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Book of Abstracts

11th Chaotic Modeling and Simulation International Conference

Editor

Christos H. Skiadas

Rome, Italy, 5-8 June 2018

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Preface

11th Chaotic Modeling and Simulation International Conference

5 – 8 June 2018, Rome, Italy

It is our pleasure to welcome the guests, participants and contributors to the 11th International Conference (CHAOS2018) on Chaotic Modeling, Simulation and Applications. We support the study of nonlinear systems and dynamics in an interdisciplinary research field and very interesting applications will be presented. We intend to provide a widely selected forum to exchange ideas, methods, and techniques in the field of Nonlinear Dynamics, Chaos, Fractals and their applications in General Science and in Engineering Sciences.

The principal aim of CHAOS2018 International Conference is to expand the development of the theories of the applied nonlinear field, the methods and the empirical data and computer techniques, and the best theoretical achievements of chaotic theory as well.

Chaotic Modeling and Simulation Conferences continue to grow considerably from year to year thus making a well established platform to present and disseminate new scientific findings and interesting applications. We thank all the contributors to the success of this conference and especially the authors of this *Book of Abstracts* of CHAOS 2018.

Special thanks to the Plenary, Keynote and Invited Presentations, the Scientific Committee, the ISAST Committee and Yiannis Dimotikalis, the Conference Secretary Mary Karadima and all the members of the Secretariat.

May 2018
Christos H. Skiadas
Conference Chair

Honorary Committee and Scientific Advisors

International Scientific Committee

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Universita` di Roma 1, Rome, Italy
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Friction and irreversibility in Navier-Stokes fluids: nonequilibrium ensembles

Giovanni Gallavotti
Universita` di Roma 1

It is presented a fluid obeying a reversible version of NS equations

The Paradox of Vito Volterra's Predator-Prey Model

Jean-Marc Ginoux[1,2]
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The aim of this article is to propose on the one hand a brief history of modelling starting from the works of Fibonacci, Robert Malthus, Pierre Francis Verhulst and then Vito Volterra and, on the other hand, to present the main hypotheses of the very famous but very little known predator-prey model elaborated in the 1920s by Volterra in order to solve a problem posed by his son-in-law, Umberto D'Ancona. It is thus shown that, contrary to a widely-held notion, Volterra's model is realistic and his seminal work laid the groundwork for modern population dynamics and mathematical ecology, including seasonality, migration, pollution and more.

Keywords: Vito Volterra · Population dynamics · Predator-prey model · Alfred Lotka · Henri Poincaré · Malthusian growth · Holling function

Do we Need a New Physics to Describe Human Behaviour? Phenomenological Standpoint

Ihor Lubashevsky
University of Aizu, Japan

First, by this talk I pose a challenging question about which mathematical formalism and possibly new physical notions should be developed for quantitatively describing human cognition and behavior, in addition to the ones already developed in the physical and cognitive sciences. Indeed, physics is widely used in modeling social systems, where, in particular, new branches of science such as sociophysics and econophysics have arisen. However, many if not most characteristic features of humans like willingness, emotions, memory, future prediction, and moral norms, to name but a few, are not yet properly reflected in the paradigms of physical thought and theory. The choice of a relevant formalism for modeling mental phenomena requires the comprehension of the general philosophical questions related to the mind-body problem. Plausible answers to these questions are analyzed and illustrated.

As the key point the two-component approach is discussed in detail. This approach may be treated as a particular implementation of the general research direction called phenomenology in philosophy of mind. The gist of two-component approach is introduction of objective and subjective components possessing individual properties and governed by own laws. The interaction of these components endows human behavior with complex properties not met in the physical realm.

Second, the concept of “strong emergence via constitutive fields” is developed as a plausible solution to the challenging problem of how novel properties appear in complex systems where none of their elements possesses them. This class of phenomena arising in physical systems has to be categorized as weak emergence, however other points of view are not excluded. Here I propose an original mechanism of strong emergence assuming that the superposition principle can be inapplicable to describing mental phenomena.

Third, I discuss a number of emergent phenomena that can arise only in physics of human mind. Actually they illustrate that physic of human mind should be regarded as an individual branch of science with its own laws and subjects of investigation. The main attention is focused on the theory of non-equilibrium phase transitions caused by the bounded capacity of human cognition. Because humans cannot recognize the difference between time moments separated by short time intervals and the difference between states of given system close in properties (concept of complex present) the notion of stationary point of dynamical systems becomes implacable to describing human actions. The concept of dynamical traps is proposed as a generalization of stationary point for this description. A number of non-equilibrium phase transitions of novel type

that can be found in systems with dynamical traps are analyzed. I also present experimental and theoretical results of our experiments on inverted pendulum balancing and car-driving. In these experiments human intermittent control--modern paradigm of human actions in controlling unstable systems--plays the leading role. As demonstrated the effects and regularities found in the presented experiments are direct consequences of complex present properties.

Keywords: Phenomenology, Mind, Intentionality, Two-component description, Emergence, Complex dynamics, High-dimensional phase space

Alternative Approach to Treatment of Separatrix Chaos: 10 Years of Development

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We will survey a relatively new approach to the theoretical treatment of chaotic layers replacing separatrices of Hamiltonian systems when they are periodically perturbed. The approach, originated by some of us 10 years ago, is based on an incorporation of the resonant Hamiltonian method into the separatrix map dynamics. The approach is of a considerable scientific and practical value. As for the former, it has already led to two major achievements. Firstly, it has allowed us to crack a problem that lay unsolved for 40 years – to accurately describe the maximum width of the chaotic layer as function of the perturbation frequency and, moreover, to show that former qualitative estimates were incorrect for a broad class of systems: the maximal width is much larger than was thought before, due to a resonant region being involved in chaotic motion. Secondly, the approach gave a possibility to explicitly describe the phenomenon of the drastic facilitation of the inter-separatrix transport (IST) onset discovered in 2003 (Phys. Rev. Lett. 90, 174101 (2003). This general phenomenon may play a particularly important role for example in the cross-jet transport in meandering flows, e.g. such as Gulf Stream or the high atmosphere jet stream, many major features of which (such as chaotic advection) may be described at least qualitatively by Hamiltonian models like von Karman vortex street. The practical importance of the approach is that it suggests very general explicit formulas allowing one to immediately predict optimal conditions for the IST onset and other relevant features of chaotic advection while such an analysis by means of computer simulations takes huge time.

The survey will be based on two major papers in Physical Review (PRE 77, 036221 (2008) and PRE 80, 066212 (2009)), a major chapter in the book “Hamiltonian Chaos Beyond the KAM Theory” (ed. A. Luo, Higher Education Press, Bejing, 2010, Chapter 2), a brief review in Fluct. Noise Lett. 11, 1240002 (2012), a brief paper in proceedings of the ICNF-2013 conference (art. no. 6578911) and two new unpublished works.

Keywords: Separatrix Chaos, Separatrix Map, Resonant Hamiltonian Dynamics, Chaotic Layer, Inter-Separatrix Transport, cross-jet transport, advection, von Karman vortex street

Influence of Topology and Absorption on Properties of Quantum Graphs and Microwave Networks

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In this talk we will discuss the experimental and numerical studies of the influence of topology and dissipation on properties of microwave networks and quantum graphs.
We present the results for the long-range fluctuation properties in the spectra of quantum graphs and microwave networks with chaotic classical dynamics and preserved or broken time-reversal invariance [1-3]. We demonstrate that the properties of microwave networks are extremely sensitive on the boundary conditions.
We also analyze the dependence of the enhancement factor and the distributions of the imaginary and the real parts of the Wigner reaction K matrix on absorption of the microwave networks with broken time reversal symmetry.

Acknowledgements: *This work was partially supported by the Ministry of Science and Higher Education Grant No. UMO-2016/23/B/ST2/03979.*

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On the Structure of the Solutions of a Resource Optimal Model

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The purpose of this paper is to illustrate that stable limit cycles represent a possible, generic equilibrium strategy in a renewable resource model. Following Wirl (2004), Bella (2010) Kogan, Venturi and Shnaiderman (2017), Bella, Mattana and Venturi (2017), we consider an optimal system with an increasing pollution and a quick depletion of the reserves. We put our model in a reduced form: a system of three first order non –linear differential equations with one pre-determined variable (a combination of the state variables) and two no pre-determined variables (related to the control variables). By using instruments of the global analysis: bifurcation theory, we are able to show as a stable cycle and complex dynamics can occur in a set for a change in some parameters. A numerical simulation is given to support our theoretical results.

Keywords: multiple steady states, oscillating solutions, double scroll.
JEL classifcation: C61, C62, E32

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Invited Talks

Modelling the Brain: From Dynamical Complexity to Neural Synchronisation, Chimera-like States and Information Flow Capacity

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In this talk, I will present a review of my recent work on the study of the brain, aiming to reveal relations between neural synchronisation patterns and information flow capacity, namely the largest amount of information per time unit that can be transmitted between the different parts of the brain networks considered. I will start with the working hypothesis, presented in Ref. [1] and supported by numerical evidence, that brains might evolve based on the principle of the maximisation of their internal information flow capacity. In this regard, we have found that synchronous behaviour and information flow capacity of the evolved networks reproduce well the same behaviours observed in the brain dynamical networks of the Caenorhabditis elegans (C.elegans) soil worm and humans. Then, I will talk about the verification of our hypothesis by showing that Hindmarsh-Rose (HR) neural networks evolved with coupling strengths that maximise the information flow capacity are those with the closest graph distance to the brain networks of C.elegans and humans. Finally, I will present results from a recently published paper [2] on spectacular neural synchronisation phenomenon observed in modular neural networks such as in the C.elegans brain network, called chimera-like states. I will show that, under some assumptions, neurons of different communities of the brain network of the C.elegans soil worm equipped with HR dynamics are able to synchronise with themselves whereas others, belonging to other communities, remain essentially desynchronised, a situation that changes dynamically in time.

Keywords: Brain modelling, Neural synchronisation, Chaotic behaviour, Information flow capacity, C.elegans, Hindmarsh-Rose neural dynamics, Chimera-like states.

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Instability of Flow between Rotating Disks

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Multi-phase flow between dynamically spaced and inclined rotating disks leads to instabilities that have significant implications in industrial operations. As the rotating speed increases, there is a potential for a catastrophic increase of the hydrodynamic and contact torque that is not well understood due the chaotic motion of the disks.

Typically, the system of multiple rotating disks is exposed to atmospheric pressure, thus at high rotational speeds air is entrained in the gap between the disks. In a pack with multiple disks, there is also a random movement of the disks in the axial direction, and a deviation from parallel alignment, thus the gap between any two disks is a stochastic variable. Finally, when an external force is applied to the system, the fluid is squeezed out of the gap resulting in solid contact. This leads to rapid temperature rise, the porous media on the disk surface are deformed squeezing fluid out of the gap, and the disks become engaged. The reverse process takes place when the disks are released, returning to the original chaotic open state.

During this process of alternating states, i.e. open, engagement, release, the flow remains laminar due to the small size of the gap. However, at high rotational speeds, uneven pressure distribution is created in the grooves of the porous material covering the disks, which results in flow reversal at the boundaries, and air entrainment. Because the gap between the disks changes at a time scale much smaller than the engagement-release cycle, a chaotic formation and collapse of air bubbles occurs. Bubbles may be forming in one pair of disks while collapsing in another and the flow is characterized by complete disorder. Gravity also plays a significant role, as fluid jets exiting the grooves are deflected. This results in submergence of the lower groves while the higher ones are exposed to atmospheric pressure. Inevitably, due to the high speed of rotation, all grooves are exposed to a periodic submergence at yet another time scale.

A multi-physics model is presented that attempts to simulate the phenomena that take place inside a disk pack during the engagement-release cycle. Visualization tests are also presented along with direct drag measurements. Although it is possible to validate the time-averaged results of the model, it is difficult to produce repeatable simulations. The flow behavior is erratic, and depends highly on the initial perturbations given to the flow. No instability is noticed in single-phase flow, thus the problem is believed to be a result of the chaotic formation of air bubbles in the variable gap between the rotating disks.

Keywords: Numerical modeling, Squeeze-film flow, Rotating disks, Asperity contact.

Complex Dynamics in the Generalized Lorenz System

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Dynamics of magnetized fluids is much more complex than expected from the standard magnetohydrodynamics. Besides chaotic behavior that often appears in nonlinear dynamical systems, hyperchaotic motions are also possible on new strange attractors [3], with bifurcations which could result in irregular intermittent behaviour [4]. Surprisingly, all this complex dynamics can be studied by analyzing a simple set of four ordinary differential equations describing hydromagnetic convection [2], which is the generalization of the famous Lorenz system for the case of turbulent convection in a fluid layer with the embedded magnetic field [1]. This provides a novel contribution to chaos theory that could be of interest to the dynamical systems community.

Keywords: Chaos, Hyperchaos, Convection, Magnetohydrodynamics, Intermittency, Turbulence.

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Intelligence (Life) as a Universal Transformation Mechanism of Chaos into Harmony

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Despite intensive development of modern science, intelligence (life) remains an unformalized and undefined scientific concept. This fundamentally delegitimizes all scientific research and development both as intellectual phenomena and science in general as a product of an exclusively intellectual activity. For the first time universalization allows to solve this problem by way of transition from the system of particular scientific concepts (axioms, dogmas) to the only single universe's axiom and derived universal system of concepts and to reveal fundamental intellectual properties that have utmost importance for all fundamental research and applied developments as a new general scientific paradigm. The initial concept of universalization made it possible to derive from the

highest universe's concepts the meta- definition, structure and formalism of the intelligence as a universal nature's property that is independent of material realization. The intelligence meta-purpose is substantiated as an active transformation of chaos into harmony for eliminating disharmony and ensuring the effective existence and development of universe's phenomena. The obtained formalism is universal for the entire living world and becomes a new standard of intelligence instead of the former unformalized concept of a human being with the widest range of application. The formalization is carried out by means of the simplest modified ER-diagrams allowing an exact set-theoretical interpretation. The basic properties of the meta-formalism of intelligence as a universal mechanism for chaos transformation into harmony and its main applications for various realizations, including the protozoan vegetable and animal worlds, humanitarian sphere, artificial intelligence, and thinking machines are investigated in this paper. The intellectual meta-formalism is completely consistent with the universal theory and is an important part of the universal cosmogony that, for the first time, externally and internally consistently explains our universe. The application of the universal chaotic theory of intelligence promises to obtain fundamentally new results in many applications.

Keywords: Universalization, Intelligence, Life, Chaos, Harmony

Numerical Methods for Approximating Long-Time Statistical Properties of Chaotic Systems

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We consider numerical algorithms that are able to capture long-time statistical properties of large dissipative turbulent systems. It turns out that the key is for the convergence of long-time statistical properties is to have the numerical algorithms mimic the dissipative properties of the original dynamical system in some appropriate fashion. We arrive at a Lax-equivalence type theorem which states that dissipativity and convergence of the algorithm would guarantee the convergence of the long-time statistical properties. Applications to classical dissipative fluid models will be presented as well.

Special Sessions Talks & Contributed Talks

Gravitational Waves, Relic Photons and Higgs Boson in a Fractal Models of the Universe

Valeriy S. Abramov
Donetsk Institute for Physics and Engineering named after A.A. Galkin, Ukraine

Models for describing of the separate large-scale fractal structures of the Universe are proposed. The relationships between the parameters of gravitational waves, relic photons and the Higgs boson are established. Estimates of these parameters are given on examples: merging of two black holes; binary neutron stars; "Cold relict spot" (supervoid). The behavior of deformation fields on the fractal index for a number of quantum model systems with variable parameters is investigated. It is shown that the presence of nonlinear oscillations is characteristic for a fractal layer without a quantum dot. Stochastic behavior for the boundaries of the quantum dots cores is observed, an anisotropy effect is possible.

Keywords: Large-scale fractal structures of the Universe, Higgs boson, Gravitational waves, Relic photons, Black holes, Binary neutron stars.

Effect of Ordering of Displacement Fields Operators of Separate Quantum Dots, Elliptical Cylinders on the Deformation Field of the Coupled Fractal Structures

Olga P. Abramova, Andrii V. Abramov
Donetsk National University, Ukraine

By the numerical modelling method the behavior of the deformation field of the coupled fractal structure with quantum dots and elliptical cylinders was investigated. It is shown that the resulting deformation field of the structure with a number of quantum dots larger than two essentially depends on the ordering of the displacement fields operators for separate quantum dots. The coupled fractal structure with elliptical cylinders is characterized by the presence of a complex deformation field. Using zero operators for pairs of quantum dots makes it possible to obtain information about separate quantum dots. Transposition of pairs of operators allows us to transfer information from one quantum dot to another.

Keywords: Coupled fractal structures, Quantum dots, Elliptical cylinders, Deformation field, Ordering of operators, Numerical modelling.

Some Points about Kulbak-Leibler Entropy Evolution in Stochastic Dynamic Systems

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This paper continues the theme regarding applications of statistical radiophysics concepts to dynamical systems theory, some results have been described at work [1]. Let's have a look at the stochastic system of ordinary differential equations of the form:

$$\dot{x} = f(x) + \xi(t), \quad x, \xi \in R^d,$$

$$\xi(t) >= 0, \quad \xi_i(t) \cdot \xi_j(t') >= 2 \cdot D_{ij} \cdot \delta(t - t'), \quad i, j = \overline{1, d} \tag{1}$$

corresponding Fokker-Planck-Kolmogorov (FPK) equation for the probability density $p(x,t)$ can be written as:

$$\frac{\partial p}{\partial t} + \sum_{i=1}^{d} \frac{\partial (p \cdot f_i)}{\partial x_i} = \sum_{i=1}^{d} \sum_{j=1}^{d} D_{ij} \cdot \frac{\partial^2 p}{\partial x_i \partial x_j}, \quad p(x,0) = p_0(x) \tag{2}$$

Kulbak-Leibler entropy corresponding to solution on equation above can expressed as [2]:

$$K[p_0; t] = \int p(x,t) \cdot \ln \frac{p(x,t)}{p_0(x)} \cdot d^d x \tag{3}$$

As is known [2], the Kullback-Leibler entropy is the distance in the metric space of multidimensional normal distributions with the same covariance matrices, therefore the functional (3) can be treated as some time-dependent measure of the proximity of the FPC equation solution (2) to its initial probability distribution $p_0(x)$.

In order to estimate the Kullback-Leibler entropy (3) of the dynamical system (1), it is necessary to know the function $p(x,t)$, which can be found by means of Krasovsky's series method [3]. According to this concept, the solution of the FPK equation (2) can be found as:

$$\ln p(x,t) = A_0(t) + \sum_{k=1}^{\infty} \sum_{i_1, \ldots, i_k = 1}^{d} A_{i_1 \ldots i_k}(t) \cdot x_{i_1} \cdot \ldots \cdot x_{i_k} \tag{4}$$

We are able to restrict series (4) to members of power N in order to approximate function $A_{i_1 \ldots i_k}(t)$ this case we obtain a self-consistent system of ordinary differential equations, initial conditions for such system

will be able to obtained from the expansion of $\ln p_0(x)$. Since the order of this system grows rapidly depending on d и N, for numerical solution this system we can use technology CUDA for parallel computing presented by Nvidia company [4]. Kullback-Leibler entropy can be estimated by means of Monte Carlo method [5]. In order to show usefulness using of the Kullback-Leibler entropy for the dynamical systems theory current paper shows dependence of the functional (3) on time for various simple systems with dimension d = 1 and different initial conditions.

Keywords: Statistical radiophysics, Fokker-Planck-Kolmogorov equation, Kullback-Leibler entropy, dynamical systems theory, differential equations.

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Transition to Chaos by Intermittency Related to the Nonlinear Dynamics of Non-Concentric Multiple Double Layers in Low-Temperature Plasma

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When a complex space charge structure in form of simple or multiple double layers generated in in low-temperature plasma passes into a dynamic state, nonlinear oscillations of the plasma parameters (such as plasma potential, discharge current, plasma density, etc.) are recorded and chaotic states often develops in certain experimental conditions. Here we report on the experimental observation of a transition to chaos by intermittency, related to the nonlinear dynamics of a non-concentric multiple double layer structure (multiple fireballs) developed in front of a positively biased supplementary electrode immersed into discharge plasma. The intermittency scenario was identified in the time series of the oscillations of the current collected by the supplementary electrode, when the discharge voltage was modified and its value is close to that of the

voltage applied on the supplementary electrode. A fractional theoretical model able to describe the intermittency phenomenon in plasma was developed, based on the continued fraction mathematical formalism. The chaotic behavior of the plasma is obtained as an overlapping of the fractal behaviors of the double layers composing the multiple structure.

Keywords: Intermittency, Fireball, Instability, Continued fraction.

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Effect of Inclination and Number of Prandtl on Chaotic Roads in Different Cavities

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We have studied numerically the transitional laminar natural convection in four ratio cavities of form A = 1. The first cavity contains air, the second cavity contains the water for these two cavities we have highlighted the effects of the number of Prandtl
The third cavity and the fourth contain a nanofluid which is copper oxide (CuO). One of the cavities is inclined with 30 degrees. The horizontal walls are adiabatic and the vertical walls are composed of two regions of the same size maintained at different temperatures. The transfer equations are solved using the (flow function - volatility formulation). We analyzed the effect of the Rayleigh number on heat transfer and on the roads to the chaos borrowed by the system for the four cavities. The systems constituted by the cavities mentioned are deterministic. the multiplicity of solutions is represented by attractors in the phase spaces. We compared the results obtained between the four cavities.

Keywords: Natural convection, Bifurcation, Attractor, Nanofluid.

A Nonlinear Behavior of Robert Disc Dynamo with Fractal Property

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In this article, the complex dynamics of a Robert disc dynamo is studied in detail. A Robert disc dynamo is a self-existing disc dynamo that is supposed to be the cause of the magnetic field of Earth, Sun and stars. The stability analysis and chaoticity of the attractor of the Robert disc dynamo is discussed in detail. With the help of fractal process based on the Robert disc dynamo, a multi-wing modified disc dynamo is obtained

that gives a 2n-wing modified disc dynamo system. The Hamiltonian energy function of modified disc dynamo concludes that the energy is decreased as the number of multi-wing increases.

Keywords: Robert Disc dynamo, Nonlinear Behavior, Fractal property, Hamiltonian energy function

Simulation of Streeter-Phelps Model with Missing and Extreme Reading of Biochemical Oxygen Demand

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Biochemical oxygen demand (BOD) is the amount of oxygen required for microbial and Environmental Life in water. BOD is more significance to water quality. The measurement of (BOD) in waste water was and still the most importance measurement, any un true reading for (BOD) make highly pollution levels in water.

This research deal with (discovery ,estimate and forecasting) of (missing and extreme) values in (BOD) reading by design a non-linear simulation system with an efficient estimation method and Streeter-Phelps model , The extremes effects in parameters readings in both BOD (Biological Oxygen Demands) and DO(Dissolved Oxygen) can caused error estimating for the model's parameter that using in determine ratio of de oxygenation and re oxygenation of the dissolved oxygen(DO),then that will caused launch big amounts of the sewage pollution

Experimental results show the effects of parameter estimators and the mean square errors (MSE) that belong to it by the different ratio of extremes. Further work with other estimating methods (robust, Bayesian) can make a different mean square error values and different (BOD, DO) estimators especially more than (20%) ratio.

Keywords: Biochemical Oxygen Demand, Dissolved Oxygen, Mean Square Error, Streeter-Phelps Model, Simulation Experiments, Missing Values, Extreme Values.

Chaos Beyond Observability

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The presented work resulted from the investigation of a generic class of models which appear naturally when dealing with PWM controlled DC/AC and AC/DC power converters. By contrast to DC/DC converters, whose dynamics is governed by one high frequency (switching frequency), a generic feature of DC/AC and AC/DC converters is the presence of two vastly different frequencies governing their dynamics, namely the high switching frequency and the low frequency of the AC signal. This leads to a number of unusual dynamic effects, such as transitions to chaos via irregular cascades of border collisions [1], structures formed by persistence border collisions inside the domain of a regular dynamics [2], and a global alignment of boundaries related to smooth bifurcations [3]. In addition, it has been already reported in [4] that chaotic attractors in the considered class of models may undergo a number of unusual transformations: some bands of multi-band attractors suddenly disappear, bifurcations are observed at parameter values quite different from what can be expected, and so on. All these effects motivate us to investigate chaotic attractors in the considered models, even if they do not correspond to the normal working regimes of the underlying circuits.

We demonstrate that the unusual effects mentioned above can be explained considering the invariant density of the chaotic attractors. Indeed, it turns out that some parts of involved chaotic attractors are associated with extremely low invariant density which is far beyond observability in physical experiments and in many cases in numerical experiments as well. Introducing first a stroboscopic map related to the period of the high frequency oscillations, and eventually a secondary stroboscopic map related to the period of the low frequency oscillations, we explain the mechanism leading to the appearance of such low density parts of chaotic attractors. Since this mechanism is related to the ratio between the frequencies governing the dynamics, we conclude that similar effects can be expected in other systems with two vastly different frequencies.

Keywords: Chaotic attractors, invariant density, power converters.

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A Secure OFDM Transmission Coding Scheme Based on 3-Dimensional Chaos Shift Keying OQPSK Modulation

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Chaotic communication system is one of the latest information transmission method. Because of rapid advancement in the wireless communications, applications of chaos coding is becoming very important. Because of chaotic phenomenon being non-periodic and having wide bandwidth, it can be used in the transmission of the ultra-wideband(UWB) communication signals.
Due to the weaknesses of one dimensional chaotic operators where an introdore can decode the trasmitted information by controlling parameter sesitivity range of the transmitted signal.For this reason we have considered a new coding(encryption) scheme where a three dimensional chaotic generator based on the Loren'z model have been considered.To do that orthogonal modulated signals(OFDM) are coded using OQPSK modulation.The encoding is carried out using discritization of the Loren'z model based on the principal of wavelet (OFDM)modulation.This method provides robust coding scheme where the simulations are also is given to verify the new type of chaos coding.

Keywords: Secure chaos coding, OQPSK modulation, 3-dimensional transmission, wavelet OFDM, robust.

Chaos-Based SoC for Securing Fingerprint Authentication Systems

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In this paper, a new chaos-based System on Chip (SoC) approach to protect fingerprint biometric models against cryptanalysis attacks is analyzed and presented. This one is based on the co-design methodology hardware/software (HW/SW) by using Field Programmable Gate array technology (FPGA). The proposed stream cipher cryptosystem is based on the chaotic sequence of the Lorenz system. Simulation and experimental results clearly demonstrate that the proposed biometric authentication system provide a good trade-off between security and performance requirements in terms of efficiency and cost. This system could therefore be integrated into a secure access control system in real time embedded applications.

Keywords: Chaos, Lorenz, biometric, Fingerprint, security, authentication, FPGA, SoC.

Solvable Probabilistic Cellular Automaton on Bethe lattice with Smooth Transition between Exponential and Inverse-Power Distribution of Avalanches

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We introduce and investigate the Random Domino Automata model – an extension of Drossel-Schwable forest-fire model – on Bethe lattice.
Introducing appropriate notion, we derive a set of equations describing a stationary state of the automaton in mean-field approximation and solve the system for specific family of parameters. We found that there exist two bounds, related to inverse power and exponential distribution of avalanches, between which there is a smooth transition between those distribution and outside which the system is unstable.
We point out a generalization of Motzkin numbers recurrence based on obtained equation for cluster size distribution for the proposed system.
Keywords: Random Domino Automata, Drossel-Schwabl forest fire model, stochastic process, self-organized criticality, Motzkin numbers recurrence, stationary state, inverse power and exponetial distribution.

Perturbation Effect of Aliphatic Alcohols on the Dynamical Regime of a Briggs-Rauscher Reaction

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The present study pertains to the dynamic evolution of the oscillatory Briggs Rauscher (BR) reaction in presence of various alcohols added singly (methanol, ethanol, n-propanol, isopropanol, tert-butanol) as well as in binary mixed form (sec-butanol + ethanol and isobutanol + ethanol) at 30oC under CSTR conditions. The oscillations were observed due to the change in $[I^-]$ with time. The results obtained reveal that the course of the BR reaction is significantly perturbed, depending on the concentration of the alcohol. The mechanism of alcohol perturbation has been interpreted on the basis of spectroscopy and electroanalytical techniques.

Keywords: Briggs Rauscher reaction, Dushmann reaction, perturbation, iodination

Sliding Mode Control with Fuzzy Boundary Layer for Chaotic Dynamical System

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Chaos is a very interesting and important phenomenon in the nonlinear systems. The basic characteristic of a chaotic system is to be sensitive depending on initial conditions. Over the past decades, many well known and new chaotic sytems have been proposed. Although, in recent years, much more complicated control methods have been studied to control the chaos, the objective of this paper is to use a relatively simple SMC technique with a boundary layer is obtained by fuzzy technique which the thickness of the layer is considered in a thin range to avoid the chattering effect. This method applied some chaotic models and the simulation results are appropriate the aim of this study and convenient to the future researches in this area.

Keywords: Sliding Mode Control, Dynamical Chaotic, Modeling, Fuzzy Boundary Layer, Simulation Study, Relatively Simple Method

Study of the Dynamic of FMO Complex with the Chaos Theory and the Temperature Effect on the Conductivity of Exciton

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Photosynthesis is a fundamental biological process that provides the primary source of energy for almost all terrestrial life. In this process, photons are absorbed by Chlorosome in an antenna complex, leading to the formation of excitons. These then migrate by hopping procedure through chromophores to a reaction center where the exciton's energy is used to release an electron. Remarkably, these processes often have a quantum efficiency of almost 98 % and uncovering the underlying biological design principles could inspire important new developments in artificial light-harvesting technologies. We use the Frenkel approach to model the exciton transition. The natural complex systems are open quantum systems and we can't eliminate environment effect.
Our aim is to investigate the photosynthetic system with the phonon bath, and the effect of temperature on exciton transfer. Using the quantum chaos, dynamic analysis of the system is possible. The photosynthetic system is a complex system that can be described in the form of random matrix theory (RMT). RMT systems follow three statistical distributions: GOE, GUE, and GSE that can be characterized by the quantum chaos.

The relation of random matrix theory and statistical distributions made by standard level spacing distributions (P(s)).
For a particular natural complex, FMO, our desire is to show that this system is stable against the exciton transition and with the temperature rising, it becomes more conductive.
For this aim, we use the quantum chaos approach: random matrix theory (RMT) and to check the stability of the system, and the transfer matrix theory and the localization length to study the conductivity of exciton.
The results confirm that FMO, despite the environmental effects has a stable statics for exciton transition.

Keywords: exciton, FMO, quantum chaos, random matrix theory, level spacing distributions, transfer matrix, localization length, conductivity

A Quantum Chaos Approach for Localization in Disordered Single-Walled Carbon Nanotube

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Carbon nanotubes (CNTs) are a kind of tubular structure that was discovered and produced by Iijima in the early 90s. Since then, the unique structure of nanotubes has been attracting the interest of the nanoscience communities. It was predicted that these nanostructures would have interesting and extraordinary properties, and then find many applications in nanotechnology. An SWNT can be viewed as a hollow cylinder obtained by rolling a single layer of graphite sheet and it can be determined by the vector (n,m). SWNTs can be divided into three chirality types by the different vectors (n,m): armchair nanotube (m,m), the zigzag nanotube (n,0) and chiral nanotube (n,m). Theoretical studies of electrical properties have displayed that these quasi-one-dimensional (1D) nanostructure materials can be either semiconducting or metallic. Electronic transport in one dimensional (1D) systems is very sensitive to the presence and type of disorder. Nitrogen (N) and boron (B) chemical substitutions are popular since they only slightly modify the atomic structure of the carbon network. In this study, we consider a single-walled armchair carbon nanotube in the presence of a random distribution of the substitutional boron atoms. We try to explore the effect of boron atoms on the localization properties of Hamiltonian eigenstates by applying level spacing distribution as a cornerstone of random matrix theory (RMT). Random matrix theory provides a striking signature for the appearance of quantum chaos (QC). By comparing the obtained results with the predictions of random matrix theory we find that for pure graphene (no disorder) the obtained P(s) is independent of the size and fits into a curve which has mixed chaotic and integrable states. It is similar to the intermediate statistics. For weak disorder we obtained large localization lengths that it can exceed the

system size and can diffuse from one end of the system to the other, then the corresponding energy level-spacing distribution is chaotic (Wigner). For strong concentration one can observers the flow of the spacing distribution towards Poisson –like.

Keywords: Canon nanotube, Random matrix theory, Quantum chaos, Wigner distribution, Poisson distribution, Impurity.

Many-Body Quantum Chaos in Strong Nuclear Force Analysis

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In interacting quantum Many-Body systems, like an atomic nucleus, quantum chaos theory is a suitable method to investigate of spectral statistics information. Statistical properties of energy levels give integrability or chaotic properties of Hamiltonian. Random matrix theory (RMT) is one of the appropriate methods for statistical analysis of energy levels in quantum Many-Body systems. With calculate eigenvalues and eigenvectors of nuclei based on nuclear shell model Hamiltonian, Brody distribution Pβ(s) and Σ2(L) provide further information about the nuclear force nature. For more conclude the results one can use also localization length (LH). In this study, we choose two different isotopes Ca46 and Ti46 with a different type of particles in valance space. These cases are in the Fp model space with Ca40 as an inert core. The fit of the computed energy levels is in general quite good with the experimental spectrum. The obtained eigenvalues should be unfolded and this procedure is done for each j state. The Ca46 with proton closed shell and six neutrons in valance space is a neutron- rich isotope and in the Ti46, both neutron and proton exist in the valence shells. With statistical analysis, Ca46 shows behavior close to Poissonian ensemble but the strong residual interaction between neutron-proton in Ti46 destroys the regularity and the behavior of the system move to Wignarian distribution. Results show that the nucleon motion in the spherical mean field is regular but residual interaction can destroy this regularity.

Keywords: Quantum chaos, Many-Body system, Nuclear shell model, Nuclear structure, Strong force.

Transport Properties of a DNA Transistor in the Presence of a Thermal Bath

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Device miniaturization is an inevitable trend in semiconductor industry development. Due to the nanometer scale and other potential superiorities, organic molecules have become one of the favorable candidates to substitute silicon-based semiconductor devices such as transistors in the future. Transistors, regardless of their size, rely on electrical gates to control the conductance between source and drain contacts. In atomic-scale transistors, this conductance is sensitive to single electrons hopping via individual orbitals. The electronic transport properties of molecules have attracted more and more attention for their possible uses as functional units for electronic devices. The most common and important function of single molecular devices is to control the charge transport properties effectively. In this work, we have tried to study the transport properties of a DNA based transistor in the presence of a thermal bath. Here, DNA chain is attached to the left and right metal leads as source and drain. DNA chain is in the contact of a thermal bath. The effect of bath parameters is studied via the quantum chaos tools. One of the most applied tools in the study of spectral correlations is the nearest-neighbor spacing distribution (P(s)). In the metallic phase, P(s) is very close to the random matrix theory developed by Wigner and Dyson. In the insulating regime, the electron levels of the strongly localized states fluctuate like random stochastic variables and the levels are distributed according to the Poisson law. Using this method, we can determine that the bath is an important factor on conducting properties of a DNA templated transistor. On the other hand, one can choose the appropriate bathe parameters for improvement of efficiency of a DNA transistor. Therefore, we can design a molecular transistor based on DNA chains with improved efficiency.

Keywords: DNA based transistor, Thermal bath, Quantum chaos, Nearest-neighbor spacing distribution.

Chaotic Transport of Interacting Particles in a Stokes Flow

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This study investigates the chaotic transport of suspended particles pumped periodically through a periodically modulated micro-tube. In a previous work, the motion of a single particle displays different non-linear

mechanisms where symmetry-breaking and routes to chaos lead to either a chaotic transport or, in contrast, a synchronized transport with the pumping. In this paper, we consider two particles interacting via the Stokes flow perturbation. We focus on the influence of this interaction when chaotic transport exists for a single transport. The particles interaction may trigger the chaotic behavior but also destroy it leading to a synchronized transport. The analysis of these phenomena is performed using continuation of periodic states and bifurcation analysis.

Keywords: Chaotic transport, synchronization, intermittency, Bifurcations, particle sorting, Stokes flow.

A New Robust Chaotic Map-Based RFID Authentication Scheme

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Radio Frequency identification technology (RFID) is one of the modern widespread emerging technologies proven to advance a wide variety of applications in both academia and industry. Yet, potential security risks, privacy issues and efficiency are still open challenges that must be considered seriously to accomplish higher protection. In this paper, we proposed a new robust solution based on chaotic map one way function that could satisfy the challenging constraints of passive RFID tags. Analyses show that the proposed solution can effectively resist the most common attacks and is more efficient than the existing protocols in the literature.

Keywords: RFID, Chaotic map, authentication, security

Dynamical Models Reconstructed from Time Series in Application to Revealing Structure of Oscillatory Ensemble

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Reconstruction of model equations from chaotic time series of experimentally observed quantities provides a researcher with a number of opportunities. A researcher can assess adequacy of available concepts about an object and its dynamical mechan-isms, predict its temporal behavior and variations under changes of parameters, etc. Experience in modeling from data series shows that an algorithm of obtaining a feasible

model should take into account specific features of an object, while universal approaches are usually less fruitful [B.P. Bezruchko, D.A. Smirnov, “Extracting Know-ledge From Time Series: An Introduction to Nonlinear Empirical Modeling” (Springer, Berlin – Heidelberg, 2010)]. This work gene-ralizes results of our research in that direction in application to spa-tially extended systems, in particular, to estimating structure and directionality of couplings in oscillatory ensembles under different limitations and deficiencies of data series.

Keywords: chaotic time series, reconstruction of equations from time series, structure of oscillatory ensembles, coupling estimation
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Equivalence (or Lack thereof) of Non-Equilibrium Ensembles in Multiscale Chaotic Systems

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Understanding under which conditions it is possible to construct equivalent ensembles is key to advancing our ability to connect microscopic and macroscopic properties of non-equilibrium statistical mechanical systems. In the case of fluid dynamical systems, this issue boils down to testing whether different mathematical models for the dissipation of energy at small scale lead to the same macroscopic properties of the fluid systems, in the limit of very large Reynolds number. Such models include both the standard choice of constant viscosity, and more exotic ones that have, nonetheless, the merit of maintaining the time-symmetry of the evolution equations, which holds in the microscopic description.
We present an investigation of the equivalence of non-equilibrium ensembles for toy models of fluid flows that have the merit of featuring multiple scales of motion, namely the two-level Lorenz '96 model and a shell model for turbulence. We find that the equivalence between the statistical ensembles emerging from the time-irreversible and time-reversible equations of motion holds to a high degree of accuracy only when the reversible model is subject to a constraint impacting preferentially the smallest and fastest scales of the system. In this case, the dynamics of the large, energy containing, scales is virtually indistinguishable between the two ensembles.

Keywords: Non-equilibrium, Time reversal symmetry, Statistical ensemble equivalence, Fluid dynamics models.

Percolation Process in the Presence of Velocity Fluctuations: Two-loop Approximation

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Critical behaviour of directed bond percolation is studied in presence of turbulent mixing [1,2]. The turbulent advecting velocity field is assumed to be incompressible and generated by the Kraichnan model. The model is studied by means of field-theoretic approach. The renormalization group (RG) method is used in order to analyse asymptotic large-scale behavior of the model near its critical point and to calculate perturbatively all fixed RG points and critical exponents in the framework of double-expansion scheme [3]. The renormalization procedure is performed to the next-to-leading order of the perturbation theory. The emphasis is placed on the details of two-loop calculations of the Feynman diagrams.

Keywords: Directed bond percolation process, Kraichnan model, Perturbative renormalization group.

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Phase Transition in Incompressible Active Fluid: Effect of Long-Range Interactions

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Phase transitions in active fluids attracted a significant attention in the last two decades. In the recent work [L. Chen et al., New J. Phys. 17, 042002 (2015)] authors showed that an order-disorder phase transition in incompressible active fluid belongs to a new universality class and suggested a potential experimental realization to be systems with long-range(LR) interactions. In this work, we study these effects by introducing non-local stress into the microscopic description of the model. Using methods of field-theoretic renormalization group we investigate the large-

scale properties around the critical point. In contrast to the suggestions of previous authors, we found that the effect of LR interactions can change the universality class to the Model A class with LR interactions or destroy the relevance of the non-linearities completely which leads to the mean-field values for critical exponents.

Keywords: active fluid, anomalous scaling, field-theoretic renormalization group, long-range interactions

Robustness of Chimera Order in Spin Systems

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Following up on a recent report about chimera ordering in Monte Carlo simulations of a three-dimensional Ising spin model [R. Singh, S. Dasgupta and S. Sinha, \Chimera order in spin systems" , EPL 95 10004 (2011)], further simulations are performed to ascertain the validity and the applicability of the initial report. In the initial work only nearest-neighbour interactions were considered within a cubic lattice of spins, having ferromagnetic interaction between spins within the same plane and anti-ferromagnet interaction between spins in adjacent planes. This model was proposed as a prototype for certain layered magnetic systems, such as FeCl2. In the present work two main deficiencies in the initial work are corrected. First, use is made of parallel tempering to explore whether or not the reported chimera ordering could be real, as opposed to being a simulation artefact. This step is necessary because it is well known that, at lower temperatures, Monte Carlo simulations can get stuck in local minima of the energy landscape. Furthermore, in the case of frustrated systems, such as the one that was considered, the energy landscape is known to be much more complicated, usually containing many local minima. Second, the initial model of a cubic lattice of spins is extended to the layers of a face-centered cubic structure, that is more appropriate for FeCl2. Our extended model thus takes into account the more correct coupling topology and exchange interactions. We find that, even with the application of parallel tempering, and in the extended model, the chimera ordering that was previously reported, persists. Chimera ordering that is more reminiscent of classic chimera states is also observed.

Keywords: Chimera states, Self-organized systems, Lattice theory and statistics, Quantized spin models, quantum spin frustration.

The Positive Influence of Allee Effect on Synchronization of von Bertalanffy' Models

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The main purpose of this work is to study the relationship between the Allee effect and the synchronization. In general it was believed that, due to competition for resources, a population will have a reduced growth rate at higher densities and increased growth rate at lower densities. However, Warder Clyde Allee introduced the idea that the reverse is true. When the population density is low, the growth rate is reduced and there is a critical population size, the Allee point, below which the population becomes extinct. The extinction of a population is not always a fact to avoid. Indeed, extinguishing a population of cancer cells is an important goal. When the population density is low, individuals require the help of others to survive, and it has been repeatedly reported by some biologists that there is a positive influence of the Allee effect on the cooperation, or synchronism, of populations. Using Bertalanffys models, in which were introduced Allee effect factors, we study the evolution of the synchronizability when the Allee point increases. In fact, our numerical results show that, when the Allee effect gets stronger, the synchronization improves. Considering any fixed network, coupling several nodes having in each one the same dynamical system modeled by Bertalanffy's equation, we observed that the synchronization begin at a lower value of the coupling parameter and the amplitude of the synchronization interval becomes larger. These results confirm the experimental observations of biologists.

Keywords: Synchronization, Allee effect, von Bertalanffy's models, Symbolic dynamics, Chaotic region.

Structural-Phase Weakly Stable States of CuZn and NiAl Alloys with Antiphase Boundaries Complexes

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Using the Monte Carlo method [1-3], it is shown that the presence of a dual defect in the form of a pair antiphase boundaries in an ordered FCC-alloy with B2 supersructure (CuZn and NiAl alloys taken as an example) gives rise to considerable structural-phase variations in the system during the order – disorder transition compared to the defect-free system. Were shown structural changes of the alloy near the antiphase boundaries with

increasing temperature, leading to their smearing and faceting. The structural violation of the order at low temperatures are observed only in the areas of the border crossing in the alloys with AFG complex in the <110> direction. Antiphase boundaries in the <110> direction positively affect the stability of the NiAl alloy during heating. In an alloy with a complex of antiphase boundaries in the <100> direction, the first structural changes in the CuZn alloy always appear near the Zn-Zn boundary and in the NiAl alloy - near the Al-Al boundary.

Keywords: Monte-Carlo, phase transformations, order, disorder, superstructure, B2, BCC-alloys, crystal.

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Stochastic Space-Time: A New Perspective on the "Ether-Drift" Experiments

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Basic theoretical aspects of both quantum physics and relativity indicate that space-time might have the fundamental stochastic nature of a turbulent fluid. This picture, if taken seriously, might have phenomenological implications for the “ether-drift” experiments (since Michelson-Morley to the modern ones with optical resonators). Indeed, if there were a preferred reference frame, e.g. the system where the Cosmic Microwave Background (CMB) is exactly isotropic, the microscopic velocity field, which determines light anisotropy at the laboratory level, may differ sizeably from the macroscopic velocity field, as directly determined by the earth's cosmic motion. This would produce deviations from a standard Fourier analysis of the data (with only smooth time modulations produced by the earth's rotation and its orbital revolution) and, therefore, make non trivial to separate a genuine physical signal from spurious instrumental noise. With these premises, we have considered a theoretical framework where Lorentz covariance is exact when light propagates in an ideal vacuum and where, within the analogy of a turbulence which becomes homogeneous and isotropic at small scales, one allows for random fluctuations of the drift around the average earth's motion. Our numerical simulations, performed in this scheme, show that

the small irregular residuals observed in the various experiments can now become consistent with the same earth's motion indicated by the direct CMB observations with aircrafts and satellites. In view of the substantial implications, for both physics and the history of science, new dedicated experiments and new methods of analysis are needed for a definite clarification.

Keywords: Stochastic space-time; numerical simulation of turbulent flow; ether-drift experiments; Cosmic Microwawe Background.

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Complex Networks Tools for the Analysis of Diagnostic Time Series in Nuclear Fusion

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The control of instabilities, such as Edge Localized Modes (ELM) and sawteeth, is considered an important ingredient in the development of reactor-relevant scenarios. One route for controlling these instabilities is by pacing, which consists of triggering them frequently enough that they do not have time to reach harmful proportions. Both instabilities are quasi periodic and therefore, after the perturbation induced by the control systems, if enough time is allowed to pass, the instabilities are bound to reoccur. This makes quite difficult the task of assessing whether an ELM or sawtooth crash has been really triggered by the control system or is the consequence of the natural evolution of the plasma dynamics. Given the nature of the experiments and the quality of the available signals, it is not possible to determine the causality at the level of individual perturbations and instability crashes, using only data analysis tools. Since quantitative and adequately validated physics models are not available, an alternative strategy consists of determining, on the basis of time series analysis, the time horizon over which the perturbations are effective in triggering the instabilities. Various advanced statistical techniques, such as Transfer Entropy, Granger Causality and Recurrence Plots, have been recently

proposed to address this issue. In the present work complex networks are used to assess the causal-effect relationship between time series. The transformation of the experimenatl time series into complex network representations allow to characterize their underlying dynamics by means of topological features. Cross-Visibility Networks, an extension of the Visibility Graphs, is used for determining the coupling between time series. The definition of connection weighting allows the use of the Geodesic Distance on Gaussian Manifolds, a metric explicitly developed for the analysis of experimental data corrupted by additive Gaussian noise. In cases affected by particularly high levels of noise, compromising the traditional treatments, the use of the Geodesic Distance on Gaussian Manifolds allows deriving quite encouraging results. In a different approach the synchronization is detected by identifying changes in the community structures of complex networks by means of the WalkTrap algorithm.

Keywords: Time series, Complex networks, Tokamaks, Plasma instabilities control, Cross-Visibility Network.

The Asymptotic Coupling Method in the Study of Ergodicity of Equicontinuous Markov Operators

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Our main goal is to establish a slight, but, as it turns out, practically useful generalization of the criterion on asymptotic stability for equicontinuous Markov operators given by T. Szarek in [4].

By *Markov operator* we mean any operator $f \mapsto Pf$ acting on bounded, measurable real-valued functions which is generated by the transition probability kernel $(x, A) \mapsto P(x, A)$ of a Markov chain (evolving in a Polish space), so that $Pf = \int f(y)P(\cdot, dy)$. Such an operator P is called *equicontinuous* whenever the family $\{P^n f : n \in \mathbb{N}\}$ is equicontinuous for every Lipschitz function f. This property can be viewed as a weaker form of the non-expansiveness with respect to the dual bounded Lipschitz distance (known as the Fortet–Mourier metric).

The aforementioned result of T. Szarek ([4, Theorem 3.3]) says that, if P is an equicontinuous Markov operator with the Feller property (i.e. P preserves continuity of functions), and there exists a state z such that

$$\inf_x \liminf_{n\to\infty} P^n(x, U) > 0$$

for every open neighbourhood U of z, then P is *asymptotically stable*, i.e. it has a unique invariant distribution π, and the sequence $\left\{\int P^n(x,\cdot)\mu(dx)\right\}_{n\in\mathbb{N}}$ is weakly convergent to π for each probability measure μ. We strengthen this result (see [1]) by relaxing the equicontinuity condition. To be precise, instead of requiring the equicontinuity of P itself,

we assume the existence of an equicontinuous sub-stochastic transition kernel $(x, A) \mapsto Q(x, A)$ such that $Q \leq P$ and $Q^n(\cdot, X)$ does not vanish on some open neighbourhood of z, as $n \to \infty$.
The idea behind the proof of this result, which will be an important component of the talk,
is inspired by paper [3], and pertains to the asymptotic coupling method, introduced by Hairer in [2].
The result proves to be useful in verifying the asymptotic stability of some specific classes of random iterated function systems with place-dependent probabilities. It enables us, for instance, to provide a stability criterion for a system which is not necessarily contracting on average and can contain non-Lipschitz unbounded transformations.

Keywords: Markov e-chain, equicontinuous Markov operator, Invariant measure, Asymptotic stability, Ergodicity, Asymptotic coupling, Iterated Function System.

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Dynamical Systems & Psychology: Mind as Machine

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This essay introduces a radical departure from current cognitive psychological models, and introduces an information processing approach to human behavior development or Mind as Machine. The new cognitive behavior model is theoretically grounded by the research of Thelen & Smith (1994) on non-linear dynamic systems, and integrates the terms; attractors, control parameter and bifurcation. Briefly, Thelen and Smith identified human behavior as complex patterns or sub-systems emerging from a few simple rules, and the interdependency and interaction between these patterns, emerging behavior and these rules. The new Model's dynamic, round and interconnected framework addresses the concepts of emergent order and complexity by demonstrating how patterns of behavior arise from the interaction between the three elements of the Model which together operate as an information processing unit. A basic introduction of non-linear dynamics systems and Jung's two-step Model of the psyche are provided as the empirical backdrop grounding the Model's dynamic framework. Terminology

relative to dynamic system theory and the new Model are introduced, as well as the assumptions and the rules that bindand structure the Model. Then, the three components that operationalize the new Model are defined and accommodated within the Model's framework, consisting of three interconnected circles, that together function as a unit or Mind as Machine. Human behavior an information processing unit is then introduced as a simple mathematical model which represents the dynamics of the system as an ordinary differential equation. Showing the causal linkage between the three elements of Mind as Machine, as a cognitive next generation cognitive framework where the "confluence of the observed with the observer" can be evidenced (Blackerby, 1998, p. 4)

Discontinuity of Light Scattering According to the Size of the Droplets and the Suspended Particles of the Atmosphere. The Transferred Energy Packs

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Rigorous scattering of the sunlight can be take place, if, in global atmosphere there are droplets and particles of arbitrary sizes. Large particles call for a way of looking at scattering problems that is quite different from the way of treating small particles. The basic and fundamental fact is that the incident beam of light, which forms a plane wave front of infinite extent, may be thought to consist of separate rays of light which pursue their own path.
According to Maxwell's theory, light carries momentum as well as energy pacs.
Momentum = Energy/C where c is the velocity of light. The momentum removed from the original light beam is proportional to Cext. The total energy scattered in all directions called Csca. The energy that is absorbed inside the particle called Cabs.
The law of conservation of energy gives: Cext = Csca + Cabs.
The quantities Cext, Csca, Cabs, are called, the cross sections of the particle for extinction, scattering and absorption, respectively.
They are functions of the orientation of the particle and the state of polarization of the incident light.

Reduced-Order Modeling of the "Fluidic Pinball"

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The “fluidic pinball” is a geometrically simple configuration with three rotating cylinders on the vertex of an equilateral triangle. Yet, it remains physically rich enough to host a range of interacting frequencies and to allow testing of control laws within minutes on a laptop. The system has multiple inputs (the three cylinders can independently rotate around their axis) and multiple outputs (downstream velocity sensors).
Investigating the natural flow dynamics, we found that the first unsteady transition undergone by the wake flow, when increasing the Reynolds number, is a Hopf bifurcation leading to the usual time-periodic vortex shedding phenomenon, typical of cylinder wake flows, in which the mean flow field preserves axial symmetry. The second bifurcation breaks the symmetry of the mean flow field, and results from a secondary pitchfork bifurcation. In this work, we propose reduced-order models (ROMs) for both bifurcations.
We have extracted dynamically consistent modes from the flow data in order to build our ROMs. The main dynamical features of the primary Hopf bifurcation can be described by three degrees of freedom. For the secondary bifurcation, at least five degrees of freedom are expected to be required in order to model the flow dynamics. Beyond their ability to identify the main physical features of the flow dynamics, ROMs also provide a simplified (and therefore faster) framework to train machine learning (flow) control.

Keywords: fluid mechanics, flow control, reduced-order modelling, transition to chaos.

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Lie Transform Normalization of Hamiltonian System with Quartic Perturbation

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The Lie Transform method is applied to find the normal form of Hamiltonian system with 2 degree of freedom including quartic nonlinearity. The unperturbed frequencies of the Hamiltonian System obey 1:1 resonance relation. The algorithm used to find the normal forms also yields us formal integrals approximately. Furthermore, the KAM Curves

obtained numerically are compared with analytical results obtained from series expansion.

Keywords: Lie Transform method, Normal form, Hamiltonian system, Poincare Section

Bifurcation Analysis of Dynamical Complexity of Signals During Antinociceptive Effect Emergence

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We examine the changes in dynamical complexity of nociceptive signals during the emergence of the antinociceptive effect. To answer the question how the patterns of impulse activity of nociceptive neurons are modified during the suppression of neuropathic pain by comenic acid (5-hydroxy-γ-pyrone-2-carboxilic acid) we apply methods of bifurcation analysis for the dorsal root ganglion (DRG) neuron model. Unlike with our previous work [Dick, 2016], where the ectopic bursting activity in the DRG model was determined by the dynamics of two slow variables, we exclude the influence of the channels of the delayed potassium current. It is connected with the evidence that neuronal trauma decreases mRNA expression of these channels and, vice versa, the significance of the TTX-resistant NaV1.8 sodium channels enhances. Under the conditions of potassium blocking we have found the mechanisms of changes of oscillation modes after the analgesic modification of the NaV1.8 sodium channels.

Keywords: Bifurcation analysis, Bursting discharge, Suppression of neuropathic pain, Potassium blocking

Self-Modulated Oscillations in the Dynamics of a Hollow Grid Cathode Discharge Plasma

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Nonlinear dynamics of a low-temperature discharge plasma obtained by using a spherical grid cathode with a hole was experimentally investigated by analyzing the oscillations of the discharge current. The experimental results revealed the presence of phenomena like bistability, hysteresis, patterns formation, but also the self-modulation of the discharge current due to the interaction between complex space charge structures existing inside and around the hollow grid cathode. In order to describe the experimentally observed phenomena, a theoretical model was developed in the frame of Scale Relativity Theory, by introducing the scale resolution

in the expression of all physical variables and the dynamics equations and admitting that the fractalization is of stochastic type, given by means of Markovian processes. The dynamics of the electrons is described using a forced damped oscillator model, leading to a Riccati type differential equation. The results provided by the model are in good agreement with the experimental findings.

Keywords: Self-modulated oscillations, Scale Relativity Theory, Complex space charge structures, Fractality.

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Simulation of Rating Data Distribution Using Entropy Analytics

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In this paper the distribution of rating data is investigated using simulated and real data. Starting from Multinomial distribution and max entropy distribution principle the related Geometric and Binomial distributions are the limiting distributions of rating data. The behavior of those limiting distributions derived by nonlinear constrained optimization explained presenting the iso-entropic lines and limiting curves. Real data of five star ratings (k=4) from TripAdvisor and movies ratings from IMDB (k=9) used as real data samples.

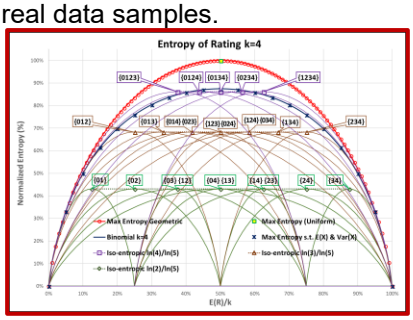

Fig 1: Five Star Rating Entropy

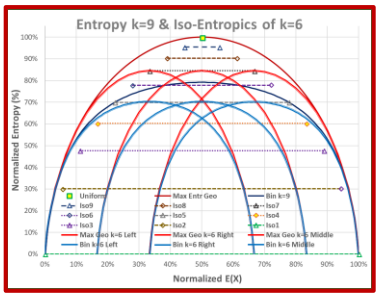

Fig 2: 10 scale Rating Entropy

Keywords: Simulation, Maximum Entropy, Multinomial Distribution, Geometric Distribution, Binomial Distribution, Rating Data, Nonlinear Constrained Optimization, TripAdvisor, IMDB.

Generation, Analysis and FPGA Implementation of Multi-Wing Chaotic System with Complex Topological Structure

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This paper firstly constructed a two dimensional continuous autonomous system with many equilibria points, which can produce multi-wing three dimensional chaotic system with complex topological structure after dimension raising. The dynamic mechanism of this system was explained from the perspective of index theory. Moreover, in order to verify the existence of the chaotic system, the analysis of basic dynamics and topological horseshoe are done to the system, thus the chaotic system is strictly verified from the mathematical point of view. Finally, to verify the chaos behavior of the system, DSP Builder is used to design a circuit and convert it to VHDL language program which was download to hardware circuit through Quartus II software. The experimental results are in good agreement with the computer simulation results.

Keywords: Index theory, Topological structure, Multi-wing chaotic attractor, Lyapunov exponents, Topological horseshoe, FPGA.

Perpetual Points in Nonlinear Dynamical Systems

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Perpetual points (PPs) are a new type of critical points in dynamical systems, defined as the ones in which the magnitude of acceleration of system becomes zero, while velocity remains non-zero. We describe properties of perpetual points and their potential use in finding co-existing attractors in considered models, especially the ones exhibiting unpredictable dynamics, like hidden or rare oscillations. The concept of PPs is introduced in different types of representative systems, i.e. damped oscillators, pendula and discrete-time maps, suggesting that perpetual points may become a general tool in studies of complex nonlinear problems.

Keywords: perpetual points, hidden attractors, multistability

Stability and Chaos in Fractional (with Power-Law Memory) Systems

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Systems with power-law memory are relevant to the research in many areas of natural and social sciences. In this presentation we propose a method to define the range of stability of fixed points for a variety of discrete fractional systems of the order $0<\alpha<2$. The method is tested on various forms of fractional generalizations of the standard and logistic maps. Based on our analysis we make a conjecture that chaos is impossible in the corresponding continuous fractional systems. We also review various forms of fractional attractors including a new type of attractors – cascade of bifurcations type trajectories and present examples of two-dimensional bifurcation diagrams for discrete fractional systems.

Keywords: Power-law menory, Fractional attractors, Cascade of bifurcarions, Asymptotic stability, Fractional/fractional difference α-families of maps.

Spectral Statistics for Double-Spherical Cavity Resonators through the Mode Decomposition Method

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We develop a theoretical method for solving Maxwell's equations to obtain the frequency spectra of inhomogeneous and asymmetric cavity resonators using a couple of effective Debye-type potentials. The structure we study specifically is the layered spherical cavity resonator with symmetrically or asymmetrically inserted inner dielectric sphere. The comparison of the exact numerical results obtained for the frequency spectrum of layered cavity resonator with centrosymmetrically inserted sphere and the spectrum found from the suggested theory reveals good agreement at the initial part of the frequency axis. The coincidence accuracy depends on the number of trial resonant modes that we use while approving our method numerically.

Keywords: Wave chaos, Cavity microwave resonator, Electromagnetic oscillations, Debye potentials.

Hidden Bifurcations in Chen System Multiscroll

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We present a new technique to discovery hidden bifurcations in the multispiral Chen attractor. This technique is based on the mind idea of the genuine Leonov and Kuznetov way for seeking hidden attractrs but applied in a extremely different manner where the numbers of spiral happening is discrete.
In this case such hidden bifurcations are dominated by a parameter ε, this added which is not exist from the initial problem is completely made to reveal the real structure of multispiral attractor. We study perfectly the multispiral Chen attractor, generated via function series, and a verification numerically our technique for unusual and equal the number of spiral from 1 to 6.

Keywords: system, multispiral attractors, hidden bifurcations

Linear Structures Extraction in Chaotic Time Series under Neural Network Teräsvirta Test

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Despite the relative simplicity of the governing equations, chaotic dynamical systems can generate signals characterized by extremely divergent and complex patterns. For analytic or more practical purposes (e.g. forecasting), they are usually studied, with different degrees of success, within the framework of the non-linear paradigm. In fact, linear equations are generally confined to an ancillary role - or ruled out altogether - given their inadequacy in effectively describing the dynamics involved and the provided outcomes, which are necessarily sub-optimal.
However, indubitably the class of linear models shows not negligible advantages over its non-linear counterpart, e.g. in terms of interpretation, easier mathematical formulation, selection, estimation as well as computational speed. All the more so, the uncertainty - typically self-generated in large amounts in chaotic dynamical systems - makes their meaningful investigation hard also for non-linear models, often regardless of the level of sophistication of those entertained. The scope of this paper is to present and theoretically justify a complexity reduction method which is capable to capture the largest amount of linear information, which is therefore suitable for a linear-type analysis. The extraction of such a portion of the signal - conducted under minimum information loss conditions - is made possible by a double stage Wavelet driven signal deconstruction and aggregation procedure. In essence, the signal is first

decomposed into sets of sub-signals and then sequentially aggregated in an exhaustive fashion, so that an optimal subset, maximizing the energy carried by the linear part of the signal, is found. The quantity of energy present in the (sub{)signal(s) is measured by the amount of variability embedded, which is estimated by a WANOVA-type (Wavelet Analysis of Variance) method, whereas the presence of non-linearity is detected through an Artificial Neural Network-based test (Teräsvirta test for neglected nonlinearity). That very many types of non-linearity can be found in real-life signals, makes practically unfeasible the definition of a unique test for the detection of each of them, therefore it is stressed that all the outcomes presented in this paper are to be referred to this test. Finally, the linearity assumption is assessed through a test of the type Portmanteau (Ljung-Box).

Keywords: Chaotic dynamical systems, multiresoluion analysis; Ljung-Box test; Teräsvirta Neural Network test; Wavelet theory; WANOVA method.

On the Exponential Decay of Solutions in Dual-Phase-Lag Porous Thermoelasticity

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In the last years, a big interest has been developed to understand the time decay of solutions for the porous thermoelasticity with different thermal mechanisms. We here want to consider the problem of the one-dimensional porous thermoelasticity when the heat conduction is given by means of the dual-phase-lag theory. We want to give suitable conditions in order to guarantee that the decay of solutions is controlled by a negative exponential. We also want to provide conditions for the slow decay of the solutions.

Keywords: One-dimensional porpous thermoelasticity, time decay, dual-phase-lag, semi-group arguments, spectral arguments.

Attractor for a Semi Discrete Fractional Klein Gordon Schrödinger System

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We consider a semi discrete in time Crank-Nicolson scheme to discretise a weakly damped forced nonlinear fractional Klein Gordon Schrödinger system

$$iu_t + (-\Delta)^{\alpha}u + i\kappa u = uv + f, \quad \textbf{(1)}$$

$$v_{tt} - \Delta v + v + \lambda v_t = \Re u_x + g \quad \textbf{(2)}$$

with $\alpha \in \left(\frac{1}{2}, 1\right)$, considered in the the whole space $\Re$. We prove that such semidiscrete system provides a discrete infinite dimensional dynamical that possesses a global attractor. We show also that if the external force is in a suitable weighted Lebesgue space then this global attractor has a finite fractal dimension.

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Limit Cycle Bifurcations and Chaos Transition in Polynomial Dynamical Systems

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We carry out the global qualitative analysis of low-dimensional polynomial dynamical systems. First, using our new bifurcational and geometric methods, we solve Hilbert's Sixteenth Problem on the maximum number of limit cycles and their distribution for the 2D Liénard polynomial, Holling quartic, and Kukles cubic dynamical systems. Then, applying a similar approach, we study 3D polynomial systems and complete the chaos transition scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of its limit cycles which is related to Smale's Fourteenth Problem.

Keywords: Polynomial dynamical system, Bifurcation, Limit cycle, Chaos transition.

The Role of the Spontaneous Breaking Symmetry Mechanism in the Mental Processes Dynamics

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Nowadays, the main difficulty of the neuroscientists is the prejudice to study only the neuronal, neuroglial and neurotransmitters structure. Starting from the quantum theory according to which every particle has a corresponding wave, and taking into account that starting from the newest cell structures, the neurofibriles, down to the cell, tissues and organs, one

can observe the existence of a strong wave spectral activity. This spectral wave component has been understudied, even if it is contained in the quantum physics theories, but also in the neurophysiological concepts and is rudimentary highlighted at the level of overall cerebral activity through EEG and EMG. This spectral component associated and related to the material, corpuscular one (the neuronal and non-neuronal structures of the brain) is at least as important as the corpuscular part. This approach of the structure and activity of the brain from a spectral perspective allows the study of the brain from the complex systems theory perspective. One can try to identify the unstructured, chaotic, stochastic component, along with the structured, causal component with linear dynamics, as a dynamics between the two components on the phase space, in which there is a permanent exchange of energy, but also of information. In this sense, one emphasizes the crucial role of the spontaneous breaking symmetry mechanism. In consequence, one can use principles, properties and characteristics from nonlinear dynamics to better understand the mental.

Keywords: Spontaneous breaking symmetry, Information, Mental, Spectral, Wave, Corpuscle, Complex system.

Torus Breakdown and Homoclinic Chaos in a Glow Discharge Tube

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Starting from historical researches, we used, like Van der Pol and Le Corbeiller, a cubic function for modeling the current-voltage characteristic of a direct current low-pressure plasma discharge tube, i.e. a neon tube. This led us to propose a new four-dimensional autonomous dynamical system allowing to describe the experimentally observed phenomenon. Then, mathematical analysis and detailed numerical investigations of such a fourth-order torus circuit enabled to highlight bifurcations routes from torus breakdown to homoclinic chaos following the Newhouse-Ruelle-Takens scenario.

Keywords: dc glow discharge; torus breakdown; homoclinic chaos; route to chaos.

An Example of Chaotic Dynamics in Magnetic Field

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In this paper we have discussed chaotic dynamics of charged particles inside an idealized magnetic field, suggested by a tokamak type configuration. Our model is based on a periodically perturbed Hamiltonian system in a half-plane. For the proof we have used the theory of Linked Twist Maps along with the method of stretching along the paths. A key step in our argument is based on the monotony of the period map associated with the unperturbed system.

Appropriate CFD Model and Impact Scale for Non-Submerged Spur Dikes

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Spur dike is a common hydraulic structure, which is widely used in hydraulic engineering such as channel regulation, flood prevention and river diversion for preserving the desired water depth, deflecting the main current and protecting river banks. In this study, the flow field around non-submerged spur dike groups was selected as the research object to assess the performance of CFD models based on flume experiments. A set of flume experiment was conducted in laboratory, based on its observed data, the numerical simulations were carried out through Finite Volume Method (FVM) and three turbulence models, i.e., standard k-ε model, Reynolds Stress Model (RSM) and Large Eddy Simulation (LES). The comparisons between computation results from CFD and observed data from flume experiment showed that the three turbulence models can simulate the three-dimensional flow around spur dike group in a certain extent with different cost justifications. Based on the numerical results, the conceptions of spacing threshold of non-submerged double spur dikes with ipsilateral layout and alternate layout in a straight rectangular channel is proposed respectively, and further quantitative investigation are also carried out. This work can provide a reference for the simulation of the turbulent flow with free surface. The two proposed empirical formulas can be used to determine the impact scale of spur dikes to river systems, which are useful for assessing river health in macro-scale and cumulative effect to river systems.

Keywords: Spur dike, CFD Model, Impact scale, Flume experiment, Dike spacing, Threshold.

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Lévy Fluctuations and Dynamic Response-Towards Understanding Processing of Biological Signals

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A generalization of Gaussian white noise to its non-Gaussian, Lévy stable counterpart serves as a model of impulsive, large scale variations observed e.g. in turbulent heat flow and solar flare fluctuations, hole transport in semiconductors, transmission of light in inhomogeneous materials and anomalous diffusive transport.
Apart from financial mathematics where the Lévy fluctuations became an attractive model for price variations, there is also an accumulating evidence from biological experiments showing that production of mRNA and proteins occurs in a pulsatile manner and creates non-Gaussian noise in individual cells. Those bursty events result in high transcriptional activity followed by long periods of inactivity and are characterized by heavy tailed distributions and Lévy-like statistics. Moreover, the cytoplasmatic mechanical activity has been documented to be far from equilibrium and the total intensity of cytoskeletal noise has been estimated to exceed the level of thermal noise. Since the intrinsic stochastic excitations may play a crucial role in transcriptional regulatory systems, it can well be that non-Gaussian Lévy noise should be a proper model of choice for underlying fluctuations in biological systems.
The lecture will cover consequences of non-Gibbsian character of the stationary states in systems driven by Lévy white noises, analysis of relaxation and dynamic response to Lévy outbursts and various applications in biophysics.

Keywords: Nonequlibrium fluctuations, Lévy noises, generalized fluctuation-dissipation theorem, anomalous diffusion, biological signal.

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Renormalization Group Approach to a Passive Scalar Advection for Turbulent Compressible Velocity Field: Two-loop Approximation

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We investigate the model of passive scalar quantity advected by the turbulent velocity field which is described by the Navier-Stokes equation for compressible fluid. The field theoretic renormalization group is used to analyze anomalous scaling of the system in two-loop approximation. Previous calculation of anomalous scaling was done in the leading one-loop order [N. V. Antonov, M. M. Kostenko, Phys. Rev. E 90, 063016 (2014), N. V. Anotonov et. al., Phys. Rev. E 95, 033120 (2017)]. The knowledge of higher order of perturbation theory is important to reconsider on the validity, accuracy and convergence properties of the expansion in the perturbation theory. The renormalization procedure is performed in a formal expansion scheme in y, where y is the exponent describing scaling behavior of random force. The aim of a contribution is to present the calculation of Feynman diagrams up to second order and obtain a knowledge about scaling regime and anomalous scaling.

Keywords: Passive scalar sdvection, Navier-Stokes equation, Renomalization group

Stochastic Navier-Stokes Equation for a Compressible Fluid: Two-Loop Approximation

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A model of fully developed turbulence of a compressible fluid is reviewed. Fluid dynamics is governed by stochastic version of Navier-Stokes equation. We show how corresponding field theoretic-model can be obtained and further analyzed by means of the perturbative renormalization group (RG). In this approach, scaling properties are related to the fixed points of the RG equations. The perturbation theory is

constructed within formal expansion scheme, actual calculations for scaling exponents are performed in two-loop order. Permissible scaling regimes and their difference from ones found at one-loop level are discussed.

Keywords: stochastic Navier-Stokes equation, anomalous scaling, field-theoretic renormalization group, compressibility

EXPAR Model to Model Chaos and Cyclical Time-Series Data

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Time-series data in various fields like agriculture, fisheries, and meteorology generally depict cyclical fluctuations. The well-known Box-Jenkins' Autoregressive Integrated Moving Average (ARIMA) methodology is generally considered for modelling and forecasting of time-series data sets. However, this approach has some serious limitations, viz. assumptions of stationarity and linearity and also it is not able to properly describe the chaos and cyclical fluctuations. In the class of parametric nonlinear time-series models, Exponential Autoregressive (EXPAR) family of models may be employed to describe data sets that depict such chaos and cyclical fluctuations. Parameter estimation algorithm is developed by employing genetic algorithm (GA) stochastic optimization procedure for EXPAR family of parametric nonlinear time-series models and its performance through simulation study is also discussed. To this end, relevant computer program in Matlab Ver. 7.4 (R2007a) software package is developed.

Keywords: Chaos, Cyclical fluctuations, EXPAR, Genetic Algorithm.

Chaoticity of Transient Current Behavior and Stretched Exponential Parametrization in As_2Te_3(In) at Different Temperatures

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Thin films have irregular current behavior under constant voltage. The time evolutions of transient current data for As_2Te_3(In) thin films were evaluated when the electric field was applied at temperatures of 296 °K, 313 °K, 333 °K and 353 °K. Firstly, the linear auto-correlation function and

mutual information technique were used to see delay times with respect to long-range correlations and short-range correlations. These methods give us different time scales and thus a more reliable delay time can be calculated by using the mutual information technique. False-nearest neighbor's method give us convenient embedding dimensions using estimated delay times. We examined the chaoticity by calculating the maximal Lyapunov exponents related to embedding dimensions and delay times with non-linear time series analysis techniques. The current mechanism of glass substrates has variable Lyapunov exponents spectrum at different temperatures. Hurst Analysis which describes the evolution of transient current in time and Detrended Fluctuation Analysis (DFA) which evaluates long range power of correlation exponents support the chaoticity of conduction mechanism and the presence of multiple regimes. DFA indicates the varying transition of the crossovers based on different temperatures. It was shown that when temperature increases, the crossovers increase, and the fact remains that the scaling exponents change. Stretched Exponential models are also studied. The current behavior as a function of time at constant different temperatures can be moderately modelled by using the appropriate stretched exponential model.

Keywords: Chalcogenides, Lyapunov Exponents, Correlation and delay time, Time Series Analysis, Hurst Analysis, Detrended Fluctuation Analysis, Scaling Exponents, Stretched Exponential dynamics

Dynamical Invariant Calculations Involving Evolution Equations with Discontinuities

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Many models of physical systems involving electronic circuit elements [1], population dynamic [2] involve evolution equations with discontinuities. The key to understand such systems is to hope that the discontinuity does not adversely affect the integration process. There are also three variable chaotic dynamical system examples, such as the Sprott systems for deriving jerky dynamics has also become of interest [3].

In order to calculate dynamical invariants in chaotic systems such as characteristic exponents and fractal dimensions we often need to find the Jacobian; this often requires attempting to differentiate discontinuous functions. Therefore finding a suitable continuous approximation to the discontinuities becomes important. In previous communications, two example systems had been used with two parametrizations for

approximating discontinuous functions with continuous ones, one of which is the same as that used in the literature.
In this work, we will use further examples to optimize the parameters of the continuous approximation to discontinuities using different examples in order to test the degree of applicability of this approach. Where possible, the invariants calculated by this method will be compared to the corresponding invariants calculated from its time series.

Keywords: Chaotic systems, Sprott systems, Fractal dimension, Lyapunov exponents, Simulation, Chaotic simulation.

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Spatial Extent of An Attractor

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Lyapunov exponents characterize the rate of approach or recession of nearby trajectories in a dynamical system defined by differential equations or maps. They are usually taken as indicators of chaotic behavior. The density of orbits in the state space or equivalently, the Poincare map is usually taken as another such indicator. Although these indicators usually give correct results, there are instances in which they can lead to confusing or misleading information. For instance, a system of three linear differential equations can have three positive eigenvalues λi leading to a solution exp(λit). The Wolf-Benettin algorithm [1] would report three positive Lyapunov exponents, in spite of the fact that the system is not chaotic. Another example is the Khomeriki model [2] or even the usual Bloch equations with strong AC field that would report a spectrum of all negative Lyapunov exponents but produce completely full state space plots.
We will consider the class of systems proposed by Sprott [3] consisting of three-dimensional ODE's with at most quadratic nonlinearities as examples. Many of them obey two scenarios one of which is Lorenz model

like behavior where an unstable linearized fixed point is surrounded by two stable fixed points so that the unstable fixed-point leads to a throw and catch behavior. The other is Rössler-like behavior whereas the system moves away from a weakly unstable linearized fixed point, nonlinear terms return it to equilibrium with a spiral out catch in mechanism. Since the presence of an attractor may involve structural stability, these two mechanisms are expected to produce different spatial extents for the attractor. Although Lyapunov exponents indicate time dependent behavior, spatial extent would complement this as a spatial measure of localization, thus adding to the information from Lyapunov exponents that characterize horizon of predictability.
Direct numerical simulation and where feasible, the normal form approach, complemented by averaging will be used to investigate selected examples of the three degree of freedom systems.

Keywords: Lyapunov exponents, Sprott Systems, Poincare map, normal form, Simulation, Chaotic simulation.

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Gröbner Basis Method in the FitzHugh-Nagumo Model

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The FitzHugh-Nagumo model is a two dimensional system of differential equations with polynomial right-hand sides. The model describes an excitable system and explains basic phenomena in dynamics of neuron activity, for example spike generations in a neuron after stimulation by external current input. The system is slow-fast, meaning system with different time scales for each state variable. We analyze bifurcation manifolds of the FitzHugh-Nagumo system in whole parameter space using algebraic approach based on Gröbner basis.

Keywords: FitzHugh-Nagumo model; Gröbner basis; slow-fast system; Hopf bifurcation; fold bifurcation

Forecasting Chaotic Business Cycles

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In order to create business cycle chaos, a dynamical forecasting system must exist that has at least three dimensions in continuous time, which include forecasts of nominal stock returns, inflation, and the growth of real economic output. This system of differential equations requires at least one nonlinear term to capture the model's sensitive dependence on initial conditions. By applying a Sprott nonlinear dynamical system, it becomes possible to forecast the business cycle.

Keywords: Monetary Policy, Chaotic and Random Uncertainty, Sprott System Application, Langevin Equation

Two Categories n-Dimensional Discrete Chaotic Systems with Applications in Image Encryption

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This paper proposes two categories n-dimensional discrete systems that are not stable at zero-equilibrium points. The conditions that the parameters in the system should satisfy are also given. Some systems are selected to hold the conditions, and it is verified that these systems have chaos behavior through the numerical simulation and the calculation of the Laypunov exponent. As an example, a four-dimensional discrete chaotic system is given and an image encryption scheme is designed based on this system. The numerical simulation shows that the encryption sequence is extremely sensitive to system parameters. Any perturbation greater than 10 ^ -15 will invalidate the decryption, and the key space of the encryption scheme can reach 2 ^ 996. The system constructed in this paper can theoretically contain an infinite number of parameters and provide a good pseudo-random source for further encryption schemes that design large key-space to resist aggressive attack.

Keywords: n-dimensional discrete systems, CHAOS, image encryption scheme, large key-space.

Non-linear Dynamics in Biological Systems

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Biological systems have complex structure and organization. Mutual interaction of biological systems from subcellular level to whole organisms is also complex and most of them exhibit non-linear behavior. To explain and to understand behavior of biological systems many biomechanical and biophysical models have been made combining knowledge from biology, medicine, physics, mechanics, physico-chemistry, biomechanics, mechanical engineering… Non-linear dynamics, bifurcation phenomenon, strange attractors and synchronization are present in these biological systems.
In this special section will be presented results from research on some subcellular structures like mitotic spindle. Energy analysis and energy transfer between nonlinear modes will be done.

Potential Energy Analysis of Homologue Chromosome Pairs trough Mechanical Oscillatory Model of Mitotic Spindle

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Regeneration of cells, tissue and organs, creation of new organisms is generated thanks to cell ability to divide. Behind this cell property lies complex cell machinery called mitotic spindle. The cell division process is susceptible to mistakes related to aging. In this paper we analyzed potential energy of homologue chromosome pairs in metaphase plate using mechanical oscillatory model of mitotic spindle.
Assuming that molecular structures within cell change their elasticity with aging, microtubules are considered as non-linear viscoelastic elements that could behave from pure elastic material (in young cells) to pure viscous material (in old cells).
We analyzed haw moving the pair of homologue chromosomes from metaphase to anaphase affects the potential energy of the system when one part of the system changes its elastic properties (the microtubules). Energy analysis and energy transfer between nonlinear modes is discussed.

Keywords: Potential energy, Chomologue chromosomes, Mitotic spindle, Aging, Viscoelastic element, Fractional derivatives, Nonlinearity

Digital Signature: Quantum Chaos Approach and Bell States

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A mathematical scheme for demonstrating the authenticity of digital messages or documents is called a digital signature. First time, Whitfield Diffie and Martin Hellman described a digital signature scheme in 1976. The security of such classical digital signature(CDS) schemes, with the rapid development of quantum computing, is seriously challenged. Therefore, Gottesman and Chuang proposed the first Quantum digital signature protocol. The quantum mechanics applied by quantum digital signatures (QDSs) is applied to guarantee the nonrepudiation, unforgeability, and transferability of a signature. Previously, the security of QDSs schemes only depended on the length of the signature. In order to increase the security, the nonlinear dynamical systems generate a kind of deterministic random-like process, which is called chaos. Given the distinctions of quantum and classical spaces, we can use the realities of quantum spaces to increase the key space and the capabilities of the dynamic systems in cryptography. One of the popular fields of quantum chaos addresses quantum maps. Furthermore, the iteration process is one-way. In this paper, quantum digital signature schemes based on bell states and synchronization of coupled chaotic map is introduced. The dynamically designed system based on synchronization of coupled chaotic map and bell states under current attacks called denial and forgery designed for the safety of the protocol. In our schemes, using the synchronization of coupled chaotic map when signing up, security increases against repudiation and forgery. Proposed QDS scheme based on bell states sequence can be used for protection of copyright. Furthermore, by using bell states in quantum digital signature, security has improved. This signature can be used in quantum computer.

Keywords: Quantum digital signature, Synchronization of coupled chaotic map, bell states, Repudiation, Forgery.

A Scaling Behavior in Percolation: Joint Effect of Anisotropy and Compressibility

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The purpose of this study is to reveal the effect of stochastic turbulent mixing on the nonequilibrium percolation process near its critical point. In this work special attention is paid to the role of large-scale anisotropy and

compressibility of turbulent flows in scaling behaviour of the system. Our investigation is based on the quantum-field renormalization group analysis in which one-loop Gell-Mann-Low equations have been derived. Asymptotic solutions of these equations made it possible to establish four stable scaling regimes and compute corresponding critical exponents, which, however, are not universal in this system; they do depend on the parameters that quantitatively determine anisotropy and compressibility of the flow.

Keywords: Stochastic turbulence, Scaling behaviour, Renormalization, Percolation.

Large Scale Behavior of generalized Stochastic Magnetohydrodynamic Turbulence with Mirror Symmetry Breaking

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Helical magneto-hydrodynamic developed turbulence (MHD) with arbitrary interaction between velocity and magnetic field fluctuations and with arbitrary self-interaction of magnetic field is investigated by means of field-theoretic renormalization group (RG) approach. Infrared stable fixed RG point, which governs large-scale behavior of MHD, is determined in two-loop approximation. The values of parameters determining the region of stability of the fixed point are calculated. Anomalous exponents of magnetic field and corresponding response field are obtained by the calculation of divergent parts of all vertex Green functions. It turns out that the turbulent Prandtl number defined as a ratio of the response functions of magnetic and velocity fields takes the same value as was previously calculated in the framework of kinematic MHD model. Stabilization effect of mirror symmetry breaking on large scale behavior of MHD and its contribution to the turbulent Prandtl number is discussed.

Keywords: renormalization group, (magneto) hydrodynamic turbulence, helicity, response functions

Modeling Turbulence via Numerical Functional Integration

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We investigate the possibility of modeling turbulence via numerical functional integration. By transforming the incompressible stochastic Navier-Stokes equation into a functional integral we are able to calculate equal-time spatial correlation of system variables using standard methods of multidimensional integration. In contrast to direct numerical simulation, our method allows for simple parallelization of the problem as the value of the integral at any point is independent of other points. Thus the entire problem does not have to fit into available memory of any one computer but can be distributed even onto several supercomputers and the cloud. We present the mathematical background of our method and its numerical implementation. The implementation is composed of a fast serial program for evaluating the integral over a given volume and a Python wrapper that divides the problem into subvolumes and distributes the work among available processes. We use Monte-Carlo integrators of the GNU Scientific Library for integrating subvolumes. We show first results obtained with our method and discuss its pros, cons and future developments.

Keywords: Stochastic Navier-Stokes equation, numerical functional integration

A Compact Non-Differential Approach for Modelling Laser Ablation Plasma Dynamics

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Theoretical differentiable physical models are often used to describe the dynamics of laser-produced plasma plumes (e.g. kinetic models, two-fluid models). Given the complexity of all the phenomena involved in the laser-matter interactions, laser plasma interaction and plasma expansion, an alternative approach might be the use of non-differentiable physical models (fractal models). We propose a fractal approach to describe the laser ablation plasma plume expansion. In the framework of our fractal hydrodynamic model, the ejected particles move on continuous but non-differentiable curves (fractal curves) so that, all physical phenomena involved in the ablation plasma dynamics depend not only on the space-time coordinates but also on the space-time resolution scales. Additionally, the particles can be reduced to and identified with their own trajectories,

so that the ablation plasma will act similar to a fluid lacking interactions by means of geodesics in a non-differentiable (fractal) space.
We present here two approaches based on the fractal model. A generalized approach focused on laser produced plasmas splitting phenomena into multiple plasma structures and plasma oscillations, and a compact version for the investigation of the spatial and temporal evolution of some plasma dynamic variables (electron temperature, ions density, thermal velocity, Debye length). This version of our model was obtained by using normalized variables of the particle density, of the velocities, of the current density etc., and by selecting suitable scale resolutions. The influence of external factors on the ablation plasma dynamics is also considered by introducing the fractality degree as a global parameter. For the first time, clear relations are proposed between fractal model variables and some plasma parameters. These results highlight the practicality of the non-differential theoretical model for the study of laser–produced plasma dynamics.

Keywords: Fractal model, Non-differential approach, Plasma oscillations, Laser ablation plasma, Plume splitting.

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Appearance and Instability of Non-Concentric Multiple Double Layers in Low-Temperature Discharge Plasma

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Experimental results presented here are related with the appearance and instability of non-concentric multiple double layers (multiple fireballs) in front of a positively biased electrode immersed into a low-temperature discharge plasma. These complex space charge structures appear up to a threshold value of the potential applied on the electrode and become unstable when the equilibrium between the production of charged particles (by excitations and ionizations) and their loss (by recombination and diffusion) disrupts. The multiple double layer instability gives rise to oscillations of the plasma parameters, reflected in the recorded time series of the current collected by the electrode. The dynamics of each of the double layers composing the multiple structure gives a unique signature in the collected current, which is composed by the overlapping of the individual dynamics. The time series of the current displays bunches of oscillations, correlated to the number of fireballs (up to six experimentally recorded) generated in front of the electrode. A theoretical fractional model was developed in order to explain the self-structuring of the discharge

plasma as multiple fireballs, as well as the complex dynamics of the multiple structure, a good agreement being obtained with the experimental observations.

Keywords: Multiple double layer, Fireball, Instability, Fractality.

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Nonlinear Noise Reduction on TESS Simulated Light Curves

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The Kepler Space Telescope has, at most, a few more months of fuel. Luckily, its successor, the next generation Transiting Exoplanet Survey Satellite (TESS) was launched on April 18, 2018. The two telescopes are, however, very different in design and operation strategy. Kepler observed dimmer stars at distances to 3,000 ly while TESS will observe stars up to 100 times brighter to a distance of 300 ly. TESS targets will be observed for shorter times, many for only 27 days as opposed to the Kepler multi-year light curves. Kepler has 4 arcsecond pixels, TESS's are much larger covering 21 arcseconds. This results in greater “crowding”. Moreover, the depth of TESS pixels is six times greater. Therefore, where a cosmic ray in Kepler would likely impact only a single pixel, in TESS it will produce a trail which impacts many pixels. The issue of noise reduction poses different challenges for TESS light curves. In order to have the community ready for TESS data analysis, simulated light curves have been made available. Relying on our experience with nonlinear projective noise reduction on continuous Kepler Space Telescope one-month light curves, we present our initial results of nonlinear noise reduction on simulated short TESS light curves.

Keywords: Nonlinear noise reduction, simulated light curves, noise

Complex Oscilations in a Thermosyphon Viscoelastic Model

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Thermoyphons, in the engineering literature, is a device composed of a closed loop containing a fluid who motion is driven by several actions such as gravity and natural convection. In this work we consider a viscoelastic fluid described by the Maxwell constitutive equation. Their dynamics are

governing for a coupled differential nonlinear system, and in several previous work we show chaos in the fluid. This work is, in some sense, a generalization of some previous results on standard (Newtonian) fluids obtained by A. Rodríguez-Bernal and E.S. Van Vleck [1], when we consider a viscoelastic fluid

Keywords: Thermosyphon, Viscoelastic fluid, Asymptotic behaviour.

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Turbulent Prandtl Number in Two Dimensions

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Using the field theoretic renormalization group technique, the two-dimensional turbulent Prandtl number of passively advected scalar field in turbulent environment driven by the stochastic Navier-Stokes equation is calculated in the two-loop approximation

Keywords: fully developed turbulence, stochastic dynamics, renormalization group, quantum field theory, Navier-Stokes equation, Prandtl number, double expansion, scalar field

Nonlinear Dissipative Soliton Dynamics

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Nonlinear coupling between incommensurable temporal scales can result in excitation of the manifold of the new degrees of freedom that corresponds to an appearance of new frequency components with distorted mutual phase relations. As a result, a femtosecond pulse (dissipative soliton, DS) losses its coherence, and its dynamics becomes chaotic and even turbulent. We propose the innovative method based on the Weil-Wigner-Moyal representation of the stochastic nonlinear dynamics. The advantages of this approach are i) possibility of numerical simulation of quantum-mechanical nonlinear equations in the Heisenberg's operator representation, and ii) direct taking into account of the stochastic properties without a statistic gathering. To study the issue of chaotization of DS dynamics, we developed the statistical mechanics and thermodynamic approaches to critical phenomena and turbulence. These methods utilize the representation of DS as a "complex" ("blocks assembly" like structures in a ferromagnet) driven by a multiscale

hierarchy of nearest-neighbor and long-range couplings. Such approach can have far-reaching consequences for the general theory of coherent (and semi-coherent) dissipative structures.

Keywords: Dissipative solitons, Coherent and turbulent dissipative structures, Weil-Wigner-Moyal representation of stochastic dynamics.

Multiple Correlation Analysis for Chaotic Time Series

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Chaos analysis methods are utilized to show the nonlinearity state of the data in the study. 222Rn measurements were performed statistical evaluation. Multiple correlation analysis were also performed on the non-parametric tests for the series, and statistically analysis were performed to determine the power of the variables. The results between real values and the predicted value of the data were obtained as statistical. The accuracy of the results was confirmed with statistical analyses. This work, a new aspect was obtained for explaining the non-linear characteristic of 222Rn concentrations from soil.

Keywords: Chaotic time series, multiple statistical analysis, corelation analysis, 222Rn data, prediction.

Detrended Fluctuation Analysis for Variations of Radon in soil: Lesvos Island (Greece)

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Many studies have been performed to understand the correlations between earthquakes and the other geological parameters. 222Rn is one of the most common parameters which is used for the analysis of this correlations. Data analysis methods has a critical important for complex dynamical system such as earthquakes. Scale analyses technique, DFA, has similar exponents with RMS (Root Mean Square) and Hurst. This is a method for prediction of long-range power-law correlation exponents. As a product of normal distribution for the monofractal time series where the variance is obtained by the second order statistical moment alone, multifractal time series have both excessively large and small fluctuation. In the current study, DFA analysis was applied for 222Rn data which was collected from Lesvos Island, Greece. The results of Detrended Fluctuation Analysis have a strong correlation for each 222Rn time series.

Keywords: DFA, Radon, time series, correlation.

Synchronization Patterns and Chimera States in Dynamical Networks with Adaptive Couplings

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In this paper we discuss the phenomenon of self-organization in dynamical networks with adaptive couplings. A feature of such systems is co-evolution of the states of network components and the properties of inter-element interaction. We have developed a method for analyzing complex synchronization patterns such us cluster and chimera states in oscillatory networks with adaptive couplings. We find several scenarios of the organization of synchronous behavior in adaptive network and also analyzed the dynamic and structural changes accompanying these processes. The influence on the self-organization processes of various schemes of coupling adaptation and initial connections structure of the network is studied.

Keywords: Dynamical networks, Adaptive couplings, Complex topology, Processes of self-organization, Synchronization, Chimera states, Simulation.

Color Image Encryption Based on Reality Preserving Fractional Hartley Transform and Chaos

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The evolution of Internet technology has led to expansion of multimedia services specially in case of Social Networking Services (SNS) and also in the field of satellite imaging, medical imaging, military purposes and other security services. All these applications require online sharing of digital images and videos. It obviously becomes the necessity to make the transmission of such information secure from unauthorized access to protect it from being misused or tampered. One of the ways to realize this is image encryption and hiding methods. Due to high correlation among adjacent pixels, bulk data and high redundancies in images, the classical number theory based methods of encryption like DES, AES, etc. are not suitable.

On the other hand, the optical information technologies offer high speed, parallel data processing and thus for image encryption, it provides greater flexibility for manipulating parameters such as wavelength, polarization, amplitude or phase. Due to this, image encryption based on optical processing has attracted much attention in recent years specially due to their superior performance in real time transmission. Most of the optical encryption methods use a DRPE (Double Random Phase encoding) that is done by using a random phase masking in spatial domain and another in the frequency domain. Such algorithms use fractional Fourier transform (FrFT), Fresnel transform (FrT), Gyrator Transform (GT), fractional angular transforms and other integral transforms. The major weakness of all above mentioned transforms is that they give complex coefficients as outcome. These complex values need a holographic technique to record two images: one for spectrum and another for phase. This makes the storage and transmission less efficient. To overcome, there are some phase retrieval algorithms proposed in the literature but again this process is time consuming and also lead to inaccuracies. In order to address this issue, we propose to use a reality preserving fractional order Hartley transform (RPFHT), which maps a real input image to a real transform output. In general the optical image encryption does not provide the desired security against various cryptanalytic attacks due to a very small key space. To make the proposed optical encryption robust against various attacks, we propose the mixing/blending in both the spatial and transform domains using chaotic dynamical systems, which have been considered a preferred choice to introduce desired level of confusion and diffusion in image encryption due to their extreme sensitivity to initial conditions/parameters, mixing and ergodicity etc.

The flow of operations in the proposed colour image encryption method based on reality preserving fractional Hartley transform and chaos will be as follows: Initially all three colour layers of input image are separately passed through a robust chaos based blending realized with a piecewise linear chaotic map. Then we apply the reality preserving fractional Hartley transform (RPFHT) on the blended image layers. The real outcome of this optical transform is then subjected to a 2D Chirikov standard chaotic mapping which will introduce further diffusion in the image data in the transform domain. The initial conditions and parameter values of piecewise linear chaotic map (different for all three colour layers), fractional order of the Hartley transform and initial conditions and system parameters of the 2D standard map together will constitute the secret key of the proposed image encryption method. Finally the three intensity images are concatenated as a single ciphered image! The decryption procedure will be simply the inverse of the encryption procedure and can be done correctly if all the secret key values are exactly same. To prove the correctness and robustness of the proposed image encryption method, we will present the result of our analysis based on correlation,

differential analysis, key sensitivity analysis, information entropy analysis etc.

Keywords: Chaotic blending, Piecewise chaotic map, robust chaos, reality preserving fractional Hartley Transform (RPFHT), chaotic standard map

The FitzHugh-Nagumo Model and Spatiotemporal Fractal Sets Based on Chaos Functions

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It is presented firstly that a one-dimensional (1-D) time-dependent logistic map for population growth is derived from the chaos solution consisting of a time-dependent chaos function, and the logistic map has the dynamics of coherence and incoherence in time, which are the so-called chimera states or chaotic transients discussed in the field of complex systems, by introducing a time-dependent system parameter for the 1-D map as one of non-equilibrium open systems. Secondly, the 2-D time-dependent solvable chaos map corresponding to the FitzHugh-Nagumo model for neural phenomena is obtained on the basis of time-dependent chaos functions to have chaotic transients in time under the assumption of a time-dependent system parameter, and to give spatiotemporal fractal sets defined by initial values as the dynamic stability region for neural cells.

Keywords: Chaos function, Time-dependent logistic map, Time-dependent system parameter, Non-equilibrium open system, Chimera states, FitzHugh-Nagumo model, Chaotic transients, 2-D solvable chaos map, Spatiotemporal fractal set, Dynamic stability region.

Stochastic Elasticity of Variance and Drivatives Pricing

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The elasticity of variance of risky assets is observed to be fast fluctuating around a level. The level itself may vary depending on the corresponding economic situation at the time of consideration. In particular, it is extraordinary during the peak period of financial crisis like the Global Financial Crisis. Based on the stochastic elasticity of diffusions, this paper develops an asset price model in a multiscale form and applies to the pricing of financial derivatives.

Modeling of Turbulent Processes in Economy of Macrosystems

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The economies of different countries even more often appear under the influence of negative tendencies and the crisis phenomena, which are caused by economic turbulence. The defining characteristics of turbulence in the nature are aggression, unpredictability and chaotic. These same characteristics are peculiar to turning points in phases of a business cycle. At the same time, the condition of turbulence in economic community is considered also perspective. At the successful maneuvering, instability creates potential and opens new opportunity for development before national economy.

The condition of turbulence of economy is usually determined as chaotic. But such interpretation of this process is not accurate enough. Turbulence is shown as a speed field that is the movement with a different speed of elements of earlier uniform economic system. These elements move not like particles in brownian motion, but show the spatial organization, forming whirlwinds.

Research of changeability of economic environment and related to it the problem of origin and development of processes of economic turbulence allowed to get next results. It is proved that autowave process is the integral component of the active economic environment and is caused by the different speed of course of economic processes. If the difference of relative speeds of economic processes not big, in economic system extends a flat wave, which leads to usual fluctuations of the main economic indicators. In a case, when the difference of speeds is higher than an admissible limit, balance of processes is broken and in economic system is formed the rotating spiral wave – a whirlwind, which leads to emergence of turbulent processes in economy.

The methods of the theory of chaos constructed model of emergence of whirlwinds and, respectively, processes of turbulence in the conditions of the active economic environment. Research of economy of a macrosystem is executed, conditions of emergence of turbulence and a trajectory of distribution are defined.

Keywords: Models Behavior of Economic Systems, Turbulent Processes in Economy, Autowave Process, Whirlwinds, Chaotic modeling, Model of Emergence of Whirlwinds.

Generalizations of Weighted Rotation Operators and their Spectra

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We consider weighted composition operators, $T = WU$, on a Banach space X of measurable or analytic functions under the assumption that there is a "rich" set $\mathcal{M}$ of multiplication operators on X such that for any $M \in \mathcal{M}$ we have $U^{-1}MU = \lambda_M U$, where λ_M is a complex number. Under some mild assumptions about the space X we describe the spectrum and essential spectra of T.

Keywords: weighted composition operators, spectrum, essential spectra.

An Iterative Averaging Approach for Describing Self-Sustained Oscillations and Rotations of an Aerodynamic Pendulum

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The mathematical model of an aerodynamic pendulum is represented in the form of an autonomous dynamical system of the second order. The system doesn't contain a small parameter. A numerical-analytic approach is proposed that provides iterative approximations of self-sustained oscillations and rotations of the pendulum. This approach is related to the Pontryagin approach for near-Hamiltonian systems, in the similar way as the Samoilenko numerical-analytic approach is related to the Krylov-Bogolyubov averaging method. At each iterative step, the averaging along the previous approximation curve is performed and the formal criterion is applied to ensure that the next approximation curve is closed. Conditions of convergence of the iterative procedure are discussed. Bifurcation diagrams of periodic motions of the pendulum are constructed.

Keywords: numerical-analytic methods, iterative approximations, self-sustained oscillations.

Magnus Type Propeller Wind Turbine as an Engine for a Wind Car

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The mathematical model of a wind car driven by the Magnus type propeller wind turbine is introduced. It is supposed that there is no slipping between the wheels and the ground. The model is represented as an autonomous dynamical system of the second order. The parametrical analysis of steady motions is performed. The possibility of the upwind motion is shown. The maximal value of the upwind speed is estimated (41% of the wind speed). For comparison, the upwind speed of the wind car driven by the torque of the traditional Savonius rotor is about 15 % of the wind speed. The optimal value of the gear ratio is derived.

Keywords: propeller wind turbine, Magnus effect, Savonius rotor, wind car, upwind motion.

Quasi–Periodic Operation Modes of a Counter-Rotating Darrieus Wind Turbine

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A small-scale counter-rotating vertical axis wind turbine in a steady wind flow is studied. The system consists of two Darrieus turbines that are intended for rotation in opposite directions. The shaft of one turbine is rigidly joined to the rotor of a generator and the shaft of the other turbine is rigidly joined to the stator. A closed few-parametric mathematical model that takes into account changeable electrical load in the local circuit of a generator is constructed. The corresponding dynamical system is a two-frequency system. The system is supposed to contain a small parameter. In order to describe quasi-periodic motions, this system is averaged over two angles under the assumption that both frequencies are bounded away from zero. It is shown that passage through resonances has no crucial effect on the behavior of the system in the desired range of the parameters of the model.

Keywords: counter-rotating Dassieus wind turbine, mathematical model, two-frequency averaging, quasi-periodic motions.

Chimera States as Drive-Response Systems

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An overview is provided of our recent work on chimera states in which we explore the inner correlations that sustain the incoherent and coherent oscillators in their respective groups [1]. Previous work along the same theme as we have presented here; namely, coupling two interacting populations of oscillators, has shown how the stability is controlled by the coupling strength, where the chimera emerges as the more stable state [2,3]. The stable state is interpreted as a result of two competing synchronization patterns [3], leading to the more stable, yet frustrated state, that we are proposing here. The drive-response theory of Pecora and Carroll [4] shows how two chaotic systems can synchronize, if they are dynamically coupled the right way. Here, we have used their theory to bring out the stability of the frustrated state over the fully synchronized state. The character of the frustrated state is better discovered in this new light, as we find out about the relative size of the coherent and incoherent sets. We gain insight into the chaotic nature of the frustrated state. This chaotic character, however, is 'reduced' as it happens in a collective state, within the chimera. A measure of this could be estimated, using the Kaplan-Yorke conjecture [5]. This conjecture equates a dimension based on the Lyapunov exponents with the geometric dimension of the attracting set. The latter is the entropy or information dimension, pointing to the interrelations within the noncoherent set, and among the set of oscillators as a whole. With many positive, but near zero exponents, this is found to be close to half the number of incoherent oscillators, and scaling in general as 0.35N, which is less than if the chaotic set were on its own [6]. We believe our analysis has provided a better understanding of the formation of the chimera, and how it is supported by its frustrated structure.

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[6]. For instance, this gives 0.7 for the Henon map, and 1.05 for the Lorentz map, compared to 1.26 and 2.07, the actual Kaplan-Yorke dimensions, respectively.

Turbulent Advection of Passive Vector Field

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The field theoretic renormalization group and the operator product expansion are applied to the model of passive vector (magnetic) field advected by a random turbulent velocity field. The latter is governed by the Navier--Stokes equation for compressible fluid, subject to external random force with the covariance ~k^{4-d-y}, where d is the dimension of space and y is an arbitrary exponent. From physics viewpoints, the model describes magnetohydrodynamic turbulence in the so-called kinematic approximation, where the effects of the magnetic field on the dynamics of the fluid are neglected. It is shown that various correlation functions of the magnetic field and its powers demonstrate anomalous scaling behavior in the inertial-convective range already for small values of y. The corresponding anomalous exponents, identified with scaling (critical) dimensions of certain composite fields (“operators” in the quantum-field terminology), can be systematically calculated as series in y. The practical calculation is performed in the leading one-loop approximation, including exponents in anisotropic contributions.

Keywords: fully developed turbulence, magnetohydrodynamic turbulence, anomalous scaling, renormalization group, operator product expansion, composite fields, compressibility, anisotropy

Energy Characteristics of a Shaker-Oscillator Model

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The purpose of our study is to study energy characteristics under interaction of the oscillating system and the shaker, which is driven by the amplifier. The amplifier as a self-exciting system of limited power is under a reverse influence of vibrational loads. The influence of oscillator vibrations on the formation of the driving force leads to a number of specific effects, in particular, to the Sommerfeld’s effect. The effects of the interaction of an electrodynamic shaker powered by a vacuum-tube amplifier of limited power and a linear oscillator, which affects the amplitude and frequency of the driving force, are studied. The total power, the supplied power and the consumed power are defined and calculated for the periodic and chaotic steady-state regimes.

Keywords: Sommerfeld-Kononenko’s effect, Supplied and consumed powers, Steady-state regimes.

Determination of Support Reaction Force of Junction between Launch Aircraft and top Mounting Upper-Stage Rocket

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Developing of conception of multistage aerospace system set a task to determine realistic force interaction between two closely-spaced vehicles. Target of research: multistage aerospace system, which consists of closely-spaced launch aircraft and upper-stage rocket. Upper-stage rocket is top mounting located on launch aircraft. Parts of aerospace system connected with each other at three junction points. Centerlines of both vehicles are coincidence. Purpose of research: determination of support reaction force of junction between launch aircraft and upper-stage rocket. There was developed mathematical model of dynamic of simultaneous motion of launch aircraft and upper-stage rocket, with taking into account of acceleration and aerodynamic loads affected to vehicles. There were defined normal and axial forces, affected to the front and rear pillar. Aerodynamic loads can be defined by wind-tunnel testing of dimensionally scaled model. Moments of all forces and loads are leaded to junction points. Obtained expressions of support reaction force are defined as function of aerodynamic loads, inertial forces, affected to both vehicles, and thrust force of launch aircraft. In software environment Maple was developed a program of dynamical motion of launch aircraft and upper-stage rocket as part of aerospace system. This program shows the changes of support reaction force in junction points of launch aircraft and upper-stage rocket at different parts of flight of aerospace system.

Keywords: multistage aerospace system, launch aircraft, upper-stage rocket, support reaction force, junction points.

Method of Synergistic Synthesis of Control Laws of Separation of two Flight Vehicle

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There is considered task of synthesis of control laws of rotation of two flight vehicles around common attachment point at the moment of separation. Aerospace system consists from two flight vehicles, which located one above the other and connected at three junction points. Lower flight vehicle perform the role of launching aircraft and lift aerospace system to the air start altitude. Upper flight vehicle is less mass and, while process of separation, starts from the upper surface of the launch aircraft. Synthesized control laws are based on nonlinear mathematical dynamic

model of motion of flight vehicle by using ADAR method. Synthesized control laws are provides asymptotic stability of closed nonlinear system "control object – autopilot", and achievement of predetermined technologic invariant.

Keywords: aerospace system, control laws, flight vehicle, ADAR method.

Investigation of Dynamical States of cosmogonical Body Formation based on the Generalized Nonlinear Schrödinger-Like Equation

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This work investigates different dynamical states of cosmogonical body formation using the generalized nonlinear Schrödinger-like equation obtained within framework of statistical theory of gravitating spheroidal bodies. The statistical theory for a cosmogonical body forming (so-called spheroidal body with fuzzy boundaries) has been proposed in our previous works [1–4]. As shown within framework this theory, interactions of oscillating particles inside a spheroidal cloud lead to a gravitational condensation increasing with the time. As a result, the generalized nonlinear time-dependent Schrödinger-like equation describing a gravitational formation of a spheroidal body has been derived. As shown here, this equation is more general than analogous equations obtained in Nelson' stochastic mechanics and Nottale's scale relativistic theory.

This work investigates different dynamical states of a gravitating spheroidal body and respective forms of the generalized nonlinear time-dependent Schrödinger-like equation. In particular, the derived time-dependent generalized nonlinear Schrödinger-like equation describes not only the state of virial mechanical equilibrium and the quasi-equilibrium gravitational condensation state, but the initial equilibrium gravitational condensation state taking place in a forming gas-dust protoplanetary cloud as well as the soliton disturbances state occurring in a spheroidal body under formation and also the gravitational instability states including the increase of gravitational compression of spheroidal body providing a formation of core of cosmogonical body. Besides, the last case involves the avalanche gravitational compression increasing (when the parameter of gravitational condensation grows exponentially with the time) among them the case of unlimited gravitational compression leading to a collapse of a spheroidal body.

The reduced model of Lorenz-type for the cubic nonlinear Schrödinger equation is obtained. In this work, we derive the system of ordinary differential equations for this model as well as a new attractor into state-space. We show that the proposed model is represented by the system of

four ordinary nonlinear differential equations with quadratic nonlinearity. We also note that the obtained attractor demonstrates the chaotic dynamics into the state-space like Lorenz one or the attractor describing flows with curvature of streamlines [5]. We apply the matrix decomposition [6] to analyze a vector function describing the obtained attractor in state-space.

Keywords: molecular clouds; initial oscillating interactions; spheroidal bodies; slow-flowing gravitational condensation; generalized nonlinear Schrödinger–like equation; reduced model; attractor; matrix decomposition.

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The Studying of Chaotic Regimes in Chua's Circuit based on Matrix Decomposition Method

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A number of studies devoted to chaotic dynamics in electronic circuits grows every year. Considerable attention is given to development of new methods for analysis of electronic circuits. An important task is to identify chaotic regimes in circuit or purposefully induct them. The scope of this study is to analyse Chua's circuit with a cubic nonlinearity using the method of matrix decomposition into a state-space. The system of Chua's nonlinear equations is expanded into a matrix series. As a result, linear, cubic and quadratic matrix terms is obtained. Numerical integration of these matrix terms makes it possible to estimate an influence of higher order nonlinearities on the Chua's circuit chaotic regime, as well as observe appearance of a chaotic attractor from linear, quadratic and cubic

matrix terms. The study was carried out using the Simulink-model, which can be ported for embedded systems as a generator of chaotic signals. It is shown that the regime of rigid self-oscillations excitation leads to appearance of a double scroll chaotic attractor into the state-space.

Keywords: Nonlinear dynamical system, Chua's circuit, chaotic attractor, matrix decomposition of Chua's circuit, rigid self-oscillations excitation

Synchronization of 0.1-Hz Rhythms in the Signals of Laser Doppler Flowmetry, Photoplethysmogram and Cardiointervalogram

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The methods are developed for the detection of interaction and synchronization between the systems of autonomic regulation of blood circulation in the human cardiovascular system from noninvasively recorded signals. Nonlinearity, variability and chaotic character of physiological systems impede the study of their interaction and require the development of specialized methods. We simultaneously recorded the signals of electrocardiogram (ECG), photoplethysmograms and laser Doppler flowmetry in healthy subjects. The cardiointervalogram was obtained from the ECG signal. Synchronization between the low-frequency components (with a basic frequency of about 0.1 Hz) of the recorded experimental signals was investigated using the proposed methods. The obtained results revealed the new mechanism of interaction between the systems of autonomic regulation of blood circulation.

Keywords: Cardiovascular system, Synchronization, Autonomic regulation, Laser Doppler flowmetry, Simulation.

Fractality of the Russian Financial Market

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The main purpose of this report is to investigate dynamics and behaviour of the financial time series for the Russian market using the fractality conception which was initially introduced by Benoit Mandelbrot [1]. The fractals have already been proved themselves as a model which describes experimental data better than previously used conventional theories in

such fields of science like radiolocation, natural resources investigations, distant sounding, navigation, meteorology, information processing from unmanned aerial vehicles (UAV) and synthetic aperture radars (SAR), medicine and biology. At the same time there is no unified theory of the financial data behaviour. Still, some worthy efforts were already done in this area for the last years [2, 3]. It was shown that price changes rather obeyed to the Levi flight rules than to the Gaussian distribution and also we could watch the evolution from the Effective Market Hypothesis to the Fractal Market Hypothesis which can better explain the market crashes especially during crises. Here we'd like to apply the fractal approach to the Russian financial time series as a very young market (about 20 years of history, 2 significant crises). So we will show which law the data distribution follows to and how similar the data are at different time scales.

Keywords: Fractality, self-similarity, financial markets, prices changes distributions, prices correlation

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Dynamics of the Double-Pendulum System with Side Stops Forced by Poly-Harmonic Excitation

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The main aim of the paper is to analyze the dynamic properties of a double-pendulum system with side stops where collisions occur. This research was motivated by a real technological problem of vibrations of bodies hanging on chains or ropes in tubes or spaces limited by walls or other bodies. The system has two degrees of freedom and is forced by poly-harmonic excitation. Its movement is governed by a set of nonlinear ordinary differential equations. As the main result it is shown that the system exhibits regular, irregular and chaotic patterns for suitable choice of parameters.

Keywords: Meachanical system; bifurcation; regular motion; chaos

A Proposed Test of Special-Relativistic Mechanics at Low Speed

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Recently, it was shown numerically for a dissipative bouncing ball system that, although the speed of the ball is low and the gravitational field is weak, the Newtonian approximation to the chaotic general-relativistic trajectory breaks down rapidly. The different Newtonian and general-relativistic chaotic trajectories could be tested in the laboratory but the parameters and initial conditions of the system must be known to very high accuracies so that sufficiently accurate trajectories can be calculated for comparison with experiment. Similarly, for low-speed non-dissipative systems where gravity does not play a dynamical role, it has been shown that the special-relativistic trajectory is not always well-approximated by the Newtonian trajectory, regardless of whether the trajectories are chaotic or non-chaotic. However, these systems are model systems, which are not realizable in the laboratory. In this paper, we present a non-chaotic system which, we show, could be used to test the different Newtonian and special-relativistic low-speed trajectories.

Magnetron Modes and Chimera States

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This paper reviews the magnetron electronic valve family development including the: the Barkhausen-Kurz oscillator, split-anode magnetron, cavity-magnetron, rising-sun cavity-magnetron, coaxial cavity-magnetron, relativistic cavity-magnetron and the packaged cavity-magnetron. The paper uses US magnetron patents and original peer-reviewed papers to provide a rich source of mathematical mode competition theory and lumped element equivalent electrical models. Synchronous and asynchronous modes are identified and quantified within the cavity-magnetron, rising-sun cavity-magnetron and relativistic cavity-magnetron. The military imperative (secrecy) that shaped magnetron development is also discussed. It is proposed that this knowledge may have a role in chaos theory when applied to natural and man-made coupled resonator networks.

Keywords: Magnetron, Mode, Dengeneracy, Instability, Chimera States.

Stability of Flows with Expanding Measures

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The notion of expansive measure for flows was introduced in [1]. In this talk, we introduce a concept of expanding measure for flows motivated by the notion of measure expansiveness for homeomorphisms in [2], and study the dynamics of measure expanding flows, invariantly measure expanding flows and measure expansive flows on compact metric spaces. More precisely, we prove that any invariantly measure expanding flow with measure shadowing has the spectral decomposition, and give an example to show that a measure expansive flow with measure shadowing does not have the spectral decomposition. Moreover we claim that an integrated flow ϕ of a C^1 vector field on a compact C^∞ manifold is C^1 stably measure expanding if and only if it is quasi-Anosov, and ϕ is C^1 stably invariantly measure expanding if and only if it is Ω stable.

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Anomalous Brownian Motion in Macromolecules and Tissues

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The use of anomalous diffusion to describe stochastic motion of particles and relaxation behavior of complex systems is well established in different fields of physics, including biophysics and physics in medicine. For example, subdiffusive transport of molecules in biological tissues and power-law fluctuations in the conformational dynamics of macromolecules are of great interest. These phenomena can be modeled by inclusion of memory effects in the standard Langevin theory of the Brownian motion (BM). In the present contribution the generalized Langevin equation for fractional BM of free and trapped particles is solved in an exceedingly

simple way to obtain the mean square displacement and other relevant correlation functions valid not only in the usual long-time diffusion limit, but at shorter times as well. The results are applied to the description of the observed subdiffusion dynamics in protein folding. New formulas for the free-induction and spin-echo NMR attenuation functions are also presented and compared favorably with experiments acquired in human neuronal tissues.

Keywords: Fractional Brownian motion, Generalized Langevin equation, Particle in a well, Protein folding, Diffusion in tissues, NMR attenuation functions.

The Delaunay Triangulation Learner

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We propose a new piecewise linear learner, called the Delaunay triangulation learner (DTL), which is a smoother duality of the 1-nearest neighbor (1-NN) leaner. Based on the data samples in a p-dimensional feature space, the Delaunay triangulation algorithm provides a unique triangulation of the space, which yields a dual graph of the 1-NN Voronoi diagram. The triangulation separates the convex hull of the samples into a series of disjoint p-simplices, where the samples are the vertices of the p-simplices. The DTL is constructed by fitting the responses through linear interpolation functions on each of the Delaunay simplices, and thus it approximates the whole functional by a piecewise linear function. We study the theoretical properties of the DTL and compare its performances with the 1-NN learner on multi-dimensional random smooth functionals. Furthermore, we propose two appropriate regularization functions to penalize the roughness of the DTL and improve its predictability on the testing data. In ensemble learning approaches, we propose the bagging DTLs, random crystal and the boosting DTL, where the DTLs are constructed on the subspaces of the features, and the feature interactions are captured by Delaunay triangle meshes. Extensive numerical studies are conducted to compare the proposed DTL and its ensembles with 1-NN and tree-based counterparts. The DTL methods show competitive performances in various settings, and particularly for smooth functionals the DTL demonstrates its superiority over other methods.

Binary Interaction Models for Random Markets

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In this communication, some economic gas-like models for random conservative markets are addressed. In these models the agents trade by pairs bringing the system toward a statistical equilibrium, this is the asymptotic wealth distribution. The time evolution of these models is given by nonlinear functional mappings. These maps are nonlinear operators in the space of wealth distributions, which are shown to conserve the total and mean wealth of the economic system. Different asymptotic results for several models are presented. The decay to the exponential distribution is found in some of them and a transition to power-like distributions is sketched when a naive bank system is suggested. Simulations and implementations of these systems in different topologies are also presented. (The talk is a review of several papers).

Keywords: Economic modeling, Random markets, Wealth distributions.

Detecting Causal Relations from Real the Data Experiments has Posed Great Challenges in Data-Driven Inference Methods

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In this work, we present the Recurrence Measure of Conditional Dependence (RMCD), a recent data-driven causality inference method using the framework of recurrence plots. The RMCD incorporates the recurrence behavior into the transfer entropy theory. We apply this methodology to some paradigmatic models and to investigate the possible influence of the Pacific Ocean temperatures on the South West Amazon for the 2010 and 2005 droughts. The results reveal that for the 2005 drought there is not a significant signal of dependence from the Pacific Ocean and that for 2010 there is a signal of dependence of around 200 days. These outcomes are confirmed by the traditional climatological analysis of these episodes available in the literature and show the accuracy of RMCD inferring causal relations in climate systems.

Keywords: Synchronization, Recurrence Plot, Extremes in Chaotic Systems, Network dynamics, nonlinear dynamics, climate.

On the Time Decay in Phase-Lag Thermoelasticity with two Temperatures

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The aim of the present communication is the study of the time decay of solutions for the one-dimensional phase-lag thermoelasticity with two temperatures. In a recent contribution [1] the authors studied the stability of the solutions determined by several models of the phase-lag heat conduction with two temperatures. Suitable conditions to guarantee the stability (instability) of the solutions were obtained there. We here want to extend the analysis to the one-dimensional thermoelastic problem.

Keywords: One-dimensional thermoelasticity, time decay, phase-lag with two temperatures, semi-group arguments, spectral arguments.

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Bifurcation Theory of Dynamical Chaos

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It will be presented that there exists a universal bifurcation scenario of transition to dynamical and spatio-temporal chaos in all kinds of nonlinear differential equations including dissipative and conservative, nonautonomous and autonomous nonlinear systems of ordinary and partial differential equations and differential equations with delay arguments. This scenario consists of several Andronov-Hopf bifurcations and then of Feigenbaum cascade of period doubling bifurcations of stable cycles or n-dimensional tori, and then of Sharkovskii cascade of subharmonic bifurcations of stable cycles or tori of an arbitrary period up to the cycle (torus) of period three, and then of Magnitskii cascade of homoclinic or heteroclinic bifurcations of stable cycles or tori up to the homoclinic or heteroclinic separatrix loop of a singular point or up to the homoclinic or heteroclinic separatrix toroidal topological structure. It is shown that this universal FShM bifurcation scenario of transition to chaos takes place in all classical dissipative and conservative systems of ordinary and partial differential equations including Lorenz, Ressler, Chua, Sprott, Duffing-Holmes, Mathieu, Croquette, Rikitaki, Henon-Heiles, Yang-Mills systems, Brusselyator, Kuramoto-Tsuzuki, Mackey-Glass equations and many others. It will be shown that transition to turbulence in Navier-Stokes equations has the same scenario.

Keywords: Dissipative and conservative systems, Transition to chaos, Universal bifurcation FShM scenario, Navier-Stokes equations, Turbulence.

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Ray and Wave Chaos in Randomly Inhomogeneous Acoustic Waveguides in the Ocean

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Sound can propagate in the ocean over distances of thousands kilometers due to the presence of the natural waveguide, the so-called underwater sound channel. However, long-range sound propagation is strongly affected by scattering of sound waves on weak random longitudinal inhomogeneity of the waveguide. In the ray approximation, this kind of scattering gives rise to Lyapunov instability and chaos of sound rays. Wavefield manifestations of ray chaos are called wave chaos. Owing to the principle of the optical-mechanical analogy, wave chaos is mathematically equivalent to quantum chaos. In the present talk the problem of wave chaos is considered in terms of random matrix theory. The finite-range evolution operator of a wavefield is introduced. The ray-based counterpart of the finite-range evolution operator is the one-step Poincaré map allowing one to find out domains of finite-time stability in the underlying phase space. Interplay between spectral properties of the finite-range evolution operator and phase space patterns of the one-step Poincaré map is examined.

Keywords: Wave chaos, Ray chaos, Underwater sound channel, Random matrix theory, Wave propagation in random media, Finite-range evolution operator.

Approximation of Slow and Fast Dynamics in Chaotic Electrochemistry Oscillators using Biorthogonal Wavelets

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In this work, we introduce a methodology that allows to approximate the slow and fast dynamics present in chaotic multiple time-scale dynamical system. Our approach is based on decomposition of the time-series using discrete wavelet transform with biorthogonal wavelets. The basic idea is that the slow and fast dynamics whose behaviors on the time-domain are characterized by big, small and fast oscillations and can be approximated with low and high frequencies, respectively, present in the analyzed time-series. In this way, a discrete wavelet transform decomposes a time-series in subbands so that it is possible to find the desired approximations by choosing specific subbands in the wavelet analysis for the process reconstruction. Our proposed approach is validated by applying it to analyze experimental time-series generated by electrochemistry oscillators in a network whose dynamic is chaotic. The results show that our approach is useful to characterize the fast and slow dynamics in time-domain.

Keywords: Chaotic Multiple Time-Scale Dynamical Systems, Wavelet Transform, Biorthogonal Wavelets, Approximations of dynamics.

The 0-1 Test for CHAOS and RQA Analysis Applied on the CML Dynamical System of Laplacian Type

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We study the dynamics of Laplacian-type coupling induced by logistic family $f_\mu(x)=\mu x(1-x)$, where $\mu\in[0,4]$, on a periodic lattice, that is the dynamics of maps of the form $F(x,y)=((1-\varepsilon)f_\mu(x)+\varepsilon f_\mu(y),(1-\varepsilon)f_\mu(y)+\varepsilon f_\mu(x))$ where $\varepsilon>0$ determines strength of coupling. Our main objective is to analyze the structure of attractors in such systems and especially detect invariant regions with nontrivial dynamics outside the diagonal. We use the 0-1 test for chaos and Recurrence Quantitative Analysis to determine the dynamics of the system in dependence of coupling parameter and directional parameter of the underlying map f.

Keywords: chaotic models, coupled map lattice, 0-1 test for chaos, recurrence quantitative analysis

Transient Chaos in the Lorenz-Type Map with Slow and Periodic Forcing

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We consider a Lorenz-type map with a boundary crisis of a chaotic attractor and study the effects of the slowly varying control parameter as well as of the periodic forcing. We extend the concept of dynamic bifurcations to chaotic phenomena [1]. As the control parameter passes through a critical value, the dynamic boundary crisis of a chaotic attractor takes place [2]. We discover the effects of delayed exit from the chaotic region and non-exponential decay of the surviving probability; the property is analyzed of the delay increase with increasing rate of the control parameter change. We discuss also what happens to the system and particularly to the transient chaotic dynamics if the control parameter periodically oscillates [3]. We study the impact of the forcing frequency and amplitude on the escape rate, analyze the phase-space image of the observed dynamics and investigate the influence of initial conditions.

Keywords: Transient chaos, Dynamic bifurcation, Periodic forcing, Boundary crisis, Lorenz-type map, Survivng probability, Escape rate.

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Dynamical Features of Acoustic Emission of Natural and Forced Stick-Slip Process

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Friction processes, including stick-slip movement, are regarded as complex phenomenons of mass and energy transfer between two surfaces through elastic and inelastic interactions of macroscopic, micro

and nano contacts. Such processes can be observed in different natural and technical systems and are subject of intense interdisciplinary investigations. For example, stick-slip motion as a model of earthquake generation, which is often analyzed for last decades. Acoustic emission (AE) accompanying the stick-slip process is a sensitive tool for revealing fine details of the complex friction process. In the present work, AE accompanying the stick-slip movement of basalt samples are investigated in laboratory slider-spring device under different experimental conditions, including weak mechanical or electromagnetic forcing of various intensity and frequency. Recurrence quantification analysis and Lempel and Ziv complexity measure methods have been used to assess changes occurred under external forcing in time series of stick-slip AE process.

Keywords: Friction processes, Stick-slip movement, Acoustic emission, External forcing, Recurrence, Complexity measure, Complex dynamics.

Transition Responses for the Timeout in TCP/RED

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TCP communication is a reliable communication protocol. However, when packets are excessively transmitted, congestion occurs in the network, and the commu-nication performance deteriorates. In addition, congestion cannot be sufficiently avoided by TCP congestion control mechanisms. The occurrence of the congestion state that causes this communication performance to deteriorate is a serious problem in the network. Therefore, Random early detection (RED) is proposed as a congestion avoid-ance method. This is one of the active queue management (AQM) systems. In this method, the congestion in the network is detected by managing the packet queue and discarding the packet based on probability. Thus, since the congestion avoidance operation can be performed earlier than the congestion occurs, it is possible to prevent degradation of the performance of the network. In recent year, mathematical modeling of congestion avoidance methods such as TCP/RED has been performed. Bifurcation phenomena of S-model which is a mathematical model of TCP/RED also has been analyzed. In the bifurcation phenomenon of the S-model, it is confirmed that the shape of timeout appears in a different form from the part with the periodic solution. However, analysis of factors that form the shape of timeout has not been done.
In this study, we discuss factors that form the shape of the timeout in the bifurcation phenomena of TCP/RED using S-model.

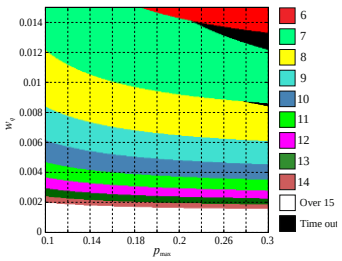

Fig. 1. Steps of transient in the p_{max}-wq space.

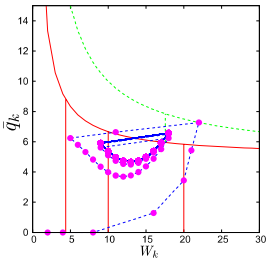

Fig. 2. Phase plane (p_{max}=0:30, wq = 0:01216).

Keywords: TCP/RED, bifurcation, Timeout.

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Stability of a Nonlinear Viscoelastic Problem Governed by Lamé Operator

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In this talk we will investigated the stability of the nonlinear viscoelastic problem governed by Lamé operator. When I' study the existence and uniqueness in [8]. We denote by Ω an open subset of $\mathbb{R}^n$ with regular boundary Γ. Let Q the cylinder $\mathbb{R}_x^n \times \mathbb{R}_t$ with Q = Ω ×]0,T[; T fini, Σ boundary of Q, L designed Lamé system define by μΔ + (λ+μ)∇div, f, $u_0(x)$ and $u_1(x)$ are functions. We look for the stabilisation of a function $u = u(x,t)$, $x \in \Omega$, $t \in]0,T[$, solution of the problem (P).

$$(P)\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - Lu + \displaystyle\int_0^t g(t-s)\Delta u(s)ds + |u|^p u = f \quad \text{in } \Omega \times]0,T[\\ u = 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{on } \Sigma \\ u(x,0) = u_0(x), \qquad \dfrac{\partial u(x,t)}{\partial t}\Big|_{t=0} = u_1(x) \qquad\qquad x \in \Omega \end{cases}$$

Keywords: Nonlinear, Priori Estimate, Stability, Viscoelastic.

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Influence of Finite Time Correlations on the Anomalous Scaling of Passive Magnetic Field

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Using the field theoretic renormalization group technique and the operator product expansion the influence of the finite time correlations of the Gaussian velocity field on the anomalous dimensions of the leading composite operators which drive the anomalous scaling of the single-time two-point correlation functions of the passively advectd magnetic field are investigated in the framework of the generalized Kazantsev-Kraichnan model. System of all scaling regimes of the model is identified and coordinates of the corresponding infrared stable fixed points are found. The anomalous dimensions of the relevant composite operators are calculated and it is shown that the leading contribution to the anomalous scaling is given by the isotropic shell in accordance with the Kolmogorov's local isotropy restoration hypothesis. It is also shown that the assumption of the finite time correlations of the velocity field of the stochastic conductive environment can have nontrivial impact on the scaling properties of advected magnetic fields.

Keywords: Kazantsev-Kraichnan model, Magnetohydrodynamics, Turbulence, Renormalization group.

Stochastic Properties of Prime Numbers Distribution

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The history of studying the prime numbers distribution of has gone a very long way from Euclid to our days. And in our time, interest in this topic remains. At this paper, a new approach to the study of this distribution is found. Using a demonstrative representation of primes in the form of four

disjointed ordered sets, the stochastic interpretation of the distribution is obtained. Based on the exact solution of the problem of quantity of primes in an arithmetic progression with the aid of the Ehrenfest sieve, estimates of the probability characteristics of all four components of the distribution are obtained.
The known exact and approximate solutions of the problem of the distribution of prime numbers in connection with the definition of its stochastic characteristics are studied. The prospects for further applications of the obtained results are considered, in particular, to the distributions of primes in other sequences of natural numbers and to certain problems in numbers theory.

Keywords: prime numbers, distribution, ordered sets, stochastic interpretation, Ehrenfest sieve, numbers theory, applications.

Chaotic Semiflows with General Acting Topological Monoids

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A semiflow is a triple consisting of a Hausdorff topological space X, a commutative topological monoid T and a continuous monoid action of T on X. The acting monoid T is usually either the discrete monoid $\mathbb{N}_0$ of nonnegative integers (in which case the semiflow can be defined as a pair (X, f) consisting of a phase space X and a continuous function $f: X \to X$), or the monoid $\mathcal{R}_+$ of nonnegative real numbers (the so-called one-parameter monoid). However, it turns out that there are real life situations where it is useful to consider the acting monoids that are a combination of discrete and continuous monoids. That, for example, happens, when we are observing certain dynamical system at discrete moments, but after some time realize that it would be beneficial to continue our observations in real time. The acting monoid in that case would be $T = \{0, t_0, 2t_0, \cdots, (n-1)t_0\} \cup [nt_0, \infty)$ with the operation and topology induced from real numbers. This partly explains the motivation for the level of generality which is pursued in our research. We introduce the psp monoids, which include all but "pathological" monoids, and most of our statements hold for them. The topic of our presentation are some recent results about chaos-related properties in semiflows, indecomposability and sensitivity of semiflows in the described general context.

Keywords: Chaos, Indecomposability, psp monoids, Semiflow, Sensitivity

Anisotropy Induced Current Reversal in Two Dimensional Driven Lattices

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Time driven lattice setups, i.e., particles in a lattice which is shaken by external time-dependent forces of zero mean, can exhibit directed transport for an ensemble of particles - a phenomenon more commonly called 'ratchet' effect. Such transport has been realized in a variety of setups leading to many intriguing effects and applications like on-site particle trapping, particle sorting and efficient velocity filters. In this context, a topic of particular interest is current-reversals which allows one to control the direction of transport of a particle ensemble by changing system parameters. We present a two dimensional driven lattice system where the lattice anisotropy is shown to induce a dynamical current reversal without any time dependent tuning of parameters. The timescale of such reversals can be systematically tuned via the lattice anisotropy. This intriguing effect can be explained by analyzing the ensemble dynamics in the underlying quasi 1D phase space along the driving direction and the diffusion properties of the lattice along the orthogonal direction. Such a dynamic transport reversal could possibly be experimentally observed using cold thermal atomic ensembles in optical lattices or colloids in optical devices.

Keywords: Driven lattices, Directed transport, Hamiltonian systems, Current reversal, Chaotic simulation.

Chaos Investment in Engineering and Robotics Applications

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Over the last few decades, the terms chaos and nonlinear dynamics are known to most engineers and scientists. Nonlinearities occur in feedback processes, in systems containing interacting subsystems and in systems interacting with the environment. It is striking to note that simple devices, such as a double pendulum, and a very complex event such as time follow the same dynamics, which can only be predicted for short time horizons. Thanks to the existence of high-speed computers, new analytical techniques and sophisticated experiments, it has become clear that the chaotic phenomenon is of a universal nature and has transverse consequences in various fields of human endeavour.

A lot of practical applications of deterministic chaos have been developed in various fields of engineering and technology. Actually, studies of nonlinear dynamics in engineering disciplines have been steadily progressing over a half century. Among these disciplines, we are going, in this paper, to deal with electrical and electronic engineering and applications of chaos in robotics.

Keywords: Chaos, Chaotic systems, Chaos engineering, Robotic applications, Chaotic simulation.

Collision of Chaotic Attractors with Repellers in a System of Two Phase Oscillators with Plastic Couplings

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Dynamics of a four-dimensional dissipative system with cylindrical phase space are considered. The system describes a behavior of two phase oscillators interacting via reciprocal couplings which depend on the oscillators' phase difference. The evolution of the coupling coefficients is described by a two-dimensional nonlinear system, i.e., the links possess the property of plasticity. We found that in the system's parameter space there exist regions which correspond to a new type of dynamics. This is so-called mixed dynamics characterized by a fundamental inseparability of attractors, repellers and conservative elements of dynamics. Such dynamics are determined in the phase space by the attractor of a new type which is called a reversible core. The properties of the reversible core differ from those of strange attractors of dissipative systems as well as those of conservative chaos in Hamiltonian systems. We discovered that the reversible core appears as a result of the collision of the strange attractor and the chaotic repeller by varying the control parameter. We found that such an attractor contains the limit of the sequence of the dissipative attractors and the sequence of the dissipative repellers.

Keywords: Chaotic attractor, Chaotic repeller, Mixed dynamics, Phase oscillator, Plastic coupling, Reversible core.

Chaos in Quaternion Blaschke Maps

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Previously, we have explored chaotic phenomena of nonlinear Blaschke products in the context of dynamical systems. In this presentation, we further extend the effort from complex mapping to quaternion mapping,

which as W. Blaschke published an article entitled “KINEMATICS AND QUATERNIONS” in 1958 and indicated that the mathematical efforts will flourish the development of geometry of motion or kinematics in physics with the use of the quaternions that L. EULER introduced to that end in 1748 and coined by W. R. Hamilton in 1843.
Blaschke products are mathematical generalization of Lorentz formalism, and hereby we define the functional sets in momentum space and as previous efforts, we characterize the divergent and transitional regions in conjunction with parametric space. Chaotic phenomena are further compared with the results in complex mapping.

Keywords: Quaternion, Blaschke, Momentum space, complex

On Chaotic Behaviour of an Orbiting Slender Body with Controllable Mass Distribution

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Attitude dynamics of a spacecraft as well as controllability of its orientation has been subject of numerous studies since the beginning of the Space Age. Ability to orient a space station according to a given law plays an important role in the design of space missions. In present paper dynamics of three-dimensional slender spacecraft with controllable mass distribution in an elliptic orbit is studied. Equations of motion are derived. Steady motions are investigated within appropriate assumptions related to the mass redistribution prescribed law. Chaotic behaviour and stable periodic motions are examined via numerical experiments.

Keywords: Orientation of space station, Beletsky's equation, chaotic motion, non-conservative mechanical system

Extreme Events versus Extreme Random Matrices

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Using the introduced by us "thinning method", we explain the link between classical Fisher-Tippett-Gnedenko classification of extreme events and their free analogue obtained by Ben Arous and Voiculescu in the context

of free probability calculus. In particular, we present explicit examples of large random matrix ensembles, realizing free Weibull, free Frechet and free Gumbel limiting laws, respectively. We also explain, why these free laws are identical to Balkema-de Haan-Pickands limiting distribution for exceedances, i.e. why they have the form of generalized Pareto distributions. Finally, we derive a simple exponential relation between classical and free extreme laws.

The Hydrogen Atom in a Circularly Polarized Microwave Field: Hopf Bifurcation and Chaos

Merce Olle, Juan Ramon Pacha
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We consider the CP problem, i.e., the Rydberg electron in a rotating electric field, whose dynamics is described by a Hamiltonian depending on one parameter, K>0.
We analyse the Hopf bifurcation appearing around one of the equilibrium points when K crosses a critical value K_{crit}. The effect of this bifurcation, focusing on regular and bounded motion versus chaotic one is also discussed.

Keywords: Hamiltonian dynamical systems, Hopf bifurcation, periodic orbits and tori, chaotic regions, invariant manifolds

On the Stability and Ultimate Boundedness of Solutions of Certain Third-Order Nonlinear Non-autonomous Delay Differential Equations

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Convergence or extreme stability of solutions of play a very important role in characterizing the behavior of solutions of nonlinear physical system like delay differential equations which are important tools in scientific modeling of some practical problems which often arise in many fields of science and technology such as after effect, nonlinear oscillations, biological systems and equations with deviating arguments.
We present in this paper the problem of convergence behavior of solutions of certain third-order nonlinear delay differential equation and obtained the sufficient conditions involved under which the solutions of the delay differential equation are convergent. The new result obtained extends and improves on earlier results on delay differential equations.

Spatiotemporal Chaos and Intermittency in Nematic Electroconvection

Iuliana Oprea, Gerhard Dangelmayr
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Spatiotemporal chaos (STC), a state that is unpredictable in both space and time, can be found in physical systems when many spatial degrees of freedom contribute to the dynamics, like thermal convection, nematic electroconvection, various pattern-forming chemical reactions, parametrically forced surface waves and other large sustained nonequilibrium systems. Unlike low dimensional systems, where different scenarios for the transition to chaos have been established, in spatially extended systems the transition from regular stationary patterns to complex spatiotemporal regimes is only partially understood, and the problem of finding general approaches for the characterization and classification of STC, as well as the identification of instability mechanisms generating it, are still open questions in nonlinear science. The electroconvection in nematic liquid crystals is a paradigm example in anisotropic extended systems, allowing theoretical studies in terms of reduced amplitude or envelope equations of STC, which can arise directly at the onset of electroconvection. In this research we identify and characterize the instability mechanisms generating the spatiotemporal complex patterns, such as spatiotemporal intermittency and chaos, in the numerical simulations of a system of amplitude and phase equations extracted from the continuum equations, for parameters corresponding to the nematic I52. Comparison with experiments at the Liquid Crystal Institute, Kent, OH, are also discussed.

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On Stochastic Approximation Techniques in the Study of a Class of Systems

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In the tentative to find new possibilities to characterize the process of generation of the words by sequences of intermediate words we have adopted a stochastic point of view involving Markov chains. But such sequences of intermediate words (called *derivations*), by which the words are generated, are finite such that it results that finite Markov chains will be connected to the process.

Now in order that our discussion should be as general as possible, the derivations are considered according to the general class of formal grammars from the so-called *Chomsky hierarchy*, namely those that are free of any restrictions and are called *phrase-structure grammars*.
In some previous papers we have introduced numerical functions able to characterize classes of derivations according to a given generative system (as the *phrase-structure grammars* known in formal languages) up to an equivalence. They are referred to as *derivational functions*.
Some new properties of symmetry and invariance are established. Also we shall refer to some combinatorial properties of the difference operator defined on the equivalence classes of derivations and finally we refer to the characterizations up to an equivalence just obtained,

Keywords: equivalence classes, random variables, Markov chains, Markov processes, stochastic calculus.

Chaos Based Substitution Boxes as a Cryptographic Primitives: Challenges and Opportunities

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Our world is undergoing a rapid transformation. In our daily lives, many services are being moved to digital environments. This transformation provides many facilities. But it has been emerged in many problems that we need to solve together. One of the most challenging topics of these problems is information security since the information in the digital environment poses a great risk. It is estimated by experts that in 2020 it will be a requirement to ensure that more than 50% of the information in the digital environment is secured. Therefore, researchers have been working on the design and analysis of next generation encryption algorithms.
Chaotic systems are an important field of study that has influenced many areas of science and engineering. In computer science, determining of optimization parameters, neural networks, prediction of time series, simulation, modelling, and secure communications are some of application areas of these fascinating systems. In the last two decades, chaos-based cryptology studies have been a hot research topic. One of these research topics has been chaos based substitution boxes design studies. In this study, chaotic systems have been used as randomness sources in designing substitution boxes.
Firstly, this study has been explained the historical development of chaos based substitution box literature. This summary of the literature has resulted in significant results. These results are as follows: (i) the complexity of the chaotic system class has no effect on the substitution box performance criteria. (ii) The main factor affecting the success of the

performance criteria is the conversion function used in the substitution box design algorithm. (iii) Both the conversion function and the chaotic system, the best performance measures for chaos (random selection) based designs are 106.75 for nonlinearity and 10 for the DP table.
Another important contributor to the study is listing various challenges and opportunities of chaos based substitution box studies. Some problems of chaos based design are the effects of digital deterioration and the number of logic elements required for implementation on the integrated circuit. Some opportunities of chaos based designs is that the they may be used as a measure against side channel analysis and as a key generator algorithm.

Keywords: Chaos, cryptography, substitution box.

The Effects on Performance of Using Chaotic Systems in Entropy Source of Deterministic Random Number Generators

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The concept of randomness is needed both in science and engineering applications. One of the main design approaches of random number generators is the deterministic random number generators. The deterministic random number generator starts with a seed value and generates algorithmically random numbers. Deterministic random number generators have many advantages. It is low cost, does not require a devoted device and can be implemented in software. But there are also disadvantages. The output can be determined completely from the kernel value. The output sequences are not really independent. A strong entropy or noise source is needed to solve these problems.
The most prominent application areas of chaotic systems are random number generators. Because chaotic systems are a powerful entropy or noise source. This strong relationship between the two disciplines is based on the precise dependence of the chaotic systems on the initial conditions and the control parameters.
In this study, chaos based deterministic random number generators are investigated. New designs have been obtained by using chaotic systems as entropy sources of designs commonly known in the literature. Statistical tests have been used to analyze the performance of new modified designs. Analysis result shows that output of this designs have been used as may application area such statistics, game theory, cryptography, and so on

Keywords: Chaos, random number generator, deterministic randomness

A Model for Storage and Recall of Images in the Human Brain using Coupled Maps

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Storage and recall of images (and in general, memories) in the human brain is still an active area of research. We propose a simple mathematical model using coupled maps that provides one possible solution to this problem. We start by theoretically investigating targeted synchronization in coupled map systems wherein only a desired (partial) subset of the maps is made to synchronize. A method is introduced to specify coupling coefficients such that targeted synchronization is ensured. The principle of this method is extended to storage/recall of images using coupled Rulkov maps that are often used to model neuronal networks in the brain. The process of adjusting coupling coefficients between Rulkov maps to store a desired image mimics the process of adjusting synaptic strengths between neurons to store memories. This model is novel in the sense that it uses both the following mechanisms that have been observed in the functioning of the human brain: synaptic modification and synchronization. The stored image can be recalled by providing an initial random pattern to the dynamical system. The storage and recall of the standard image of Lena is explicitly demonstrated.

Keywords: Targeted synchronization, Rulkov maps, Storage and recall of images in the brain.

Embedding-Dependent, full Scale Characterization of Sample Correlation Integrals

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The assessment of the correlation dimension D2 out of time series is based on the evaluation of the distribution of small-scale distances between vectors generated by the embedding procedure. Here, going beyond the small-scale limit, we investigate the whole distribution of embedded vector distances. We compared, via Kolmogorov-Smirnov statistics, embedding-dependent distributions of distances generated by a system of interest with those generated by a known reference, namely Gaussian white noise. The resulting set of embedding-dependent Kolmogorov-Smirnov statistics is a distinguishing portrait of the underlying system and can provide a new tool for tackling key issues as the determination of the optimal sampling period or stationarity assessment.

Keywords: correlation integrals, embedding, chaos, noise.

Chaotic Model in the Hilbert Spaces

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The report is devoted to investigation of necessary and sufficient conditions of the existence of chaotic behavior in nonlinear model in the form of the following system of differential equations in the Hilbert space *H*:

$$\begin{cases} \varphi'(t,\varepsilon) = \varphi(t,\varepsilon) + \psi(t,\varepsilon) + \varepsilon f_1(\varphi(t,\varepsilon),\psi(t,\varepsilon),\varepsilon), \\ \psi'(t,\varepsilon) = \varphi(t,\varepsilon) + \varepsilon f_2(\varphi(t,\varepsilon),\psi(t,\varepsilon),\varepsilon), t \in J \end{cases} \quad (1)$$

with boundary condition

$$l(\varphi,\psi)(\cdot,\varepsilon) = \alpha, \quad (2)$$

where $\varphi,\psi, \in C^1(J,H)$, $C^1(J,H)$ is the Banach space of continuously differentiable vector-functions on the interval *J* with valued in the Hilbert space *H* (or from the space $W_2^1(J,H)$); vector-functions f_1, f_2 are strongly differentiable; *l* is linear and bounded operator which translates solutions of (1) into the Hilbert space H_1. We find solutions of (1), (2) which turn into one of the solutions of the generating problem ($\varepsilon = 0$). It should be noted that considering problem can be solved with using of the operator equation for generating elements [1] which for the periodic case in the finite-dimensional spaces:

$$l(\varphi,\psi)(\cdot,\varepsilon) = (\varphi(T,\varepsilon) - \varphi(0,\varepsilon), \psi(T,\varepsilon) - \psi(0,\varepsilon)) = \alpha = (0,0)$$

has the form of the well-known equation of generating amplitudes [2]. We show that equation of generating elements in the finite-dimensional case is equal with the well-known Melnikov [3] function and gives condition of homoclinic chaos [4] (well-known Palmer's lemma).

Keywords: Chaotic modeling, Moore-Penrose pseudoinvertible operator, Palmer's lemma, Melnikov function.

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Recovery of Couplings and Parameters of Elements in Networks of Oscillators from Time Series

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The structure of interactions between elements in a network of oscillators determines the network dynamics. Understanding the structure of interactions is necessary for solving a variety of problems in physics, climatology, physiology, and other sciences. In this paper we propose a method for reconstructing the couplings and parameters of elements in networks of diffusively coupled ring time-delay oscillators. The method is based on the minimization of an objective function characterizing for each network element the distance between the points of the reconstructed nonlinear function, and separation of the recovered coupling coefficients into significant and insignificant ones. The method efficiency is illustrated for chaotic time series generated by model equations of diffusively coupled time-delay systems and for experimental chaotic time series gained from diffusively coupled electronic oscillators with time-delayed feedback. Developing this method, we illustrated its efficiency for a network of neuron-like oscillators, including the case of delays in couplings.

Keywords: Networks of oscillators, Parameter estimation, Time series analysis.

Thematic Course: Statistical Theory of Fractal Radar

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Fast development of the fractal theory in radar and radio physics led to establishing of the new theoretical direction in modern radar [1-8]. It can be described as «Statistical Theory of Fractal Radar». This direction includes (at least at the initial stage) the following fundamental questions: 1 - The theory of the integer and fractional measure. 2 - Caratheodory construction in the measure theory. 3 - Hausdorff measure and Hausdorff-Besicovitch dimension. 4 - The theory of topological spaces. 5 - The dimension theory. 6 - The line from the point of view of mathematician. 7 - Non-differentiable functions and sets. 8 - Fundamentals of the theory of probability. 9 - Stable probability distributions. 10 - The theory of fractional calculus. 11 - The classical Brownian motion. 12 - Generalized Brownian

motion. 13 - Fractal sets. 14 - Anomalous diffusion. 15 - The main criteria for statistical decision theory in radar. 16 - Wave propagation in fractal random media. 17 - Wave scattering generalized Brownian surface. 18 - Wave scattering surface on the basis of non-differentiable functions. 19 - Difractals. 20 - Cluster analysis. 21 - Theory and circuitry of fractal detectors. 22 - Fractal-scaling or scale-invariant radar. 23 - The multi-radar. 24 - MIMO radar. 25 - Cognitive radar. This list of studied questions, of course, is supposed to be expanded and refined in the future. The author has been dealing with it for nearly 40 years of his scientific career.

Keywords: Fractals, Radiolocation, Fractal radar, Low-contrast target, Fractal detector.

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Topological or Fractal Detectors. Principles of Building, Circuitry Engineering and Its Application for Detecting Stealthy High-Altitude Pseudo-Satellite

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Issues of circuitry engineering of fractal or topological detectors of radar signals have been considered. These detectors perform at low signal-to-noise ratios [1, 2]. Its mechanism is based on utilization of absolutely new fractal approaches [3, 4]. At that they apply functionals which depend on topology and dimension of a received signal rather than use energy characteristics (when the likelihood ratio is defined exceptionally by the energy of a received signal). The experimental results of detection of low-contrast targets in the presence of high-intensity noises of different types are presented. The reported methods of synthesis of topological detectors and certain examples of building of such detectors are required for systems of information processing in modern radio location. The given types of detectors are useful while detecting stealthy high-altitude pseudo-satellites (HAPS) and their aggregations. HAPS – are aircrafts located in the stratosphere at altitude more than 20 kilometers, for long flights, lasting months and years [5]. These unmanned aircrafts may present planes, airships or aerostats and act as localized satellites. Potential applications of HAPS use a combination of the satellite stability and the plane operating availability in real time: they include providing the communication during a natural disaster or long-term observation over a certain target area.

Keywords: Fractals, Radiolocation, Low-contrast target, High-altitude pseudo-satellites, Fractal detector.

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Triple Autocorrelation Function and Bispectra of Electrical Current for Rikitake System in Chaotic Mode

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The report presents graphical estimating of the triple autocorrelation function and bispectra [1] for dynamic variables describing the Rikitake system [2]:

$$\begin{cases} \dot{x}_1 = -\mu_1 \cdot x_1 + x_2 \cdot x_3 \\ \dot{x}_2 = -\mu_1 \cdot x_2 + x_1 \cdot x_4 \\ \dot{x}_3 = 1 - x_1 \cdot x_2 - \mu_2 \cdot x_3 \\ \dot{x}_4 = 1 - x_1 \cdot x_2 - \mu_3 \cdot x_4 \end{cases} \quad \textbf{(1)}$$

equations above describe dependence of derivatives by dimensionless time from parameter vector (μ_1, μ_2, μ_3).

Numerical solution of system by means of the fourth-order Runge-Kutta method with step of time Δt within diapason $[0,2 \cdot N \cdot \Delta t]$ gives estimation of triple autocorrelation function on square $[0, N \cdot \Delta t] \times [0, N \cdot \Delta t]$:

$$Q^s_{\text{jk}} = \frac{1}{N} \cdot \sum_{n=1}^{N} x_{s,n} \cdot x_{s,n+j} \cdot x_{s,n+k}, \quad s = 1,2,3,4\ , \quad \textbf{(2)}$$

where $x_{s,n}$ - result of numerical solution (1) at the time $n \cdot \Delta t$, as it follows from (2) bispectr estimation is:

$$B^s_{\text{jk}} = \sum_{l=1}^{N} \sum_{m=1}^{N} \quad U_{\text{jl}} \cdot U_{\text{km}} \cdot Q^s_{\text{lm}}, \quad \textbf{(3)}$$

where $U_{\text{jk}} = \exp[-i \cdot \frac{2 \cdot \pi \cdot j \cdot k}{N}]$ — unitary matrix of the discrete Fourier transformation [3].

Thus, using of statistical radiophysics concepts for the theory of dynamical systems мы systems allows to offer more accurate criteria for distinguishing these two types of system behavior, advantages of the method was demonstrate on example of Rikitake system (1).

Our attention to Rikitake system due to possibility using this system as a generator of chaotic electrical oscillations in a new generation of noise-resistant information transmission systems [5], and at [1] it has been described real workable model of Rikitake system.

Equations (1) describe a coupled system consist of two disk dynamos and the dynamic variables x_1 and x_2 have the meaning of dimensionless electric currents, variables x_3 and x_4 are dimensionless angular rotational speeds [2].

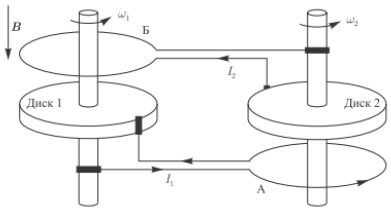

Fig. 1. Physical implementation of the Rikitake system.

Keywords: Rikitake system, chaotic mode, triple autocorrelation function, bispectrum, Runge-Kutta method, noise-resistant information transmission systems.

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A Fractional Nonlinear Schrödinger-Poisson System

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We are concerned with the following dampted fractional nonlinear Schrödinger-Poisson system,

$$\begin{cases} u_t + \gamma u + i(-\Delta)^s u + iu\phi = f, \\ \pm(-\Delta)^t \phi = |u|^2, \end{cases} \quad \textbf{(0.1)}$$

where $\gamma > 0$, $(\Delta)^a$ is the fractional Laplacian operator for = s, t ∈ (0; 1): The fractional Schrödinger equation provides us with a general point of view on the relationship between statistical properties of quantum-mechanical path and structure of the fundamental equations of quantum mechanics. The operator $(\Delta)^a$ can be seen as the infinitesimal generators of Lévy stable diffusion processes.

The study is divided into the following open problems.

- The first is to analyse how the different orders of the Laplacian operator affect the existence and uniqueness of solutions as well as the existence of a global attractor for the system (0.1). Assuming that the external force has some decay at the infinity the finite fractal dimension is proven.
- Following up, the next step to address is the discrete counterpart of the continuous dynamical systems (0.1). Actually, we consider a suitable semi-discrete fractional nonlinear Schrödinger equation introducing a suitable Crank-Nicolson scheme in time, keeping the space variable continuous. The aim is to prove that such a semi-discrete equation provides a discrete infinite dimensional dynamical system that possesses a global attractor.

Keywords: fractional nonlinear Schrödinger-Poisson system; Global Attractor; Absorbing Set; continuity; asymptotic compactness, unbounded domain.

Bifurcations of One-Dimensional One-Parametric Maps Revisited

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A parameter dependent family of maps $z \to F(z,a)$ with real or complex variable and parameter is studied. We deal with dynamics and bifurcations of iterates of this map in dependence on the parameter a and real bifurcations are analysed in a section of the phase-parameter complex hyperplane. Structure of bifurcation polynomials of polynomial maps will be presented on a logistic map.

Keywords: bifurcations of maps, period-doubling, chaos, complex dynamics, logistic map.

Chimera States in Networks of Globally Coupled Bistable Oscillators with Delayed Feedback

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The collective dynamics of oscillators is studied both numerically and experimentally in a network of identical bistable time-delayed feedback systems globally coupled via the mean field. Different ways of formation of the mean field are considered. The influence of delay and inertial

properties of the mean field on the collective behavior of globally coupled oscillators is investigated. Under a specific choice of initial conditions, two clusters containing oscillators with substantially different frequencies coexist in the network. It is shown that two different chimera states can take place in the network. In one of these states, the oscillators in the first cluster performing periodic oscillations in the fundamental mode exhibit synchronous behavior, while the oscillators in the second cluster performing chaotic oscillations at the third harmonic of the fundamental mode exhibit asynchronous behavior. In another chimera state, the chaotic oscillators are synchronized, while the periodic oscillators are desynchronized. It is shown that varying the parameters of the mean field it is possible to control the behavior of oscillators in clusters, including the formation of chimera states.

Keywords: Networks of coupled oscillators, Nonlinear time-delay systems, Clusterization, Synchronization, Chimera states.

Generation of a Hamiltonian Conservative Chaotic System with Strong Pseudo-Randomness

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Differing from the dissipative chaotic system whose orbit approaches to the invariant strange attractor with a fractal dimensional space volume, a conservative chaotic system has no strange attractor, and whose orbit fills in full dimension of phase space. Therefore, the conservative chaotic system has better ergodic property than dissipative chaotic system, which is advantageous in generating pseudorandom sequence in application to the chaos-based encryption. A conservative chaotic system preserves volume in phase space. A Hamiltonian system not just conserves volume in phase space but also preserves energy. The conservative chaotic systems in literature are rare, and Hamiltonian conservative chaotic systems are even rarer. If a conservative chaotic system preserves volume instead of energy will probably grow unbounded in long term. Therefore, a Hamiltonian conservative chaotic system is more significant. The big problem of the existing conservative chaotic systems is that they have small positive Lyapunov exponents leading to the narrow frequency spectrums, which is disadvantageous in generating pseudorandom number. On the basis of mechanics of rigid body, this paper proposes a four-dimensional Hamiltonian conservative chaotic system with a large (more than 500) Lyapunov exponent, large state amplitude and energy, and whose frequency spectrum is 50K times than the existing ones. Perturbation theory and mechanical theory are applied to analyze the Hamiltonian system in terms of the evolution of different dynamics. The

technics of Poincare map, power spectrum, etc. are used to demonstrate the richness of the Hamiltonian conservative chaotic system. The sequence number of the conservative system passes the NIST test, which is greatly useful in application of chaos-based encryption. The detail dynamical analysis is provided, which enlightens how to generate a new conservative chaotic system.

Keywords: Chaos, Hamiltonian conservative chaos, Rigid body, NIST, Pseudo-random number.

Topology and Persistent Homology of Recurrent Dynamics of Nonlinear Dynamical Systems

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The periodic nature of trajectory passing through a phase space is often studied by means of recurrence plots which in a 2-dimensional representation illustrate the collection of pairs of times at which the trajectory visits the same point in phase space. In this presentation we characterize recurrence plots and recurrent dynamics from the aspect of the high dimensional topology. Namely, the recurrence matrix, constructed from numerically or experimentally generated time series of the dynamical system, is interpreted as an adjacency matrix of the recurrent simplicial complex. The advantage of such an approach is that we can use a specific type of a simplicial complex (for example neighborhood complex or clique complex, to mention just a few) which is best adapted to the specific dynamics. We introduce a topological measure equivalent to the Sinai-Bowen-Ruelle measure and show how various topological invariants and other measures of simplicial complexes may be directly related to the statistical and dynamical characteristics of the system under study, such as Lyapunov exponents, entropy etc. Furthermore, we inspect the persistent topological features of simplicial complexes using homology and structure vectors of simplicial complex and show how it reveals new properties of recurrent dynamics which elude the standard approaches.

Keywords: Recurrence plot, recurrent simplicial complex, persistent homology, structure vectors

Reference: Slobodan Maletić, Yi Zhao and Milan Rajković, Persistent topological features of dynamical systems, Chaos 26 (2016) 053105.

Chaotic Synchronization in Richards' Models

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The bifurcation structure of Richards' models is investigated on a two dimensional parameter space. We provide sufficient conditions for the occurrence of extinction, stability, period doubling, chaos and non admissibility. Considering networks having in each node a Richards' function, we used the Lyapunov exponents to measure the system complexity, to obtain the synchronization intervals and to discuss the evolution of the synchronization level in terms of the model parameters. First we fix the network topology and change the local dynamics expressed by the models parameters, next, we fix the local dynamics and change the network topology. We obtained some results about the synchronization of these network.

Keywords: Richards' equation, bifurcations, networks, synchronization, Lyapunov exponents.

Platonic Solids and Fractals

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The Timaeus, which was considered by many early Platonists and Medieval Neoplatonists to be Plato's most important work, puts forward Plato's cosmology. In particular, the Timaeus provides a deep connexion between the five famous platonic solids and the fundamental elements of the universe. Using the Mandelbrot algorithm in a unified mathematical theory, we present how to generate, from a hypercomplex dynamical system, the following regular platonic solids: the cube (earth), the tetrahedron (fire) and the octahedron (air). Moreover, a 3D fractal transitions video between the related elements is proposed.

Keywords: Tricomplex dynamics, Generalized Mandelbrot set, Platonic solids, 3D fractals, Fractal alchemy.

Seismic Amplifications in Near-Shore Induced by Seaquakes using the Boundary Element Method

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We study seismic amplifications in near-shore due to seaquakes. For this aim, the Boundary Element Method is used. Boundary elements irradiate waves and force densities can be obtained for each element. Applying the boundary conditions a system of integral equations of the Fredholm type of second kind and zero order is obtained. Firstly, we used a numerical configuration to verify the present formulation with the ideal sea floor shape to estimate the seismic amplifications. After the verification of the formulation, simple slope configurations were studied to estimate spectra of seismic motions. It was found that P-waves can produce seismic amplifications from 1.2 to 3.9 times the amplitude of the incident wave, and that SV-waves can generate seismic amplifications up to 4.5 times the incident wave. It was also determined that the highest amplifications are observed at the shore as compared to the ones at the sea floor.

Keywords: Seaquake, elastic waves, boundary element method.

Existence of Chaos and Attractors in the Iberian Margin

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An analysis was made to the time series from the Iberian Margin, using the high resolution analysis of the marine sediment core MD01-2443 with a duration of 420,000 years. The temperature data obtained in this core, they was were taken from an alkenone record. From this series it was possible, through non-linear methods, to obtain an attractor with low fractal dimensionality and to corroborate the existence of deterministic chaos in the system, with the help of the correlation dimension and the Lyapunov exponents, respectively. This is important since few works have been developed, trying to prove the existence or not of a chaotic behavior in the climate, as well as the construction of a possible attractor. This is important since few works have been developed, trying to prove the existence or not of a chaotic behavior in the climate, as well as the construction of a possible attractor. In our case it was possible to obtain

both, at the same time that the results are with agree with some of the few works, previously published.

Key words: Deterministic chaos, fractal dimensionality, Lyapunov, climate.

One of the Simplest Chaotic Generator: Modeling, Research and Control

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In this paper one of the simplest chaotic circuit is presented. Circuit was modeling by using MultiSim software environment. System's behavior is investigated through numerical simulations, by using well-known tools of nonlinear theory, such as phase portrait, bifurcation diagram, Lyapunov exponents and Kaplan–Yorke dimension.Submitted by chaotic attractor and time distributions of two chaotic coordinates. Submitted values of capacitor C2 in which generated controlled chaotic attractors. Control 2-period and 3-period attractors and time distributions of two chaotic controlled coordinates are also submitted. For the first time was conducted control CCC circuit. For demonstration was applied modern software environment MultiSim. The values of capacitor C2 can be used as keys for masking and decryption of information carrier in modern systems transmitting and receiving information.

Keywords: simplest chaotic generator; control; MultiSim.

Memristor: Modeling and Research of Information Properties

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A memristor is the fourth basic element of the electric circuit together with capacitors, resistors and inductance coils. This is a passive two-terminal network with nonlinear volt-ampere characteristics, which has hysteresis and is capable of changing its resistance depending on the amount of charge that flows through it. In this article the practical implementation of the Chua's scheme is presented. KNOWM memristor of BS-AF-W 16DIP series based on the phenomenon of ionic conductivity was used as a nonlinear element. Temporal dependence of the system is discussed, the chaotic attractors are found and the signal spectrum is given.

Keywords: Chaos, memristor, hysteresis, Chua.

Interaction of a Propagating Vortex with a Vortex Entrapped in a Bay

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We consider a dynamical model describing an interaction of vortex moving towards the bay along the boundary with another one trapped in the bay. We presume the equal vortices strengths. First problem is to obtain the typical stationary regimes of the trapped vortex dynamics without an influence of the incident vortex. Second problem is to consider the interaction of both vortices. We have found four different regimes of two vortices dynamics. First one is the weak interaction when incident vortex passes along the bay and entrapped one continues to stay in the cavity. Second one is the case of exchange. Third one is the fast escape when the both vortices leave the bay in simple way. The last one is the intricate scattering when the vortices exhibit convoluted dynamics inside or near cavity and after long enough time, one or both vortices leave the vicinity of the bay. It is worth to mention that the vortices interaction is strongly irregular. It is difficult to establish is it chaotic dynamics or not due to finite time of interaction. The results might be useful to understand of typical regimes of eddy interaction near curved boundaries of the ocean.

Keywords: Point vortex, Vortex-topography interaction, Irregular motion, Vortex interaction.

Excitation of Discrete Breathers in ac Driven One-Dimensional Chains with Hard and Soft Type Anharmonic On-Site Potentials

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One-dimensional chain of pointwise particles harmonically coupled with nearest neighbors and placed in six-order polynomial on-site potential is considered. Power of the energy source in the form of single ac driven particle is calculated numerically for different amplitudes A and frequencies within the linear phonon band. The results for the on-site potentials with hard and soft nonlinearity types are compared. It is shown that in the case of hard (soft) anharmonicity the chain supports movable discrete breathers (DBs) with frequencies above (below) the phonon band. Our main findings are as follows. When the driving frequency is

close to (far from) DB frequency, the power of the energy source normalized to A^2 increases (decreases) with increasing A. In view of this observation we further demonstrate that when the driving frequency is close to the DB frequency, the energy source quasi-periodically emits moving DBs. Thus, these results indicate that the moving DBs can assist energy transfer from the ac driven particle to the chain.

Hyperchaotic Multimedia Stream Cipher to Secure Real-Time Video/Audio Transmission over WiFi

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In this paper, we propose Android-based application to secure real time video/audio transmission over WiFi between mobile platforms (smartphones and tablets). The application uses hyperchaotic stream cipher based on continuous 4D chaotic system with One-Time Pad (OTP) cipher encryption scheme.
At the transmitter, a hyperchaotic system generates two groups of dynamic encryption keys, characterized by high quality randomness confirmed by passing the NIST statistical tests, one for encrypting the compressed video stream and the other for encrypting the compressed audio stream. The encrypted data are exchanged in real time over WiFi between mobile platforms using UDP protocol. Note that the video and audio streams are sent simultaneously. At the receiver, the received encrypted frames will undergo the reverse operations used in transmission.
The most difficult problem to solve in real-time data transmission, especially in chaos-based transmission, is synchronization. However, for the symmetric chaos-based cryptography using OTP cipher encryption scheme, this problem can be resumed b these two questions:
1. How can we generate decryption keys, at the receiver, identical to those of the encryption used at the transmitter?
2. If the solution is found for 1, then how to decrypt with the correct key?
To solve this thorny problem, we have exploited the synchronization mechanism of the UDP protocol and we insert in the encrypted frame, an additional frame (32 bytes) which will ensure the synchronization of the two generators at the transmitter and receiver.
Through this application, we have been able to demonstrate the feasibility and robustness of the hyperchaos based encryption / decryption of the video/audio stream transmitted in real time between users through real wireless transmission channel (WiFi) from which experimental results are given. In addition, the proposed solution can be adapted to different wireless communication protocols (Bluetooth, NFC, GSM, ...).

Keywords: hyperchaotic system, OTP cipher, chaos-based cryptography, video/audio, UDP protocol, Android application, WiFi, mobile platforms.

Tests for Determining the Allowable Limit of Lead Toxicity; IN-VITRO Investigations on the Phaseolus-Vulgaris Plant

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Taking into account the harmful effects of exposure to lead, its presence in very small quantities in nature and more particularly in the human body, it has been useful to adopt an original method for estimating the highest low dose may cause symptoms of toxicity, and this by the use of a pollution bio indicator, that is the plant, iteration tests is used to confirm the results.

A New Approach about How to Make Reliable Predictions Inside Chaotic Regions

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The discovery of chaos has created new fundamental limits on the ability to make predictions. In principle the future is completely determined by the past, but in practice small uncertainties are amplified, so that even though the behaviour is predictable in the short term, it is unpredictable in the long term. In this work, we focus on a nonlinear prediction method called 'prediction by analogies' in order to look into that issue considering some high-frequency financial data. It is based on the assumption that the prediction horizon for which reliable predictions inside chaotic regions can be obtained will depend inversely on the evolution of the spatial distribution of the Lyapunov exponent and its stability. This approach involves reconstructing the state space from a single time series using Taken's theorem. So far this reconstruction method was apply considering only time series sampled at fixed time intervals. The reconstruction of the attractor using high-frequency financial data with a variable time-lapse between each observation needs several previous considerations. Additionally, we have simulated some fractional Brownian motions to evaluate our proposal obtaining better results than most of the existing methods for complex time series prediction.

Keywords: Nonlinear prediction, Deterministic Chaos, Lyapunov exponent, fractional Brownian motion, High-frequency financial data

On a Cournot Dynamic Game with Differentiated Goods and Asymmetric Cost Functions

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In this study we investigate the dynamics of a nonlinear Cournot- type duopoly game with differentiated goods, linear demand and different cost functions. The game is modeled with a system of two difference equations. Existence and stability of equilibrium of this system are studied. We show that the model gives more complex chaotic and unpredictable trajectories as a consequence of change in the parameter of speed of adjustment of the bounded rational player and in the parameter of horizontal product differentiation. A higher (lower) degree of player's adjustment or a variation of the parameter of product differentiation (weaker or fiercer competition) destabilize (stabilize) the economy. The chaotic features are justified numerically via computing Lyapunov numbers and sensitive dependence on initial conditions. Also, we show that in the case of asymmetric costs there are stable trajectories and a higher (lower) degree of product differentiation does not tend to destabilize the economy.

Keywords: Cournot duopoly game; Product differentiation; Dynamical system; Heterogeneous expectations; Homogeneous expectations; Asymmetric costs; Stability; Chaotic behavior.

The Water Dripping Dynamics under a Non-Uniform Electrical Field

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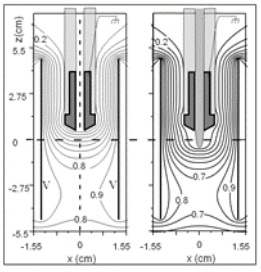
Two experimental bifurcation diagrams were obtained with two different control parameters. One was the traditional parameter, that is, the faucet opening or water flux Φ the other is an electrical positive voltage (V) applied in a metallic cylinder that surrounds the pendant water column and the metallic grounded nozzle, as illustrated in the figure below. The equipotential lines, in the (x,z) plane, were estimated by the relaxation method considering the system cylindrical symmetry. The water molecule is dipolar then the water pendant grows polarized in a non-uniform electrical field so an extra force is added in the –z direction. With a laser system we could measure the time between successive drops (T_n) to construct the experimental bifurcation diagram T_n vs. Φ which is very similar to the bifurcation diagram T_n vs. V up to V≈1.8 kV, both presenting



similar attractors (periodic and chaotic) characterized by their first return maps T_{n+1} vs. T_n. Above V≈1.8 kV there are no more similarities between the two diagrams. For the both cases of parameter control similar interior and boundary crises were observed. We also measured the mean drop mass <m> collecting and counting the number of drops (N) and the time (T) to fulfill a volume of 28.6ml. The water flux Φ=28.6/T =1.258 g/s was not affected by the applied voltage despite the mean weights of drops has varied from 53mg to 25mg in the interval [0 4.5] kV

Keywords: Dripping faucet, Experimental Chaos, Bifurcations, Crises

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Chaotic Analysis of Acid Rains with Time Series of pH Degree, Nitrate and Sulphate Concentration on Wet Samples

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Chaos theory as a new paradigm in science has attracted researchers during the last century. Chaos is observed in a variety of natural systems which are non-linear dynamic with characteristics such as sensitive dependence on initial conditions and control parameters. Acid rain is one of the environmental problems that has adverse effects on human health and ecological environment. In this study, we aim to investigate the chaotic nature of acid rains in Turkey through nonlinear defecting approaches. The implementation study of this research is based on the time series 14 years provided by Turkish State Meteorological Service which contained pH degree of rain water, concentration of sulfate and nitrate. Lyapunov exponents, bifurcation diagrams, reconstruction of the phase space and power spectrums have been used to reveal the chaotic nature of acid rains.

Keywords: acid rains, chaotic analysis, nonlinear analysis, Lyapunov exponents.

A Multi-Arc Approach for Chaotic Orbit Determination Problems

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Chaotic dynamical systems are characterized by the existence of a predictability horizon, connected to its Lyapunov time, beyond which

predictions of the state of the system are meaningless. Spoto and Milani, 2016 applied the classical least squares fit and differential correction algorithm to determine an orbit and a dynamical parameter of a simple discrete system - the standard map - with observations distributed beyond the predictability horizon. They found a time limit beyond which numerical calculations are affected by numerical instability: the computability horizon.
In this talk we aim at pushing forward such inherent obstacle to numerical calculations in chaotic orbit determination by applying the constrained multi-arc method (cf. Alessi et al, 2012) to the same dynamical system. This strategy entails the determination of an orbit when observations are grouped in separate observed arcs. For each arc a set of initial conditions is determined and all subsequent arcs are constrained to belong to the same trajectory. We show that the use of this technique in place of the standard least squares method has significant advantages, allowing to perform accurate numerical calculations well beyond the computability horizon.

CFD Simulations of Indoor Airflow in Module Room with the FCU Cooling System

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As nowadays people spend much longer time inside the buildings, creating a comfortable and healthy indoor environment with air-conditioning systems becomes more significant than ever before. CFD (computational fluid dynamics) simulation is a useful and effective way to analyze the airflow patterns and thermal environment of ventilation systems, which can guide designers to conduct the evaluation and optimization of such systems. In this study, the steady and transient CFD simulations were conducted to assess the airflow patterns and thermal environment of a ceiling-hanging FCU cooling system in Hong Kong. Base on the steady CFD simulation of the target room, the average temperature at the Plane of Y=1.2m, a typical height level for seated occupants is 23.1ºC, which is in good agreement with the set-point temperature of 23ºC. The velocity vectors illustrate airflow patterns inside the target room, and the velocity magnitude is below 0.8m/s in occupied areas, which conforms the thermal standards as well. The changing trend of average temperature and fluctuation of velocity are assessed by transient CFD simulation, and the numerical results agree well with the experimental data. Therefore, CFD simulation presents outstanding capacity to dynamically predict indoor airflow as well as relevant thermal parameters to provide useful guidance for designers and operators to construct the omfortable and healthy indoor environment.

Keywords: CFD simualtion, FCU cooling systme, Indoor airflow, Thermal comfort, Transient simulation.

Butterfly Effects of the First and Second Kinds in Lorenz Models

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Over the span of 50 years, the pioneering study using a three-dimensional Lorenz model (3DLM) in 1963 and follow-up studies in 1969 and 1972 have changed our view on the predictability of weather and climate by revealing the so-called butterfly effect. Although Lorenz's '63 and '72 studies emphasized nonlinear dynamics, researchers often apply a "simple" conceptual model that contains monotonic positive feedback but no negative nonlinear feedback to understand the characteristics of nonlinear solutions within the 3DLM. In this study, we: (1) define butterfly effects of the first and second kinds to indicate the sensitive dependence of solutions on initial conditions and the enabling role of tiny perturbations in producing an organized large-scale system (e.g., a tornado), respectively; (2) illustrate important but overlooked features (i.e., the boundedness and recurrence of solutions within the 3DLM); (3) present examples to illustrate common misunderstandings regarding butterfly effects and explain the fundamental differences between the two kinds of butterfly effects; (4) illustrate the fundamental role of nonlinearity in creating oscillatory components with incommensurate frequencies, transferring energy across scales, and providing negative or positive feedbacks; (5) discuss various types of solutions (e.g., chaotic, linearly unstable, and/or nonlinear oscillatory solutions) in Lorenz models; and (6) propose that the entire weather is a superset that consists of both chaotic and nonchaotic processes. Depending on the time-varying collective impact of heating, dissipations, and nonlinearity, specific weather systems may appear on a chaotic or non-chaotic orbit for a finite period of time.

Keywords: Butterfly Effects; Lorenz models; Recurrence; Nonlinear Feedback Loop; Chaos; Quasi-periodicity

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Coexistence of Chaotic and Non-Chaotic Orbits in a New Nine-Dimensional Lorenz Model

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In this study, a new nine-dimensional Lorenz model (9DLM) is presented to reveal (1) the coexistence of chaotic and non-chaotic orbits with moderate Rayleigh parameters; and (2) the coexistence of limit cycle/torus orbits and spiral sinks with large Rayleigh parameters. The 9DLM is derived based on the extension of the nonlinear feedback loop (NFL). This approach was first outlined using a 5DLM by Shen (2014) and then applied to drive the 7DLM by Shen (2016). The 9DLM requires larger critical value for the Rayleigh parameter (rc of 679.8) for the onset of chaos as compared to the rc of 24.74 for the 3DLM, the rc of 42.9 for the 5DLM and rc 116.9 for the 7DLM. This confirms the hypothesis that a proper selection of Fourier modes for the extension of the NFL within the high-dimensional LM can provide negative feedback to stabilize solutions. The role of the NFL in producing recurrent solutions with multiple incommensurate frequencies is revealed using a non-dissipative Lorenz models (NLMs). It is shown that governing equations for the locally linear 3D-, 5D-, 7D-, and 9D- NLMs are identical to those for the systems with one, two, three, and four masses and springs, respectively, yielding one, two, three and four (incommensurate) frequencies, respectively. As outlined above, high-dimensional LMs possess various kinds of solutions (e.g., chaotic and limit cycle/torus) within different range of Rayleigh parameters. We propose that, contrary to the traditional view that weather is chaotic, it is in fact a superset that consists of both chaotic and non-chaotic processes.

Keywords: Chaos, Limit Torus, High-dimensional Lorenz model

Hyperchaos in Oscillating Systems with Limited Excitation

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The oscillating systems of a pendulum type, nonideal in the sense of Sommerfeld-Kononenko, are considered. Such systems are used to modeling oscillations in hydrodynamics, shell theory, and other applications [1, 2].
The bifurcations "limit cycle-chaos-hyperchaos" are studied in detail. The existence of two types of hyperchaotic attractors in the system is established. A complex scenario of transition to hyperchaos is described. The revealed scenario begins with symmetric cascades of doubling the

periods of limit cycles and ends with a transition to hyperchaos through generalized intermittency with two coarse-laminar phases. This scenario is illustrated in detail by projections of phase portraits, Poincaré sections, distributions of invariant measures, and other characteristics of attractors of the system.

Keywords: Limited excitation, chaotic and hyperchaotic attractors, scenario of transition to hyperchaos

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Transition to Deterministic Chaos in Some Electroelastic Systems

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The nonideal dynamic system "analog generator - piezoceramic transducer" is considered. The mathematical model of such a system is constructed in the paper [1]. In this paper it is shown that the main reason of the emergence of deterministic chaos is the interaction between the generator and the transducer.

The existence of several types of invariant tori of the system is established. Some scenarios of transition from regular regimes to chaotic ones are described. The phase-parametric characteristics and maps of dynamics regimes of the system are constructed. The atypical alternations of the cascade of period-doubling bifurcations and intermittency in transitions to chaos are shown. Various types of chaotic attractors are described in detail. The so-called hidden attractors are discovered, both inside invariant tori and inside chaotic attractors.

Keywords: Invariant torus, chaotic attractor, scenario of transition to chaos, hidden attractors.

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Extraction and Classification of Convectively Coupled Equatorial Waves through Eigendecomposition of Koopman Operators

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We study spatiotemporal patterns of convective organization using a recently developed technique for feature extraction and mode decomposition of spatiotemporal data generated by ergodic dynamical systems. The method relies on constructing low dimensional representations (feature maps) of spatiotemporal signals using eigenfunctions of the Koopman operator. This operator is estimated from time-ordered unprocessed data through a Galerkin scheme applied to basis functions computed via the diffusion maps algorithm. Koopman operators are a class of operators in dynamical systems theory that govern the temporal evolution of observables. They have the remarkable property of being linear even if the underlying dynamics is nonlinear, and provide, through their spectral decomposition, natural ways of extracting intrinsic coherent patterns and performing statistical predictions.
We apply this approach to brightness temperature data from the CLAUS archive and extract a multiscale hierarchy of spatiotemporal patterns on timescales spanning years to days, including dominant intraseasonal mode of tropical variability (MJO) but also traveling waves on temporal and spatial scales characteristic of convectively coupled equatorial waves (CCEWs). In particular, we examine if the activity of these coherent structures is modulated by low-frequency atmospheric and oceanic variability. We discuss various properties of waves in our hierarchy of modes, focusing in particular on their across-scale interactions and temporal evolution. As an extension of this work, we discuss the deterministic and stochastic aspects of the variability of these modes.

Vector-Valued Spectral Analysis of Indo-Pacific Climate Variability

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We study Indo-Pacific variability using a recently developed framework for spatiotemporal pattern extraction called Vector-Valued Spectral Analysis (VSA). This approach is based on the eigendecomposition of a kernel integral operator acting on vector-valued observables (spatially extended fields) of the dynamical system generating the data, constructed by

combining elements of the theory of operator-valued kernels for multitask machine learning with delay-coordinate maps of dynamical systems. A key aspect of this method is that it utilizes a kernel measure of similarity that takes into account both temporal and spatial degrees of freedom (whereas classical techniques such as EOF analysis are based on aggregate measures of similarity between "snapshots"). As a result, VSA has high skill in extracting physically meaningful patterns with intermittency in both space and time, while factoring out any symmetries present in the data. We demonstrate the efficacy of this method with applications to various model and observational datasets of oceanic and atmospheric variability in the Indo-Pacific sector. In particular, the recovered VSA patterns provide a more realistic representation of ENSO diversity than conventional kernel algorithms.

Methodology on Exploring the "Limits to Human Lifespan"

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We introduce and apply several methods for exploring the human lifespan process in connection to the publication by Dong et al. 2016 on "Evidence for a limit to human lifespan" in Nature in 2016 and the debate emerging mainly published in Nature as well in 2017 along with authors replies. We found it constructive to propose and test several applied methodologies using the centenarian and supercentenarian data sets from various data bases and to give illustrative presentations along with fitting and forecasting. By analyzing these data sets various results are obtained and presented in supporting illustrations and tables. We have provided more tools for analyzing centenarian and supercentenarian data along with the existing methodologies. The findings published in our two books published by Springer recently are presented.

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Solution of the Ancient Greek Problem of Trisection of Arbitrary Angle

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A solution of the ancient Greek problem of trisection of an arbitrary angle employing only compass and straightedge that avoids the need for two marks on *Archimedes Marked Ruler* is introduced. It is argued that although *Wantzel* [1-2] 1837 theory concerning non-existence of rational roots of *Descartes-Wantzel* cubic equation is correct it does not imply impossibility of trisection of 60° angle. This is because according to the construction method being introduced the square of cosine of trisected angle $\cos^2(\alpha)$ is related to cosine of its double $\cos(2\alpha)$ thus requiring extraction of square root that is constructible rather than cubic root associated with rational solution of *Descartes-Wantzel* equation. In other words, the assumption that angle trisection only involves geometric constructions that lead to cubic equation is not valid. Therefore, *Wantzel's* proof does not rule out angle trisection because a construction method exists that does not require extraction of cubic root. The absence of rational roots connecting $\cos(\alpha)$ to $\cos(3\alpha)$ proved by *Wantzel* is attributed to the fact that the solution of trisection problem involves ratios of geometric lengths (hence arithmetic numbers) that are associated with two circles of different radii hence metrics or "measures". The earlier formulation of the problem by *Descartes* the father of algebraic geometry is also discussed. If one assumes that the ruler and compass employed in the geometric constructions are *Platonic* ideal instruments then the trisection solution proposed herein should be exact.

Keywords: The trisection problem, angle trisection, Wantzel theory, regular polygons, heptagon.

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Procedural Symbolic Musical Rhythm Patterns Generation through Sequential Dynamical Systems

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Algorithmic composition using symbolic audio has kept being as an active an wide research area among composers, mathematicians and engineers who use tools that go from neural networks and machine learning up to

abstract entities as the well known Topos of Music. In this paper an original algorithmic rhythm model for symbolic audio through sequential dynamical systems is proposed. First, a mathematical framework based on Category Theory is presented as a formal basis to represent musical rhythm elements with mathematical objects. This framework is then further extended through graphs and sequential dynamical systems. A practical implementation using the circle graph and some of it´s equivalece classes is proposed at the end of the paper. Such implementation includes also an general overview of the compositional process for Apraxia, an original electroacoustic work for multipercussion and electronics, composed by the author of this paper using the proposed model.

On the Acoustic Characteristics of Sounds in Child Speech

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It is known that speech sounds develop in children from birth to about age four. During development, the acoustic characteristics of sounds change to finally reach adult-like characteristics. The purpose of this study is to determine this change for both vowel and consonant sounds and see whether this change can be captured by a satisfactory data mining model. To examine this, a bilingual child's speech sounds in Greek and English are analyzed at four different ages: two years seven months, three years, three years five months, and three years ten months. The results obtained indicate that there is a considerable change in the first two formant frequencies of vowel sounds in English, the child's weaker language, with increasing age. Compared with the first two formant frequencies of Greek vowel sounds, it can be seen that there is interference, especially at earlier ages. As far as consonant sounds are concerned, focus is on consonant clustering in order to determine how the duration between voice offset and voice onset of the two consonants changes with increasing age. It is found that changes are similar in both languages, starting with the insertion of a short duration vowel between consonants at earlier ages, disappearing with increasing age. Moreover, the duration of the inserted vowel decreases with increasing sonority distance between consonants. Based on these results, a model is developed for a data mining technique in order to capture these changes in the acoustic characteristics. It is aimed that this study will guide similar investigations for other language-sounds in child speech.

Stabilization of Autoresonant Modes

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A mathematical model describing autoresonance phenomena in nonlinear systems with a small time-periodic external force is considered. Solutions with a significantly growing energy and a bounded phase detuning are associated with the autoresonant capture. Stability of such solutions ensures the observability of the corresponding resonant modes. We study the stabilization problem of unstable regimes, and we show that a decaying parametric perturbation of nonlinear systems can stabilize the unstable autoresonant solutions.

Keywords: nonlinear system, resonance, stability, stabilization

Integrability Analysis of Chaotic and Hyper-Chaotic Financial Models

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We consider chaotic and hyper-chaotic financial systems recently studied in the literature. They have forms of three and four first-order differential equations defined by

$$\dot{x} = z + (y - a)x, \quad \dot{y} = 1 - by - x^2, \quad \dot{z} = -x - cz, \tag{1}$$

$$\dot{x} = z + (y - a)x + w, \quad \dot{y} = 1 - by - x^2, \quad \dot{z} = -x - cz, \quad \dot{w} = -dxy - kw, \tag{2}$$

In the both systems (a, b, c, d, k) are real positive parameters. We present the complexity of these models by means of numerous methods and techniques, among others: the Poincare cross-sections, the bifurcations diagrams, the Lyapunov exponents, etc. Moreover, we show that system (1) is not integrable in the class of functions meromorphic in variables (x, y, z) for all values of parameters while hyper-chaotic model (2) is not integrable provided the condition $L := 1 + d(a + d - c) > 0$ is fulfilled. For this purpose, we use the differential Galois approach that is based on analysis of variational equations along certain particular solutions of the systems. On the other hand, we show that for $L < 0$ the hyper-chaotic finance model (2) possesses first integral that allows us to reduce the system to a three-dimensional one.

Keywords: finance models, chaos and hyper-chaos, Poincare sections, bifurcation diagrams, Lyapunov exponents, non-integrability, non-Hamiltonian systems, differential Galois theory.

Emergence of the Devil's Staircase in the Forced BVP Oscillator with a Diode

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Mixed-mode oscillations (MMOs) have been studied extensively. They are characterized by their time series waveforms consisting of L large excursions and s small peaks. In chemical experiments such as Belousov-Zhabotinsky reaction, MMO-incrementing bifurcations (MMOIBs) are known to occur. Moreover, in the research of the MMOIBs with an autonomous chemical system, it is reported that a firing number $F=L/(L+s)$ decreases stepwise every adding MMOs to have the devil's staircase emerge. Furthermore, it is found that MMOs and MMOIBs occur even in extremely simple electronic circuits such as an extended BVP oscillator and a forced BVP oscillator shown in Fig. 1.

In this study, we analyze the forced BVP oscillator including a diode and observe the devil's staircase in nonautonomous systems that occur MMOIBs. First, the circuit model is shown, and the behavior of the waveform is explained. Next, the one-dimensional Poincaré map is derived. Then, the numerical results are shown, and the devil's staircase is observed. In addition, it is also shown that a firing number irregularly increases out of the parameter range of generating staircase. Finally, the mechanisms responsible for the devil's staircase and the firing number that takes higher values in chaos-generating regions are precisely explained using the one-dimensional Poincaré map.

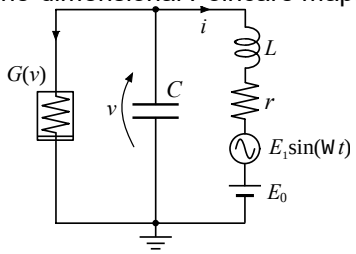

Fig. 1. The forced BVP oscillator

Keywords: forced BVP oscillator, mixed-mode oscillations, devil's staircase

Hemispherical Non-Coherent Electrical Activity as an Early Sign of Focal-Subcortical Lesions at Neurodegenerative Diseases- Wavelet Analysis of ECoG Spectrum of Rats Brain

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Excessive basal ganglia neuronal spike synchronization is also well established in 6-OHDA rat model of Parkinson's disease (PD). The role of the motor cortex in pathophysiology is less clear. In rodent models, cortical beta rhythms are excessively synchronized to basal ganglia spike discharge but it is not known if this is true in humans with PD. Further, interactions between basal ganglia spiking and higher frequency cortical activity have not been explored.

Wavelet Origin pro (Origin lab Ltd.), based analysis were applied.

Wavelet transforms are useful for analyzing signals for non-linear changes of phase and frequency, local maxima and minima, or related parameters. Wavelet transforms have been shown to have applications to a wide variety of problems like a signal smoothing, noise removal and frequency band decomposition. Origin's wavelet transform tools support continuous and discrete transformations, using algorithms developed by the Numerical Algorithms Group (NAG).

After time-frequency (wavelet) analysis 6 frequency subintervals, were set: <4 Hz; (delta), 4-8 Hz; (theta), 8-13 Hz; (alpha), 13-30 Hz; (beta), 3 Hz spikes, 30-100 Hz (gamma) oscillations.

Broadband gamma activity in primary motor cortex is elevated in PD as well as excessively coupled to the phase of low frequency cortical rhythms (phase-amplitude coupling).

Thus, analysis of the interaction of basal ganglia spiking with cortical broadband could yield great insight into basal ganglia-cortex dynamics in PD. To address these questions, we recorded primary motor cortex (M1) simultaneously with subthalamic nucleus (STN) unit discharge, in awake rats. Were recorded using subdural electrocorticography (ECoG), a technique that combines excellent spatial and temporal resolution with sufficient signal amplitude to resolve high frequency broadband activity.

Our findings support a model of the basal ganglia-thalamocortical loop in PD in which cortical 4-30 Hz rhythms are phase locked to gamma activity, and in which phase modulated "waves" of gamma activity in motor cortex drive STN hyperactivity.

Parkinson's disease (PD) is marked by excessive synchronous activity in the beta (13–30 Hz) band throughout the cortico-basal ganglia network obtained by ECoG spectral analysis.
Local field potentials (LFPs) recorded from the subthalamic nucleus (STN) of sham-treated animals (by infusion of buffered solution into caudate putamen of the brain). Implanted with stimulation electrodes for the treatment of Parkinson's disease (PD) demonstrate strong coherence with the cortical electroencephalogram (ECoG) over the beta-frequency range (13–30 Hz).

Generalization of Inversive Congruential Generator with a Variable Shift

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The inversive congruential method for generating uniform pseudorandom numbers is a particulary attractive alternative to linear congruential generators, which show many undesirable regularities. In the present paper a new inversive congruential generator with a variable shift is introduced. Exponential sums on inversive congruential pseudorandom numbers are estimates. The results show that these inversive congruential pseudorandom numbers pass serial tests on the equidistribution and statistical independence.

Keywords: inversive generator, exponential sum, discrepancy.

Non-Linear Stability Observation using Magneto-Controlled Diffraction with Opto-Fluidics

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We have developed a magneto-optical system which simulates the stability of fixed points and the trajectories of orbits present in dynamical systems. The question of stability is significant because a real-world system is constantly subject to small perturbations, and these orbits can be observed with a Ferrocell, a device using ferrofluid, which is a magnetic fluid obtained with a kind of colloid containing surfactant coated nanometer particles dispersed in a carrier liquid, and this device can be used in applications of optical effects. Our magneto-optical system is

based in a Hele-Shaw cell containing ferrofluid, illuminated with an external light source, such as LED light or lasers [1]. By injecting a light propagating along the in-plane direction of the liquid film, the orbits can be observed, in a such way that we can bend the light. The trajectories of the orbits are obtained by the diffracted light, which consists of light patterns inside the liquid film, and these light patterns are related to Faraday effect, linear dichroism, and linear birefringence. The diffraction pattern is different from that produced by a wire because there are no fringes in these light patterns, and the absence of well-defined spacing fringes indicates the existence of multiple diffraction. Under certain circumstances, these light patterns can have the same properties of the force lines of magnetic fields. The main idea of this work is to propose a device applied to non-linear systems, based on magneto-photonics. We have studied some properties of this magneto-optical device, such as light polarization and obtained hypergeometric polynomials for the light intensity of the polarization generated by the magnetic field [2], due to the Faraday effect, which usually follows a Langevin type behavior. We present the patterns obtained for different magnetic fields simulating dynamical systems. In addition to this, we present the effect of magneto-controlled illumination using the Ferrocell, based in the effect of light scattering using magnetically polarizable nanoparticles suspension.

Keywords: Non-linear system analysis, Ferrofluids, Multiple diffraction

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Rainbows, Billiards and Chaos

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Starting at the end of the last century, Chaos theory is used to explain since the dynamics of a dripping faucet [1] to the essence of black holes [2]. The main aspect of this ubiquity is because chaotic systems involve nonlinear systems, and most systems behave linearly only when they are close to equilibrium, far from this region, we can observe a myriad of behaviors. We studied some phenomena involving rays and waves in optics and acoustics, such as rainbows, fogbows, Glory effect, iridescent clouds, halos [3, 4, 5] and sound waves in acoustic billiards [6, 7, 8] from the point of view of chaotic systems. We explore the aspects of ray splitting and their relationship with Chaos theory, based on different subjects, such as Random Matrix Theory, Caustics, Interference and Geometrical Theory of Diffraction. One interesting case in such systems is that the existence

of discontinuities or singularities can lead to wave diffraction [9], which is related to additional contributions to the trace formula, with the presence o creeping orbits and caustics. This approach can be extended to quantum systems, such as nuclear rainbow. We will present scattering of light in open systems and compare them to scattering of particles. We have observed that one interesting aspect of these systems is that the whole system has to be examined all at once, as a coherent entity, and Chaos is not formless and always left some kind of signature of its presence.

Keywords: Billiard, Wave Chaos, Random Matrix, Ray Optics, Diffracted Rays

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The Many Flavours of Supergranulation

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I study the complexity of supergranular cells using the intensity patterns obtained from the Kodaikanal solar observatory in 1999 during the active phase of the Sun. The Data consist of visually identified supergranular cells, from which a fractal dimension 'D' for supergranulation is obtained according to the relation P α AD/2 where ‘A' is the area and ‘P’ is the perimeter of the supergranular cells. I find a fractal dimension close to about 1.2 which is consistent with that for isobars and suggests a possible turbulent origin. I also study the supergranular area versus latitude relation.

Keywords: Sun: granulation- Sun: activity – Sun: photosphere

Mechanical Analogy for the Wave of Nuclear Burning

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We consider a model of neutron-nuclear wave burning. The wave of nuclear burning of the medium is initiated by an external neutron source and is the basis for the new generation reactors {the so-called "traveling-wave reactors".
We develop a model of nuclear wave burning, for which it is possible to draw an analogy with a mechanical dissipative system. Within the framework of the new model, we show that two burning modes are possible depending on the control parameters: a traveling autowave and a wave driven by an external neutron source. We find the autowave to be possible for certain neutron energies only, and the wave velocity has a continuous spectrum bounded below.

Analysis of Emergency Situations on Hydraulic Structures in Central Asia and the Caucasus

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Almost the whole territory of Central Asia and the Caucasus is the huge risk region of the whole world. On its territory are located the so-called independent new countries that appeared after the USSR collapse: Kazakhstan, Kyrgyzstan, Uzbekistan, Tajikistan, Turkmenistan, Georgia, Armenia and Azerbaijan, where predominantly the muslim population of about 90 million people live today. Here the risks are caused by the main following factors [1-4]:

1. The high-mountain Tien Shan, Pamir and Caucasus massifs, located here, determine the risk nature of the large and complex hydrotechnical structures (GTS): hydro stations, their dams and huge artificial reservoirs on the located water basins of large rivers such as Irtysh, Naryn, Syr Darya, Amur Daria, Araks, Kura and others.
2. The same massifs determine the high seismic region activity, where the following natural disasters often occur: earthquakes, mudflows, landslides, slumps. The presence of the large GTS and the Semipalatinsk Testing Nuclear Poligon in the water basin of the largest river of Kazakhstan of Irtysh, significantly aggravates the situation.

3. The long-term development of various deposits of non-ferrous metals and uranium ores and the numerous large enterprises for their processing were carried out in the same territory. The availability of cheap GTS energy sources stimulated the emergence of the numerous large energy-intensive enterprises with their dangerous numerous radioactive and toxic tailing storages.
4. These GTS were built in the Soviet Union period. Their high operating wear, also including due to inadequate conditions of their exploitation by new owners, made them by the negative sources of increased risks. The different possible accidents can cause global large-scale catastrophes with significant human casualties and material damages with irreversible pollution of vast areas and the North-Arctic Ocean and, consequently, the World Ocean.
5. The Muslim regional population with its low standard of living contributes to the emergence here of centers of so-called frozen conflicts, as well as centers for the recruitment of terrorists and future soldiers of Islamic state. Everybody may see that terrorist acts all over the world realized by the representatives of the Middle East, Africa and Muslim countries, formed in result of USSR collapse.

We devoted the large cycle of our many years of work to problems of risks, including in the case of the controlled terrorist attacks with using of explosives. Part of them was performed on the main international projects, such as the ISTC, with the detailed analysis of some risk objects.

Among the internal reasons of GTS accidents are the the following: incorrect design and design solutions - 23%; poor-quality construction and installation work - 27%; violation of the rules of operation and untimely renewal of extremely worn-out equipment - 48%. For example, the probability of destruction of structural elements of concrete dams around the world for the period from 1900 - 1980 gg. was 0.45 × 10-3 / year. The significant contribution in the GTS n the accidents, is caused by the imperfection of modern methods of forecasting and reducing emergency risks. Existing prognosis procedures abuse the use of various semi-empirical coefficients.

In our communication, the problems of system forecasting and the reduction of the risk under GTS accidents are studied in detail by first presenting them in the form of diagrams of cause-effect relationships, and then computational mathematical models. These models take into account not only the reasons on the GTS accidents, but also their most likely destructive outcomes.

The cause-effect diagrams, such as "tree of incident" (IT) and "event tree" (ET) were used for the study. The IT includes equipment failures, staff errors, uncalculated external impacts on the GTS, which can form causal chains leading to the implementation of the accident in question, called the ET head event, taking into account this ET the most likely and destructive outcomes of the simulated accident. The single head event, called the central one, and several outgoing branches are allocated. As

this event, modeled on the GTS, the loss of tightness of the dam was selected as a result of the destruction of its structural elements. It was believed that this loss could be the result of extreme development of geodynamic or manmade processes. In identifying their causes, the structures of the existing dams and the typical hazards of their operation, identified by the detailed analysis of known accidents, were taken into account with the using of the deductive-axiomatic method. The authors constructed an illustrative logical-linguistic model of the emergence and destructive development of an accident, which is shown in detail on the diagram, containing 41 IT preconditioning events and 10 ET outcomes, which are distributed over 8 levels of this diagram.

All groups of IT reasons are divided into the 2 groups. In the ET construction, the classification of accidents according to the existing detailed standard was taken into account.

Our report analyzes the existing space -temporal relationship of natural and manmade disasters in according to the presented maps of global risks of the countries of the research region.

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Periodic Windows and Intermittency in the Generalized Lorenz Model

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We analyze the dynamics of the generalized Lorenz model proposed by Macek and Strumik [1, 2]. Based on bifurcation diagrams we have done the systematic studies of the dependence of this four dimensional system on control parameters. The results show the existence of period-doubling bifurcations and several periodic windows, for example period-3 windows. It is worth to underline that at the edge of selected period-3 windows other interesting phenomena occur. In particular, we have identified new sets of control parameters, for which type I intermittency exists.

Keywords: Nonlinear systems, bifurcation, intermittency.

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Useful Criteria Verifying Limit Theorems for Certain Markov Chains

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The central limit theorem (CLT) and the law of the iterated logarithm (LIL) are, along the strong law of large numbers (SLLN), the most common limit theorems. While the CLT refers to the convergence of certain random variables towards the normal distribution, the LIL describes the behavior of single, appropriately scaled, trajectories.
Some well-known results concerning limit theorems, obtained mainly due to the martingale method, are gathered in [1]. Although the asymptotic behavior of stationary and ergodic Markov chains is already well investigated, limit theorems for a wider class of Markov processes are still the subject of research.
Together with D. Czapla and K. Horbacz we have proven certain criteria on the CLT and the LIL for a quite general class of Markov chains. Our aim was to provide useful assertions that can serve biologists and physicists to study their models in terms of limit theorems. Therefore we do not require from the transition probabilities of Markov chains neither to be continuous, nor to satisfy the spectral gap property in the Wasserstein metric (cf. [2,3,4]). Instead, we propose certain conditions, relatively easy

to verify in many biological models, that yield the exponential mixing result (see e.g. [5] for the precise formulation), together with the SLLN, the CLT and the LIL. In the proofs we used the properties of an asymptotic coupling, whose construction was adapted from [5].
To justify the usefulness of stating such criteria, we decided to verify them for a particular discrete-time Markov system, for which we were not able to verify conditions proposed in [2] and [3]. The piecewise-deterministic Markov process defined via interpolation of the explored Markov chain can be used e.g. to describe a model for gene expression.

Keywords: Markov chain, random dynamical system, central limit theorem, law of the iterated logarithm, Markovian coupling, asymptotic coupling.

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Uncertainty Relation for Chaos

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A necessary condition for the emergence of chaos is given. It is well known that the emergence of chaos requires a positive exponent which entails diverging trajectories. Here we show that this is not enough. An additional necessary condition for the emergence of chaos in the region where the trajectory of the system goes through, is that the product of the maximal positive exponent times the duration in which the system configuration point stays in the unstable region should exceed unity. We give a theoretical analysis justifying this result and a few examples. We stress that the criterion suggested involves only local exponents and **is not** concerned with asymptotic defined exponents.

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Modeling Behavior of Economic Systems on the Edge of Chaos

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Researches of the socio-economic systems show that they along with characteristic system properties possess the row of distinctive properties. These include: diffuse properties, ability to self-organization, functioning in the conditions of self-organized criticality on edge of chaos or in the conditions of crossing of chaoses.

Particular interest causes property of self-organized criticality. It supposes the ability of the system to evolve in the direction of achievement of criticality (sharp change of properties of the system) and support itself in this state. Narrow areas (edges of chaos) which are on border of a zone of homeostasis and external chaos are thus formed. Their feature consists that the most minimum changes of parameters in these areas bring to is disproportionate to considerable changes in behavior of system. Thus, if such systems are especially sensitive to the factors being on the edge of chaos, that, having investigated these factors, it is possible to forecast big changes in system, using some signs in their dynamics or structure.

To achieve this goal, the methodology of modeling a problem is developed on the basis of model of dynamics growth and models of avalanches. In the space of system parameters obtained border (the critical area), below which the steady mode of functioning of system is realized, and higher there is the unsteady chaotic mode. As approaching critical area fluctuations grow in system. Growth of a susceptibility of system to external influences is observed. And also, there is a long correlation between system elements, comparable to the size of the system.

The evolution of systems on the edge of chaos is determined by processes, which got the name of weak chaos. They are the result of self-organized criticality. The weak chaos significantly differs from completely chaotic behavior. Completely chaotic systems are characterized by time interval, to go beyond which when forecasting it is impossible. In weakly chaotic systems, such characteristic is absent. Therefore, they allow long-term forecasting.

Keywords: Chaotic modeling, Edge of Chaos, Self-Organized Criticality, Model of Dynamics Growth, Models of Avalanches, Models Behavior of Economic Systems.

Route to Chaos in a Double Microresonator with Gain and Loss

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In the present study we consider chaotic dynamics of a system of two coupled ring waveguide structure with linear gain and nonlinear absorption. It can be implemented in various settings including microresonator nanostructures, polariton condensates, optical waveguides or atomic Bose-Einstein condensates of ultra-cold atoms packed into circular-shaped trap. In this setting we investigate the rich structure of period doubling bifurcations, eventually leading to chaotic behavior. We show that the so called Galerkin approximation can explain most of the system dynamics mapping the system behavior to the dynamics of few coupled oscillator modes.

Keywords: Microresonators, Coupled ring structures, BEC in a toroidal trap, Chaotic dynamics, Period doubling.

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