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Authors: **Beatrice Venturi**, **venturi@unica.it , Department of Business and Economics, Italy, Giovanni Bella, bella@unic.it , Department of Business and Economics, Italy, Paolo Mattana, Department of Business and Economics, Italy.**

Contact Information

Please specify phone / cellular of presenting author 3317655342

Title :Shilnikov chaos in the Lucas model of endogenous growth

Abstract

This paper shows that chaotic dynamics may characterize the Lucas's (1988) two-sector continuous time endogenous growth model. Our "route to chaos" exploits the existence of a family of homoclinic orbits in the effective dimension spanned by the dynamics of the model. In this situation, Shilnikov (1965) proved that if the saddle quantity is positive, the homoclinic orbit shapes and organizes the flow of trajectories in a relatively wide region around it, determining irregular transitional dynamics and high sensitivity to initial conditions. Implications are noteworthy. On the one hand, our results imply a weakened role of the intertemporal equilibrium theory in providing indications about future economic conditions, given the initial state of the economy. On the other hand, there emerges the possibility of endogenizing interesting dynamic phenomena, such as irregular cycles and bursts of volatility in a quite standard growth setting. Over the last decades, the ability of the intertemporal equilibrium theory to provide indications about future economic conditions, given the initial state of the economy, has been questioned in many relevant aspects. In the specific field of the two-sector, continuous-time, endogenous growth model, an influential literature has clearly established that in the presence of a non-competitive element (often an externality), the determinacy of the rational expectation equilibrium, namely the uniqueness of the collection of agents' actions corresponding to a given fundamentals, is not warranted. In other words, given the aggregate capital stock, and despite the standard assumption of perfect foresight, multiple equilibria can easily take place, even in plausible regions of the parameter space. The phenomenon can occur either locally (cf, inter al., Chamley, 1993; Benhabib and Farmer, 1994; 1996, Benhabib and Perli, 1994, Benhabib et al., 1994; Benhabib et al. 2000), when multiple equilibria are located in a close neighborhood of the steady state, or, as shown more recently, in the large (cf. inter al. see also Mattana and Venturi , 1999; Fiaschi and Sordi , 2002, Venturi , 2014; Bella and Mattana, 2014) over a global range of initial growth rates. A more subtle erosion of the ability of the intertemporal equilibrium theory to provide indications about future economic conditions in the same setting is the possibility of complex behavior of the Ramsey-Euler system of equations arising from intertemporal maximization. The presence of complexity implies in fact deep unpredictability (Grandmont, 1985) that has been argued to totally undermine the possibility of rational expectations in economics: infinite precision is required in the measurements of initial conditions for the motion to be fully predicted even qualitatively. In general terms, a dynamical system is complex if it endogenously does not tend asymptotically to a fixed point, a limit cycle, or an explosion (Day, 1994); on the contrary, it can give rise to a number of interesting phenomena, such as nonperiodic fluctuations, mixing cycles, switches and so on. This article is aimed at developing some results on the existence of chaotic behavior in a quite familiar context: the continuous-time standard Lucas endogenous-growth model (1988). The Ramsey-Euler conditions arising from Lucas's model imply a non-linear three-dimensional system of first order conditions which is already know to possesses a rich spectrum of dynamic behavior that goes, as the parameters of the model are tuned, from a stable equilibrium point to a Hopf cycles, either super-critical or sub-critical. Our route to chaos exploits the existence of a homoclinic orbit to a saddle-focus equilibrium. The striking complexity of the dynamics near these homoclinic orbits has been discovered and investigated by Shilnikov. This involves hyperbolic horseshoes close to the homoclinic orbit, but possibly also periodic attractors and strange attractors.

Keywords: Multiple steady states, Homoclinic bifurcations, Oscillating solutions, Chaos Attractor

JEL classification: C61, C62, E32

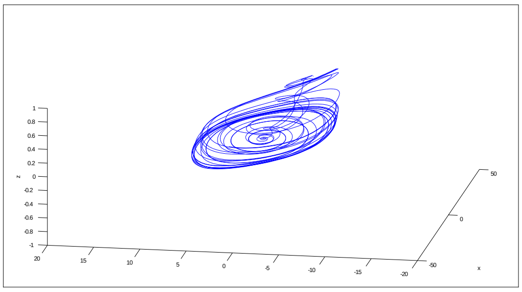

Fig. 1: The Lucas attractor

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