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The effect of uncertainty on production-inventory policies with environmental considerations

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Abstract-We consider a manufacturing firm whose production is characterized by polluting emissions, an incorporated pollution abatement process and continuous-time inventory control. Recognizing the stochastic nature of both pollution and inventory dynamics, we study the impact of consumer demand and pollution uncertainty on production-inventory policies under environmental costs/taxes imposed on the manufacturer. We find that the manufacturer, facing environmental uncertainty, reduces both inventory and pollution levels in the long run. The same effect is observed in terms of inventories under proportional and progressively growing environmental taxes but not necessarily in terms of pollution. In particular, emission taxes most impact expected steady state inventories while ambient pollution taxes combat long-run pollution levels.

Index Terms — **Inventory Control, Stochastic Demands, Production Emission, Pollution.**

I. INTRODUCTION

Industrial waste, one of the largest causes of global pollution, endangers both people and the environment. Waste produced by industrial activity contaminates many sources of drinking water, releases unwanted toxins into the air and reduces the quality of soil all over the world. Government policies and regulations to reduce pollution are commonly based on taxation and pollution permits or a combination of both. Pollution permits, (e.g., carbon trading schemes) are market-based and aimed at creating a financial incentive to pollute less by enabling the manufacturer to sell excess permits to other firms. Pollution reducing taxes involve both emission and ambient environmental quality charges. For example, Title IV of the US Clean Air Act Amendments and National Ambient Air Quality Standards, regulates SO_2 emissions and ambient SO_2 concentrations respectively. Pharmaceutical manufacturing plants, for example, generate a variety of wastes during manufacturing and the pharmaceutical industry has been subject to emission taxes as well as ambient charges for sewage effluents and receiving river pollutant concentrations. Direct emissions of active pharmaceutical ingredients (APIs) from drug manufacturing have been identified as a source of environmental discharges that, in some cases, greatly exceed toxic threshold concentrations. For example, the concentrations in the effluent from a treatment plant receiving wastewater from about 90 manufacturing units in Patancheru (India) were, for some pharmaceutical, greater than those found in the blood of patients ingesting the product itself as a medicine ([12]). Similar pharmaceutical pollution sources have been reported in the USA and Europe ([1]). Furthermore, emission concentration spikes frequently observed in pharmaceutical drying and coating processes commonly result in air pollution. According to the OECD's book, "Taxation, Innovation and the Environment", environmental taxes (e.g., in California, hazardous waste is subject to numerous fees including emission taxes, landfill taxes, fees based on the threat to water quality and bay protection as well as toxic cleanup fees), provide an ongoing incentive to abate all emission levels. To further reduce its tax liability, the firm can switch to a less-polluting fuel, add a scrubber, change disposal methods, clean its wastes or otherwise adjust its production process. Recent developments in the pharmaceutical industry have led to minimizing wastage by strict inventory control and prevalent construction of treatment plants with the aim of pre-treating the otherwise toxic effluent before sending it to the local municipal sewage treatment plant.

The effect of environmental policies on production-inventory decisions (see, for example, [18], [3], [4], [13]) has been addressed in the literature. The common approach is to employ a modification of the Arrow-Karlin deterministic model ([10]). The model's convex (typically quadratic) production costs are known to induce production smoothing and thereby increased inventories over a short planning horizon. In particular, Wirl ([18]) analyzes the inventory dynamics of production with emissions when accounting for pollution taxes. He shows that a rise in marginal production costs makes it beneficial to smooth production and to carry larger inventories and that this property does not necessarily hold if future costs are discounted. Dobos ([3]), who adds tradable emission permits to Wirl's model while dropping the discounting factor, also finds that it is optimal to smooth production along with selling a portion of the firm's emission permit (assuming it is divisible). Analogous results are obtained in [4] when it is assumed that the change in production rate is controlled rather than the production rate itself. Li ([13]) adds to the Dobos model ([3]) the option of investing in pollution abatement activities and the corresponding capital dynamics. He demonstrates with numerical examples production smoothing along with inventory increases due to emission trading. Similar dynamics to those that appear in [13]) are commonly used to present the evolution of the pollution stock under an environmental absorption capacity and manufacturer's pollution abatement activities but without inventory considerations (see, for example, [9] and [21]).

An important point in the researches cited above is that they focus on the short-term effect of environmental policies on production management in a deterministic setting by choosing a limited planning horizon and predefined seasonal consumer demand. However, environmental policies have long-term cumulative consequences. If one attempts to study these consequences by increasing the planning horizon, the optimal inventory naturally drops to zero due to the deterministic setting under quite a wide range of conditions imposed on the rate of demand fluctuations. That is, environmental concerns would have no impact on the production inventory policies. In contrast to the literature above, we recognize in this paper the stochastic nature of both dynamic pollution and inventory stocks in order to study the long-term impact of demand and pollution uncertainties on production and inventory policies.

Similar to the Arrow-Karlin model, our model is based on linear-quadratic assumptions in terms of the costs and dynamics involved. Specifically, since environmental absorption capacity is generally stochastic ([20], [19], [7]), measuring pollution involves uncertainty. Various approaches have been suggested for effectively assimilating data for monitoring and controlling environmental quality by assuming linear pollution dynamics. For example, Zhang et al. ([22]) assume that transport phenomena of pollutants in the environment are described by linear diffusion equations with a Gaussian noise process.

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Romanowicz and Young ([15]) use linear Gaussian model structures for measuring pollution associated with discharges from the repository plant of British Nuclear Fuel Ltd. In the present paper, we also adopt linear pollution stock dynamics with a Gaussian noise (Wiener diffusion process) in the proportional form ([21]). Likewise, we model stochastic consumer demand (see, for example, [11], [8], [6] and [17] for similar stochastic production models). The goal of this study is to determine: (i) how proportional and increasingly growing (convex) emission and ambient pollution taxes affect the expected steady state inventory and pollution stocks; and (ii) how the level of uncertainty alters the relationships in (i).

The range of charges paid by polluting firms in real-life is wide. For example, the fees applied to emissions of sulfur oxides (SOx), NOx, VOCs due to industrial energy consumption vary in the US from $5 per ton (in Maine for amount emitted up to 1000 tons) through $150 per ton (in New Mexico). This tax, in terms of production output with 1% (or less) and up to 10% polluting rates implies fees of $0.05- $15 per ton. Annual pollution ambient charges, such as the fees of the California Bay Protection and Toxic Cleanup Act vary from $300 for low pollutant concentrations, 0.05 mg/l, in terms of water quality, through $11,000 (up to 0.7 mg/l) to fund the toxic cleanup program. This, in terms of the concentrations defines the hourly fees ranging from $0.1 to $1.8 per each mg/l. In this paper we derive an optimal feedback solution for a polluting firm and conduct a numerical study by varying all environmental costs within the described ranges in order to understand the effect of those costs on the production-inventory policies.

We find that the manufacturer, facing environmental uncertainty, reduces both his inventory and pollution levels in the long-run. The expected long-run pollution stock, however, decreases under ambient pollution taxation but not necessarily under emission taxes. Both emission and ambient taxes decrease expected steady state inventory stocks. This is in contrast to the literature cited above that finds inventory growth due to production smoothing when consumer demand and environmental absorption dynamics are deterministic and the time horizon is relatively short.

II. Statement of the Problem and the Optimality Conditions

Consider a manufacturer whose production process, which causes polluting emissions, is characterized by demand uncertainty. Changes in the cumulative demand rate, $R(t)$, are determined by the stochastic process, $dR(t) = Ddt + \sigma dW(t)$, where D and σ are the mean and volatility of the demand, respectively, and $W(t)$ is a standard Weiner process. The classical inventory stock $X(t)$ dynamics is due to the difference between the production rate $u(t)\geq 0$ and the demand rate,

$$dX(t) = (u(t) - D)dt - \sigma dW(t). \tag{1}$$

This implies that if there is a stock, $X(t)>0$, then all demanded products have been shipped by time t and an inventory holding cost is incurred. On the other hand, if there is a shortage, $X(t)<0$, then the shortage is backlogged and supplied later which induces a backlog cost.

We assume pollution abatement activities (e.g., water treatment, solid waste incineration and desulfurization of flue gases) are incorporated into the production and the amount of labor employed in the abatement process per time unit is denoted by $z(t)$ ([2]). The resultant pollution stock $P(t)$ is proportional to the emission $eu(t)$ associated with production $u(t)$ and inversely proportional to the abatement activity $z(t)$ along with the natural environmental capacity of pollutant assimilation $P(t)(\gamma dt + \varepsilon dS(t))$, where $\gamma P(t)$ and $\varepsilon P(t)$ represent the mean and the volatility of pollutant assimilation, respectively; and $S(t)$ is a standard Weiner process not correlated with $W(t)$. Specifically, the pollution stock is described by the stochastic process (see, for example, [20]):

$$dP(t) = (eu(t) - z(t) - \gamma P(t))dt - \varepsilon P(t)dS(t), \tag{2}$$

The deterministic version of this equation obtained by setting $\varepsilon = 0$ is widely used to present either pollution dynamics or capital dynamics in pollution abatement activities ([5],[9], [13]).

We assume that all operational costs incurred by the firm are convex. In particular, in addition to the pollution abatement cost $G(z(t))$, the firm causing pollution bears emission $C_1(u(t))$, production $C_2(u(t))$ and ambient (cumulative) pollution $A(P(t))$ costs/taxes for preventing immediate and cumulative damage to human health and/or the environment ([14]). The objective is to minimize the total discounted expected cost:

$$\min_{u(.)} J = \min_{u(.)} E\left\{ \begin{array}{c} \int_0^\infty [H(X(t)) + C(u(t)) + A(P(t)) + \\ G(z(t))]e^{-\delta t}dt \end{array} \right\}, \tag{3}$$

where $H(X(t))$ is the inventory cost, $C(u(t)) = C_1(u(t)) + C_2(u(t))$ and δ is the discount rate.

Denoting partial derivatives of a function, y, with respect to x, $\frac{\partial y}{\partial x}$ by y_x, $\left.\frac{\partial y}{\partial x}\right|_{x=a}$ by $y_x(a)$ and $\frac{\partial^2 y}{\partial x^2}$ by y_{xx} , and omitting the independent variable t, where the dependence on time is obvious, the principle of optimality is

$$\delta V = \min_{u\geq 0}\left\{H(X) + C(u) + A(P) + V_X(u-D) + G(z) + V_P(eu - z - \gamma P) + \tfrac{1}{2}\sigma^2 V_{XX} + \tfrac{1}{2}\varepsilon^2 P^2 V_{PP}\right\} \tag{4}$$

where $V = V(X,P)$ is the cost-to-go function and optimal production/abatement controls are straightforwardly found with the aid of the first order optimality condition as described in the next proposition.

Proposition 1. Let $C_{uu} > 0$ and $G_{zz} > 0$.
If $C_u(0) \leq -V_X - eV_P$, then an optimal production rate is given by
$$C_u(u) = -V_X - eV_P;$$
Otherwise, u=0.
If $G_z(0) \leq V_P$, then an optimal abatement activity rate is given by
$$G_z(z) = V_P,$$
Otherwise, z=0.

Proof: Since u-dependent terms of (4) are convex, to derive an optimal production control, we differentiate the right-hand side of (4) with respect to u, which results in $C_u(u) = -V_X - eV_P$. Given $C_{uu} > 0$, the left-hand side of this equation is monotone in u and therefore its interior solution is feasible, $u\geq 0$, if $C_u(0) \leq -V_X - eV_P$. Otherwise, u=0. Similarly, an optimal control z is determined. ■

That is, an optimal production rate is found by equating the marginal production and emission cost to the marginal profit from inventories and emission while the optimal pollution abatement rate is due to equating the marginal abatement cost to the marginal profit from polluting production.

Following the convex cost of the Arrow-Karlin approach, also adopted in the aforementioned literature, we assume the second-order polynomial costs,

$$H(X) = h_1X + h_2X^2,\ C(u) = c_1u + c_2u^2,\ A(P) = a_1P + a_2P^2,\ G(z) = g_1z + g_2z^2. \tag{5}$$

Then (4) leads to the Hamilton-Jacobi-Bellman (HJB) equation,

$$\delta V = h_1X + h_2X^2 + c_1u + c_2u^2 + a_1P + a_2P^2 + g_1z + g_2z^2 + V_X(u-D) + V_P(eu - z - \gamma P) + \tfrac{1}{2}\sigma^2 V_{XX} + \tfrac{1}{2}\varepsilon^2 P^2 V_{PP}, \tag{6}$$

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where, with respect to Proposition 1, an optimal production rate is

$$u=-\frac{1}{2c_2}(c_1+V_X+eV_P), \quad (7)$$

if $\frac{1}{2c_2}(c_1+V_X+eV_P)\geq 0$, otherwise u=0; and an optimal pollution abatement rate is

$$z=-\frac{1}{2g_2}(g_1-V_p), \quad (8)$$

if $V_P-g_1\geq 0$, otherwise z=0.

III. OPTIMAL LONG-RUN PRODUCTION-INVENTORY POLICY

We next consider linear strategies with the aid of a polynomial form of the cost-to-go function

$$V(X,P)=b_1+b_2X+b_3X^2+b_4XP+b_5P+b_6P^2 \quad (9)$$

and subsequently verify that (9) satisfies the HJB (6). Substituting (7) and (8) into (6) and accounting for (9), we obtain six non-linear equations (A1) in six unknowns, b_1, b_2, b_3, b_4, b_5 and b_6 presented in the appendix. Accordingly, optimal production (7) and abatement (8) rates are a linear combination of the effect of the current inventory X and pollution P levels,

$$u=-\frac{1}{2c_2}(c_1+b_2+eb_5+(b_4+2eb_6)P+(eb_4+2b_3)X), \quad (10)$$

$$z=-\frac{1}{2g_2}(g_1-b_5-b_4X-2b_6P). \quad (11)$$

Denoting

$$\alpha_1=c_1+b_2+eb_5,\ \alpha_2=b_4+2eb_6,\ \alpha_3=eb_4+2b_3, \quad (12)$$

we conclude as follows.

Proposition 2. Let the cost functions be defined by (5), real-valued constants $b_1>0, b_2, b_3>0, b_4, b_5$ and $b_6>0$ satisfy the system of equations (A1), and $\alpha_1, \alpha_2, \alpha_3$ are defined by (12).

If $c_1+b_2+eb_5+(b_4+2eb_6)P+(eb_4+2b_3)X\leq 0$, then $u=-\frac{1}{2c_2}(\alpha_1+\alpha_2P+\alpha_3X)$,

If $(g_1-b_5-b_4X-2b_6P)\leq 0$, then $z=-\frac{1}{2g_2}(g_1-b_5-b_4X-2b_6P)$.

Proof: The proof follows from Proposition 1and substitution (9) into (4) to ensure the verification theorem holds as detailed in [11]. ■

To illustrate Proposition 2, we simulate optimal production and pollution abatement by generating Weiner processes $W(t)$ and $S(t)$ in equations (1) and (2) with the *ItoProcess* function of Maple 18. Figure 1 presents simulation results with ten sample paths (ten replications, each colored either blue or red) obtained over 50 time units with 100 time steps, when $X(0)$=10.0, $P(0)$=15.0, g_1=0, g_2=1.5, $c_1=0$, $c_2=1$, $h_1=0$, $h_2=.5$, $a_1=0$, $a_2=1.2$, $D=30$, $\sigma=2.5$, $\varepsilon=0.1$, $e=0.1$, $\delta=0.01$, and $\gamma=0.1$. For this data, $b_1=91753.28611$, b_2=-59.98612266, $b_3=0.7060194454$, $b_4=-0.1043041593$, $b_5=7.850604270$, $b_6=1.197459315$ satisfy the system of equations (A1) and result in α_1= -59.20106223, α_2= 0.1351877037, and α_3= 1.401608475.

From Figure 1 we observe that all conditions of Proposition 2 hold for a realistic production environment. In particular, the firm always produces, u>0 (with the mean value of 29.99343408 and standard error=0.01448935133 at t=50 computed based on 10^4 replications); always abates, z>0 (with the mean value of 26.99625739 and standard error=2.236590602 at t=50 based on 10^4 replications); and the pollution is kept within a positive range of values.

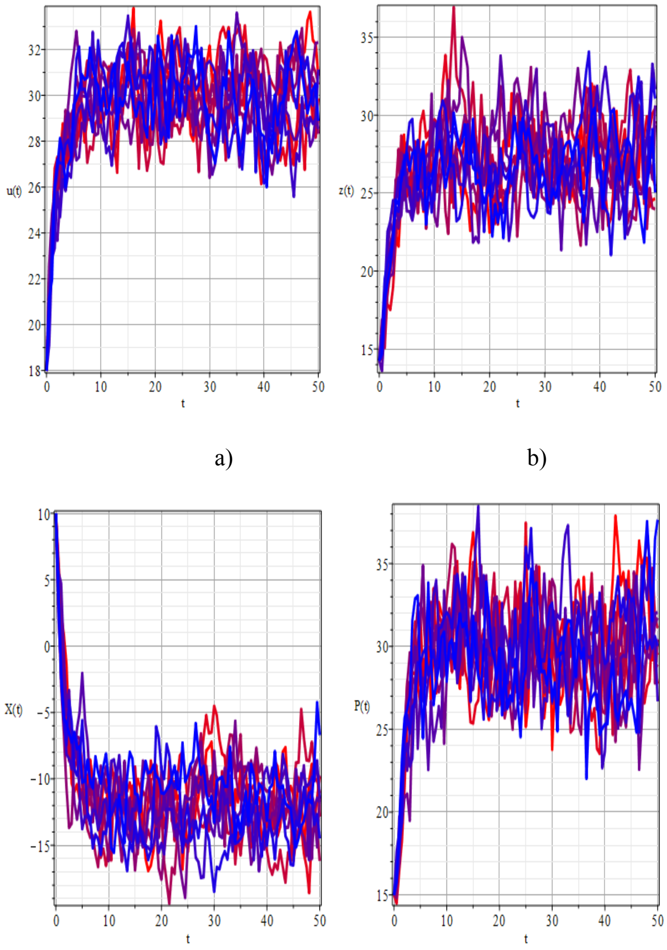

Figure 1: Ten sample paths of a) an optimal production rate, b) optimal pollution abatement rate, and the corresponding c) inventory and d) pollution stocks.

Substituting (10)-(11) into the state equations (1) and (2) and employing notations (10), we obtain the evolution of stochastic inventory and pollution stocks:

$$dX=-\left(D+\frac{1}{2c_2}(\alpha_1+\alpha_2P+\alpha_3X)\right)dt-\sigma dW, \quad (13)$$

$$dP=-\left(\frac{e}{2c_2}(\alpha_1+\alpha_2P+\alpha_3X)-\frac{1}{2g_2}(g_1-b_5-b_4X-2b_6P)+\gamma P\right)dt-\varepsilon PdS. \quad (14)$$

Next, taking expectations in (13) and (14), we have in our original notations:

$$\dot{x}=-\left(D+\frac{1}{2c_2}(c_1+b_2+eb_5+(b_4+2eb_6)p+(eb_4+2b_3)x)\right) \quad (15)$$

$$\dot{p}=-\frac{1}{2}\left(\left(\frac{e^2b_5+(b_2+c_1)e}{c_2}+\frac{b_5-g_1}{g_2}\right)+\left(\frac{2e^2b_4+eb_3}{c_2}+\frac{b_6}{g_2}\right)x+\left(\frac{2e^2b_6+eb_4}{c_2}+\frac{b_6}{g_2}+\gamma\right)p\right), \quad (16)$$

where $x=E[X]$ and $p=E[P]$.

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Proposition 3. Consider an optimal production control determined by Proposition 2. Then the expected steady state inventory and pollution stocks are given respectively by:

$$p^{ss} = \frac{2(g_2(b_4e^2+2eb_3)+c_2b_4)D+g_1(eb_4+2b_3)+b_4(b_2+c_1)-2b_3b_5}{2g_2\gamma(eb_4+2b_3)+4b_3b_6-b_4^2},$$

$$x^{ss} = -\frac{2(g_2(2b_6e^2+eb_4+2c_2\gamma)+2c_2b_6)D+2g_2\gamma(eb_5+b_2+c_1)+c_1(2eb_6+b_4)+2b_6(b_2+c_1)-b_4b_5}{2g_2\gamma(eb_4+2b_3)+4b_3b_6-b_4^2}$$

Proof: The proof immediately follows by setting $\dot{x}(t)=0$ and $\dot{p}(t)=0$ in (15) and (16), respectively, and solving the resultant algebraic system of two equations in two unknowns, x and p. ■

To examine the stability of the expected steady states that have been found, we construct the Jacobian matrix for the linear system of equations (15) and (16):

$$J = \begin{pmatrix} -\frac{1}{2c_2}(eb_4+2b_3) & -\frac{1}{2c_2}(b_4+2eb_6) \\ -\frac{1}{2}\left(\frac{2e^2b_4+eb_3}{c_2}+\frac{b_6}{g_2}\right) & -\frac{1}{2}\left(\frac{2e^2b_6+eb_4}{c_2}+\frac{b_6}{g_2}+\gamma\right) \end{pmatrix}. \quad (17)$$

The determinant and trace for the Jacobian are respectively,

$$\Delta = \frac{((eb_4+2b_3)\gamma g_2+b_6(e(b_4-2b_6)+2b_3-b_4))c_2-2eg_2(eb_6+b_4/2)(eb_4-2b_3)}{4{c_2}^2g_2}; \quad (18)$$

$$T = -\frac{((eb_6+b_4)2e+\gamma c_2+2b_3)g_2+b_6c_2}{2c_2g_2}. \quad (19)$$

Proposition 4. The expected steady state inventory and pollution levels determined by Proposition 3 are globally and asymptotically stable if

$$((eb_4+2b_3)\gamma g_2+b_6(e(b_4-2b_6)+2b_3-b_4))c_2 > 2eg_2(eb_6+b_4/2)(eb_4-2b_3),$$

$$((eb_6+b_4)2e+\gamma c_2+2b_3)g_2+b_6c_2>0.$$

Further, if $T^2-4\Delta\geq 0$, the steady state is a sink, i.e., the convergence to the stable steady state is monotonic; otherwise it is a spiral sink (the convergence is with transient oscillations).

Proof: The proof readily follows from applying (18) and (19) to the stability criteria in [16]. ■

Based on Propositions 3 and 4, it is straightforward to verify for the data of the simulation example (Figure 1) that:

(i) The expected steady states of inventory and pollution stocks are x^{ss}=-12.02685786 (while *mean* X=-11.98370862 at t=50 based on the simulation with 10^4 replications) and p^{ss}=30.01758329 (while *mean* P=30.02459034 at t=50 based on 10^4 replications).

(ii) $\Delta = 0.6366607036 > 0$, $T = -1.605869833<0$, and $T^2-4\Delta = 0.032175107 > 0$, that is, the found steady states are stable and convergence to these states is monotone (which is also observed from Figure 1 and from the mean values at t=50 being very close to the expected values x^{ss} and p^{ss}).

The system of non-linear equations (A1) is solvable only numerically. Therefore we next study numerically the effect of environmental concerns and uncertainty on stable steady-state, production-inventory policies.

IV. NUMERICAL ANALYSIS

Demand uncertainty does not impact the expected steady states, which is typical for linear-quadratic control models (see, for example, [11]). Further, the expected steady state production level is not influenced by cost coefficients due to the unchanged mean customer demand. This is also observed in our numerical computations. Therefore, we present below only graphs of p^{ss}, X^{ss} and z^{ss} for different ε and cost coefficients. In particular, using the data similar to those from the previous section: g_1=0.1, g_2=1.5, c_1 = 0.1, c_2 = 1, h_1 = 0, h_2 = 0.5, a_1 = 0.1, a_2 = 1.2, D = 30, σ = 2.5, ε = 0.1, e = 0.1, δ = 0.01, and γ = 0.1, we vary only one specific (marginal) cost coefficient and/or the environmental volatility ε for each case in point. We start off from the coefficient c_1 that represents the proportional effect of the constant marginal emission cost on the manufacturer.

Proportional emission taxes

Proportional growth of c_1 induces linear reduction in the expected steady- state inventory level (Figure 2b), while not affecting pollution abatement efforts (Figure 2c) and subsequently the expected pollution long-run stock (Figure 2a). From Figure 2a, however, we observe that the pollution stock decreases (improves) with growing environmental uncertainty ε. The positive effect of the environmental concerns on the pollution stock, p^{ss}, is accomplished by an increased steady-state abatement rate z^{ss}, as seen from Figure 2c. Consequently, though the expected steady-state production level is not influenced by the proportional emission taxes, the expected inventory level does decrease with both growing environmental uncertainty and greater emission taxation. This is to offset the fact that the same production and thereby emission rates are associated with a higher cost of emission (Figures 2b and 2c).

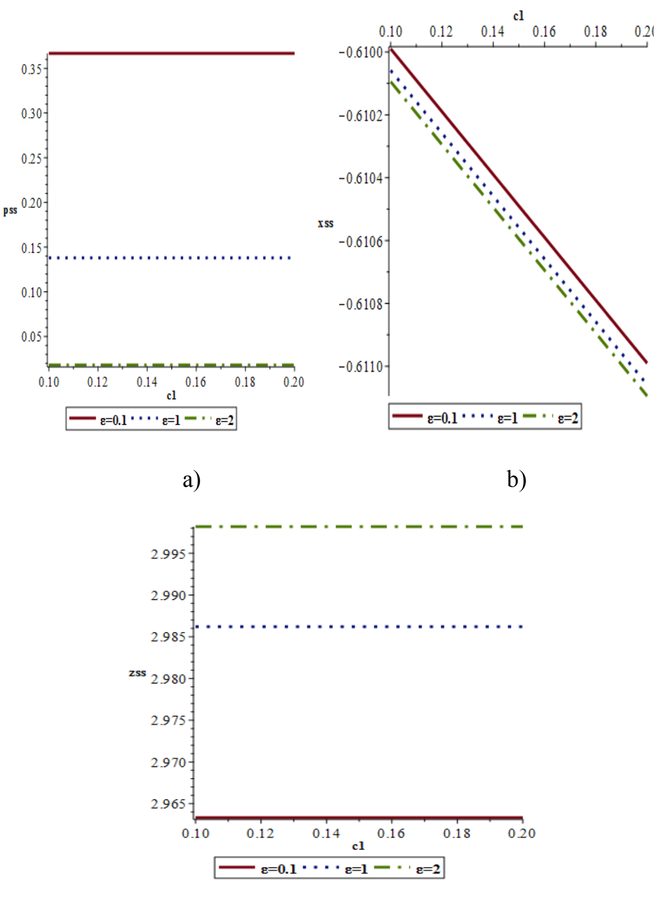

Figure 2. The effect of linear emission costs and of environmental uncertainty on a) pollution p^{ss}, b) inventories x^{ss}, and c) abatement z^{ss}.

Proportional ambient pollution taxes

Similar to the effect of the constant marginal emission cost c_1 presented in Figure 2, by varying ambient pollution marginal cost coefficient a_1, we find that unlike c_1, ambient pollution taxes linearly reduce not only steady-state inventories (Figure 3b) but also the expected steady-state pollution stock (Figure 3a). This is accomplished by increasing the pollution abatement rate (Figure 3c).

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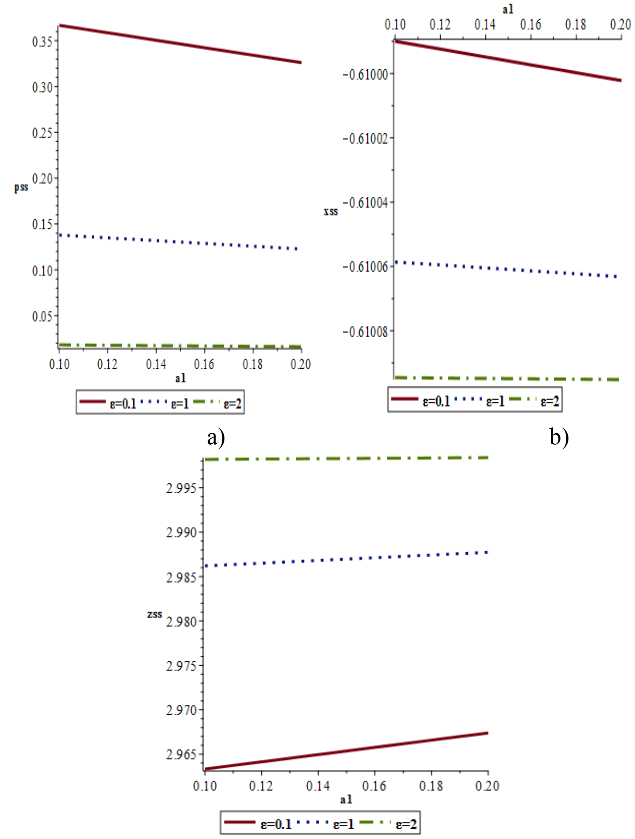
The effect of uncertainty ε naturally remains the same – the higher the environmental uncertainty, the greater the abatement effort and the lower the pollution along with inventory stocks (see Figures 2 and 3).



a) b)

c)

Figure 3. The effect of linear ambient pollution costs and of environmental uncertainty on a) pollution p^{ss}, b) inventories x^{ss}, and c) abatement z^{ss}.

Progressively growing emission taxes

Similar to the proportional emission taxation, progressively growing emission taxes impact only inventory levels that naturally reduce faster than under proportional taxation as shown in Figure 4.

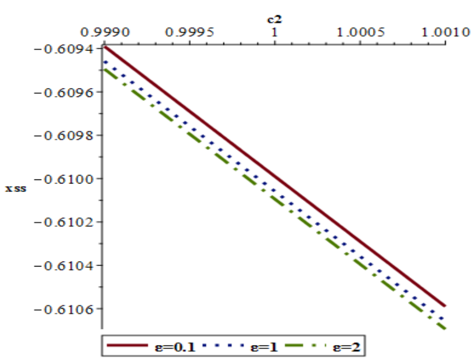
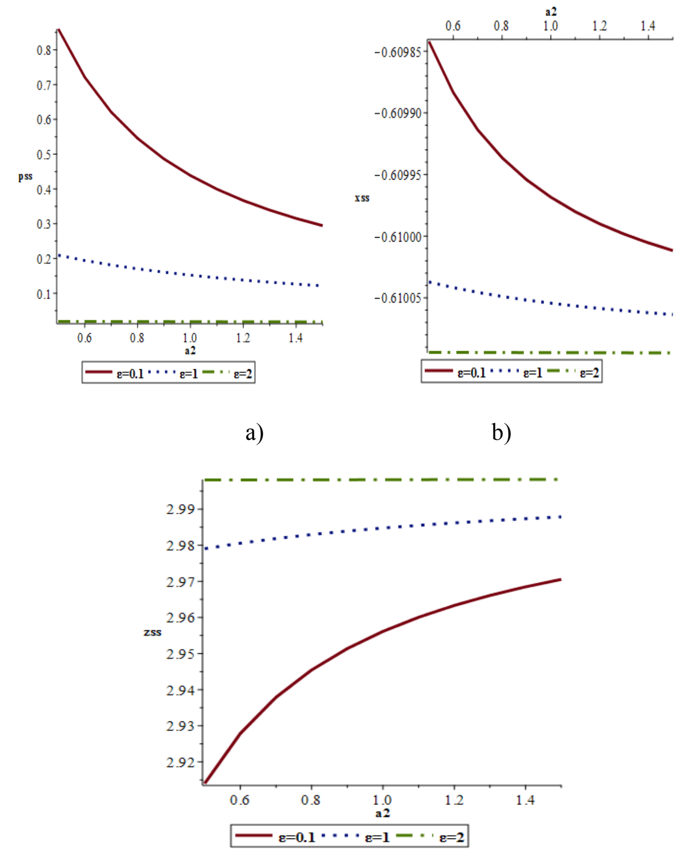


Figure 4. The effect of progressing emission costs and of environmental uncertainty on inventories x^{ss}.

Progressively growing ambient pollution taxes

Unlike progressively growing emission taxation and similar to the proportional ambient pollution costs, both pollution and inventory stocks are reduced by increasingly growing ambient pollution taxes. In this case, the effect is naturally non-linear as shown with Figure 5. Moreover, all the policies are influenced more strongly under lower pollution uncertainty.



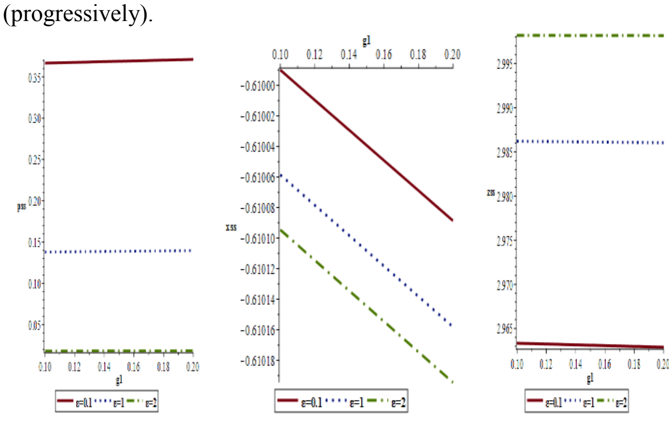
a) b)

c)

Figure 5. The effect of progressively growing ambient pollution costs and of environmental uncertainty on a) pollution p^{ss}, b) inventories x^{ss}, and c) abatement z^{ss}.

Pollution abatement costs

The effect of pollution abatement costs is quite expected (see Figures 6 and 7): the inventories still drop while pollution increases slightly (significantly) when the abatement cost grows proportionally (progressively).



a) b) c)

Figure 6. The effect of proportional abatement costs and of environmental uncertainty on a) pollution p^{ss}, b) inventories x^{ss}, and c) abatement z^{ss}.

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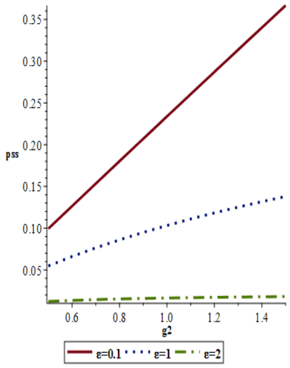
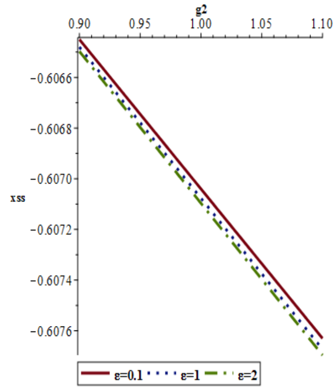
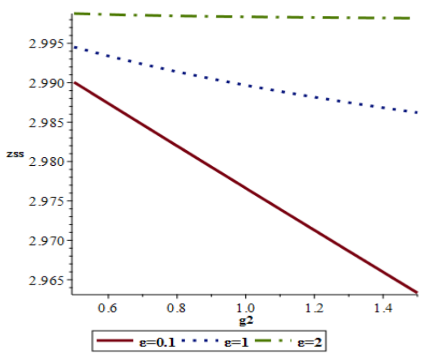


a)



b)



c)

Figure 7. The effect of progressively growing abatement pollution costs and of environmental uncertainty on a) pollution p^{ss}, b) inventories x^{ss}, and c) abatement z^{ss}.

V. Conclusions

We study the effect of environmental policies on production-inventory decisions under environmental and customer demand uncertainties. Assuming that the manufacturing process causes polluting emissions that can be reduced with an incorporated pollution abatement process, we find that environmental uncertainty leads to both lower expected steady-state pollution and inventory stocks while not affecting the expected steady-state production rate. The reduction in pollution is accomplished by increased pollution abatement efforts.

Although the emission taxes imply more costly production, which causes production smoothing, thereby increasing inventories and slowing down of the emission in the short-run, in the long-run we observe no impact on the expected steady-state pollution stock. This sustains the well-known claim that the industry fully exploits demand while simply passing on increased production costs to its consumers. The ambient pollution costs, however, do reduce the long-run expected pollution stock. The reduction is especially efficient with progressively growing taxation compared to proportional ambient pollution costs. Furthermore, in contrast to the short-run effect on inventories of production smoothing (described in deterministic studies), we find that environmental taxes always decrease long-run, inventory stocks. The strongest impact is observed with progressively growing emission taxes. The result is due to the fact that under growing production costs induced by environmental taxes, the manufacturer prefers to produce more to order rather than to stock thereby minimizing surpluses. Growing pollution abatement costs similarly decrease long-run inventory stocks. Naturally these costs are detrimental to the environment since they lead to lower abatement efforts and thereby higher pollution.

Although the computational experiments we conducted are in accordance with the US taxation system, a wider numerical analysis about the possible effects of various taxation approaches, including those under consideration as well as the hypothetical methods would be an important extension to the current research. As with the literature we cited in this study, we employed a simple Arrow-Karlin framework for our model based on linear-quadratic assumptions in terms of the dynamics and costs (including symmetric inventory-related costs) involved. More general assumptions can be considered to model the costs and environmental absorption properties and their impact on production-inventory policies which is a challenging direction for future research. Further, when there are a number of industrial firms which pollute the same aria, then a game-theoretic approach can be used to study the consequences of such a competition which is also an important direction for future research.

Appendix

A1. The system of equations for the coefficients of the cost-to-go function:

$$\left(4c_2(\sigma^2 b_3 - Db_2 - \delta b_1) - (eb_5 + b_2 + c_1)^2\right) g_2 - c_2(b_5 - g_1)^2 = 0$$

$$\left(2c_2(h_1 - \delta b_2 - 2Db_3) - (eb_5 + b_2 + c_1)(eb_4 + 2b_3)\right) g_2 - c_2 b_4 (b_5 - g_1) = 0$$

$$\left(2c_2\left(a_1 - Db_4 - b_5(\delta + \gamma)\right) - (2eb_6 + b_4)(eb_5 + b_2 + c_1)\right) g_2 + 2c_2 b_6 (g_1 - b_5) = 0$$

$$\left(4(h_2 - \delta b_3)c_2 - (eb_4 + 2b_3)^2\right) g_2 - c_2 b_4^2 = 0$$

$$\left(4c_2\left((\varepsilon^2 - \delta - 2\gamma)b_6 + a_2\right) - (2eb_6 + b_4)^2\right) g_2 - 4c_2 b_6^2 = 0$$

$$\left(2c_2(\delta + \gamma)b_4 + (2eb_6 + b_4)(eb_4 + 2b_3)\right) g_2 + 2c_2 b_4 b_6 = 0$$

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