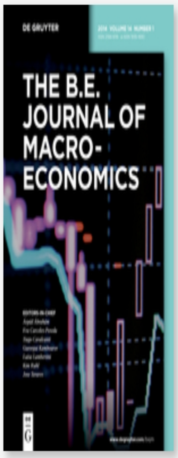

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Relative-Preference Shifts and the Business Cycle

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Relative-Preference Shifts and the Business Cycle*

William Addessi and Francesco Busato

Abstract

This paper develops a two-sector dynamic general equilibrium model in which intertemporal fluctuations (and sectoral co-movement) are driven by idiosyncratic shocks to relative preferences among consumption goods. This class of shocks may be interpreted as shifts in consumer tastes. When shifts in preferences occur, consumers associate a new and different level of satisfaction to the same basket of consumption goods according to their modified preferences. This paper shows that if the initial composition of the consumption basket is sufficiently asymmetric, then a shift in relative preferences produces a "perception effect" strong enough to induce both intersectoral and intrasectoral positive co-movement of the main macroeconomic variables (i.e., output, consumption, investment, and employment). Furthermore, by extending the theoretical framework to a multisector model and introducing a more flexible structure for the relative-preference shock, we show that the parameter restrictions needed to observe sectoral co-movement after a relative-preference shock are much less severe. In particular, co-movement among most of the sectors emerges under general conditions, without requiring high levels of asymmetry in the consumption basket's composition and/or high aversion to risk. It is a welcome result that these findings are reached without introducing aggregate technology shocks, input-output linkages, or shocks perturbing the relative preference between aggregate consumption and leisure.

KEYWORDS: demand shocks, two-sector dynamic general equilibrium

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1 Introduction

The co-movement of economic activity across different sectors is one of the most important regularities of all business cycles (Lucas, 1977). Burns and Mitchell (1946) include intersectoral co-movement in their definition of business cycles, and many empirical studies demonstrate pro-cyclical behavior of cross-sector measures of employment, output, and investment (Christiano and Fitzgerald, 1998; Huffman and Wynne, 1999). It is difficult to identify reasonable aggregate disturbances that are capable of explaining historical business cycles, so a vast literature investigates the transmission mechanisms from sectoral shocks to aggregate fluctuations. This approach has dealt with two main challenges: (i) explaining how sectoral fluctuations spread over the entire economy; and (ii) explaining why sectors *move together*.

The multisector dynamic general equilibrium literature uses productivity shocks and technological linkages to explain sectoral co-movement. In fact, the input–output structure grants that fluctuations in each sector take the same direction after an idiosyncratic productivity shock.[1] Cooper and Haltiwanger (1990) highlight the role of demand. These authors suggest that the normality of demands for consumption goods is the channel through which sectoral shocks spread over the economy; meanwhile, the main mechanism of shocks' intertemporal transmission relies on the low number of sectors holding inventories. In this framework, an increase in inventories immediately reduces both the production and the income in sectors holding inventories. This, in turn, reduces the demand for goods produced in the other sectors, and hence the expected positive co-movements of employment and output emerge.

Departing from the existing economic mechanisms implemented by the cited literature, this paper develops a framework without introducing either exogenous changes to productivity or input–output linkages and without relying on the "income effect" as in Cooper and Haltiwanger (1990).

In this paper, fluctuations are induced by exogenous shocks to the structure of preferences. In particular, shocks affect consumers' relative preferences among consumption goods. We show that this class of preference shock is capable of explaining the positive co-movements of output, consumption, investment, and employment among sectors. It is important to emphasize that this kind of shock affects only relative preferences among consumption goods; it does not directly modify the preference relation between the composite consumption good and leisure time.

[1] Selected references are Long and Plosser (1983), Murphy et al. (1989), Hornstein and Praschnik (1997), Horvath (1998, 2000), and Huffman and Wynne (1999).

Thus our mechanism differs from the one addressed by Wen (2006, 2007) and Bencivenga (1992), who investigate the effects of variations in the marginal rate of substitution between consumption and leisure.

The stylized economy is characterized as follows. Consider a two-sector economy where, in each sector, a distinct output is produced by using labor services (freely mobile across sectors) and a sector-specific capital stock; the sector-specific output yields one type of consumption good and one type of investment good that can be used as capital but only in the same sector. This assumption precludes sectoral co-movement induced by complementarity in the production process. Utility is defined over leisure and a consumption index that includes the consumption goods of both sectors. Next, assume that the consumption index is a Cobb–Douglas (homogeneous of degree 1) function over the consumption goods. Hence, the modeled preference shifts constitute the only exogenous source of intertemporal, intersectoral, and intrasectoral dynamics of employment, consumption, output, and investment.

We interpret the dynamics by focusing on different ways of perceiving the same consumption basket according to the state of the relative preferences. The balance of the paper is organized as follows. Section 2 details the benchmark economy. Section 3 presents the theoretical mechanism and selected numerical results. Section 4 discusses and extends model results, and Section 5 concludes. Finally, the Appendix includes all proofs and derivations.

2 A Two-Sector Model with Relative-Preference Shifts

This section presents the baseline dynamic equilibrium model with relative-preference shocks. Because there are no restrictions to trade, we solve the dynamic planning problem of a benevolent planner.

2.1 The Benchmark Economy

The benchmark model is structured as a two-sector, two-good economy with endogenous labor supply choice. There is a continuum of identical households of total measure 1. The relative demands for goods are driven by autonomous changes in preferences of the representative household. Capital goods are sector specific, whereas labor services can be reallocated across sectors without bearing any adjustment cost.

2.1.1 Preferences

Define a Cobb–Douglas consumption index as follows:

$$C_t = c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}, \tag{1}$$

where $c_{1,t}$ and $c_{2,t}$ denote (respectively) the consumption of good 1 and good 2 at time t; $s_{1,t}$ and $s_{2,t}$ denote the preference weights and follow exogenous stochastic processes (to be defined shortly).[2] In this framework, a positive shock to $s_{1,t}$ changes the instantaneous structure of preferences in favor of good 1. In order to analyze aggregate consequences of only the relative-preference shifts among consumption goods, we preserve the homotheticity (of degree 1) of preferences and assume that $s_{1,t} + s_{2,t} = 1$ for all $t = 1, 2, ...$. It is then sufficient to specify that s_1 follows an autoregressive process: $s_{1,t} = \rho s_{1,t-1} + (1-\rho)\, s_1 + \varepsilon_t$, where $0 \leq \rho \leq 1$ and s_1 indicates the steady-state value. Quantity ε_t is a random variable normally distributed with zero mean and variance σ_ε^2. A relative-preference shock $\{\varepsilon_{1,t}\}_{t=1}^{\infty}$ is transitory but has persistent effects because of the preference structure. Finally, it is important to note that C_t is not the aggregate consumption reported in national accounts at time t; in other words, C_t is not a macroeconomic aggregate but rather an index that represents the structure of preferences. We will return to this subject later.

Preferences over consumption index C_t and leisure ℓ_t are described by a state-dependent felicity function $U(C(\mathbf{c}_t), \ell_t; \mathbf{s}_t) : \mathbb{R}_+^2 \times \mathcal{S}^2 \times [0,1]^2 \to \mathbb{R}$:

$$U(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma} - 1}{1-\gamma} + B\ell_t, \tag{2}$$

where γ measures the degree of risk aversion and is inversely proportional to the elasticity of intertemporal substitution; ℓ_t denotes leisure hours. In order to better understand the behavior of demands for consumption goods, we assume that the marginal utility of leisure is constant and equal to B.[3] Leisure hours are defined as

$$\ell_t = 1 - n_{1,t} - n_{2,t}, \tag{3}$$

where $n_{1,t}$ and $n_{2,t}$ denote working hours in sector 1 and 2, respectively. The structure implies that available hours are normalized to 1 and that labor services shift across sectors without adjustment costs.

---

[2] Also Stockman and Tesar (1995) use the Cobb–Douglas aggregator for tradable consumption goods in a two-country framework.

[3] In a later section we show that linearity in leisure is not a necessary condition. Our assumption here simplifies the explanation of the mechanism underlying the relative-preference shifts in consumption goods.

2.1.2 Production Technologies

Each good is produced by physical capital and labor using a sector-specific Cobb–Douglas production function:

$$y_{1,t} = \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} \text{ and } y_{2,t} = \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2}. \tag{4}$$

Here $y_{j,t}$, $k_{j,t}$, and λ_j denote (respectively) output, capital stock, and technology level in sector j for $j = 1, 2$ ($j = 1, 2$ hereafter) and where α_j measures the elasticity of output to capital in sector j. Production is not subject to exogenous technology changes (i.e., the λ_j parameters are constant over time). As remarked in the Introduction, this assumption strongly differentiates our model from the traditional approach that focuses on the effects of idiosyncratic productivity shocks.

The allocation constraint is specific for each sector and is given by

$$c_{1,t} + i_{1,t} = y_{1,t} \quad \text{and} \quad c_{2,t} + i_{2,t} = y_{2,t}, \tag{5}$$

where $i_{j,t}$ denotes the investment flows at time t.

In each sector, capital accumulation follows the standard formulation

$$k_{1,t+1} = (1 - \delta_1)k_{1,t} + i_{1,t} \text{ and } k_{2,t+1} = (1 - \delta_2)k_{2,t} + i_{2,t}, \tag{6}$$

where δ_j denotes the depreciation rate of capital stocks at time t. Equations (4)–(6) dictate that the capital stock used in sector j be produced entirely in sector j. This hypothesis makes capital goods fixed across sectors and thereby rules out input–output transmission mechanisms. Hence we can isolate the way preferences drive intersectoral co-movements with no influence from production processes.

2.1.3 The Model's Solution and Equilibrium Characterization

The planner maximizes the expected present discounted value of the return function $\mathcal{V}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(\mathbf{c}_t, \ell_t; \mathbf{s}_t)$, where β ($0 < \beta < 1$) is a subjective discount factor. The maximization problem is subject to the allocation constraints (equation (5)), the capital accumulation constraints (equation (6)), and the total-hour constraint (equation (3)). The state of the economy at time t is represented by a vector $\chi_t = \langle k_{1,t}, k_{2,t}, s_{1,t}, s_{2,t} \rangle$. Controls for the problem are consumption flows $\mathbf{c}$, investment flows $\mathbf{i}$, and labor services $\mathbf{n}$. Once we introduce dynamic multipliers

$\phi_{1,t}$ and $\phi_{2,t}$, forming the Langrangean $\mathcal{L}_0$ yields

$$\begin{aligned}\max_{\{c_{j,t},n_{j,t},k_{j,t+1}\}_{j=1}^{2}} \mathcal{L}_0 \;=\; & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{\left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{1-\gamma} - 1}{1-\gamma} + B\left(1 - n_{1,t} - n_{2,t}\right) \\ & + \phi_{1,t}\left[\lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} - c_{1,t} + \left(1-\delta_1\right) k_{1,t} - k_{1,t+1}\right] \\ & + \phi_{2,t}\left[\lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2} - c_{2,t} + \left(1-\delta_2\right) k_{2,t} - k_{2,t+1}\right] \Bigg\}, \end{aligned} \tag{7}$$

where $\mathbb{E}_0$ is the conditional expectation operator on time-0 information. First-order conditions with respect to jth consumption flow and working hours (FOC($c_{j,t}$), FOC($n_{j,t}$)) read:

$$\begin{aligned} c_{1,t} &: \quad s_{1,t} c_{1,t}^{s_{1,t}-1} c_{2,t}^{s_{2,t}} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{-\gamma} = \phi_{1,t}, \\ c_{2,t} &: \quad s_{2,t} c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}-1} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{-\gamma} = \phi_{2,t}; \end{aligned} \tag{8}$$

$$\begin{aligned} n_{1,t} &: \quad B = \phi_{1,t}\left(1-\alpha_1\right) \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{-\alpha_1}, \\ n_{2,t} &: \quad B = \phi_{2,t}\left(1-\alpha_2\right) \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{-\alpha_2}. \end{aligned} \tag{9}$$

Here $\left(1-\alpha_j\right) \lambda_j k_{j,t}^{\alpha_j} n_{j,t}^{-\alpha_j} = w_{j,t}$ is the marginal productivity of labor in sector j.

Combining the previous equations, we can rewrite the optimality conditions for sectoral consumptions and working hours as

$$\begin{aligned} s_{1,t} \frac{C_t^{1-\gamma}}{c_{1,t}} w_{1,t} &= B, \\ s_{2,t} \frac{C_t^{1-\gamma}}{c_{2,t}} w_{2,t} &= B. \end{aligned} \tag{10}$$

Optimality conditions (equation (10)) indicate the standard equality between the weighted marginal utility of consumption, $s_{j,t}\left(C_t^{1-\gamma}/c_{j,t}\right)$, and the weighted marginal utility of leisure, $B/w_{j,t}$.

Optimal investment dynamics are determined by the Euler equations

$$\begin{aligned} \beta \mathbb{E}_t \left[\frac{s_{1,t+1} \frac{C_{t+1}^{1-\gamma}}{c_{1,t+1}}}{s_{1,t} \frac{C_t^{1-\gamma}}{c_{1,t}}} \left(\alpha_1 \lambda_1 k_{1,t+1}^{\alpha_1 - 1} n_{1,t+1}^{1-\alpha_1} + \left(1-\delta_1\right)\right) \right] &= 1, \\ \beta \mathbb{E}_t \left[\frac{s_{2,t+1} \frac{C_{t+1}^{1-\gamma}}{c_{2,t+1}}}{s_{2,t} \frac{C_t^{1-\gamma}}{c_{2,t}}} \left(\alpha_2 \lambda_2 k_{2,t+1}^{\alpha_2 - 1} n_{2,t+1}^{1-\alpha_2} + \left(1-\delta_2\right)\right) \right] &= 1, \end{aligned} \tag{11}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on the information available at time t. Observe that the pricing kernel

$$\Pi_{j,t} = \left(C_{t+1}^{1-\gamma}/C_t^{1-\gamma}\right)\left(\left(s_{j,t+1}c_{j,t}\right)/\left(s_{j,t}c_{j,t+1}\right)\right)$$

depends on the dynamics of the preference parameters, the consumption of goods, and the consumption index.

The system of optimal conditions and resource constraints determines the deterministic steady state; then, the log-linearization of the model around this steady state describes the dynamics.[4] Next we describe the model's parameterization before presenting the simulation results.

2.2 Parameterization

The system of equations that defines the dynamic equilibrium of the model depends on a set of twelve parameters. Six pertain to technology (the capital share α_j, the capital stock quarterly depreciation rate δ_j, and the level of technology λ_j in both sectors). The other six pertain to consumer's preferences (the subjective discount factor β, the relative risk aversion coefficient γ, the marginal utility of leisure B, the relative preferences for good 1 s_1 and s_2 for good 2, and the autoregressive coefficient of the preference process ρ).[5]

The sectors are characterized by the same technology so that all differences between the equilibrium values of the sectoral variables will derive from consumer preferences. This assumption makes it easy to associate the parameterization of the relative preferences among consumption goods to the composition of the initial consumption basket. In fact, under the symmetric hypothesis on the supply side, we have $c_1 \gtreqless c_2$ if and only if $s_1 \gtreqless s_2$. Assuming differences in the supply side would complicate our exposition of the mechanisms with no significant added value in the understanding of the role of preferences. The model is parameterized for the postwar U.S. economy, except for relative-preference parameters that are used to set up the theoretic investigation.

Technology parameters $(\delta_j, \alpha_j, \lambda_j)$: These are set to commonly used values in the real business cycle (RBC) literature. In particular, we consider a symmetric economy from the supply side, so $\delta_1 = \delta_2 = 0.025$, $\alpha_1 = \alpha_2 = 0.36$, and $\lambda_1 = \lambda_2 = 1$.

Consumer preferences $(\beta, \gamma, B, s_j, \rho)$: The quarterly subjective discount factor β is set to correspond to an annual real interest rate of 4%; it yields $\beta = 0.99$. The

[4]Following Uhlig (1999), Appendix A and Appendix C report the steps used to determine the steady-state values and the dynamic equations of the model.

[5]Recall that s_2, the preference parameter for good 2, is set equal to $1 - s_1$.

relative risk aversion γ is equal to 5. The relative preference for good 1, s_1, varies in the range $0 < s_1 < 1$. The autoregressive coefficient of the preference process ρ is 0.99. The marginal utility of leisure, B, is endogenously calibrated to generate $n_1 + n_2 = 0.3$.[6]

3 Results

3.1 Structure of the Simulations

This section investigates how the stylized economy responds to an increase in the relative preference for good 1. Anticipating a result, the initial composition of the consumption basket is a key element for determining the sign of intersectoral co-movements; for this reason, the model is simulated for different compositions of the consumption basket in steady state.

The simulations show how the dynamics change according to the relative weight of each good in the consumption basket. In order to explain the emerging results, we introduce a "perception effect" that describes how a consumer's satisfaction, for a given consumption choice, changes according to the state of her preferences. We then use sensitivity analysis to investigate the role of selected parameters.

3.2 Baseline Simulations

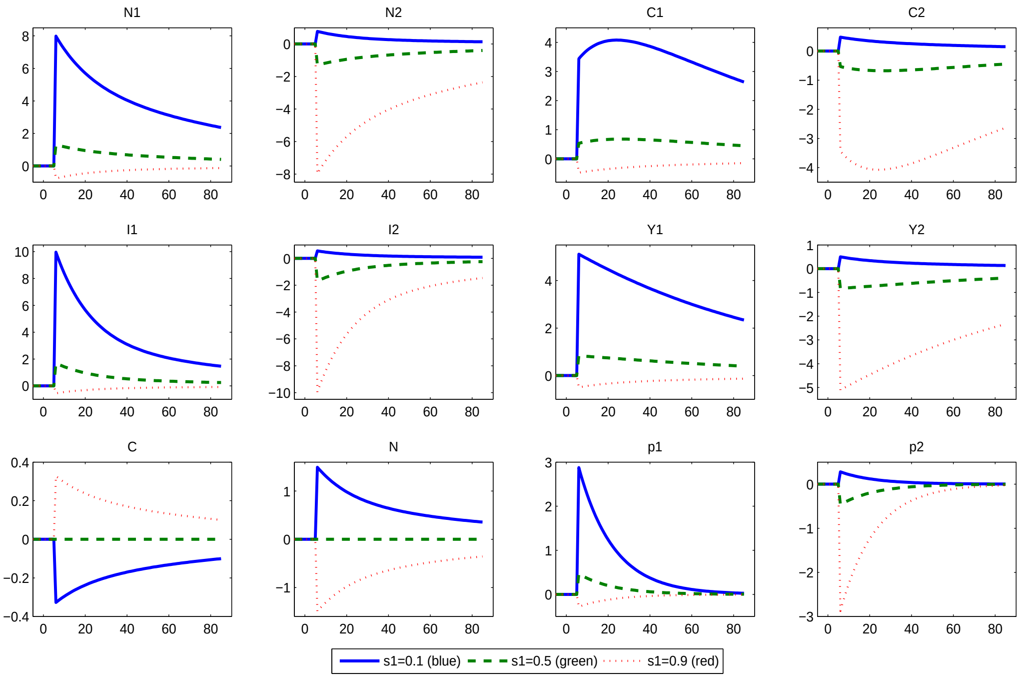
This paper examines three different pre-shock scenarios: a fully symmetric consumption basket with $s_1/s_2 = c_1/c_2 = 1$ ($s_1 = 0.5$); and two asymmetric consumption baskets with $s_1/s_2 = c_1/c_2 = \frac{1}{9}$ ($s_1 = 0.1$) or $s_1/s_2 = c_1/c_2 = 9$ ($s_1 = 0.9$).

Technically, we run three sets of simulations while maintaining the baseline calibration —except for the steady-state value of s_1. We set s_1 equal to 0.1, 0.5, and 0.9; we then report the impulse response functions of the sectoral variables in Figure 1 with (respectively) blue, green, and red lines. To ease the comparison between consequences of the different settings, we impose that the preference shocks always have the same magnitude: 1% of 0.5. Thus, immediately after the shock, s_1 moves from 0.1 to 0.105 (blue line), from 0.5 to 0.505 (green line), and from 0.9 to 0.905 (red line).

Figure 1 describes (as do most of the remaining figures) the impulse response functions of labor services (n_1, n_2), consumption (c_1, c_2), investment (i_1, i_2), and output (y_1, y_2) of both sectors; C is the consumption basket as defined in equation (1), and $N = n_1 + n_2$ is the total employment. The last two boxes refer to the ratio

[6] See Appendix B.

Figure 1: Impulse Response Functions, benchmark version



between the marginal utility of each consumption good and the marginal utility of leisure ($p_1 = U_{c_1}/U_l$, $p_2 = U_{c_2}/U_l$), that we may consider the price of each good when the marginal utility of leisure is the numéraire. The figure shows the first 80 quarterly percentage deviations from a scenario in which all innovations are set to 0.

The green lines (with $s_1 = s_2 = 0.5$) in Figure 1 show an economy whose input factors (n_2 and i_2) have been withdrawn from the production of good 2 (y_2 and c_2 decrease) and allocated to sector 1. Intrasectoral co-movements among consumption, investment, employment, and output are positive in both sectors, but intersectoral co-movements are negative. In fact, the sector characterized by an increase in preference (sector 1) goes through an expansive phase while the other sector goes through a recessive phase. Prices follow the direction of the preference weights.

The blue lines (with $s_1 = 0.1$, $s_2 = 0.9$) show an economy with both sectors in expansion. Both intersectoral and intrasectoral co-movements are positive. Prices co-move, and p_2 increases even if s_2 falls.

The red lines (with $s_1 = 0.9$, $s_2 = 0.1$), similarly to the previous case, show an economy characterized by positive intersectoral and intrasectoral co-movements; however, the dynamics are completely reversed. The stylized economy experiences a recession in both sectors and in all variables (consumption, investment, employment, and output). Prices co-move with different dynamics for p_1 and s_1.

3.3 Source of Intersectoral Co-movement

Roughly speaking, the representative agent chooses among the consumption of two different goods and leisure. Under the preference structure reported in equations (1) and (2), the marginal rate of substitution between sector specific consumption and leisure depends on the consumption of both goods. In fact, the influence of c_j on consumer utility (i.e., the marginal utility of c_j) is given by the effect of c_j on C and by the effect of C on the utility function. If preferences shift then both effects vary, but we shall demonstrate that only the latter can induce positive intersectoral co-movement.

In the previous three simulations (reported in Figure 1), following the positive shock to s_1 the ceteris paribus effect of c_1 on C increases while the effect of c_2 on C decreases.[7] This produces a "substitution" effect that reduces consumption of good 2 and increases consumption of good 1. After a preference shift, then, the C_{c_j} change in a way that does not sustain positive intersectoral co-movements. On the contrary, the ceteris paribus change of C affects the optimal choice in both sectors in the same direction. In fact, if C decreases (increases) then the marginal impact of C on the utility function increases (decreases), which in turn pushes up (down) the marginal utility of both goods. We call this change the *perception effect*. It has no impact on the marginal rate of substitution between c_1 and c_2, but it does affect the marginal rate of substitution between each consumption good and leisure.[8]

The blue-line case describes a context where the perception effect increases the marginal utility of both consumption goods. The reverse occurs in the red-line case: the marginal utility of consumption basket has fallen, so sector 1 also experiences a recession phase. In both cases, it is evident that positive intersectoral co-movements are driven by sectoral (and not aggregate) preference shocks. Finally, the green lines report dynamics where the perception effect is absent; in this case, only the substitution effect matters.

[7] In Appendix D and E (respectively) we prove that $C_{c_1 s_1} > 0$ and $C_{c_2 s_1} < 0$.

[8] The preference shock generates a sort of "real wealth" effect, if we assume that real wealth is measurable by the level of utility a consumer can reach. Indeed, after the shock, satisfaction varies because the consumer associates revised levels of satisfaction to the same goods. Hence satisfaction changes even when consumption is fixed. However, even if we adopt the previous definition, this mechanism would be quite different from what is reported in microeconomics manuals. In fact, the first element to change is not the budget constraint but rather the indifference curve.

Table 1 summarizes the possible scenarios after a positive shock to s_1.

Table 1: Possible dynamics of consumption after a positive shock to s_1.

Before the shock	Perception	Substitution	Final result
$c_1 < c_2$	$c_1 \uparrow$ $c_2 \uparrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \uparrow\downarrow$
$c_1 = c_2$	$c_1 \leftrightarrow$ $c_2 \leftrightarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$
$c_1 > c_2$	$c_1 \downarrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow\downarrow$ $c_2 \downarrow$

Now let's address the argument from an analytical perspective. The starting point is the optimal condition that determines the choice between consumption and leisure in each sector:

$$U_{c_j} w_j = U_C C_{c_j} w_j = B. \tag{12}$$

Equation (12) imposes that, in equilibrium, the marginal utility of consumption of good j, U_{c_j}, when weighted with the marginal productivity of labor in sector j, w_j, must be equal to the marginal utility of leisure, B.[9] The marginal utility of c_j can be decomposed as the product of the first derivative of the utility function with respect to the consumption basket, U_C (i.e., the marginal utility of C), and the first derivative of the consumption basket with respect to the single consumption good, C_{c_j}. The key question concerns what happens to U_C after an increase in s_1 (as in our simulations). In order to provide an answer, we must focus on the signs of two derivatives. The first is the sign of the derivative of the marginal utility U_C with respect to the consumption basket (i.e., the second derivative of the utility function with respect to the consumption basket). This sign is negative; in fact, $U_{CC} = -\gamma C^{-\gamma-1} < 0$ for $\gamma \geq 0$.

The second sign applies to the partial derivative of the consumption basket with respect to the exogenous shock, C_{s_1}.[10] The sign of this derivative depends on the ratio between the consumption goods that make up the consumption basket: $C_{s_1} = c_1^{s_1} c_2^{1-s_1} \ln(c_1/c_2)$.[11] In the blue-line version (reported in Figure 1), $s_1 = 0.1$ and so

[9] If we assume that B is constant, then the dynamics of equation (12) can be expressed as $\tilde{s}_{j,t} - \widetilde{c}_{j,t} + (1-\gamma)\,\widetilde{C}_t + \alpha_j \widetilde{k}_{j,t} - \alpha_j \widetilde{n}_{j,t} = 0$, where the tilde indicates the growth rate. This way of representing the dynamics characterizing equation (12) may facilitate understanding the mechanism described in this section.

[10] We are interested in the direct effect of s_1 on C, taking the rest as given. This is why we do not consider the indirect effect generated by variations in consumption composition.

[11] Recall that in our model the supply sides of each sector are perfectly symmetric; it follows that

$c_1/c_2 < 1$; hence $C_{s_1} < 0$. If C falls then U_C rises.[12] In this case, the direct effect of a positive shock to s_1 is a reduction in C and then an increase in U_C. Notice that, according to the optimal conditions of equation (12), the product $C_{c_j} w_j$ must fall in both sectors. This can occur either by an increase in c_j (since the derivative of C_{c_j} with respect to c_j is negative; in fact, $C_{c_1 c_1} = s_1 (s_1 - 1) c_1^{s_1 - 2} c_2^{1 - s_1} < 0$ and $C_{c_2 c_2} = -s_1 (1 - s_1) c_1^{s_1} c_2^{-s_1 - 1} < 0$) or by an increase in employment n_j (to reduce w_j). The reverse occurs in the case with $s_1 = 0.9$ (red lines in Figure 1).[13]

This mechanism helps to explain the positive intersectoral co-movements reported in the cases with blue and red lines. It also explains why economic booms (dooms) occur after a preference shock when s_1 is set low (high).

Finally, the green lines represent a perfectly symmetric economy: $s_1 = 0.5$, $c_1/c_2 = 1$, $C_{s_1} = 0$. The direct effect of preference shifts on U_C is null, so the dynamics of the stylized economy are driven only by substitution effects among sectoral goods.

We have not described all the forces at work because we are mainly interested in the mechanisms that induce positive intersectoral co-movements. Therefore, as indicated by the arrows in Table 1, the initial composition of the consumption basket represents a necessary (but not sufficient) condition for observing positive intersectoral co-movements in response to relative-preference shifts.

Before extending the analysis to addressing the role of some selected parameters, a further clarification of our results is needed. As mentioned previously, C does not equal aggregate consumption. To build aggregate macroeconomic variables we should define the aggregation technique, and to test our model we should introduce a more realistic structure with linkages in the supply side. But this is beyond our purpose. Our aim is simply to describe a possible new source of intersectoral co-movement, so it suffices to focus on sectoral variables. Yet it is still worth noting that, in the cases with intersectoral co-movement (blue and red lines in Figure 1), sectoral real variables and prices move in the same direction. This suggests that emerging sectoral dynamics (expansion or recession) should characterize the paths of aggregate variables in the same way. Thus, the results reported in Figure 1 should be consistent with a positive correlation between aggregate variables.

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the relative dimensions of steady-state values of the sectoral variables depend only on consumer preferences. Then $c_1 \gtreqless c_2$ if and only if $s_1 \gtreqless s_2$.

[12]If we assume a CES function for the consumption index, then the economic mechanism does not change significantly. In fact, if $C = (s_1 c_1^q + (1 - s_1) c_2^q)^{1/q}$ then $\partial C/\partial s_1 = (1/q)(s_1 c_1^q + (1 - s_1) c_2^q)^{1/q - 1} (c_1^q - c_2^q)$; hence the sign depends again on the relative dimensions of the consumption goods.

[13]We point out that $C_{c_1 s_1} > 0$ while $C_{c_2 s_1} < 0$. Consequently (not considering the behavior of the marginal productivity of labor), if $C_{s_1} < 0$ then a rise in c_1 is surely needed to produce a fall in C_{c_1}, whereas the equivalent statement is not always true for c_2.

3.4 What about Investment Choice?

The behavior of investments depends on the shock's persistence. It is therefore useful to run another set of simulations with a lower value of the autoregressive coefficient: $\rho = 0.92$; the results are graphed in Figure 2. Hereafter, we focus our explanations on the case represented by blue lines (i.e., $s_1 = 0.1$) but the same mechanisms are at work in the other cases—although with different results.

Some evidence emerges clearly. First, the impulse responses are less persistent. Second, the intrasectoral co-movements change. In particular, reducing the persistence of the preference shock causes the responses of sectoral investments to change direction. Such mechanisms are much the same as those discussed by Wen (2006, 2007) in a one-sector model: in the absence of increasing returns to scale (as in this case), nonpersistent changes in consumption preferences crowd out investment. It follows that only persistent increases in consumption demand can sustain investment by prompting a further increase in labor supply.[14] Finally, it must be remarked that the persistence of preferences does not affect the sign of intersectoral co-movements because it does not affect the marginal rates of substitution between consumption goods and leisure.

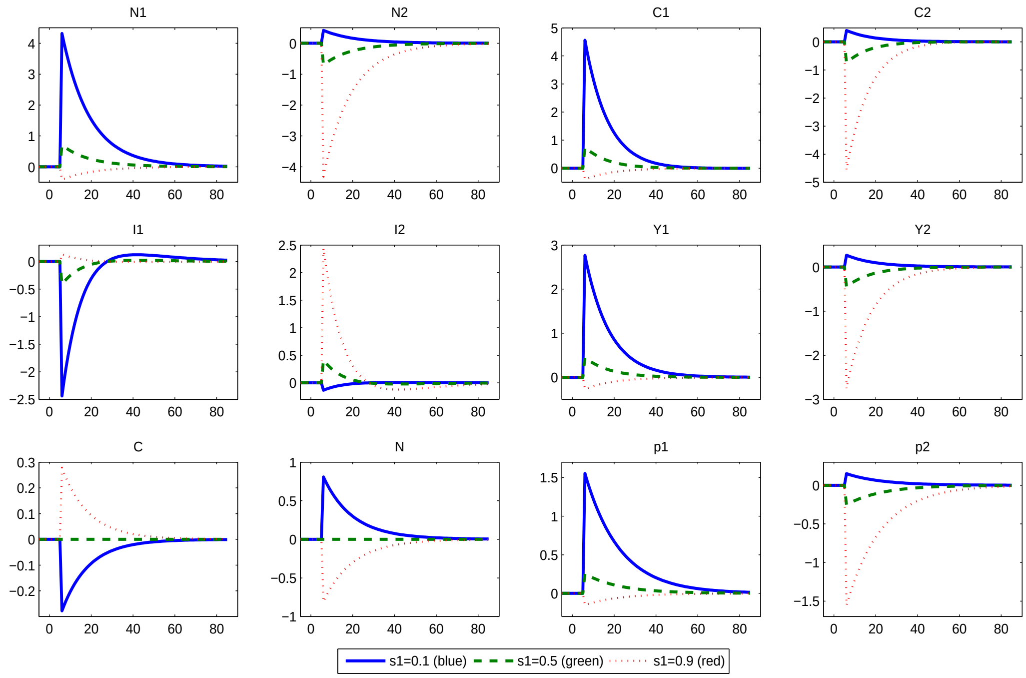
3.5 Role of the Relative Risk Aversion Coefficient (Consumption)

Roughly speaking, the relative risk aversion coefficient determines how much the marginal utility of consumption varies after a change in the consumption basket. It is thus reasonable to assume that this parameter may be relevant in the present framework. In order to illustrate the effects of a change in γ, we replace $\gamma = 5$ with $\gamma = 1.5$ and then simulate the usual preference shock. The resulting dynamics are illustrated in Figure 3.

The figure shows that, with low values of γ, the positive co-movements between sectors vanish. The reason is that the significance of the perception effect has strongly decreased. By the dynamic equation of the first derivative of the utility function with respect to the consumption basket ($\widetilde{U}_C = -\gamma\widetilde{C}$; see also the dynamic equation reported in note 9), it is clear that γ is a scale factor of the effect of C on the marginal utility of consumption. So if γ is low then variations in C have little effect on the relative preference between consumption goods and leisure, which means that intersectoral co-movements are infrequent.

[14] Just as in Wen (2006), the impulse responses show that, when ρ is low, the response of c_j is relatively higher than that of n_j and y_j.

Figure 2: Impulse Response Functions, ρ=0.92



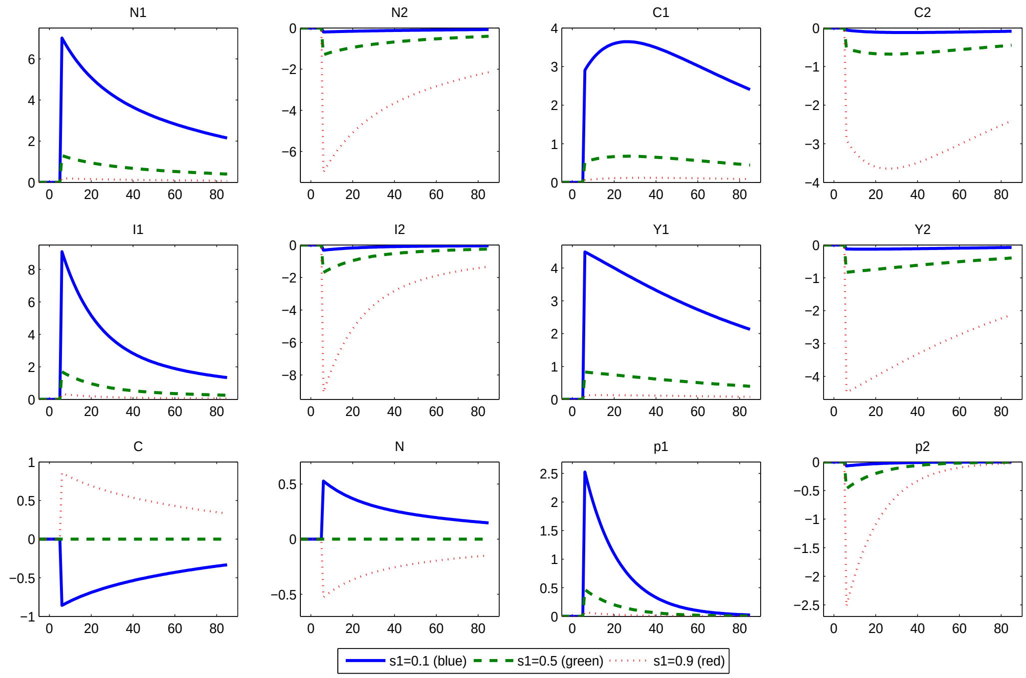
3.6 Role of the Intertemporal Elasticity of Substitution (Leisure)

In the previous simulations we assumed that the marginal utility of leisure was constant. This assumption allowed us to observe how the impact of both the substitution effect and the perception effect changes without the influence of variations in the marginal utility of leisure.

Now let's remove this assumption and solve the model with the following utility function: $u(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma}-1}{1-\gamma} + \frac{1}{v}B(1 - n_{1,t} - n_{2,t})^v$, where $1 - v$ controls the degree of risk aversion and is inversely proportional to the elasticity of intertemporal substitution in leisure. We set $v = -1$ and report the impulse response functions in Figure 4.

Results indicate that positive intersectoral co-movements are less frequent when v decreases. This parameter does not directly modify the perception effect or the substitution effect among consumption goods. The different dynamics emerge owing to the behavior of the marginal utility of leisure, which is positively related to the labor supply when $v < 1$. In fact, the higher labor supply in sector 1 increases the marginal utility of leisure and then reduces the incentive to increase the labor

Figure 3: Impulse Response Functions, γ=1.5



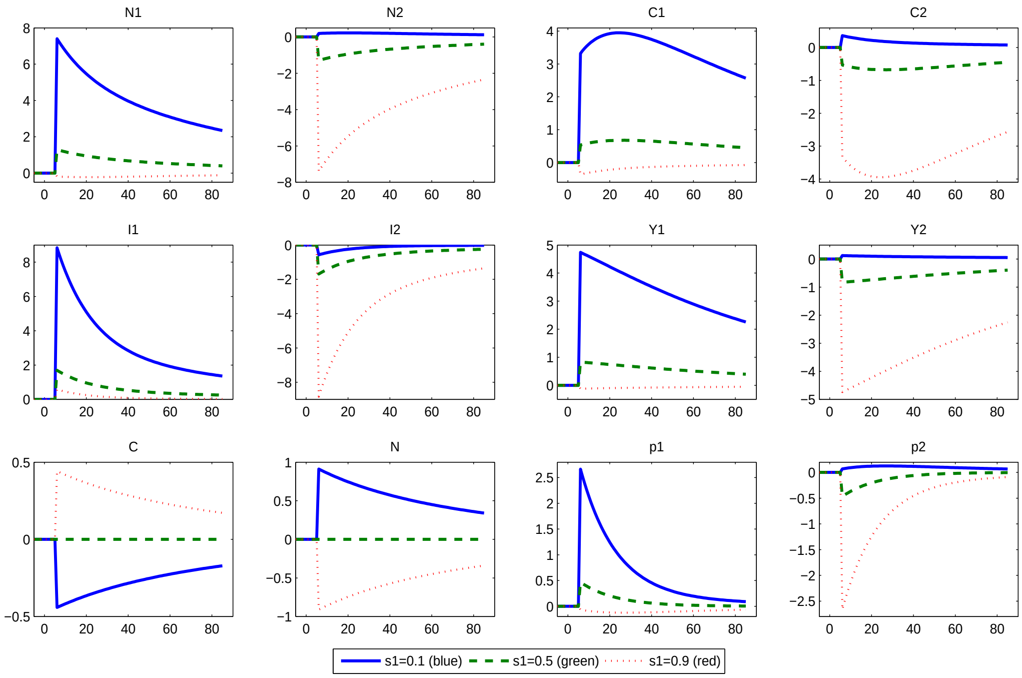
supply in sector 2 as well. Under this parameterization, the perception effect is still able to generate positive co-movements between sectoral consumption and employment, but the effect is not strong enough to support investment in sector 2. Total labor supply is less reactive.

4 Discussion and Extensions

An important implication of the analysis so far is that a relative-preference shift is more likely to induce sectoral comovement when the asymmetry of the composition of the consumption index is strongly asymmetrical and risk aversion is high. Indeed, the simulations had to be run under a quite "comfortable" parameterization in order for clear impulse response functions to appear in the figures. This fact mandates a discussion of the robustness and relevance of the proposed mechanism.

We begin by reporting the binding constraints required, when setting s_1 and γ, to observe intersectoral and intrasectoral co-movement. When $\gamma = 5$, we need $s_1 \leq 0.179$ (for an upturn; $\frac{c_1}{c_2} \leq 0.218$) or $s_1 \geq 1 - 0.179$ (for a downturn;

Figure 4: Impulse Response Functions, υ=-1.0



$\frac{c_1}{c_2} \geq 4.587$). Otherwise, keeping $s_1 = 0.1$ or $s_1 = 0.9$, we require $\gamma \geq 2.1$, which falls to 1.56 if $s_1 = 0.05$ or $s_1 = 0.95$.

The linkage between the level of risk aversion and the sectoral relative preferences emerges from simulations and is consistent with the suggested economic intuition: the former measures the effect of a change in C on the marginal utility of consumption, whereas the latter determines the magnitude of the variation of C after the preference shock. The point is that we must evaluate whether such constraints allow the proposed mechanism to be relevant. In order to address this issue, we reduce and make constant the value for risk aversion (next we run other simulations with $\gamma = 2$) and then focus on interpreting of the asymmetry in the consumption index. We propose two arguments to support the notion that constraints on the parameterization setting do not vitiate the relevance of this contribution.

The first argument concerns identification of the kinds of goods. Because we consider a general equilibrium framework, goods 1 and 2 represent all the available goods in the economy. It follows that their relative sizes can vary considerably according to what they represent. For example, "recent" goods may represent a small share of the economy and yet be subject to positive preference shocks.

The second argument concerns the range of the co-movement that has to be explained. As well documented by Hornstein (2000), most but not all industries of the economy co-move. This implies that, in the presence of many kinds of goods, it is important to identify a mechanism capable of explaining the co-movement among most of them. We therefore extend our model to analyze how the proposed mechanism works in an economy with m different goods, and we show that a relative-preference shock can induce the co-movement of many sectors without imposing relevant constraints on parameters. Let's set a more general definition of the consumption index:

$$C_t = \prod_{j=1}^{m} c_j^{s_{j,t}}, \tag{13}$$

where $\sum_{j=1}^{m} s_{j,t} = 1$ for all t. A shift ε_t in the relative preferences may change the preference structure in the following way: $s_{j,t} = s_{j,t-1} + \mu_j \varepsilon_t$ for all j with $\sum_{j=1}^{m} \mu_j = 0$. In this case, the partial derivative of the consumption index with respect to the preference shift is[15]

$$C_\varepsilon = C \sum_{j=1}^{m} \mu_j \ln c_j. \tag{14}$$

Notice that, as in the two-sector case, the preference shock's effect on the composite consumption index influences the consumption–leisure choice in the same direction in each sector. This connection opens endless possibilities in terms of the composition of C and how the shock affects preferences for the different goods. In this generalized case, the perception effect does not emerge only if $\sum_{j=1}^{m} \mu_j \ln c_j = 0$.

Let's focus on the case where the preference shock concerns only two kinds of goods. For example, assume $\mu_j = 0$ for $j \neq 1, 2$ and $\mu_1 = -\mu_2 = 1$. From equation (14) it follows that

$$C_\varepsilon = C \ln \left(\frac{c_1}{c_2} \right). \tag{15}$$

Observe that this time c_1 and c_2 do not represent the whole economy. Equation (15) permits us to anticipate an important result that will be confirmed by the next simulations: Under the hypotheses characterizing this stylized economy, a preference shift between two kinds of goods generates co-movement among all the other

[15] Hereafter we omit the time indices.

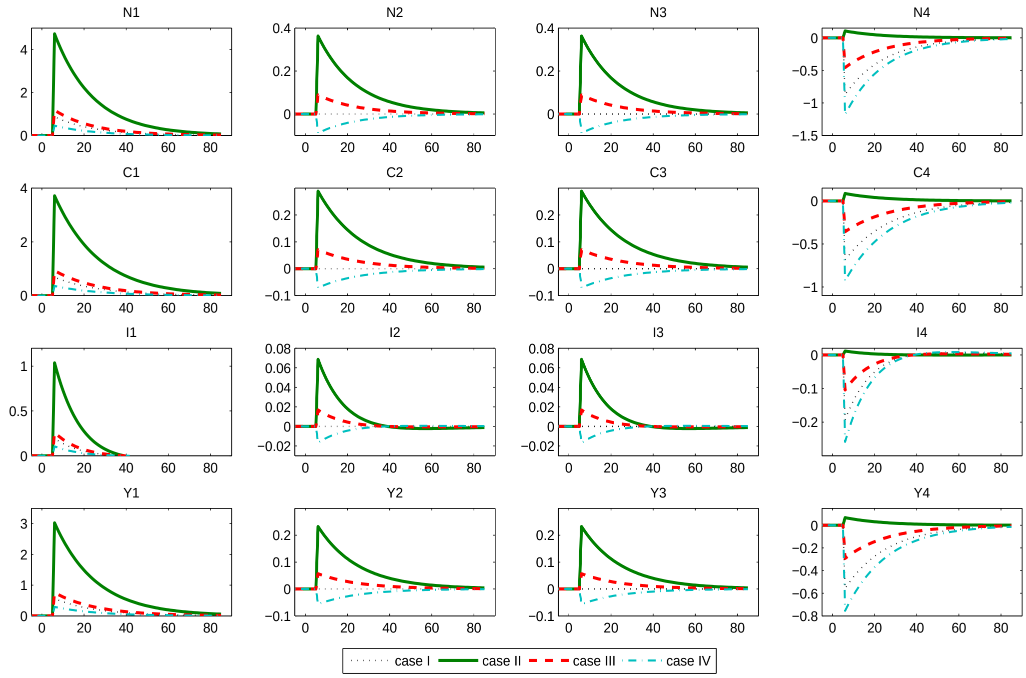
goods. The implication is that, even if the perception effect is not strong enough to induce co-movement between the sectors directly affected by the shock (sectors 1 and 2), the preference shift pushes the other $m-2$ sectors in the same direction (which changes according to the sign of $\ln(c_1/c_2)$).

As an example, we solve a four-goods model with the same characteristics as the two-goods model analyzed in previous sections. The only changes involve the value of the relative risk aversion, $\gamma = 2$ (previously $\gamma = 5$), and the value of the persistence coefficient of the preference shock, $\rho = 0.95$ (previously $\rho = 0.99$), which is consistent with the empirical findings of Foster et al. (2008). We run two sets of simulations. In the first set we consider a preference shift between two sectors—mainly to show that, if the aim is to explain co-movement among most (but not all) of the economic sectors, then the proposed mechanism is not significantly constrained by the parameterization of the preferences. In the second set of simulations we analyze a preference shift among three sectors. Specifically, we assume that an increase in preference for one sector may occur at the expense of two other sectors. We find that if the shock is split among more than two sectors, then the proposed mechanism is able to generate co-movement among sectors whose preferences move in opposite directions with fewer binding restrictions on the parameter setting.

The first set of simulations of the impulse response functions is graphed in Figure 5; Table 2 summarizes the setting of the preferences and reports the sectoral outcomes. The common hypothesis is that $\mu_1 = -\mu_4 = 1$ and $\mu_2 = \mu_3 = 0$. The analyzed cases differ in the parameterization of the relative preferences. In Case I, the sectors directly affected by the shock have the same weight in the consumption index; in this case, then, there is no perception effect. The sector with the positive shock grows, the sector with the negative shock falls, and the other sectors remain stable. In Case II, there is high asymmetry among the weights of the sectors directly affected by the shock. The result is a strong push toward co-movement, and all the sectors move in the same direction (up)—including the sector with the decreasing preference. Case III and Case IV confirm that, for the sectors whose relative preferences do not change, the dynamics depend on the perception effect. Indeed, it is worth noting that, given s_1 and s_4: (a) the values of s_2 and s_3 do not affect the dynamics of sector 2 and sector 3; and (b) that the dynamics of these sectors in Case III are exactly the inverse of those in Case IV, since parameters have been chosen to produce the same perception effect but with the opposite sign.

For the second set of simulations (see Table 3 and Figure 6) we assume that $\mu_1 = -\mu_4 - \mu_3 = 1$ and $\mu_2 = 0$. Case V and Case VI are characterized by the same structure of relative preferences but have different values of μ_3 and μ_4. The perception effect is strong and pushes toward an increase in consumption in both cases: in Case V, all the sectors experience an upturn; in Case VI, sector 4

Figure 5: IRFs after a shock that affects s_1 positively and s_4 negatively



experiences a downturn because it is the most negatively affected by the shock.[16] The results in Case VII are similar to those in Case VI but with less asymmetry in the size of the sectors. This indicates that the theoretical mechanism presented here can explain co-movement also between sectors whose relative preferences go in opposite directions (sector 1 and sector 4) without resorting to extreme parameter settings. Finally, Case VIII represents a special combination of parameter values that yields an almost null perception effect. This combination induces dynamics similar to Case I (i.e., the sectors with stable relative preferences do not move).

5 Conclusions

The main intent of this paper is to suggest a new source of sectoral co-movements during business cycles. In our model, relative-preference shocks between consump-

[16]Consistently with the parameterization, Figure 6 shows that sectors 1 and 2 have the same dynamics in Case V and in Case VI, whereas sector 3 experiences a greater increase in Case VI than in Case V.

Table 3: Sectoral dynamics after a shock that affects s_1 pos. and s_4 neg.

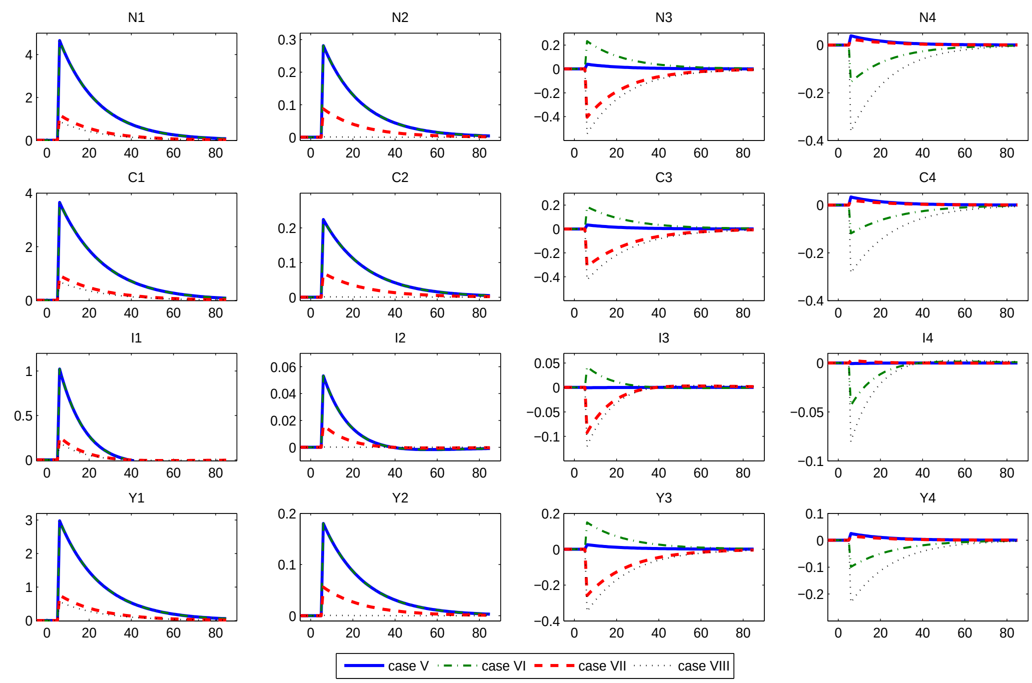
	I	II	III	IV
s_1	0.25	0.05	0.2	0.4
s_2	0.4	0.05	0.3	0.2
s_3	0.1	0.05	0.1	0.2
s_4	0.25	0.85	0.4	0.2
μ_1	1	1	1	1
μ_2	0	0	0	0
μ_3	0	0	0	0
μ_4	-1	-1	-1	-1
Sectors in upturn	1	$1, 2, 3, 4$	$1, 2, 3$	1
Sectors in downturn	4	$\varnothing$	4	$2, 3, 4$
Stable sectors	$2, 3$	$\varnothing$	$\varnothing$	$\varnothing$

tion goods—which may be interpreted as shocks to households' tastes—are the only mechanism generating fluctuation. The results indicate that, in order to induce intersectoral and intrasectoral co-movement in a stylized economy with only two kinds of goods, constraints on the parameter settings (especially for risk aversion and relative preferences) are necessary but are not, in our opinion, so binding as to render the theoretical point moot. In the last section we showed that, by extending the analysis to a multisector model and introducing a more flexible structure for the propagation of a preference shock, constraints on setting the parameters become much less severe. Consistently with the empirical evidence reported in Hornstein (2000), our model generates co-movement among most but not all industries of the economy under a wide range of parameterizations. In this case, both the values of the relative preferences and the spread of the preference shock determine the direction of the co-movement and the number of sectors involved.

In fact, the entire economy tends toward expansion (recession) when preferences shift toward goods that represent a minor (major) share in the consumption basket. This is because households now obtain less satisfaction from the consumption basket chosen before the shock (perception effect). Consequently, they increase labor supply in order to restore the optimal balance between leisure and each kind of consumption good.

Our model also gives clear indications about the role of some parameters. First, the persistence of the preference shift strongly affects the responses of investments. This is because ρ determines the duration of the perception effect and therefore affects the inter-temporal optimal path of consumption. Second, the coefficient of

Figure 6: IRFs after a shock that affects s_1 positively and affects s_3 and s_4 negatively



risk aversion, γ, determines the relevance of the perception effect in the trade-off between consumption and leisure in each sector. Finally, if the marginal utility of leisure is increasing in labor supply, then the perception effect must be higher in order to generate positive intersectoral co-movements.

The mechanism sketched here is quite new in the economic literature. In fact, multisectoral models generally tend to explain positive co-movements of economic sectors by positing an input–output structure that transmits sectoral shocks over the entire economy.

It should be emphasized that our model differs from models proposed by Bencivenga (1992) and Wen (2006, 2007), which consider direct variation in relative preferences between consumption and leisure *independently* of the composition of the consumption basket. Such variation can be due to the use of leisure time for other activities (e.g., homework production; see Benhabib et al., 1991), it can also be induced by alternative phases of the "urge to consume" (see Wen, 2006) that vary the importance of consumption. In contrast, our model focuses on two other

Table 4: SDs after a shock that affects s_1 pos. and affects s_3 and s_4 neg.

	V	VI	VII	VIII
s_1	0.05	0.05	0.2	0.25
s_2	0.05	0.05	0.05	0.3
s_3	0.45	0.45	0.4	0.3
s_4	0.45	0.45	0.35	0.15
μ_1	1	1	1	1
μ_2	0	0	0	0
μ_3	-0.5	-0.1	-0.9	-0.75
μ_4	-0.5	-0.9	-0.1	-0.25
Sectors in upturn	1, 2, 3, 4	1, 2, 3	1, 2, 4	1
Sectors in downturn	$\varnothing$	4	3	3, 4
Stable sectors	$\varnothing$	$\varnothing$	$\varnothing$	2

elements: the starting composition of the consumption basket and the shifts in relative preferences among consumption goods. The latter element is studied in Phelan and Trejos (2000), but they use it in order to correlate aggregate fluctuations with negative co-movement in sectoral employment.

Particularly interesting implications and extensions of the model involve the analysis of advertising and innovation. Both factors can produce externalities strongly resembling those analyzed in the paper. This opens up other research possibilities in fields that lie outside this model's current range.

Appendix

A. Steady State

From the Euler equations (11), we have

$$\frac{k_1}{n_1} = \left(\frac{\alpha_1\lambda_1}{1/\beta - 1 + \delta_1}\right)^{\frac{1}{1-\alpha_1}},$$

$$\frac{k_2}{n_2} = \left(\frac{\alpha_2\lambda_2}{1/\beta - 1 + \delta_2}\right)^{\frac{1}{1-\alpha_2}}.$$

Combining the first-order conditions for consumption and labor then yields

$$c_1 = \left(s_1 TT^{s_2(1-\gamma)} \frac{(1-\alpha_1)\,\lambda_1}{B} \left(\frac{k_1}{n_1}\right)^{\alpha_1} \right)^{\frac{1}{\gamma(s_1+s_2)+1-s_1-s_2}},$$

$$c_2 = TTc_1,$$

$$C_t = c_1^{s_1} c_2^{s_2},$$

where $TT = \frac{s_2}{s_1}\frac{(1-\alpha_2)\lambda_2 k_2^{\alpha_2} n_2^{-\alpha_2}}{(1-\alpha_1)\lambda_1 k_1^{\alpha_1} n_1^{-\alpha_1}}$. Define $Q_j = \left(\lambda_j \left(\frac{k_j}{n_j}\right)^{\alpha_j - 1} - \delta_j\right)^{-1}$. The feasibility constraint then implies that

$$k_1 = Q_1 c_1,$$

$$k_2 = Q_2 c_2.$$

Finally, we use the capital accumulation process to obtain:

$$i_1 = \delta_1 k_1,$$

$$i_2 = \delta_2 k_2;$$

$$n_1 = \left(\frac{k_1}{n_1}\right)^{-1} k_1,$$

$$n_2 = \left(\frac{k_2}{n_2}\right)^{-1} k_2.$$

B. Calibration of B

We assume that

$$n_1 + n_2 = N = 0.3.$$

We can use equations (8) and (9) to express working time as follows:

$$N = \left(\left(\frac{k_1}{n_1}\right)^{-1} Q_1 + \left(\frac{k_2}{n_2}\right)^{-1} Q_2 TT \right) c_1.$$

After substituting the steady-state value of c_1 and then finding the value of B according to the parameterization of n_j, we have

$$B = \left(\begin{array}{c} \left(\left(\frac{k_1}{n_1} \right)^{-1} Q_1 + \left(\frac{k_2}{n_2} \right)^{-1} Q_2 TT \right)^{\gamma} \\ \times \left(s_1 TT^{s_2(1-\gamma)} \left(1-\alpha_1\right) \lambda_1 \left(\frac{k_1}{n_1} \right)^{\alpha_1} \right) N^{-\gamma} \end{array} \right).$$

C. Log-Linearization

- $\tilde{s}_{1,t} - \widetilde{c}_{1,t} + (1-\gamma)\, \widetilde{C}_t + \alpha_1 \widetilde{k}_{1,t} - \alpha_1 \widetilde{n}_{1,t} = 0$
- $\tilde{s}_{2,t} - \widetilde{c}_{2,t} + (1-\gamma)\, \widetilde{C}_t + \alpha_2 \widetilde{k}_{2,t} - \alpha_2 \widetilde{n}_{2,t} = 0$
- $y_1 \widetilde{y}_{1,t} = c_1 \widetilde{c}_{1,t} + i_1 \widetilde{i}_{1,t}$
- $y_2 \widetilde{y}_{2,t} = c_2 \widetilde{c}_{2,t} + i_2 \widetilde{i}_{2,t}$
- $k_1 \widetilde{k}_{1,t+1} = (1-\delta_1) k_1 \widetilde{k}_{1,t} + i_1 \widetilde{i}_{1,t}$
- $k_2 \widetilde{k}_{2,t+1} = (1-\delta_2) k_2 \widetilde{k}_{2,t} + i_2 \widetilde{i}_{2,t}$
- $\widetilde{y}_{1,t} = \alpha_1 \widetilde{k}_{1,t} + (1-\alpha_1)\, \widetilde{n}_{1,t}$
- $\widetilde{y}_{2,t} = \alpha_2 \widetilde{k}_{2,t} + (1-\alpha_2)\, \widetilde{n}_{2,t}$
- $r_1 \widetilde{r}_{1,t} = \alpha_1 (\alpha_1 - 1)\, k_1^{\alpha_1 - 1} n_1^{1-\alpha_1} \widetilde{k}_{1,t} + \alpha_1 (1-\alpha_1)\, k_1^{\alpha_1 - 1} n_1^{1-\alpha_1} \widetilde{n}_{1,t}$
- $r_2 \widetilde{r}_{2,t} = \alpha_2 (\alpha_2 - 1)\, k_2^{\alpha_2 - 1} n_2^{1-\alpha_2} \widetilde{k}_{2,t} + \alpha_2 (1-\alpha_2)\, k_2^{\alpha_2 - 1} n_2^{1-\alpha_2} \widetilde{n}_{2,t}$
- $\widetilde{C}_t = s_1 \widetilde{c}_{1,t} + s_2 \widetilde{c}_{2,t} + s_1 \ln(c_1) \widetilde{s}_{1,t} + s_2 \ln(c_2) \widetilde{s}_{2,t}$
- $\widetilde{N}_t = \frac{n_1}{N} \widetilde{n}_{1,t} + \frac{n_2}{N} \widetilde{n}_{2,t}$
- $s_2 \widetilde{s}_{2,t} = -s_1 \widetilde{s}_{1,t}$
- $\widetilde{s}_{1,t} - \widetilde{s}_{2,t} - \widetilde{c}_{1,t} + \widetilde{c}_{2,t} + \alpha_1 \widetilde{k}_{1,t} + (1-\alpha_1)\, \widetilde{n}_{1,t} = 0$
- $\widetilde{s}_{2,t} - \widetilde{s}_{1,t} - \widetilde{c}_{2,t} + \widetilde{c}_{1,t} + \alpha_2 \widetilde{k}_{2,t} + (1-\alpha_2)\, \widetilde{n}_{2,t} = 0$
- $-\widetilde{p}_{1,t} + \widetilde{s}_{1,t} + (1-\gamma)\, \widetilde{C}_t - \widetilde{c}_{1,t} - (1-v)\, \frac{N}{1-N} \widetilde{N}_t = 0$
- $-\widetilde{p}_{2,t} + \widetilde{s}_{2,t} + (1-\gamma)\, \widetilde{C}_t - \widetilde{c}_{2,t} - (1-v)\, \frac{N}{1-N} \widetilde{N}_t = 0$

Forward equations

- $1 = \widetilde{s}_{1,t+1} - \widetilde{s}_{1,t} + \widetilde{c}_{1,t} - \widetilde{c}_{1,t+1} + (1-\gamma)\,\widetilde{C}_{t+1} - (1-\gamma)\,\widetilde{C}_{t} + \widetilde{r}_{1,t+1}$
- $1 = \widetilde{s}_{2,t+1} - \widetilde{s}_{2,t} + \widetilde{c}_{2,t} - \widetilde{c}_{2,t+1} + (1-\gamma)\,\widetilde{C}_{t+1} - (1-\gamma)\,\widetilde{C}_{t} + \widetilde{r}_{2,t+1}$

D.

Proof. In this appendix we prove that $\frac{\partial^2 C}{\partial c_1 \partial s_1} > 0$. From (1) it follows that

$$\frac{\partial C}{\partial c_1} = s_1 c_1^{s_1-1} c_2^{1-s_1},\ 0 < s_1 < 1,$$

and the corresponding steady-state equation is

$$\frac{c_1}{c_2} = \frac{s_1}{1-s_1}.$$

The derivative of $\frac{\partial C}{\partial c_1}$ with respect to s_1 is given by

$$\frac{\partial^2 C}{\partial c_1 \partial s_1} = c_1^{s_1-1} c_2^{1-s_1}\left(1 + s_1 \ln\left(\frac{c_1}{c_2}\right)\right).$$

therefore,

$$1 + s_1 \ln\left(\frac{c_1}{c_2}\right) > 0,$$

from which we obtain

$$e > \left(\frac{c_2}{c_1}\right)^{s_1}.$$

Now substituting the steady-state values of c_1 and c_2 leads to

$$e^{1/s_1} s_1 + s_1 - 1 > 0.$$

Because $e^t > t$ for $t > 1$, the following always holds:

$$\frac{e^t}{t} + \frac{1}{t} - 1 > 0.$$

E.

Proof. Here we prove that $\frac{\partial^2 C}{\partial c_2 \partial s_1} < 0$. By equation (1),

$$\frac{\partial C}{\partial c_2} = (1 - s_1)\, c_1^{s_1} c_2^{-s_1},\ 0 < s_1 < 1;$$

and, in steady state,

$$\frac{c_1}{c_2} = \frac{s_1}{1 - s_1}.$$

The derivative of $\frac{\partial C}{\partial c_2}$ with respect to s_1 is

$$\frac{\partial^2 C}{\partial c_2 \partial s_1} = -c_1^{s_1} c_2^{-s_1} \left(1 - (1 - s_1) \ln \left(\frac{c_1}{c_2}\right)\right),$$

which proves that

$$1 - (1 - s_1) \ln \left(\frac{c_1}{c_2}\right) > 0.$$

Therefore,

$$e > \left(\frac{c_1}{c_2}\right)^{1 - s_1},$$

and substituting the steady-state values of c_1 and c_2 leads to

$$e^{1/(1-s_1)} (1 - s_1) - s_1 > 0.$$

Setting $t = \frac{1}{1 - s_1}$, we again conclude that:

$$\frac{e^t}{t} + \frac{1}{t} - 1 > 0.$$

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