



Multiple-steps scenario optimisation for pumping plants activation in water supply systems

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ABSTRACT

Economic aspects concerning the high costs related to energy requirements for managing complex water supply systems need a robust strategy, particularly considering the activation of pumping plants. Considering hydrological uncertainties, the definition of strategic rules can ensure energy savings and the well-timed activation of costly water transfers for shortage risk alleviation. The modelling approach has been developed aiming at defining strategic rules of pumps activation thresholds. It considers the need for seasonal variations of activation and the different costs of energy in diverse time slots, according to the usual cost rules adopted by the authorities. Starting with the traditional scenario analysis approach, a new algorithm has been developed considering a multiple-steps scenario optimisation implemented using GAMS interfaced with CPLEX solvers. The results should allow the water authority to establish a robust strategy for pumping activation to guarantee the fulfilment of water demands and to ensure an energy-saving policy.

Key words: complex water supply system, energetic time slots, pumping optimisation, scenario analysis

HIGHLIGHTS

- This research aims to develop a new algorithm considering a multiple-steps scenario optimisation.
- The modelling approach defines optimal activation pumping thresholds associated with stored water levels in reservoirs, in order to guarantee a fulfilment of water demand and ensure an energy-saving policy.
- This approach allows the water system authority to establish a robust strategy to manage complex water supply systems.

1. INTRODUCTION

Optimisation modelling aimed at energy saving and the definition of strategic rules for costs minimisation in water systems are an interesting and current research topic (Pasha & Lansey 2014; D'Ambrosio *et al.* 2015; Nault & Papa 2015; De Paola *et al.* 2017; Sechi *et al.* 2019; Hurford *et al.* 2020). Specifically, the definition of policies concerning the effectiveness of the activation of emergency and costly water transfers to alleviate droughts is relevant for the water system authorities (Kasprzyk *et al.* 2013; Asefa *et al.* 2014; Mateus & Tullós 2016; Napolitano & Sechi 2020).

For the definition of management policies, the optimal strategy for costly water transfers activation is a decision problem conditioned by hydrological uncertainties. As is well known, problems related to the optimisation of complex multi-reservoir systems affected by uncertainty have been addressed by implementing specific modelling approaches, and in particular, applications of stochastic dynamic programming have been documented in this field (Labadie 2004; Delipetrev *et al.* 2015; Menke *et al.* 2016). Alternatively, to depict the uncertainty, different scenarios can be considered in the optimisation procedure, following the Scenario Analysis methodological approaches (Rockafellar & Wets 1991; Pallottino *et al.* 2004; Guivarch *et al.* 2017).

In the case of pessimistic forecasts, variability in hydrological inputs can generate scenarios characterised by water excess and unnecessary water transfers, determining spilling from reservoirs and causing losses resulting as regrets costs (Kang & Lansey 2014). Moreover, shortages can occur in the supply system and resulting deficits can generate economic losses and social problems. To avoid these occurrences, robust emergency transfer activation policies in system management are

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needed. This should be done taking advantage of lower energy prices in specific time slots (Alvisi & Franchini 2017). These measures can achieve significant economic savings when defining procedures aimed to avoid the risk of water shortages. Therefore, transfer activation needs to consider the organisation of energy time slots in the periods examined (Quintiliani & Creaco 2019) since energy tariffs may be characterised by different prices in the time slots established by the electricity authority.

Robust definition of strategic rules for costs minimisation should ensure, simultaneously, both energy savings and water shortage risk alleviation. The model formulation needs to highlight simultaneously both these requirements: to guarantee an adequate fulfilment of water demand while respecting an energy-saving policy.

This study aims to deal with the uncertainties involved in addressing these problems: both the effectiveness of the activation of emergency measures requiring energy costs, and water shortage alleviation. The adopted solution methodology develops a multiple-steps scenario optimisation (MSSO) approach, in order to define optimal trigger rules for the emergency activation of pumping stations located in complex multi-reservoir water supply systems. The optimisation model has been written to ensure a cost–risk trade-off (Gaivoronski *et al.* 2012; Xie & Huang 2014; Yuan *et al.* 2016) under different hydrological scenarios. Emergency transfer activation policies obtained using MSSO should be expressed in terms of operative rules defining seasonal trigger activation values for water pumping stations. These trigger values will provide the water system authority with strategic system management information.

The proposed MSSO approach has been applied to a real case study, characterised by a multi-reservoir and multi-user water supply system in a drought-prone area, located on the island of Sardinia (Italy).

2. MSSO APPROACH

Treating a water management problem, the graph theory (Diestel 2005) is considered as an efficient support for the mathematical modelling. This approach allows schematising a complex water supply system problem through a flow network on a graph. In order to obtain a coherent description of the behaviour of the water supply system along the time horizon of investigation, the network model has been organised following the multi-period dynamic optimisation approach described in Pallottino *et al.* (2004) and Sechi & Zuddas (2008). The water supply system has been preliminarily schematised through a single-period flow network graph (static representation of the system) and subsequently by its replicates, obtaining the dynamic multi-period graph representation drafted in Figure 1.

The dynamic multi-period network is generated by replicating the basic graph for each period of the time horizon $t \in T$. Moreover, in the network, it is possible to add some dummy nodes and arcs to represent not only the physical components but also events that may occur in the system. The dummy nodes could represent a possible external source or sink acting to satisfy demands to receive spilling flows. In Figure 1, the node named ‘sea node’ gives this opportunity and agrees with the satisfaction of continuity constraints. The dummy arcs from the sea node to demand nodes are activated only to represent deficit flows in case of shortages. These additional flows assure a correct balance of flows at each demand node of the system. The flow on these arcs must be heavily penalised and they highlight water system’s deficits and the necessity to make recourse to external water resources. Moreover, inter-period connection arcs have been inserted in the multi-period network between nodes that represent storage capacities in the system. They allow carrying water stored at the end of each period to the next one; therefore, reservoir nodes are connected between them to transfer unused water at the end of the period t to the following $t + 1$ period.

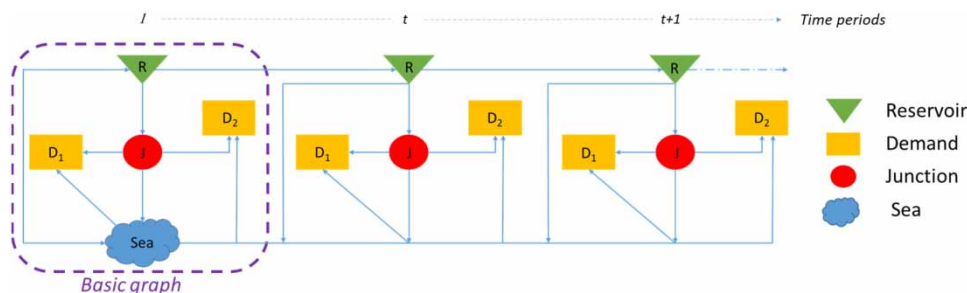


Figure 1 | Dynamic multi-period graph representation.

As is well known, problems related to the optimisation of complex water systems are affected by uncertainty related to hydrological and demand behaviours. To depict the uncertainty, different scenarios are considered in the scenario optimisation approach (Rockafellar & Wets 1991; Pallottino *et al.* 2004; Guivarch *et al.* 2017). Following the network representation approach, each possible scenario corresponds with a dynamic multi-period graph, following a particular sequence of water availability or supply requirement (i.e. hydrological or demands series). Each dynamic multi-period graph represents the possible realisation of these data for the system in an examined scenario.

Each scenario corresponds to a dynamic multi-period network, associated with a hydrological and demand sequence, as previously defined and shown in Figure 1.

In Figure 2, the scenario aggregation of the dynamic multi-period network is represented by a sequence of dots, where a dot represents the system in a time period. This representation is frequently named ‘Scenario tree’.

To model the scenario aggregation for the water system, some general rules (Rockafellar & Wets 1991; Pallottino *et al.* 2004) must be followed to correctly organise the predefined set of scenarios in the scenario tree and to manage the large amount of data. To perform scenario aggregation, branching times and stages are defined. A branching time identifies the time at which some scenarios, that are identical up to that time, begin to differ. A stage corresponds to the sequence of time periods between two branching times. Stage 1 corresponds to the initial hydrological characterisation of the system up to the first branching time. This represents the ‘root’ of the scenario tree. To perform scenario analysis, location of branching times and the extension of time periods between them must be defined. Each ‘stage’ considers the extension between branching times and should be defined in order to ensure a correct description of the hydrological variability.

When modelling water resource systems, the single time-period extension is frequently equal to 1 month, in order to properly represent hydrological inputs, water storage availability, and seasonal supply requirements. The total time horizon considered in the scenario optimisation could be equal to several decades in order to represent the system behaviour over a sufficiently large time extension. Time stages could correspond to single or multiple of hydrological years and are given by the sequence of time periods between two branching times. Herein, Stage 1 corresponds to the root of the scenario tree with an initial characterisation of the system up to the first branching time and, starting from Stage 2, each scenario assumes an independent hydrological behaviour.

Once the scenario tree has been completed and all necessary data have been defined, it is possible to perform the MSSO. The procedure has been drafted in the flowchart given in Figure 3.

The first step of MSSO develops through a *calibration procedure* of the main parameters of the optimisation model, testing the robustness of the evaluations and calibrating the cost/benefits attributions for the considered process (i.e. spilling costs, storage benefits, etc.). The scenario optimisation model can be expressed as the collection of one deterministic model for each scenario $g \in G$ plus a set of congruity constraints. Taking into account all scenarios $g \in G$, the scenario optimisation model

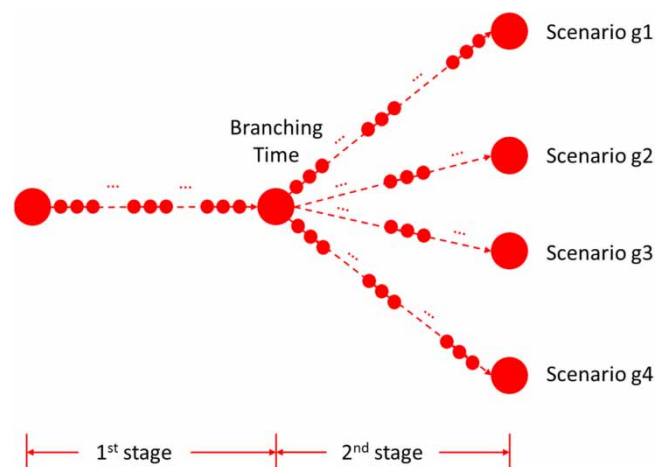


Figure 2 | Scenario-tree aggregation procedure.

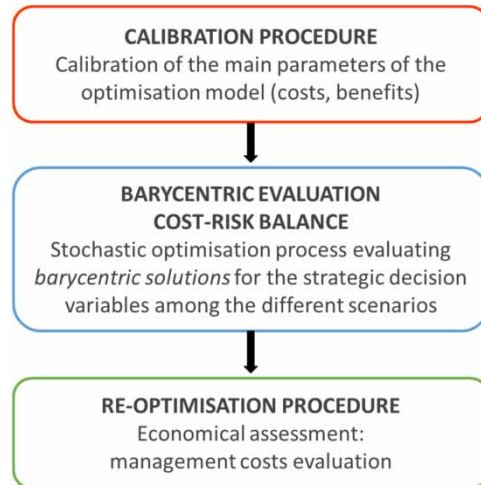


Figure 3 | Main modelling steps.

can be written using the following structure (Rockafellar & Wets 1991; Pallottino *et al.* 2004):

$$\text{Minimise } \sum_{g \in G} p^g c^g x^g \quad (1)$$

subject to:

$$A^g x^g = b^g, \quad \forall g \in G \quad (2)$$

$$l^g \leq x^g \leq u^g, \quad \forall g \in G \quad (3)$$

$$x^* \in \Phi, \quad (4)$$

All decision variables in the model (1)–(4) are scenario-dependent, then they are identified by index $g \in G$.

The objective function (OF) (1) is defined as the sum of the cost objectives of all scenarios weighted by p^g . Each scalar p^g describes a weight that could be assigned to each scenario in order to characterise its relative importance. The c^g is a cost vector and its components can represent costs, benefits, or penalties assigned to each variable x , which describes the water flow along a certain arc of the network.

The following parameters are needed to define constraints (2)–(3):

- A : is the coefficient matrix of the constraints;
- b : it is the RHS (right-hand side), this vector appears in the constraints system and its components can represent a supply or a demand associated with a single node. Additional flows (deficit flows) should be considered in order to assure the fulfilment at each node of these continuity constraints;
- u and l : these represent respectively the lower and upper bound vectors, whose components impose limits on variables x by physical, technological, or environmental requirements. Evaluation of these bounds in a detailed formulation of the model requests appropriate analysis of the system.

The general linear structure of the optimisation model is additionally characterised by constraints (4) that oblige the scenario-tree model to be built to comply with a set Φ of congruity constraints. These ‘non-anticipative constraints’ impose that, along the scenario tree, the subsets of decision variables, corresponding to the indistinguishable part of each scenario, must be equal to themselves. Dealing with reservoirs, this means that, in all scenarios, the amount of resource stored in the reservoir at the end of the time t to transfer in the period $t + 1$, must be the same. By introducing these constraints, the model will be redundant and the components of the model (variables and constraints) that are associated with overlapped scenarios can be reported just once. Non-anticipative constraints can be written in explicit forms using relations (5)–(8) and considering the scenario-tree scheme with the four branches drawn in Figure 2. The constraints take into account flows in conveyance

arcs (5); inter-period water storage transfers in reservoirs (6); water spilling from reservoirs (7); and water deficits in demand fulfilments (8):

$$x_{nt}^{g1} = x_{nt}^{g2} = x_{nt}^{g3} = x_{nt}^{g4}, \quad t = 1, \dots, T \quad (5)$$

$$xv_{nt}^{g1} = xv_{nt}^{g2} = xv_{nt}^{g3} = xv_{nt}^{g4}, \quad t = 1, \dots, T \quad (6)$$

$$xs_{nt}^{g1} = xs_{nt}^{g2} = xs_{nt}^{g3} = xs_{nt}^{g4}, \quad t = 1, \dots, T \quad (7)$$

$$x_{dnt}^{g1} = x_{dnt}^{g2} = x_{dnt}^{g3} = x_{dnt}^{g4}, \quad t = 1, \dots, T \quad (8)$$

where x_{nt}^g represents the water flowing along the n -th arc of the system during the t -th period in the g -th scenario; xv_{nt}^g represents the stored water in the n -th reservoir transferred by the inter-period connection in the t -th period and g -th scenario; xs_{nt}^g is the water spilled by the n -th reservoir to the sea node in the t -th period and g -th scenario; and x_{dnt}^g is the occurrence of water deficit in the demand node n -th during the t -th period and g -th scenario.

According to constraints (5)–(8), the flow variables will assume the same values for all scenarios in each single period until the branching time T ; up to that period (Stage 2) they will develop to each leaf independently.

The second step of MSSO develops a stochastic optimisation procedure in order simultaneously to achieve the minimisation of energy costs and to ensure an adequate fulfilment of the water demand. The OF of the model has been modified using a cost-risk balancing trade-off and it will examine the set of scenarios, looking for a barycentric solution for some strategic decision variables which are related to operative rules to be considered in the occurrence of the scenarios.

In this optimisation step, additional terms will be added to the OF (1) in order to guarantee a balanced trade-off between costs and risks elements. The scenario analysis follows the risk/cost approach detailed in Gaivoronski *et al.* (2012) and Napolitano *et al.* (2016). Costs refer to the amount of money that the water system authority should pay to manage the supply system (e.g. storage, pumping, delivery costs, and so on); while risks mainly refer to the water deficit costs that arise from occurrences of water scarcity.

In a symbolic form, we can write the new OF as follows:

$$\text{Minimise}_{x^g, \hat{S}^{gf}, S^{bf}} (1 - \lambda)\{\text{COST ELEMENT}\} + \lambda\{\text{RISK ELEMENT}\} \quad (9)$$

In the mathematical model, the cost element of the OF (9) (reported in Equation (1)) tries to find the system flow configuration that minimises the costs supported by the water system management, while the risk element should be considered as quadratic and minimise the Euclidean distance between the barycentric value (S^{bf}) and the single scenario trigger value (\hat{S}^{gf}) for each time slot $f \in F$.

The risk element of the OF (9) can be expressed as:

$$\sum_{g \in G} \sum_{i \in P} \sum_{f \in F} p^g \left[w^g \left\| \hat{S}_i^{gf} - S_i^{bf} \right\|^2 \right] \quad (10)$$

where F is the set of energy time slots $f \in F$; G is the set of considered scenarios $g \in G$; and P is the set of considered pumps $i \in P$.

During the optimisation process, this step aims to identify the barycentric optimal seasonal pump activation schedules (S^b), using the reservoirs' storage volumes as trigger values in the decisions. These values represent a barycentric solution among all the variables in the scenario decision problem. Scalars w^g are costs related to the risk occurrence in each scenario $g \in G$. They are the weights assigned to the quadratic function $\|\hat{S}_i^{gf} - S_i^{bf}\|$ for each scenario.

Barycentric solutions are balanced by the λ parameter. Low values of parameter λ imply that the cost terms prevail over the risk elements and that energy-saving management policy prevails; otherwise, for λ values closer to the unit, the risk elements prevail and a careful management policy should be adopted to ensure the fulfilment of water demand and avoid the occurrence of droughts.

The activation of pumping stations is here assumed to be dependent on the stored volume levels in reservoirs that in ordinary conditions supply the downstream demand nodes by gravity. Therefore, in order to model the pumps activation, a binary variable for each i -th pumping station should be assigned. This variable represents the on/off condition for each single pumping station as it can assume one (active) or zero (no-active) values.

Moreover, in this MSSO step, a set of new constraints must be implemented to define the flows that can be withdrawn along the arcs where the pumping plants are located. Each energy slot $f \in F$ can be modelled as a specific arc and will be characterised by a different maximum pumping capacity, proportional to the number of hours of operation over the daily 24 h.

$$x_{if}^g = (1 - h_{if}^g)f, \quad \forall f \in F, \forall i \in P, \forall g \in G \quad (11)$$

where $i \in P\{1, \dots, n_P\}$ summarises all the pumping stations (n_P) located in the water supply system; $f \in F\{1, \dots, n_F\}$ summarises all the considered energy slots (n_F); and $h_{if}^g \in \{0, 1\}$ are binary variables representing on/off conditions.

This new constraint (11) guarantees the activation of the i -th pump if the value of the water volumes stored in the reservoirs of reference is lower than the optimised seasonal activation thresholds. Moreover, in the case of plant operation, the flow conveyed along that arc will be equal to the maximum pumping capacity assigned to each energy time slot. This management rule ensures (if activated) a complete use of the pumping capacity of each single pumping plant.

In order to identify the pumps activation thresholds, moreover, a seasonal analysis has been performed: two seasons' activation thresholds have been identified, in order to correctly define management rules. The procedure refers specifically to Mediterranean countries where flows can be divided into a sequence of dry and wet months, and consequently, two activation values have been evaluated for each pumping station for the wet and dry seasons.

The third step of MSSO develops a re-optimisation procedure to evaluate the economic efficiency of the system management when applying the obtained values, as also described by [Napolitano et al. \(2016\)](#) and [Sechi et al. \(2019\)](#). Having evaluated the optimal barycentric values for pump activation rules using the scenario analysis, this step of the MSSO can be considered as the implementation of an economic post-processor, by which unplanned deficit costs and pumping costs are evaluated for each scenario. An economic evaluation of the reliability of the system can be performed based on the network's flows obtained in the previous step for each scenario configuration. Through this economic evaluation, it is possible to verify the expected outputs determined by the assumptions about the barycentric values adopted for the pump activation trigger values.

All the steps of the MSSO procedure have been implemented using the software [GAMS \(2008\)](#) interfaced with CPLEX ([IBM 2017](#)) solvers specifically designed for modelling mixed integer optimisation problems.

3. MSSO APPLICATION

3.1. Pranu Antoni water supply system description and first step of MSSO

An application of the proposed MSSO methodology has been developed considering a real water supply system located in a drought-prone area in Sardinia (Italy), characterised by a South Mediterranean climate. This water system is shown in [Figure 4](#).

[Figure 4](#) simplifies the real complexity of the system, nevertheless, it can adequately be used to get the flow configuration and to provide strategic information about management rules for activation of the pumping station in order to avoid water deficits.

The water sources are the reservoirs listed in [Table 1](#) with their storage capacity. To achieve an easier and faster solution, the optimisation process has been developed neglecting the physical characteristics represented by the evaporation components.

Pumped water transferred from the smaller Pranu Antoni reservoir (R2) could enhance the stored volume in the larger capacity Cantoniera reservoir (R1). The evaluation of the hydrological inputs to the reservoirs is given by the Sardinian Region Water Plan ([RAS 2006](#)) for a 71-year time horizon. These hydrological records have been used to define four hydrological scenarios and to construct the scenario tree with four branches which is drafted in [Figure 2](#). These four scenarios were established considering climatological trends in the last decades characterised by different criticality in different periods.

All scenarios extend for 20-year periods and are composed of 240 monthly periods with one branching time located at the 120th period. The scenario-tree scheme has been organised with a common root of 10 years (first decade of the hydrological

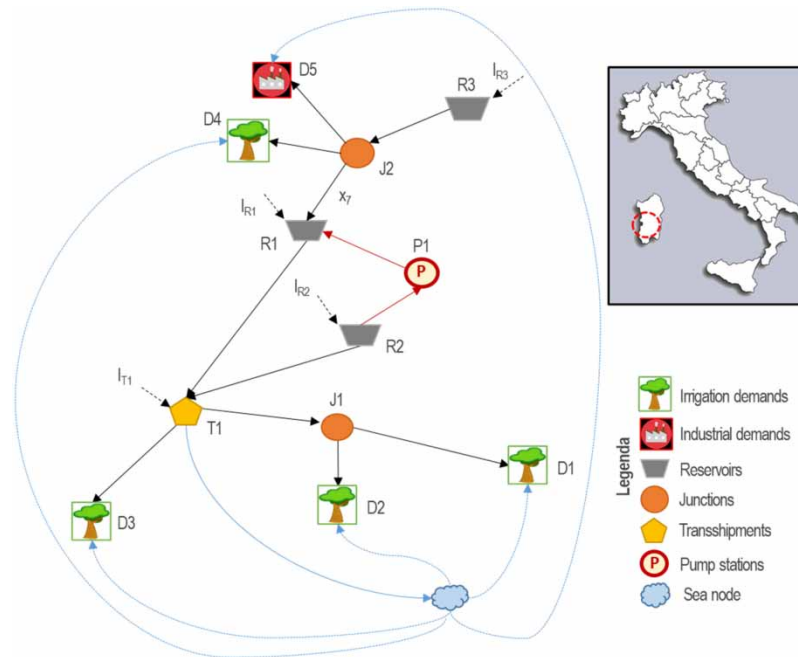


Figure 4 | Pranu Antoni water supply system.

Table 1 | Reservoir main features

Code	Reservoir	Capacity (10 ⁶ m ³)	Storage benefit (€/10 ⁶ m ³)
R1	Cantoniera	366	0.01
R2	Pranu Antoni	8.9	0.0001
R3	Taloro	70	0.001

database) and the following data diversified in 10 years scenarios (e.g. the scenario g1 is composed with the root of 10 years and the second series decade, g2 is composed with the same common root of 10 years and the third series decade, and so on). In this way, each scenario takes into account different hydrological criticism. Anyway, in the model, the same weight has been provided to each scenario (0,25), in order to satisfy the sum: $\sum_{g \in G} D^g = 1$.

The main statistics of the four hydrological scenarios are given in Table 2.

The water demand in the system has been grouped into five main centres, according to two different uses: irrigation and industrial.

The occurrences of deficits in water demand satisfaction have been categorised according to the usual management rules of the Water Authority, in two classes: planned and unplanned deficits. Planned deficits are shortages that can be forecast in advance, at least at the beginning of the irrigation season, which starts in Mediterranean regions at the beginning of April

Table 2 | Main statistics of hydrological scenarios

Reservoir scenario	R1				R2				R3			
	g1	g2	g3	g4	g1	g2	g3	g4	g1	g2	g3	g4
Mean (10 ⁶ m ³)	160.5	126.9	139.5	153.4	110.4	87.6	90.1	104.9	69.7	52.2	63.2	68.4
Variance	13,306	10,498	8,427	7,449	5,112	3,055	3,342	3,659	1,820	1,312	1,240	885
St. Dev.	115.4	102.5	91.8	86.3	71.5	55.3	57.8	60.5	42.7	36.2	35.2	29.8
Median	114.7	91	98.7	117.8	79.6	73.4	68.7	82.1	60.1	44.9	51.7	65.0

(dry periods). Meanwhile, unplanned deficits can arise in periods when the water availability given by hydrological scenarios leads to unpredictable scarcity affecting water uses. The cost values for planned and unplanned deficits are given in Table 3 together with the yearly water demand and rate cost for water transfers in normal conditions. These costs have been evaluated using regional studies considering the water annual rates for unit of volume applied to the different kind of use (RAS 2006). The adopted cost values for planned and unplanned deficits roughly change by one order of magnitude in each cost-class.

The diversion dams and junction nodes do not have a significant storage capacity; therefore, by using a monthly time step, the incoming flow to these nodes must be diverted downstream to demand centres or to larger capacity reservoirs (Tables 4 and 5).

Pumping station P1 can provide additional water inflow to the reservoir R1, determining an additional economic cost mainly caused by energy costs. The pumping flow capacity of transfer from reservoir R2 to reservoir R1 is given in Table 6.

In defining the pumping station P1 activation rules, we assume that the activation will be referred to critical threshold levels in the stored volume in the reservoir R1 that supplies the demand nodes.

In the first step of MSSO, all the main parameters needed to characterise the water system have been evaluated through a *calibration phase*, then solving the multi-period and a multi-scenario optimisation model (1)–(4). The results of this first optimisation step are used in order to calibrate the storage benefits, deficit and spilling costs, considering the hydrological uncertainty and to highlight criticality in system management.

3.2. Second step of MSSO: defining barycentric activation thresholds

In order to define the barycentric activation values between scenarios, the pumping energy needed must be assumed according to different cost values in different time slots. Moreover, we need to obtain an optimised balance between the cost term (related to energy) and the risk term (related to water scarcity).

Table 3 | Features of demand centres

Code	Demand centre	Water demand (10 ⁶ m ³ /year)	Rate cost (€/m ³)	Planned deficit cost (€/m ³)	Unplanned deficit cost (€/m ³)
D1	Irrigation Sud Sardegna	130	0.006	0.06	0.6
D2	Irrigation Oristano 2	100	0.006	0.06	0.6
D3	Irrigation Oristano 1	79	0.006	0.06	0.6
D4	Irrigation Media Valle	9.4	0.006	0.06	0.6
D5	Industrial Ottana	5.2	0.256	2.56	25.6

Table 4 | Diversion dams codes

Code	Diversion dams	Spilling cost (€/10 ⁶ m ³)
T1	Santa Vittoria	1

Table 5 | Junctions codes

Code	Junctions
J1	Sant'Anna
J2	Taloro

Table 6 | Pumping station flow capacity

Code	Pump station	Flow capacity (m ³ /s)
P1	Pranu Antoni	5

In looking for barycentric values in activation thresholds, a compromise λ value in relation (9) has been assigned equal to 0.5, thus considering an equal balance between the cost and risk terms.

Regarding energy costs, as shown in Figure 5, it is possible to identify three different time-cost slots: F1, F2, and F3, used by the Italian electricity authority for each hour in a day of consumption. Each time-cost slot in a transfer between R2 and R1 can be modelled considering a specific conveyance arc in the system graph, as depicted in Figure 6.

These arcs are subject to upper bounds given by the maximum volume of water that can be transferred monthly in each time-cost slot: upper bounds refer to the conveyance capacity evaluated considering the total number of hours in each time-cost slot in the month.

Table 7 summarises the capacity in the time slots for the pumping station. Each time slot is characterised by a different energy cost (c_{F1} , c_{F2} , c_{F3}), upper bound capacity, and maximum monthly transferred volume. The total transfer capacity can be obtained by adding the capacities of the three time slots.

The main results in the second step of MSSO are the *barycentric activation thresholds* evaluated among the considered hydrological scenarios. Two seasonal values have been obtained related to the storage value at the beginning of each period in the dry and wet seasons. The rules obtained for activation impose the activation of the pumping station if the R1 storage at the beginning of each period in the dry and wet seasons is below these threshold values.

Constraints have also to be considered in order to ensure a correct relationship between the wet and dry values in the threshold levels S^{wet} and S^{dry} for reservoir R1. The following relation was assumed:

$$S^{wet} > S^{dry} \tag{12}$$

According to this assumption, the pumping plant will be switched on mostly during the wet period in order to transfer water to the larger capacity reservoir for storage, aiming to ensure the fulfilment of the water demand of users during the next dry semester (proactive management). The pump activation aims to protect the downstream users mainly during drought years, to avoid the risk of drought.

The obtained activation thresholds in the wet and dry seasons are given in Table 8.

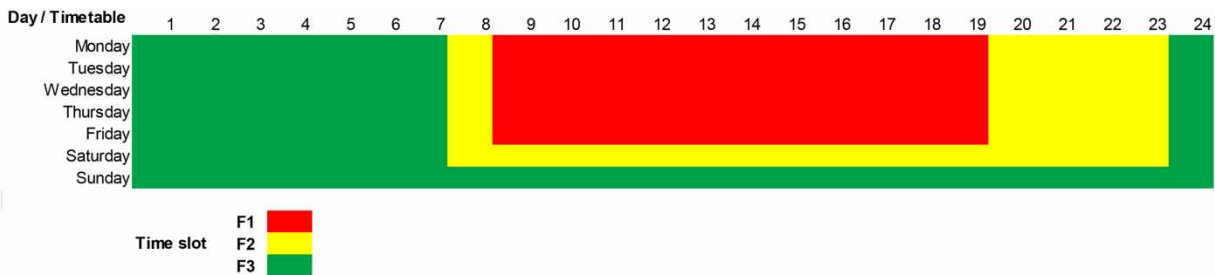


Figure 5 | Organisation of weekly time slots: F1, F2, and F3 used by the Italian electricity authority.

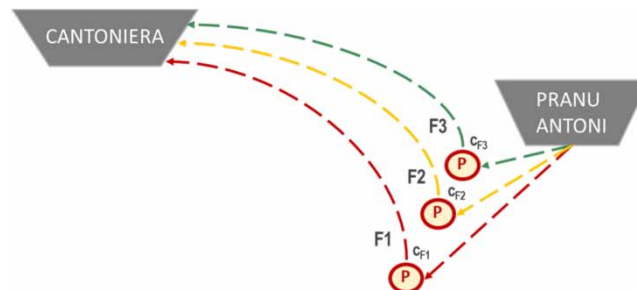


Figure 6 | Pumping transfers from Pranu Antoni to Cantoniera Dam.

Table 7 | Time slots features and pumping costs

Code	Time slot	% on 24 h	Pumping cost (€/kWh)	Capacity (m ³ /s)	Monthly volume (10 ⁶ m ³ /month)
F1	08:00–19:00	46	0.03878	2.29	6.02
F2	07:00–08:00 19:00–23:00	21	0.04378	1.04	2.74
F3	23:00–07:00	33	0.04838	1.67	4.38

Table 8 | Optimised activation thresholds – proactive management

Code	S^{wet} (10 ⁶ m ³)	S^{dry} (10 ⁶ m ³)
F1	93.595	0.630
F2	93.292	0.378
F3	8.312	0.159

3.3. Third step of MSSO: re-optimisation procedure

In the third step of MSSO, the obtained thresholds values have been assigned as fixed parameters during the re-optimisation procedure (Napolitano *et al.* 2016). This re-optimisation step aims to evaluate for each considered scenario the economic efficiency of the adopted rules.

Table 9 gives the average annual costs for the most critical scenario g2. The costs are divided between each energy-cost time slot and considering the rules for pumping activation previously defined.

The management rules for pump activation determine significant energy costs particularly during the most critical 3-year period in the considered scenario. These years refer to the most critical drought along the considered time horizon. Correctly, the pumping station has been switched on during the time slots characterised by lower energy-costs F1–F2, and a zero value was obtained for the highest energy-cost time slot F3. The pumped volumes behaviour during these drought periods is shown in Figure 7.

The adopted rules can guarantee proactive management of the water supply system: the transfer of water from R2 to R1 is activated during the wet semesters before the critical dry seasons occur. The optimised activation rules obtained will simultaneously minimise the energy costs and reduce the possible damage caused by water deficit.

Table 10 shows the amount of water deficits that affect the demand centres. These deficits concern only the use for irrigation, protecting the industrial demand requirements, which represent the major priority user of this water supply system. The optimised management ensures acceptable behaviour in terms of expected water shortages, while fulfilling the demand from higher priority users even during drought periods.

3.4. Sensitivity analysis

Testing the effectiveness of the MSSO approach, a sensitivity analysis has been done on parameter lambda. Changing the lambda value between 0 and 1, optimal thresholds values, flows configuration and consequently the OF results can vary significantly. Namely, as shown in Figure 8, considering lower values of λ , the cost term importance in the OF prevails over the risk term and cost values remains close to zero that suggest an energy-saving oriented management policy. Otherwise, for

Table 9 | Pumping costs (scenario g2) – third step

Time slot	Mean costs (€/year)
F1	58,364
F2	23,991
F3	0

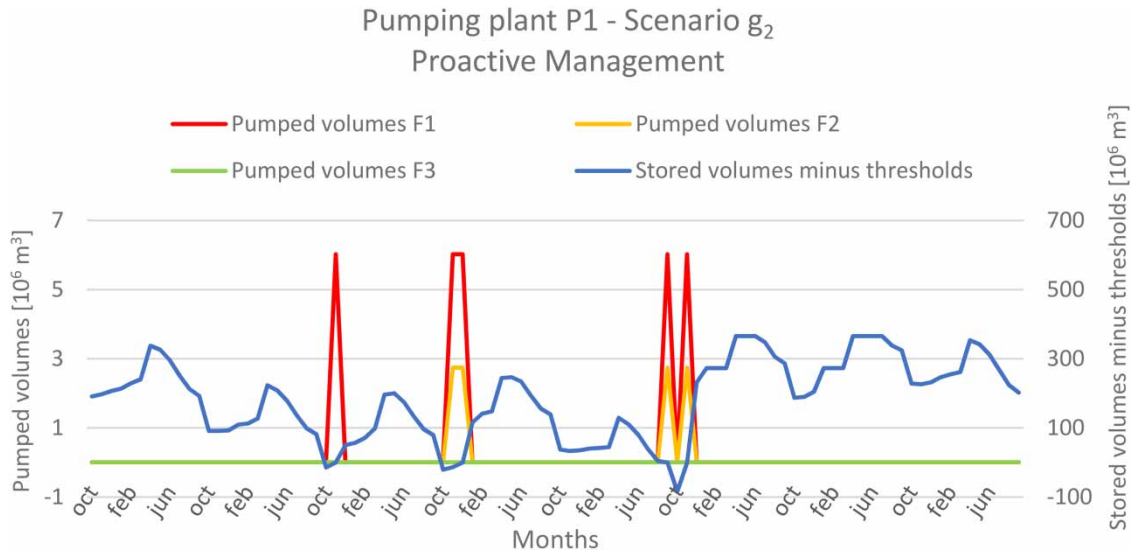


Figure 7 | Pumps activation management.

Table 10 | Deficit costs – optimised management

Code	Water demand	Mean costs (M€/year)
D1	Irrigation Campidano	0.121
D2	Irrigation Oristano 2	0.195
D3	Irrigation Oristano 1	0.180
D4	Irrigation Media Valle	0
D5	Industrial Ottana	0
	Total	0.497

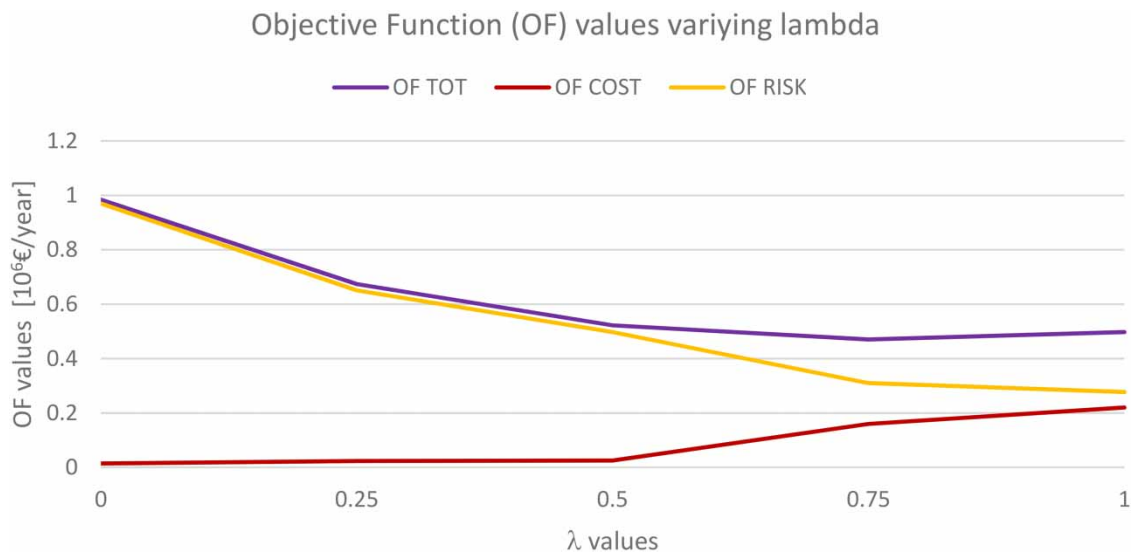


Figure 8 | Sensitivity analysis on lambda value. Please refer to the online version of this paper to see this figure in colour: <https://doi.org/10.2166/hydro.2021.082>.

higher λ values, the risk term importance increases obtaining an early pumps activation. The OF behaviour is reported in Figure 8, where the purple line represents the total value of the OF, evaluated as the sum of cost and risk terms. The lambda value which minimises the total OF is located between 0.5 and 0.75.

4. CONCLUSION

The proposed MSSO modelling approach has been described and applied to a real water resource system, evaluating the activation triggers of water pumping plants related to storage levels in reservoirs.

The MSSO application has been developed considering different scenarios in hydrological series and the expected demand requirements. Barycentric solutions for all the scenarios were retrieved by a cost–risk trade-off evaluation to allow the management of emergency and costly water transfers in the event of drought-risk.

For each energy-cost time slot, two seasonal activation thresholds have been identified. Emergency and proactive management approaches have been considered for definition of the pumping plant rules.

An economic evaluation of costs and penalties has been retrieved through an economic post-processor in the third step of the procedure, considering the water shortage penalties associated with each use and the energy pumping costs pertaining to each time slot.

For the considered water scheme, the retrieved trigger values for pumps activation guarantee an acceptable reduction in terms of water deficit along the considered time horizon. The effectiveness of the obtained results is currently in a testing phase by the Regional Water Authority, and they have been considered as strategic information in order to adopt the most promising management policy.

The model has been implemented using the software GAMS interfaced with CPLEX solvers.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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