scenarios, as shown in Figure 18.

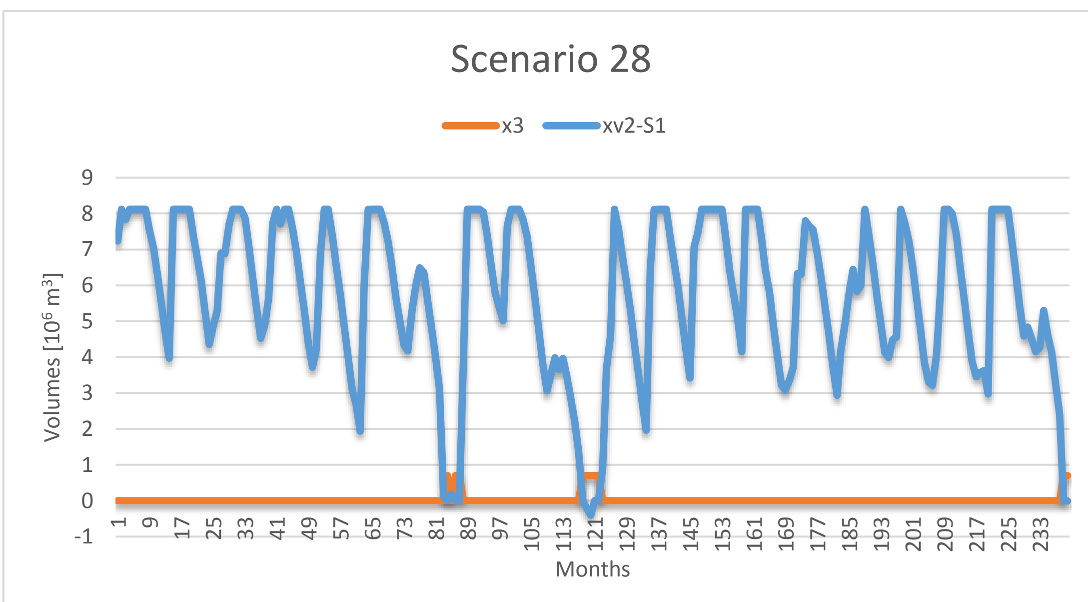
Solving the new scenario tree configuration some computational problems arise. Indeed, adding the new set of hydrological scenarios, the resulting number of branches grows up generating some complications during the algorithm resolution. Despite the existence of some computational time burden, the optimization processes can be developed up to step 3, than parametric multi-scenario optimization to evaluate the threshold storage levels of reservoir for pumping initiation. The reason is that using GAMS win32 (version 24.1.2) some computational problems arise at this step due to problems complexity and model dimensions. Specifically, the Cplex Mixed Integer Programming solver exceeds the maximum admitted computational time and, moreover, Cplex encounters memory problems while solving the multi-scenario optimization and barycentric value definition of threshold levels S^b. Namely, as showed in Table 25, the dimensions of the new model are increased considerably.

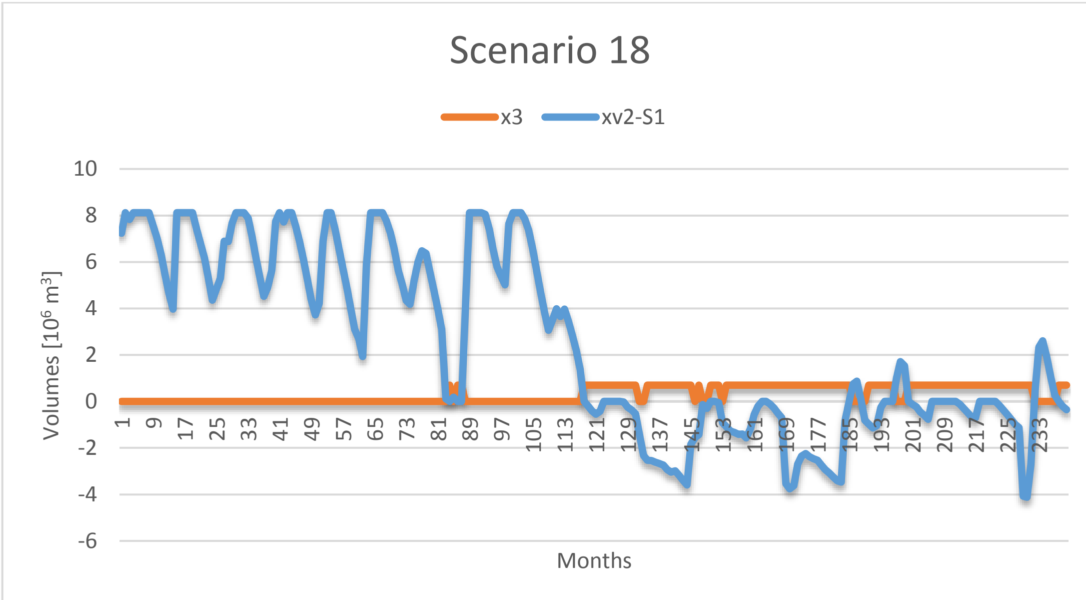
Table 25: Model Dimensions – Synthetically Scenarios

Constraints	Variables	
	Binary	Continuous
7668	216	2214

Therefore, overtaking this computational problem, in order to compare the historical and synthetic scenario optimization results, step 3 was developed by appointing the barycentric values, reported in Table 22, obtained from the historical scenario optimization.







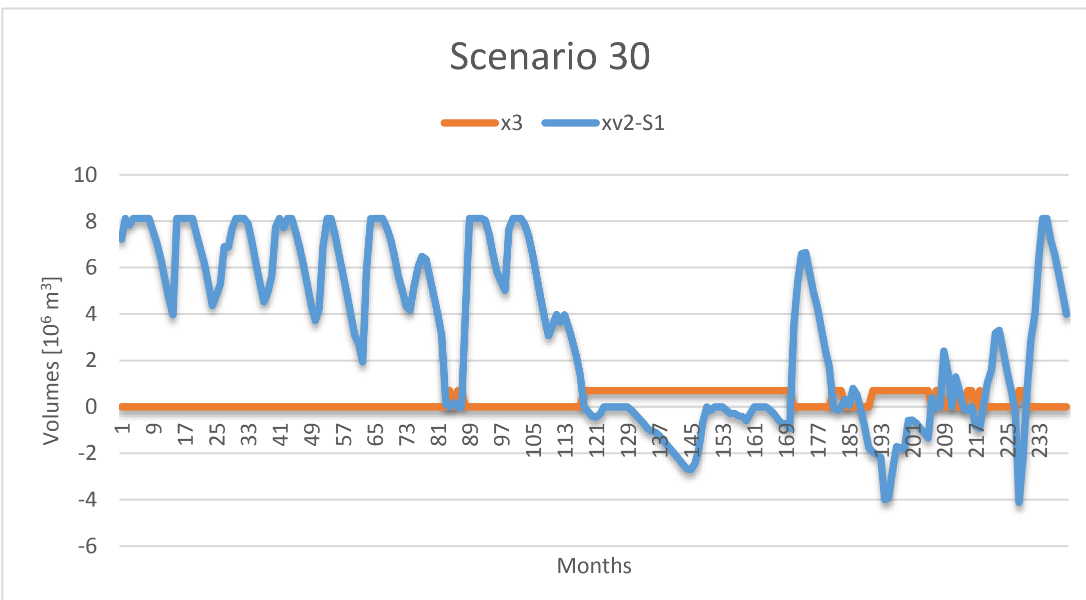


Figure 23: Synthetically Scenarios: Pumped Volumes by P1 and Storage Volumes [10^6 m^3] (minus threshold value S1)

Figure 23 highlights the operation of pump station P_1 in four selected synthetic hydrological scenarios.

These selected scenarios are characterized by different water resource availabilities: namely, the first two are the richest, while the final two are the poorest in terms of input flows to reservoirs. Even considering these critical scenarios, pumps activation allows satisfying about completely demand requests. Nevertheless, introducing the new set of synthetically scenarios it is possible to consider a more critical hydrology, taking into account longer drought phenomena. Herein, as in the previous section, orange line represents the storage volume, reduced by threshold values, while the blue line represents the pumped volumes.

As recognized from historical scenarios analysis, the cost-risk balancing approach has contextually restricted deficit risks for users and has minimized the costs of the system in condition of scarcity.

The results obtained using the synthetic scenarios were finally analyzed using the *economic post-processor*: unplanned deficits and pumping costs are considered as in the previous case. Table 26 shows the mean annual values of unplanned deficits and pumping costs. More critical scenarios generated synthetically justify higher costs.

Table 26: Cost Function Results - Synthetically Scenarios

Annual Average Costs	$C_{deficit}$	$C_{pumping}$	C_{TOT}
[10^6 €/year]	0.191	3.371	3.562

Nevertheless, it must be noted again that the barycentric activation values are retrieved by the optimization on historical scenarios due to Cplex computational problems and model dimensions of using synthetic scenarios.

6. Stochastic Quasi Gradient Methods for the Water Pumping Schedules Thresholds Optimization

6.1. SQG for a water resource management problem

To overcome the computational problems, a new optimization model has been considered, based on the Stochastic Quasi-Gradient methods (SQG). The optimization problem remains the same: trying to define optimal threshold values for pumping activation. The use of an SQG approach can be considered as an innovative proposal for this kind of problem, considering that, until now the SQG approach has been adopted mainly to face on to econometric and logistical problems.

In these problems, where the level of uncertain is high, to adopt decision-making solutions is necessary finding a correct future system management and improving the security level in the decision-making process. Specifically, it is possible to look for the optimal set of parameters q that describe the pumping rules activation, minimizing the average monthly costs, which are sums of all costs supported in the water system management and, at the same time, reducing the shortage risk occurrences for system's users and activities. The network state v^t (water volumes in reservoirs) evolves in discrete time $t = 1, \dots, T$ (months). At each t arrives a water demand d^t and inflow r^t. Therefore, the pumping schedules are defined by pumping rules with parameters q and at each t; the network flows x^t are obtained from minimization of the following objective function (22).

$$C^T(q, v^t, d^t, r^t) = \min_{x \in X} C(x, q, v^t, d^t, r^t) \tag{22}$$

Subject to constraints (23) (flow continuity, bounds, etc.).

$$\Phi(x, q, v^t, d^t, r^t) = 0 \tag{23}$$

The state of the system between two consecutive time periods is regulated by v^{t+1} at the beginning of period $t + 1$, which is obtained from state equation (24).

$$v^{t+1} = \Psi(x^t, q, v^t, d^t, r^t) \tag{24}$$

Where the functions $C(\cdot), \Phi(\cdot), \Psi(\cdot)$ are linear with respect to (x, v).

The problem tries to find the set of parameters $q = (q_1, \dots, q_n)$ in order to minimize the average steady state costs, thus solving the optimization problem with infinite time horizon (25).

$$\underset{q \in Q}{\text{Minimise}}\, F(Q), F(q) = \lim_{t \to \infty} \frac{1}{t} C^t(q, v^t, d^t, t^t) \tag{25}$$

Where $Q \subseteq R^n$ is some feasible set for parameters q.

This problem can be solved using SQG methods, properly a stochastic approximations and gradient projection methods of nonlinear programming.

Relating to water resources, the application of this method is a novelty, in previous researches was developed to optimization of general simulation models of discrete event dynamic system in *Dupačová et al.* (1991) and *Gaivoronski* (2005).

As shown in the modules interaction scheme reported in Figure 24, the approach to the problem solution goes ahead by a concurrent interrelation among simulation, optimization and cost evaluation steps.

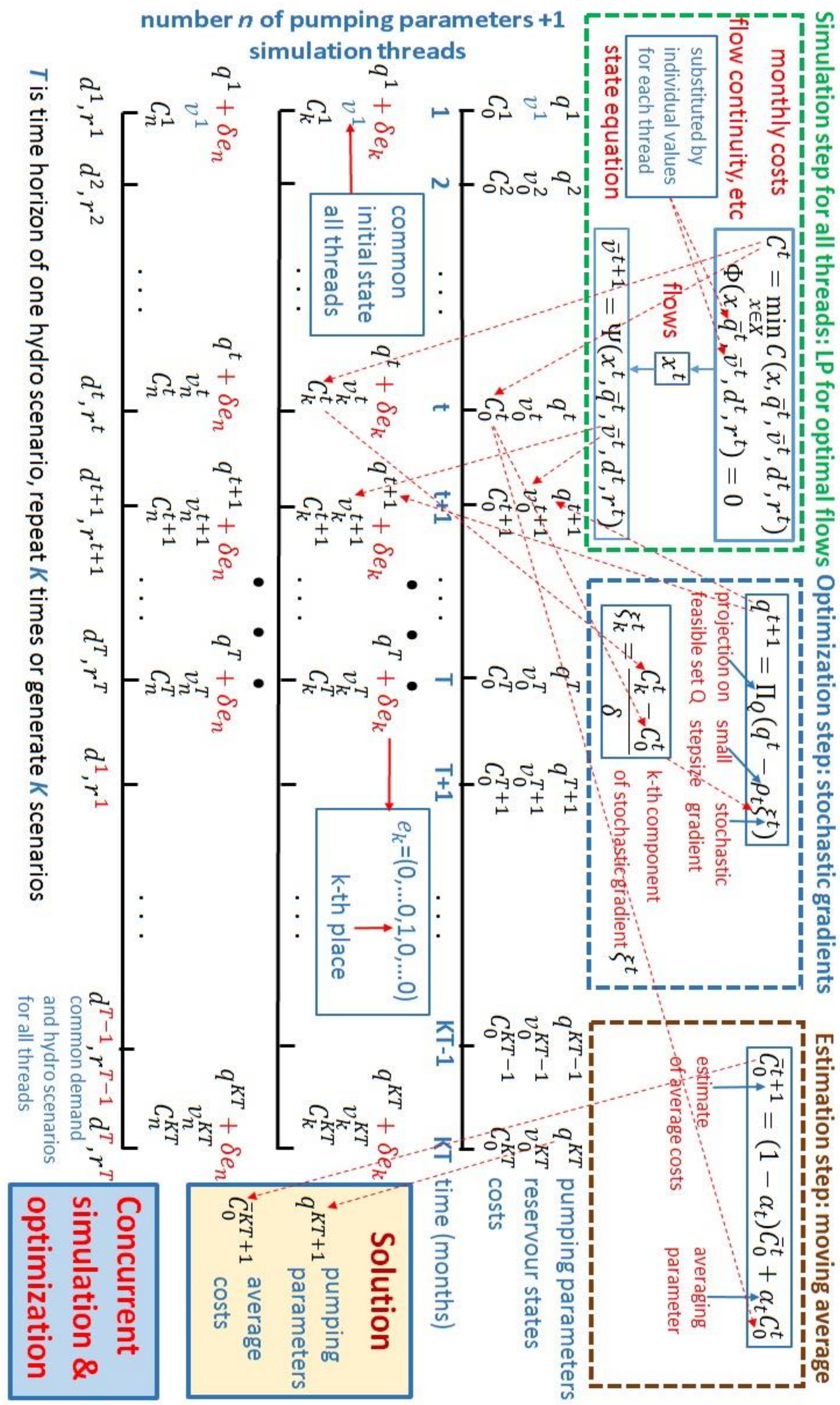


Figure 24: Current Simulation and Optimization Process

In the following, a brief description of the concurrent interactions among them is given:

- *Simulation step for all threads with LP searching for optimal flows*: the simulation process is referred to each single period t of the time horizon and it is characterized by *n*-processes simultaneously. Each process has different sets of pumping activation threshold parameters q according the (26).

$$q^t + \delta e_k \quad (26)$$

$\delta > 0$ is a small positive value and e_k is a vector of zeros with one value in the *k*-th position.

Here, will be minimize the objective function of costs referred to the single period t (22), in order to obtain an optimal configuration of the network water flows x^t.

- *Optimization step searching for stochastic gradients*: the optimization process will be applied between two consecutive periods and the configuration will be evaluated according to the equation (27).

$$q^{t+1} = \prod_Q (q^t - \rho_t \xi^t) \quad (27)$$

Where $\rho_t > 0$ is the step size and $\prod_Q(\cdot)$ is the projection operator on feasible set q. The *k*-th component of stochastic gradient $\xi^t = (\xi_1^t, \dots, \xi_n^t)$, will be estimate as (28).

$$\xi_k^t = \frac{C_k^t - C_0^t}{\delta} \quad (28)$$

- *Evaluation step*: simultaneously there is the estimation step based on a moving average (29).

$$C_0^{t+1} = (1 - \alpha_t)\bar{C}_0^t + \alpha_t C_0^t \quad (29)$$

The estimate costs C_0^{t+1} are dependent on the average costs $\bar{C}_0^t$, referred to all periods, and the costs evaluated in the previous step C_0^t. The relationship between these terms is regulated by an averaging parameter α_t, which usually assumes the 0.3 value.

6.2. Mathematical Model statement

The mathematical statement should be built searching for an optimal water system network configuration. The water system is defined by the basis network $\Theta = (N, L)$, where N identifies the set of nodes and L represents the set of direct links. This network can assume several configurations $\Theta_m = (N_m, L_m)$, $m \in M$. The subsequent configuration of the system is defined by some configuration rules (30), which are function of the system state v^t and the vector of threshold parameters q.

$$\Theta^t = f(q, v^t) \tag{30}$$

The decisions, involved in the network management of the water system at the period t, are divided in two different classes:

- Θ^t: *network configuration*, this decision is about the structure of the network, depending on the selection of parameters q;
- x^t: *network flows definition*, referring to the links on the selected configuration Θ^t.

The performance of the system in the single period t is described by the cost function $C^t(\Theta^t, x^t, v^t, d^t, r^t)$, which depends on the current configuration of network flows, demands and inflows.

All these decision variables are evaluated by a cost function optimization, through a two levels procedure: on the *first solution level*, the network flows are retrieved at every time period $t \in T$. On the *second level* the network configuration parameters will be upgraded.

6.2.1. Definition of Network Configuration

At each period $t \in T,$ the cost function $C^t(\Theta^t, v^t, d^t, r^t)$ depends through (30) on the threshold parameters $q,$ as shown in (31).

$$C^t(q, v^t, d^t, r^t) = C^t(\Theta^t, v^t, d^t, r^t) = C^t(f(q, v^t), v^t, d^t, r^t) \tag{31}$$

It means that the optimal values of the network configuration parameters q should be evaluated by minimizing the average steady state costs $C(q),$ such as reported in (32):

$$C(q) = \lim_{t \to \infty} \frac{1}{t} C^t(q, v^t, d^t, r^t) \tag{32}$$

or in an expected finite horizon costs (33).

$$C(q) = \sum_{t=1}^{T} \gamma_t \, \mathbb{E} C^t(q, v^t, d^t, r^t) \tag{33}$$

Here γ_t represent appropriate discontinuing coefficients. Therefore, the control of parameters q is regulated solving the problem (19) for a feasible set $Q \subseteq R^n$.

$$\min_{q \in Q} C(q) \tag{34}$$

6.2.2. Network Flows Selection

After a definition of the network configuration Θ^t, the flows along the links in the network can be evaluated according two different ways:

- *Optimization of the single period costs*: in a single period t, the network flows x^t are obtained minimizing the cost function (35a).

$$C^t(\Theta^t, v^t, d^t, r^t) = \min_{x \in X(\Theta^t)} C^t(\Theta^t, x, v^t, d^t, r^t) \tag{35a}$$

Subject to

$$\Phi^t(\Theta^t, x, v^t, d^t, r^t) \leq 0 \tag{35b}$$

where $\Phi^t(\Theta^t, x, v^t, d^t, r^t)$ is a vector function able to define the flow continuity constraints and different technological and economical restrictions. The additional feasible set of solutions $X(\Theta^t)$ is usually characterized by a simple structure. Furthermore, the functions C^t and Φ^t are both linear, consequently, the problem (35) can be solved as a *linear programming model* of small to medium dimensions.

- *Optimization of discounted multi-period costs, considering a moving window horizon*: fixing a moving window of length $\tau > 1$, in this configuration the inflows r^t are known, but inflows in the subsequent time periods r^s, where $s = t + 1 : t + \tau$, are random vectors with known distributions.

The flows x^s can be evaluated solving the problem (36a-36d).

$$\min_{\bar{x}^s \in X(\Theta^t), s=t:t+\tau} [C^t(\Theta^t, \bar{x}^t, v^t, d^t, r^t) + \textstyle\sum_{s=t+1}^{t+\tau} \eta_s \mathbb{E}\, C^s(\Theta^t, \bar{x}^s, v^s, d^s, r^s)] \tag{36a}$$

Subject to

$$v^{s+1} = \Psi^s(\Theta^t, \bar{x}^s, v^s, d^s, r^s),\ s = t{:}\,t + \tau - 1 \tag{36b}$$

$$\mathbb{E}\bar{\Phi}^s(\Theta^t, \bar{x}^s, v^s, d^s, r^s) \leq 0,\ s = t{:}\,t + \tau - 1 \tag{36c}$$

$$\Phi^s(\Theta^t, \bar{x}^s, v^s, d^s, r^s) \leq 0,\ s = t{:}\,t + \tau - 1 \tag{36d}$$

here, η_s are discounting coefficients and $\bar{\Phi}^s(\cdot)$ are some known functions. The set of constraints (36c) and (36d) can be split in two parts: constraints (36c) are satisfied in average, while constraints (36d) are satisfied for almost all (in probabilistic sense) values of random vector r^t, in the case of network flow constraints.

The problem (35) is considerably more complex than the problem (36), because for $s > t$ its solution x^s is a function of random variables r^l, with $l = t + 1{:}\,s$. Its complexity grows fast with the number of states and the length of time window τ. At the beginning of time period $t + 1$, the next network configuration Θ^{t+1} can be evaluated solving the problem (21a-21d) on a new moving window $t + 1{:}\,t + \tau + 1$, in order to obtain water flows at time period $t + 1$.

6.3. Optimization of water pumping thresholds

In the previous chapter, has been shown how, modelling this problem through a traditional *Scenario Analysis Approach* with a two stages stochastic programming model, some computational difficulties arise. In this section, the same stochastic optimization problem has been modelled using the SQG approach, which allows to solve substantially larger models and to provide optimal solution under high uncertainty.

Managing a water network optimization, reported in the Figure 11, the fundamental problem is to optimize the water system network configuration. In a water pumping optimization problem, some links of the water system can have nonzero flows only if the pumps are installed on these links. Hence, a link becomes operational when the corresponding pump is switched on and it pumps water, while, if a pump is switched off there is not flow along the corresponding link and, consequently, it does not exist in a current network configuration. Thus, different configurations are selected by switching on and off the considered pumping plants.

6.3.1. Water system network modelling

The water system has been studied as a network flow problem according the graph theory. As extensively explained in the *Chapter 3*, the set of network nodes are characterized by different types of elements:

$$N = (R, U, D, S) \tag{37}$$

where R is the set or reservoirs nodes, which can store water, D represents the water demands and S the sea node. The transhipments nodes U are able to connect different reservoirs among them, but do not have their own capacity to store resource.

$L(i, j)$ is the set of directed links, with $i, j \in N$. If $(i, j) \in L$ this means that the water flow at the time period t (x_{ij}^t) can exist from node i to node j, then node i is a *parent* of node j and node j is a *child* of node i.

The set of pumping links $P \subseteq L$ represents all links equipped with pumps. The flow on a link $(i, j) \in P$ can be different from zero if the pump on this link is switched on and is pumping water. Moreover, if it happens the amount of water pumped is equal to the pumping plants capacity (reported in Table 10).

Let $\bar{P} \subseteq P$ be an arbitrary and possibility empty operating subset of the set P of pumping links. The pumps on $\bar{P}$ are switched on and the pumps on $P \setminus \bar{P}$ are switched off. Any such set defines a configuration $\Theta(\bar{P}) = (N, L(\bar{P}))$ as follows:

$$N = (R, U, D, S),\ \ L(\bar{P}) = L \setminus (P \setminus \bar{P}) \tag{38}$$

Thus, there is one to one correspondence between network configurations and sets $\bar{P}$ of operating pumping links. The characteristic function of set $\bar{P}$ is regulated by $\chi_{ij}(\bar{P})$.

$$\chi_{ij}(\bar{P}) = \begin{cases} 1 & if\ (i,j) \in P \\ 0 & otherwise \end{cases} \tag{39}$$

In order to obtain a correct management of the system, flow balance equations for reservoirs (40a), transhipments (40b) and pumping nodes has been included in the constraints.

$$v_i^{t+1} = (1 - \varsigma)v_i^t + r_i^t + \sum_{k \in K^-} x_{ki}^t - \sum_{k \in K^+} x_{ik}^t\,,\ \ i \in R \tag{40a}$$

$$\sum_{k \in K^-} x_{ki}^t - \sum_{k \in K^+} x_{ik}^t = 0,\ \ i \in U \tag{40b}$$

Where v_i^t and r_i^t are respectively the stored volume of water and hydrological inflow in reservoir $i\ \in R$ at the beginning of the period t. the parameter ς_i^t represents the fraction of water, which is evaporated from reservoir $i\ \in R$, during the time period t, thus the evaporated volume from this reservoir could be evaluated multiplying $\varsigma_i^t v_i^t$.

K_i^- represents the set of parent nodes of node i: $K_i^- = \{j: (j, i) \in L\}$. Conversely, K_i^+ is the set of children nodes of i: $K_i^+ = \{j: (i, j) \in L\}$; $K_i^+ = \emptyset$ if $i \in \{D \cup S\}$.

Moreover, there are capacity constraints on nodes and links, which take the following form:

$$v_i^{t+1} \leq V_i^{max},\ i \in R \tag{41a}$$

$$0 \leq x_{ij}^t \leq g_{ij},\ i,j \in R \cup U,\ (i,j) \in L \setminus P \tag{41b}$$

$$0 \leq x_{ij}^t \leq \chi_{ij}(\bar{P}) g_{ij},\ (i,j) \in P \tag{41c}$$

where g_{ij} represents the capacity of the link $(i,j) \in L$, regulating the maximal water flow which can pass along this arc. Constraint (41a) implies that the amount of stored water in a reservoir can not exceed the reservoir capacity $V_i^{max}, i \in R$. There are also some technological constraints such as the lowest admissible amount of water stored in reservoirs $V_i^{min}, i \in R$.

$$\sum_{k \in K_i^-} x_{ki}^t + u_{pdi}^t + u_{ndi}^t = d_i^t,\ i \in D \tag{42a}$$

$$0 \leq u_{pdi}^t \leq \beta_i d_i^t,\ u_{ndi}^t \geq 0,\ i \in D \tag{42b}$$

The constraints (42a) and (42b) aim to manage the water demands flows. Due to scarcity conditions, sometimes the total water demand d_i^t, referred to the node i at the period t, can be satisfied only partially. Hence, the observed deficit could be categorized in two classes: planned and unplanned. Planned deficits u_{pdi}^t can be forecasted and communicated in advance to consumers, while the unplanned ones u_{ndi}^t arise when the realization of water inflows follow hydrological scenarios of unpredictable scarcity, affecting and harming several consumers. Therefore, it is possible to apply the inequality: $c_{nd}^i > c_{pd}^i$, which means that costs related to unplanned deficit c_{nd}^i are higher than the costs of planned one c_{pd}^i. In the constraint

(42a) appears a parameter β_i that represents the maximal fraction of planned deficit allowed from demand value at each node. When this maximal fraction value is overcome, the observed deficit values should be considered as unplanned.

Some of the considered costs are actually real costs, rather than others that are opportunity costs or penalties introduced guaranteeing a correct operation of the system. The costs c_p^{ik} are needed to manage the pumping links $(i,k) \in P$, while the opportunity costs c_w^i are referred to the spilled water x_{iS} to the sea from node $i \in V \cup U$.

Among the penalty costs are included c_v^i and c_y^{ik}, which respectively represent a penalty for violating lower bound constraint on the volume of reservoir $i \in V$ and a penalization if amount of pumped water on link (i,k) is less than g^{ki}, consequently $c_y^{ik} > c_{pd}^{ik}$. This last penalty is due to technological considerations: if a pump is switched on, it should operate with full pumping capacity if the available resource is enough. Nevertheless, in some periods, could happen that there is not enough water for pumping during the whole period. Therefore, in some cases the corresponding pumps will operate only during some part of the period considered.

The total costs expression is written as follow (43):

$$C^t(\Theta^t, x^t, v^t, d^t, r^t) = C^t(\bar{P}, x^t, v^t, d^t, r^t) = \sum_{(i,k)\in\bar{P}} c_p^{ik} x_{ik}^t + \sum_{i\in D} c_{pd}^i u_{pdi}^t +$$

$$+ \sum_{i\in D} c_{nd}^i u_{nd}^t + \sum_{k\in K_S^-} c_w^k x_{kS} + \sum_{i\in V} c_v^i v_{i-}^{t+1} + \sum_{(i,k)\in\bar{P}} c_y^{ik} (g^{ik} - x_{ik}^t) \quad (43)$$

This yields the following linear programming problem (44), which solution provides water flows able to minimize the costs incurred during a single period t.

$$\min_{\substack{v_i^{t+1}, v_{i-}^{t+1}, v_{i+}^{t+1} \geq 0, \\ x_{ik}^t, u_{pdi}^t, u_{ndi}^t \geq 0}} \begin{bmatrix} \sum_{(i,k)\in \bar{P}} \left(c_p^{ik} - c_y^{ik}\right) x_{ik}^t + \sum_{i\in D} c_{pd}^i u_{pdi}^t + \sum_{i\in D} c_{nd}^i u_{ndi}^t + \\ + \sum_{k\in K_S^-} c_w^k x_{kS} + \sum_{i\in V} c_v^i v_{i-}^{t+1} \end{bmatrix} \tag{44}$$

This objective function (44), subject to constraints (42 - 40), represents a specification of problem (22 - 23) defining the network configurations searching for optimal pumping thresholds. All components of this problem are defined by linear function, in order to manage a linear programming problem of small to medium dimension.

6.3.2. Pumping Schedules Constraints

In order to get an efficient and fast solution, each pump station operation has been managed by a class of *linear pumping rules* using volume fraction values. To each pump station $p \in P$ corresponds a single volume fraction q_p, that can continuously varying between 0 and 1.

Hence, the pump p is put into operation during the time-period t when, at the beginning of this period, the water volumes v_i collected in the reservoir i and weighted by the volume matrix elements h_{ip} (Table 11) are smaller than the maximal reservoir capacity v_i^{max} (reported in Table 6), weighted by the same matrix multiplied by the corresponding volume fraction (45).

$$\sum_{i\in V} h_{ip}\, v_i^t \leq q_p \sum_{i\in V} h_{ip}\, v_i^{max} \tag{45}$$

h_{ip} summarizes the functional dependences between volumes in reservoir i and pump p. Therefore, the pumping activation are defined and at each t step by pumping rules (29) with parameters q.

The set of operating pumping plants $\bar{P}(q, v^t)$ at time period t, which defines the water flow network configuration $\Theta(\bar{P}) = \Theta(q, v^t)$, is the set of all pumps, for which conditions (45) are satisfied.

$$\bar{P}(q, v^t) = \{p \mid p \in P,\ \sum_{i \in V} h_{ip}\, v_i^t \leq q_p \sum_{i \in V} h_{ip}\, v_i^{max}\} \quad (30)$$

The optimal values of thresholds are obtained by minimization of these costs averaged over the whole time horizon and the minimization of the steady state costs $C(q)$ as in equation (22).

Even introducing these linear constraints it is easier to solve huge stochastic problems, achieving a fast problem solution. In addition, through activation of thresholds parametrization, the problem related to a high number of decision variables can be efficaciously solved.

6.4. SQG Computational supports

The SQG model has been implemented using MATLAB *(MathWorks, 2017)* environment as main developing support. In MATLAB language has been written large part of the routine programming tasks, furthermore it has been interfaced with Cplex *(IBM, 2017)*, concerning the optimization tools, and Excel, about the input/output data management. Moreover, a user-friendly support has been build on the same spreadsheet of Excel, in order to guarantee an easier and intuitive process management. Further information about the model development are available in *Gaivoronski (2005)*.

6.5. SQG Recursive Simulation and Optimization

The SQG optimization approach has been applied to the extended time horizon in order to obtain the optimized threshold values. A relevant aspect has to be highlighted: SQG succeeds managing this extended time horizon, which was impossible with the scenario analysis approach due to excessive computational requirements, as reported in the *Chapter 5.4.2*.

This problem has been solved by a concurrent simulation and optimization, as previously described. The process has been implemented on the synthetically generated series, assigning as starting initial values the activation thresholds obtained by historical scenarios reported in Table 22. The South Sardinia water system configuration has been kept equal to the one used with scenario analysis approach. Running SQG, the same network configuration, hydrological series, unit costs were considered. The number of adopted iterations in SQG optimization is set equal to 1000, it means that the length of the total hydrological database will be equal to 636×10^4 periods.

The result is a new set of activation threshold values evaluated using the SQG method managing an operative cost minimization, a risk occurrences reduction and providing a new final configuration Θ^t of the water flows along the system's links, which are also dependent on the selection of parameters q and system state v^t.

The retrieved set of activation thresholds for pumping transfer activation, obtained using SQG and the extended hydrology are given in Table 27 in terms of water volumes. These pumping schedules highlight a reduction in terms of thresholds values, it means that the previous values reported in Table 22 have been futher optimized.

Table 27: Obtained Activation Thresholds Values Considering Synthetic Scenarios

Activation Threshold	S1	S2	S3	S4
[10^6 m^3]	0.271	72.48	2.775	21.316

SQG results provide the opportunity to evaluate successfully all supported costs and the objective function value along the extended time horizon composed by historical and synthetic database. These activation thresholds have been used in order to evaluate the new objective function value.

Table 28: Objective Function Results – Current Simulation and Optimization

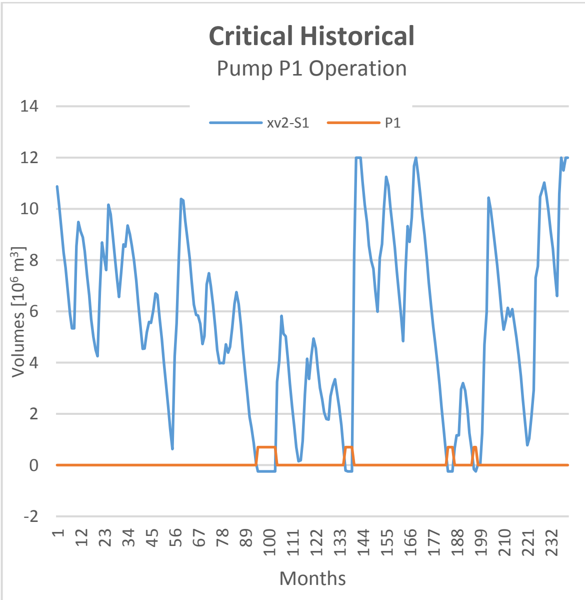
Flow element Mean Cost	10^6 € /month
Spilling	0.005
Shortage	0.231
Pumping	0.215
Total	0.451

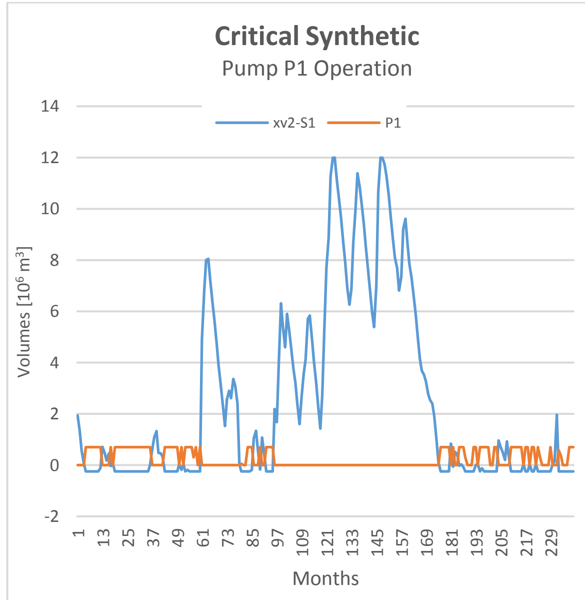
The total objective function value could be split in three main contributions: spilling, shortage (water deficit costs) and pumping. The deficit cost evaluation has been done considering both planned and unplanned deficits. Shortage and pumping costs have higher values, providing main contributions, which are reported in the Table 28. Moreover, as shown in the Table 29, historical and synthetic hydrological series are characterized by different criticality in terms of water resource availability. Considering the driest 20 years, the total average water input to reservoirs in synthetically generated series is equal to 293.83 10^6 m^3/year, significantly smaller than the average volume referred to the historical one, that equals 345.69 10^6 m^3/year. Comparison between input series at each reservoir are given in Table 29.

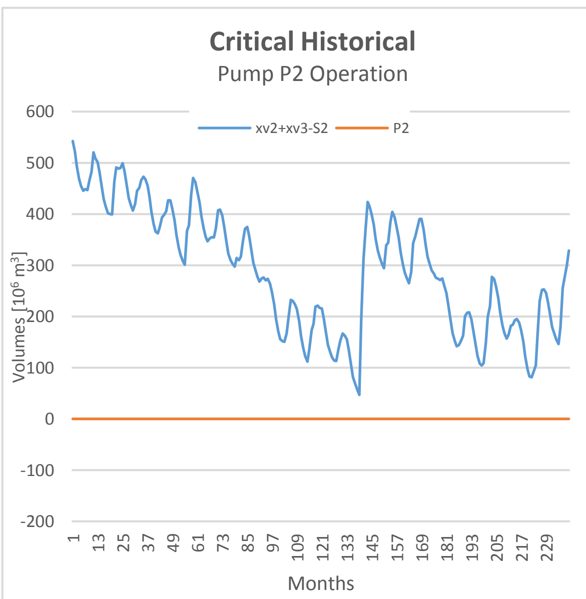
Table 29: Comparison of More Critical 20 Years between Historical and Synthetic Hydrologic Series

Average Annual Input to Reservoir [10^6 m^3]	Critical Historical						Critical Synthetic					
	R1	R2	R3	R4	R5	Σ	R1	R2	R3	R4	R5	Σ
	124.4	11.4	177.0	2.7	30.1	345.7	137.9	7.6	124.8	1.9	21.6	293.8

These more critical occurrences depict previsions in future climatic changes, taking into account heavier drought occurrences. Hydrological features can strongly affect the water system management and, specifically, the pumping behavior, as synthetically represented in Figure 25.


Critical Historical
Pump P1 Operation
xv2-S1
P1
Volumes [10⁶ m³]
14
12
10
8
6
4
2
0
-2
1 12 23 34 45 56 67 78 89 100 111 122 133 144 155 166 177 188 199 210 221 232
Months


Critical Synthetic
Pump P1 Operation
xv2-S1
P1
Volumes [10⁶ m³]
14
12
10
8
6
4
2
0
-2
1 13 25 37 49 61 73 85 97 109 121 133 145 157 169 181 193 205 217 229
Months


Critical Historical
Pump P2 Operation
xv2+xv3-S2
P2
Volumes [10⁶ m³]
600
500
400
300
200
100
0
-100
-200
1 13 25 37 49 61 73 85 97 109 121 133 145 157 169 181 193 205 217 229
Months

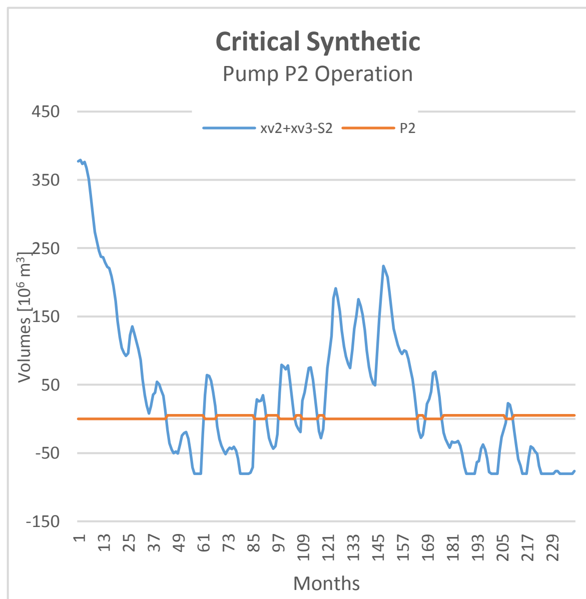

Critical Synthetic
Pump P2 Operation
xv2+xv3-S2
P2
Volumes [10⁶ m³]
450
350
250
150
50
-50
-150
1 13 25 37 49 61 73 85 97 109 121 133 145 157 169 181 193 205 217 229
Months

Figure 25: SQG – Pumped and Storage Volumes [10^6 m^3] (Minus Threshold Values)

Figure 25 highlights operation of the pumping stations in these more critical 20 years of historical and synthetic series. The blue lines represent the storage volumes, reduced by threshold values, while the orange line represent the pumped volumes, main differences in pump stations management are referred to P2 and, even in less measure, to P4. These pumping plants are always switched off during historical scenario, while along the synthetic time slot these pumps work during several months. The occurrences of more emergencies require activation of water pumping plants, which supposed to be used in order to move an additional flow to priority demand centers during droughts. Therefore, expanding the model inserting a synthetically generated hydrology allows to take into account drought effects and to test the efficiency of pump stations operation.

Water flows along the arcs referred to the thresholds values reported in Table 27 have been used in order to evaluate a real economic response of the system. Therefore, another economic post-processor has been constructed (such as has been done after the Scenario Analysis procedure) considering only the costs related to unplanned deficits and pumping operations. As before, the unplanned deficit elements refer to the additional costs supported during shortage periods, when demand requests can not be satisfied.

Table 30: Economic Post-Processor Results

Annual Average Costs	$C_{deficit}$	$C_{pumping}$	C_{TOT}
[10^6 €/year]	0.277	2.578	2.855

The costs reported in Table 30 are shown in terms of average annual values. Almost the total amount is due to energy contribution (pumping costs), while just a low contribution depends on the unplanned deficit occurrences, which means that almost the totality of system's users have been satisfied during the considered time horizon.

Moreover, comparing these SQG's results, reported in Table 30, to the cost values obtained using the Scenario Analysis Approach shown in Table 26, it is possible to highlight a reduction in terms of total costs, improving the water system's performance and saving around 0.7 million of euros/year. Overall, SQG succeed to solve efficaciously a large size water resource problem, which was not possible using the Scenario Analysis.

In Table 31 and Table 32 are reported all consumptions obtained by means of SQG, they are referred to each pump station and demand nodes (in terms unplanned deficit costs).

Table 31: Economic Post Processor – Pumping Costs

Pumping costs	P_1	P_2	P_3	P_4	Σ
[10^6 €/year]	0.181	0.854	1.315	0.228	2.578

Table 32: Economic Post Processor - Unplanned Deficit Costs

Unplanned deficit costs	D_1	D_2	D_3	D_4	D_5	D_6	Σ
[10^6 €/year]	0.0616	0.1085	0.0076	0.0493	0.0494	0.0002	0.27673

These results highlight a huge reduction in term of costs supported by the water system's Authority managing this real water system respect the current configuration, saving more than 1 million of euros/year and considering hydrological input variability and climate change impact.

6.6. Sensitivity Analysis

A system's sensitivity analysis has been then developed in order to measure the dependence of costs by the volume thresholds used defining pumping activation. Therefore, for each pump station, the obtained volume fraction q_p can be evaluated considering reservoir threshold values reported in Table 27 with relation to pump stations by dependences given in Table 11.

Volume fraction are expressed in terms of values varying between 0 and 1 and all the optimal values of volume fractions $q_p \in [0,1]$ are given in Table 33.

Table 33: Optimal Volume Fraction Values

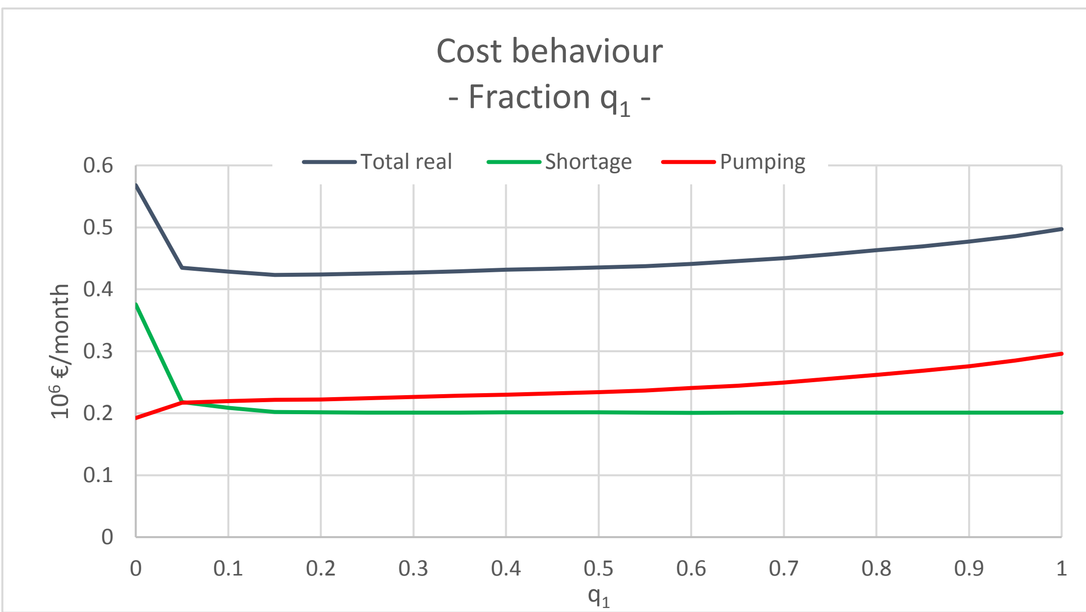
Volume Fractions [-]	q_1	q_2	q_3	q_4
	0.0201	0.1264	0.3061	0.0109

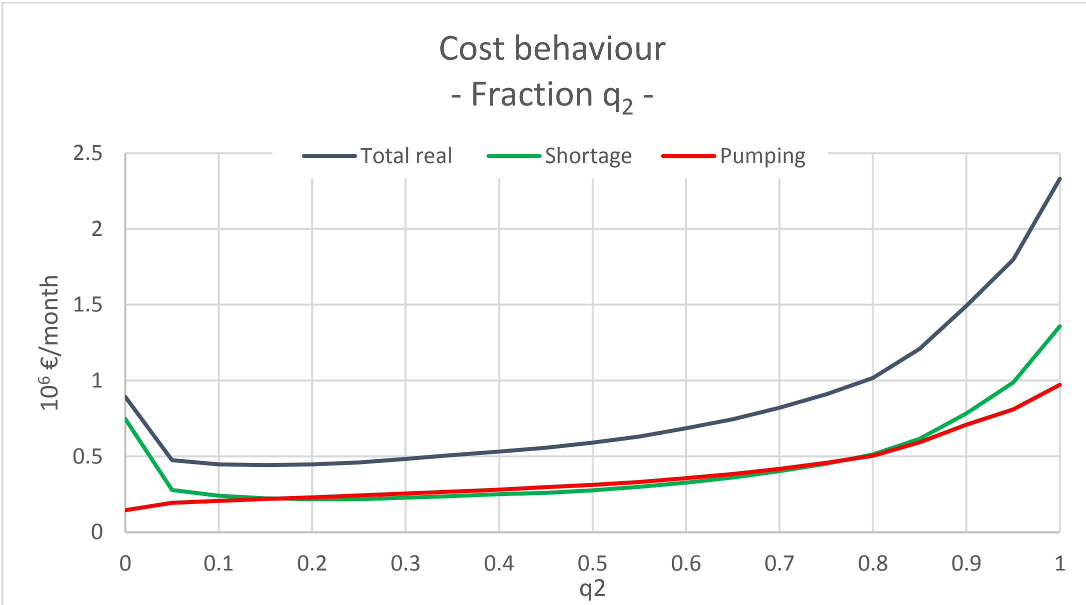
In order to check the additional cost that can be expected if the management authority modify the proposed optimal activation rule, an additional simulation process has been implemented varying, for each pump station $p \in P$, the single volume fraction q_p. During this process, the volume fractions for all other pumps will be kept constant and equal to the optimized values. Simulations related to selected pump has been varied between theoretical lower and upper bounds 0 and 1.

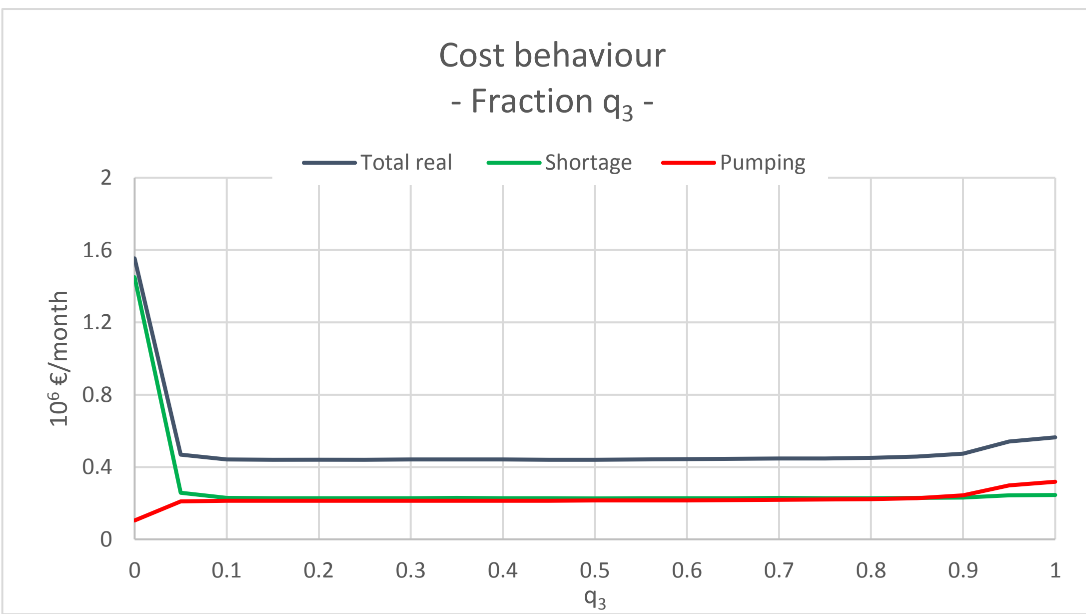
The costs behavior are reported in Figure 26: the blue line highlights the total costs varying pump fraction activation rules. As previously discussed, these costs should mainly be considered as sum of two contributions pumping costs (red line) and shortage costs (green line). Average monthly cost variations are reported in Figure 26.

The total cost functions have generally the tendency to increase smoothly using higher volume fraction values, related to optimal ones. Instead, this increasing behavior is stressed if the authority should use lower volume fraction values for pumps activation.

These behaviors can be easily justified, the right increasing values are, in a significant range, proportional to the energy-pumping cost and do not significantly varies from a gently slope and linear trend. The left increasing values are mainly related to shortages penalization that are generally growing with higher slope. This behavior of green line (shortage costs) for pump P4 is related to deficit occurrences in demand D2 if an excessively increasing water resource is diverted to other demands from reservoir R1.




Cost behaviour
- Fraction q_2 -
Total real
Shortage
Pumping
10^6 €/month
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
q2


Cost behaviour
- Fraction q_3 -
Total real
Shortage
Pumping
10^6 €/month
2
1.6
1.2
0.8
0.4
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
q_3

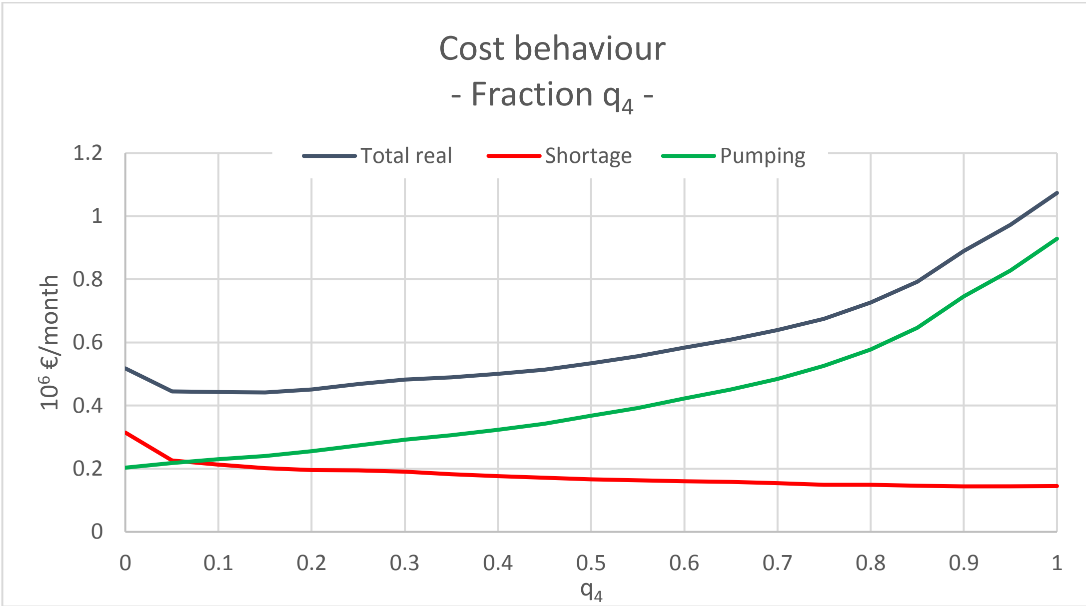


Figure 26: Real Costs Varying the Pumping Rules

These obtained results have a crucial importance in terms of decision support to the water system's Authority, giving an efficient tool for a correct the water pumping plants management.

7. Conclusions and Perspective

7.1. Conclusions

The main motivation of this thesis arises from economical and management issues by taking some strategical decisions about pumps activation rules under uncertainty. Therefore, providing an efficient DSS to the water system's authority in order to get a *robust decision policy* tool and referring to future climatic behavior.

The problem of optimizing pumping activation rules is related to a multi-reservoir water supply systems and has been analyzed using two optimization methodologies: *Scenario Analysis Approach* and *Stochastic Gradient Methods*. A trade-off between cost and risk elements has been considered: testing the effectiveness of emergency transfers to alleviate droughts requires early warning and activation and, on the other hand, the high operating costs of pump stations require a robust approach to define activation rules. The *Cost-Risk Balancing Approach* was developed in order to balance energy cost minimization requirements and the reduction of damage caused by water shortages. Both these optimization methodologies have been developed considering a historical and generated hydrological scenario occurrences.

A real-case application optimizing a water system in the South Sardinia (Italy) region was then developed. The considered scenario-tree was evaluated by composing 50 synthetic scenarios of equal length with the historically observed scenarios as common root. The historical database was also used during the optimization processes to define reference threshold values, while, in a subsequent step, managing uncertainty, generated scenarios were considered.

The *Scenario Optimization* provided *barycentric values* that define pumping activation thresholds on historical hydrological series. These results arise from a *two stage stochastic programming* and they are able to obtain an optimal solution taking into account historical series, defining efficient activation rules.

This approach highlights as GAMS could be considered as an excellent support during the model development. This software allowed writing easily optimization models, interfacing with Cplex solvers and Microsoft Excel for an efficient input and results representation. Nevertheless, extending the scenario-tree dimensions, when considering a synthetic database some computational problems occurred. They were due to some problems of complexity and model dimensions solving MIP problems.

The recursive simulation and optimization process based on the *SQG Methods* confirmed its potentialities even when applied to the management of water resource systems. The application on this field of research is a novelty because until now this method have been applied to supply chain problems, energy generation and financial applications.

This methodology allowed solving substantially larger models, extending the range of hydrological scenarios considered. Moreover, it provides an enhancement for the optimal solutions under large uncertainty defining a significant larger number of synthetic scenarios.

By considering an extended hydrological synthetically generated scenario, the SQG approach managed successfully the larger dimension model.

Both these approaches achieved a set of optimized pumping activation thresholds. In a post processor phase, pumping costs and unplanned deficit costs were evaluated in order to compute the real costs supported in water system management. This evaluation highlighted a significant costs reduction, when compared to the actual management of the system. The cost-risk balancing model could contextually reduce conflicts between users in shortage conditions. These methods guarantee almost the complete fulfilment of the barycentric values of new targets in water demands. Some unplanned deficit still remain but only for few periods of the considered time horizon.

Furthermore, SQG was able to provide an improvement in the values of total costs and lower computational time compared with the Scenario Optimization Approach developed in for the same type of problem.

In conclusion, the research developed in this thesis wants to provide a contribution in achieving some strategical information aimed to energy saving and to correct pumping stations management.

7.2. Future research

In prospect, on the basis of the achieved results and the limitations found in the development of this work, mainly the following possible improvements aspects in future research could be highlighted:

- to vary the weight p^g given to each hydrological scenario, increasing the weights assigned to the more critical scenarios, that are characterized by low hydrological input availability;
- to take into account in both models the evaporation losses;
- to identify two possible activation thresholds for each pumping plant located in the water system, in order to define a seasonal value: one related to the dry period and one to the wet period;
- to increase the mathematical modeling development using the Scenario Analysis Approach, adopting specialized decomposition techniques, in order to overcome the computational problems highlighted in this work;
- to improve these models adding some new constraints able to better describe the real behavior of the South Sardinia water supply system;
- to extend and generalize the modelling approach to water system networks in order to easily apply it to other zones, beyond the Sardinian region.

Particularly, an important aspect could be to interact with the water system's Authorities in order to evaluate obtained results by thorough comparison with the current management behavior, building a model more adherent to the reality.

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