Dyonic black holes in nonlinear electrodynamics from Kaluza-Klein theory with a Gauss-Bonnet term

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Abstract

Five-dimensional Kaluza-Klein theory with an Einstein-Gauss-Bonnet Lagrangian induces nonlinear corrections to the four-dimensional Maxwell equations, which however remain second order. Although these corrections do not have effect on the purely electric or magnetic monopole solutions for pointlike charges, they affect the dyonic solutions, smoothing the electric field at the origin for positive values of the Gauss-Bonnet coupling constant. We investigate these solutions in flat space, and then extend them in the presence of a minimal coupling to gravity, obtaining exact charged black hole solutions that generalize the Reissner-Nordström metric.

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1. Introduction

It is well known that Maxwell equations can be generalized in a non-linear way, adding to the Lagrangian higher powers of the invariants constructed from the electromagnetic field. Well-known examples are the corrections due to quantum electrodynamics that were proposed by Heisenberg and Euler [1] or the highly non-linear Born-Infeld Lagrangian [2] and their generalizations by Plebański [3]. These generalizations still yield second order field equations, but can give rise to solutions with regular electric or magnetic field [3].

Nonminimal coupling of the Maxwell equations to the gravitational field is instead more difficult, if one requires that the field equations remain second order and linear in the second derivatives of the electromagnetic potential and of the metric tensor. This problem has been studied in general in [4]. It is notable that a simple example of a model obeying this property can be obtained by dimensional reduction of a Kaluza-Klein (KK) theory containing a Gauss-Bonnet (GB) contribution [5-8].

We recall that KK theories [9,10] provide a unification of general relativity with electromagnetism based on the assumption that spacetime is five-dimensional and the fifth dimension is not observable because it is curled in an extremely small circle. However, in higher dimensions the Einstein-Hilbert Lagrangian is not unique, and one may add to it a GB term, which would not be effective in four dimension, since in that case it reduces to a total derivative. GB terms were shown in [11] to give the most general corrections to higher-dimensional gravity leading to second order field equations and compatible with some natural assumptions.

The introduction of this term in the five-dimensional Lagrangian permits to obtain by dimensional reduction to four dimensions a model whose predictions differ from those of the Einstein-Maxwell (EM) theory, giving rise to the possibility of an indirect evidence of the existence of a fifth dimension. In fact, the dimensionally reduced theory contains corrections to the EM coupling that are of the kind discussed in [4]. Moreover, they provide nonlinear modifications of the pure electromagnetic lagrangian, that give rise to corrections of the standard electrodynamics [6]. Although these corrections can be considered as a special case of Plebański's nonlinear electrodynamics [3], and in particular of its simplified version proposed in [12], their properties are rather peculiar, due to the particular combination of coefficients in the Lagrangian. For example, the purely electric or magnetic solutions of the Maxwell equations are not modified. It follows that, although regular solutions can be obtained for more general quadratic electrodynamics coupled to gravity [13-15], this is not the case for this model.

These facts are particularly relevant in relation with uniqueness and no-hair theorems for black holes. These theorems state that the only spherically symmetric asymptotically flat solution of the EM theory is the RN metric [16]. However, if nonlinear electromagnetic terms, like those of Born-Infeld [17] or Plebański [13-14], or extra fields with nonminimal coupling, like the dilaton [18-19], are added to the standard EM action, the theory will exhibit different solutions. Also the generalization to Yang-Mills fields can give rise to nontrivial solutions [20].

In fact, the solutions of the five-dimensional Einstein-GB theory have been studied from a higher-dimensional point of view in ref. [8], where it was shown that the effect of the GB term is only detectable through the coupling of electrodynamics to the gravitational field. However, the case of dyonic solutions was disregarded in that paper. As we shall see, dyonic solutions of the standard Maxwell equation are modified, even in the absence of the gravitational field, due to the nonlinear terms present in the field equations. Dyons were introduced in ref. [21] and have found many application in grand unified theories. Especially interesting are also their implications on the properties of charged black holes, in particular in relation with uniqueness and no-hair theorems. The coupling of our dyonic solution with gravity of course modifies the standard RN black holes, giving another example of the failure of the uniqueness theorems in case of nontrivial couplings.

In this paper, we describe exact flat-space dyonic solutions of the nonlinear Maxwell equations derived from GB-KK theory and show that solutions with everywhere regular electric field are possible. We then investigate the effect of these configurations on the black hole solutions of general relativity if the electromagnetic field is minimally coupled, obtaining an exact solution. We briefly discuss its properties and thermodynamical parameters.

However, we shall not consider the nonminimal couplings with the gravitational field arising from the dimensional reduction of the GB Lagrangian, since this problem is more involved and will therefore be studied separately [22].

2. The dyonic solution in flat space

We consider a five-dimensional Einstein-Gauss-Bonnet theory, with action

$$I = \int \sqrt{-g}\; d^5x(R + \alpha S), \tag{1}$$

where α is a coupling constant of dimension (lenght)^2, R is the Ricci scalar and S the Gauss-Bonnet term, $S = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$.

We use the simple ansatz[†] [5]

$$g_{\mu\nu} = \begin{pmatrix} g_{ij} + g^2 A_i A_j & gA_i \\ gA_j & 1 \end{pmatrix}, \tag{2}$$

where A_i is the Maxwell potential and g a coupling constant. Discarding total derivatives, the action (1) reduces to [1-3]

$$I = \int \sqrt{-g}\; d^4x \left[R - \frac{g^2}{4} F^{ij} F_{ij} + \frac{3\alpha g^4}{16} \Big[(F^{ij} F_{ij})^2 - 2F^{ij} F_{jk} F^{kl} F_{li} \Big] - \frac{\alpha g^2}{2} L_{int} \right], \tag{3}$$

where

$$L_{int} = F^{ij} F^{kl} (R_{ijkl} - 4R_{ik}\delta_{jl} + R\, \delta_{ik}\delta_{jl}), \tag{4}$$

and $F_{ij} = \partial_i A_j - \partial_j A_i$. This model of electromagnetism modified with nonlinear terms has been previously considered in ref. [4]. The Einstein-Maxwell coupling (4) has also been investigated in ref. [8].

In this section we shall consider the electromagnetic field in flat spacetime, neglecting gravity, since we are mainly interested in the nonlinear modifications of the Maxwell theory. The solutions of the EM theory (3) will be discussed in the following section.

The electromagnetic sector of the action (3) then reduces to [2]

$$I_{em} = \int d^4x \left(-\frac{g^2}{4} F^{ij} F_{ij} + \frac{3\alpha g^4}{16} \Big[(F^{ij} F_{ij})^2 - 2F^{ij} F_{jk} F^{kl} F_{li} \Big] \right). \tag{5}$$

The field equations derived from (4) read

$$\left(1 - \frac{3\alpha g^2}{2} F^2 \right) \partial_j F_{ji} + 3\alpha g^2\, \partial_j (F_{jk} F_{kl} F_{li}) = 0. \tag{6}$$

and contain derivatives of the potential A_i not higher than second order. Of course, the field F_{ij} also satisfies the Bianchi identities, $\partial_{(i} F_{jk)} = 0$.

It is easy to see that for purely electric or magnetic solutions the terms coming from the GB correction give no contribution [2,4], in contrast with most nonlinear models of electrodynamics [3]. However, let us consider a spherically symmetric dyonic solution, whose potential is given in spherical coordinates by

$$A = a(r)\, dt + P \cos\theta\, d\phi. \tag{7}$$

In an orthogonal frame one has

$$F_{01} = a'(r), \quad F_{23} = \frac{P}{r^2}, \tag{8}$$

where $' = d/dr$. Clearly, F_{23} satisfies (6).

To find the solution for the electric potential $a(r)$, it is convenient to write the action in terms of it and perform the variation. After integration on the angular variables, the action is proportional to

$$I_{em} = \int r^2 dr \left[a'^2 - \frac{P^2}{r^4} + 3\alpha g^2 \frac{P^2 a'^2}{r^4} \right], \tag{9}$$

[†] Greek indices run from 0 to 4, Latin indices from 0 to 3.

and its variation gives

$$\left[r^2\left(1+3\alpha g^2\frac{P^2}{r^4}\right)a'\right]'=0. \tag{10}$$

Therefore,

$$a'=F_{01}=\frac{Q}{r^2\left(1+3\alpha g^2\frac{P^2}{r^4}\right)}=\frac{r^4}{r^4+3\alpha g^2P^2}\,\frac{Q}{r^2}, \tag{11}$$

with Q an integration constant, that can be identified with the electric charge. The potential can be obtained by integration.

It follows that in this model the electric field of a point charge is distorted in the presence of a magnetic monopole; in particular, for $\alpha<0$ it diverges at $r=(3|\alpha|g^2P^2)^{1/4}$, while for $\alpha>0$ it is regular everywhere. However, the magnetic field (7) is still singular at the origin. In the limits $P=0$ or $Q=0$ one recovers the standard solutions.

3. The coupling to gravity

We now introduce gravity, to see the effects of nonlinear electromagnetism on spherically symmetric black hole solutions. However, we neglect the nonminimal EM coupling (4), since in this way we can obtain exact solutions. This is a good approximation as long as $F^2\gg R$. The inclusion of the nonminimal coupling will be investigated elsewhere [22].

Moreover, contrary to ref. [4], we consider the solutions of the four-dimensional effective theory, rather than those of the five-dimensional one, since they admit a more transparent interpretation. In the following, in order to obtain the standard normalization, we shall set $g^2=4$ and define $\bar\alpha=\alpha/4$, so that $\alpha g^2=\bar\alpha$.

We seek for spherically symmetric solutions of the form

$$ds^2=-e^{2\nu}dt^2+e^{2\mu}dr^2+e^{2\rho}d\Omega^2, \tag{12}$$

$$A=a(r)\,dt+P\cos\theta\,d\phi. \tag{13}$$

Like in the flat case, we calculate the field equations by substituting this ansatz into the action and performing the variation. We obtain

$$I=2\int dr\left[(2\nu'\rho'+\rho'^2)\epsilon^{\nu-\mu+2\rho}+e^{\nu+\mu}+a'^2e^{-\nu-\mu+2\rho}-P^2e^{\nu+\mu-2\rho}-3\bar\alpha P^2a'^2e^{-\mu-\nu-2\rho}\right]. \tag{14}$$

In the gauge $e^\rho=r$ the field equations stemming from (14) read

$$2\frac{\nu'}{r}+\frac{1}{r^2}-\frac{e^{2\mu}}{r^2}+a'^2e^{-2\nu}+\frac{P^2}{r^4}e^{2\mu}+3\bar\alpha a'^2\frac{P^2}{r^4}e^{-2\nu}=0, \tag{15}$$

$$-2\frac{\mu'}{r}+\frac{1}{r^2}-\frac{e^{2\mu}}{r^2}+a'^2e^{-2\nu}+\frac{P^2}{r^4}e^{2\mu}+3\bar\alpha\,a'^2\frac{P^2}{r^4}e^{-2\nu}=0, \tag{16}$$

$$\left[r^2e^{-\nu-\mu}\left(1+3\bar\alpha\,\frac{P^2}{r^4}\right)a'\right]'=0. \tag{17}$$

Combining (15) and (16), we get

$$\nu'+\mu'=0. \tag{18}$$

Hence, for asymptotically flat solutions, $\mu=-\nu$ and, integrating (17),

$$a'=\frac{Qr^2}{r^4+3\bar\alpha P^2}\approx\frac{Q}{r^2}\left(1-\frac{3\bar\alpha P^2}{r^4}+\dots\right), \tag{19}$$

with Q an integration constant. Substituting in (15), one can rearrange as

$$(re^{2\nu})'=1-\frac{P^2}{r^2}-\frac{Q^2r^2}{r^4+3\bar\alpha P^2}\approx1-\frac{Q^2+P^2}{r^2}-\frac{3\bar\alpha Q^2P^2}{r^6}+\dots \tag{20}$$

which displays order-$\bar\alpha$ corrections to the corresponding equation for the RN metric function.

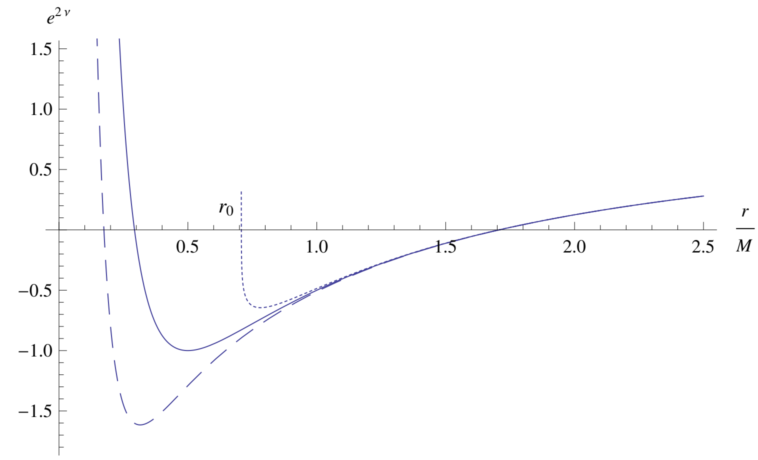

Fig. 1: The metric function $e^{2\nu}$ for generic black holes, with $Q = P = \frac{M}{2}$. The continuous line shows the RN solution, the dashed line the $\alpha > 0$ solution (21) and the dotted line the $\alpha < 0$ solution (40).

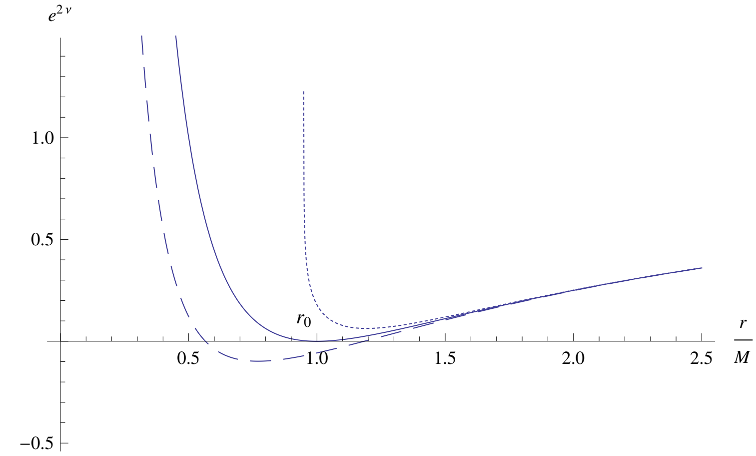

Fig. 2: The metric function $e^{2\nu}$ for near-extremal black holes with with $Q = P = \frac{M}{\sqrt{2}}$. The continuous line shows the RN solution, the dashed line the $\alpha > 0$ solution (21) and the dotted line the $\alpha < 0$ solution (40).

Eq. (20) can be solved exactly. Let us first consider the case $\alpha > 0$. Setting $\gamma = \sqrt{3\bar{\alpha}P^2}$ and choosing boundary conditions such that the term in parentheses vanish at infinity, the solution is

$$e^{2\nu} = 1 - \frac{2M}{r} + \frac{P^2}{r^2} + Q^2 f(r), \tag{21}$$

with

$$f(r) = \frac{1}{2\sqrt{2\gamma}\, r}\left[\pi + \arctan\left(1 - \frac{\sqrt{2}\, r}{\sqrt{\gamma}}\right) - \arctan\left(1 + \frac{\sqrt{2}\, r}{\sqrt{\gamma}}\right) + \frac{1}{2}\log\frac{r^2 - \sqrt{2\gamma}\, r + \gamma}{r^2 + \sqrt{2\gamma}\, r + \gamma}\right]. \tag{22}$$

This metric exhibits some similarity with the so-called geon solution of the Born-Infeld nonlinear electromagnetism coupled to gravity [11].‡ For small γ, it gives a slight correction to the RN solution, which however is relevant in the context of uniqueness theorems. In Fig. 1 and 2 some generic solutions are depicted together with the corresponding RN metric function and corresponding solutions with negative $\bar{\alpha}$.

The asymptotic behavior of (21) reproduces that of the RN metric up to order r^{-2},

$$e^{2\nu} = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} - \frac{\gamma^2 Q^2}{5r^6} + o\left(\frac{1}{r^7}\right), \tag{23}$$

‡ Curiously, a similar solution has also been obtained in a rather different nonlinear EM model [23].

and one can therefore identify M with the mass, Q and P with the electric and magnetic charge, respectively. Instead for $r \to 0$ the behavior is different from that of RN,

$$e^{2\nu} \sim \frac{P^2}{r^2} - \left(2M - \frac{\pi Q^2}{2\sqrt{2\gamma}}\right)\frac{1}{r} + o\,(1)\,. \tag{24}$$

In particular, the $\frac{1}{r}$ term becomes repulsive near the origin for $M < \frac{\pi Q^2}{4\sqrt{2\gamma}}$. The departure from the RN behavior are therefore greater for small r. The term proportional to $\frac{1}{\sqrt{\gamma}}$ arises because we have fixed the boundary conditions so that M is the mass of the solution. It is useful to define an effective mass near the singularity as $m = M - \frac{\pi Q^2}{4\sqrt{2\gamma}}$.

The curvature scalar is given by $R = \frac{4\gamma^2 Q^2}{(r^4+\gamma^2)^2}$ and is regular everywhere, but, in contrast with the RN solution does not vanish. However also in our case a curvature singularity occurs at the origin, since

$$R_{ijkl}R^{ijkl} \sim \frac{56P^4}{r^8} - \frac{96mP^2}{r^7} + \frac{48m^2}{r^6} + o\left(\frac{1}{r^5}\right). \tag{25}$$

The leading order term in (25) depends only on P and not on Q.

The causal structure depends on the values of the parameters M, P and Q that characterize the solution. Due to the nontrivial form of the metric, a general discussion can be made only numerically. It turns out that the causal structure is analogous to that of RN: for M greater than an extremal value that depends on P and Q, one has two horizons, while for M smaller a naked singularity occurs. If $\gamma \ll 1$, as arguable on physical grounds, the solution should not differ much from the RN metric and one can resort to a perturbative expansion in γ. However, one must be careful, because this fails at small r, since, as follows from (24), in this regime $\sqrt{\gamma}$ appears at the denominator.

The RN metric is known to exhibit a singularity at the origin and two horizons at

$$\tilde{r}_\pm = M \pm \sqrt{M^2 - Q^2 - P^2}. \tag{26}$$

Unfortunately, it is not possible to obtain an exact expression for the location of the horizons of the metric (21). We can obtain an approximation by expanding in the small parameter γ,

$$e^{2\nu} = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} - \frac{\gamma^2 Q^2}{5r^6} + o(\gamma^4). \tag{27}$$

This expression essentially coincides with the large-r expansion. The leading-order corrections are proportional to γ^2, and we can compute the zeroes of the metric as $r_\pm = \bar{r}_\pm + \gamma^2 \Delta r_\pm + o(\gamma)$, where

$$\Delta r_\pm = \pm \frac{Q^2}{5\,\bar{r}_\pm^4\,(\bar{r}_+ - \bar{r}_-)}. \tag{28}$$

It follows that extremal black holes with $r_+ = r_-$ occur for

$$M^2 = P^2 + Q^2 - \frac{\gamma^2 Q^2}{2(Q^2+P^2)^2} + o(\gamma^4). \tag{29}$$

However, our approximation works well only near the external horizon, while it breaks down near r_-, where one should consider also higher-order contributions in order to obtain reliable results, especially in the case of negative $\bar{\alpha}$, that we shall consider in the following. An exact condition for extremality can nevertheless be obtained in terms of the horizon radius, namely

$${r_+}^2 - P^2 - \frac{Q^2 {r_+}^4}{{r_+}^4 + \gamma^2} = 0, \tag{30}$$

which can be written as

$$ {r_+}^2 = r_-^2 = Q^2 + P^2 - \frac{\gamma^2 Q^2}{(Q^2+P^2)^2} + o(\gamma^4). \tag{31} $$

Notice also that from the metric (21) one can obtain the value of the mass corresponding to an horizon located at r_+. At leading order in γ this gives

$$ M = \frac{1}{2}\left(\frac{{r_+}^2 + Q^2 + P^2}{r_+} - \frac{\gamma^2 Q^2}{5{r_+}^5}\right) + o(\gamma^4). \tag{32} $$

4. Black hole thermodynamics

The thermodynamical quantities can be computed in the standard way: the entropy can be identified with one fourth of the area of the external horizon, namely

$$ S = \pi r_+^2, \tag{33} $$

while the temperature can be calculated as

$$ T = \frac{1}{4\pi}\frac{d\, e^{2\nu}}{dr}\bigg|_{r=r+} = \frac{1}{4\pi}\left(\frac{1}{r_+} - \frac{P^2}{r_+^3} - \frac{Q^2 r_+}{r_+^4+\gamma^2}\right) \approx \frac{1}{4\pi}\left(\frac{r_+^2 - Q^2 - P^2}{r_+^3} + \frac{3\bar\alpha Q^2 P^2}{r_+^7}\right) + o(\bar\alpha^2). \tag{34} $$

Both temperature and entropy are increased with respect to the Reissner-Nordström black hole of same mass and charges. In Fig. 3, the behavior of the temperature vs. the horizon radius is compared with that of the RN black hole.

Alternatively, the thermodynamical quantities can be written in terms of $\bar r_\pm$,

$$ S = \pi\left(\bar r_+^2 + \frac{6\bar\alpha Q^2P^2}{5\bar r_+^3(\bar r_+ - \bar r_-)}\right) + o(\bar\alpha^2), \qquad T = \frac{1}{4\pi}\left(\frac{\bar r_+ - \bar r_-}{\bar r_+^2} + \frac{6\bar\alpha Q^2P^2}{5\bar r_+^7}\,\frac{2\bar r_+ - \bar r_-}{\bar r_+ - \bar r_-}\right) + o(\bar\alpha^2). \tag{35} $$

The latter expressions are good approximations except for near-extremal black hole. It is remarkable that at this order of approximation, both entropy and temperature are symmetric for the exchange $Q \leftrightarrow P$.

As in the RN case, the temperature vanishes at extremality and for $r_+ \to \infty$, and has a maximum at the critical point $r = r_c$. The black hole is thermodynamically stable for $r_+ < r_c$ and becomes unstable for $r_+ > r_c$. For the RN black hole $\bar r_c^2 = 3(P^2+Q^2)$, i.e. $M_c^2 = \frac{4}{3}(Q^2+P^2)$. This can be checked by calculating the specific heat, that in r_c diverges and changes its sign [24]. In fact, the specific heat at constant charges of the RN black hole is given by

$$ C_{Q,P} = T\,\frac{\partial S}{\partial T}\bigg|_{Q,P} = -2\pi r_+^2\,\frac{r_+^2 - (Q^2+P^2)}{r_+^2 - 3(Q^2+P^2)}. \tag{36} $$

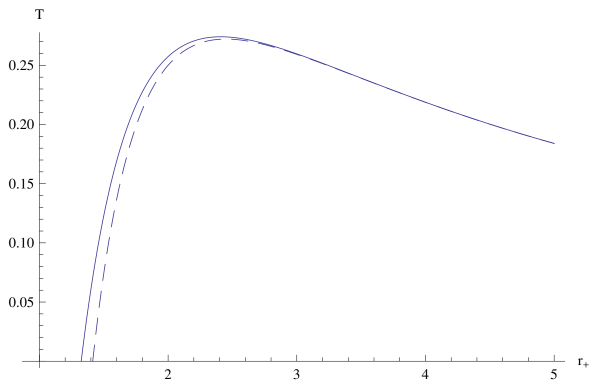

Fig. 3: The temperature T as a function of the black hole radius r_+. The continuous line corresponds to our solution with $Q = P = 1$, $\gamma = 1$, the dashed line to the RN solution with identical charges.

In our case, one has instead

$$C_{Q,P} = -2\pi \frac{{r_+}^2({r_+}^4+\gamma^2)({r_+}^6-(Q^2+P^2){r_+}^4+\gamma^2{r_+}^2-\gamma^4)}{{r_+}^{10}-3(Q^2+P^2){r_+}^8+\gamma^2(Q^2-6P^2){r_+}^4-3\gamma^4P^2}, \tag{37}$$

which diverges for

$${r_+}^2 = 3(Q^2+P^2) - \frac{9\gamma^2}{Q^2+P^2} + o(\gamma^4). \tag{38}$$

Hence, the critical point occurs for a slightly smaller value of r_+ with respect to the RN case, but the qualitative behaviour of the specific heat is the same.

It may also be interesting to evaluate the Helmholtz free energy $F = M - TS$ associated to the solution (21). A short calculation gives

$$F = \frac{1}{4}\left(\frac{{r_+}^2+3(Q^2+P^2)}{r_+} - \frac{21\bar\alpha}{5}\frac{Q^2P^2}{{r_+}^5}\right) + O(\bar\alpha^2). \tag{39}$$

It is evident that for positive $\bar\alpha$ the free energy is always smaller than the one associated to the RN black hole, indicating a greater thermodynamical stability at fixed charges, while the opposite happens for $\bar\alpha < 0$. A numerical plot of F using the exact solution is shown in Fig. 4.

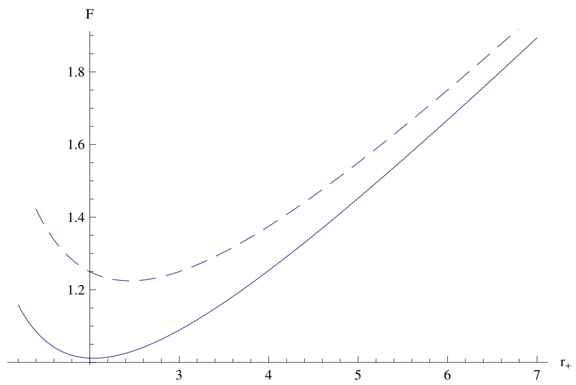

Fig. 4: The free energy F as a function of the black hole radius r_+. The continuous line corresponds to our solution with $Q = P = 1$, $\bar\alpha = 0.1$, the dashed line to the RN solution with identical charges.

To conclude, let us briefly comment on the case $\alpha < 0$. The metric function has a simpler form,

$$e^{2\nu} = 1 - \frac{2M}{r} + \frac{P^2}{r^2} + \frac{Q^2}{2\sqrt{\gamma}\,r}\left[\frac{\pi}{2} - \arctan\frac{r}{\sqrt{\gamma}} - \frac{1}{2}\log\frac{r-\sqrt{\gamma}}{r+\sqrt{\gamma}}\right], \tag{40}$$

where now $\gamma = \sqrt{3|\bar\alpha|P^2}$, but has the same asymptotic behavior (23) as the previous solution, except for the sign of the term proportional to γ^2. Also the expansion for small γ has the same form as (27). The curvature scalar is $R = \frac{4\gamma^2Q^2}{(r^4-\gamma^2)^2}$. Now a curvature singularity occurs at the surface $r_0 = \sqrt{\gamma}$, while the horizons are located at

$$r_\pm \approx \bar r_\pm \pm \frac{3\bar\alpha P^2Q^2}{5\,\bar r_\pm(\bar r_+ - \bar r_-)}. \tag{41}$$

If $r_0 > r_-$, a single horizon is present and the causal structure is similar to that of the Schwarzschild solutions. Otherwise, the properties are analogous to those of the solution with positive α and all the previous formulas still hold, taking into account that $\bar\alpha$ has opposite sign. This is true in particular for the thermodynamical quantities.

5. Conclusion

We have considered the effect of the nonlinearity of the electrodynamics induced by a five-dimensional KK model with Einstein-GB lagrangian on the dyonic solutions with a pointlike source. While it is well known that purely electric or magnetic solutions are not modified in this model, we have shown that the dyonic solutions differ from those of the Maxwell theory, and the electric field can be regular everywhere.

In our model, the field equations contain at most cubic terms in A_i, but the model can be generalized to higher powers by increasing the number of the internal dimensions and adding higher-order GB terms. Also in this case the pure electric or magnetic fields of pointlike sources maintain the standard form, but the dyonic solutions are modified and for suitable ranges of values of the coupling constants the singularity of the electric field is suppressed.

Going to higher dimensions also allows the introduction of Yang-Mills fields through the Kaluza-Klein mechanism. Of course, in this case more complicated solutions are expected.

We have also examined the coupling with gravity and have found a new class of solutions that modify the RN metric, with a Maxwell field identical to the flat space solution and a metric that deforms the RN solution. The solutions still depend on the three parameters M, Q and P, but are no longer dual for the interchange of Q and P. For positive α they exhibit a pointlike singularity, while for negative α the singularity is spherical. The horizon structure is similar to that of RN, with two horizon, but for some values of the parameters it can present one or no horizons.

We have discussed the thermodynamics of the black hole solutions and found that it does not differ significantly from that of RN. Our results are notable since they show that the introduction of nonlinear equations for the electromagnetic fields affects the results of the black hole uniqueness theorems also in case of minimal coupling to gravity, analogously to what happens in more general models [13-15].

In this paper we have not considered the nonminimal coupling between gravity and electromagnetism induced by the KK-GB model. Such coupling spoils the possibility of solving the field equations analytically, but the main properties of the solutions should be preserved. We plan to investigate this topic in a future publication [22].

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