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Exploiting the European Volatility Index Features: Anti-Persistence, Skewness, Kurtosis, and the Role of the Hurst Exponent

Michele Anelli[1], Michele Patanè[1], Stefano Zedda[2]*

[1]Department of Business and Law, School of Economics and Management, University of Siena, Siena, Italy
[2]Department of Business and Economics, University of Cagliari, Cagliari, Italy
Email: anelli2@student.unisi.it, michele.patane@unisi.it, *szedda@unica.it

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Abstract

Volatility indices are fundamental in the study of stock markets. In this paper, we analyzed the classical statistical characteristics of the main volatility index of the European stock markets (VStoxx) and evidenced some interesting connections and cause-effect relationships between the Hurst exponent and the moments of the distribution. Our results suggest that the market volatility is characterized by anti-persistence and mean reversion and that the Hurst exponent variations seem to anticipate the variations of the other moments of the distribution such as skewness and kurtosis, so that the Hurst exponent variations can possibly signal near-term market reversals.

Keywords

VStoxx, Hurst Exponent, Kurtosis, Skewness, Cointegration, Vector Error Correction Model, Granger-Causality

1. Introduction

The volatility indices in recent years raised increasing interest among the various market players. Accurate modeling and forecasting of volatility are crucial both for theoretical studies and the financial markets' investment activity. However, to our knowledge, no published and working paper studied the statistical features and linkage between the moments of the VStoxx index distribution over time with a fractal approach.

In this paper, we analyzed the classical statistical characteristics of the main volatility index of the European stock markets (VStoxx) and evidenced some interesting connections and cause-effect relationships between the Hurst exponent and the moments of the distribution.

More in detail: We analyzed the VStoxx index distribution over time for verifying:

1) The statistical and structural characteristics of the volatility indices through the study of the distribution of returns over time. As in other analysis papers (see Bagato et al., 2018), this study starts from the analysis of the basic characteristics of the distribution of logarithmic returns of the European volatility index Vstoxx, so directly from the distribution moments, to empirically describe some significant evidence;

2) The anti-persistence and mean reversion structurally present in the historical series, through the study of the behavior of the Hurst exponent. The Hurst exponent is a fundamental index for the analysis of the long-range dependence features of observable time series (Resta, 2012). We analyzed the Hurst exponent behavior in the historical time series of the European index of equity volatility. In this analysis, we verified that the Hurst exponent value of logarithmic returns is mainly concentrated between 0 and 0.5. This confirms the anti-persistence of the European volatility index Vstoxx, and its mean reversion characteristic. In the literature, a Hurst exponent that structurally moves between 0 and 0.5 has led to the definition of volatility as "rough" (see Gatheral et al., 2014). This characteristic of anti-persistence of the historical series of logarithmic returns also brings with it the adoption of *Fractional Brownian Motion* models (see Neuman & Rosenbaum, 2018) used in the logic of analysis and pricing when the Hurst exponent assumes values that are structurally different from 0.5, the typical value of the classic Brownian Motion;

3) The relationships between the various moments of the distribution and the Hurst exponent. We have focused more on this last aspect even if our analysis is not exhaustive, but opens the way for further and in-depth analysis both in the field of risk management, but also in terms of pricing of derivative instruments with non-linear payoffs linked to stock volatility indices. Nevertheless, we believe it is an additional useful tool for portfolio managers besides classical evaluation metrics (i.e. P/E, P/S, Dividend Yield, etc.) to signal a potential near-term market excess (increasing probability of cyclical market inversion point) when it is close to its interval extremes.

The paper is organized as follows: Section 2 reports a literature review, Section 3 describes the data and model, Section 4 reports its results, and Section 5 concludes.

2. Literature Review

Although there exist different approaches aimed at volatility forecasting, historical and implied volatility models are the most exploited. Siriopoulos and Fassas (2019) test and document the information content of all publicly available implied volatility indices regarding both the realized volatility and the returns of the underlying asset. Their findings suggest that implied volatility includes information about future volatility beyond that contained in past volatility but, at the same time, they show that implied volatilities in commodities, bonds, and currencies, react differently from equities to underlying price changes. Implied volatility models are based on the Black-Scholes Model (BSM) the standard model for options pricing. Volatility is one of not deterministic inputs available for an immediate application in the formula, which is assumed to be constant over time and coupled with the assumption of normally distributed log returns. Iqbal (2018) reports a usual leptokurtic distribution of the log returns of spot prices rather than a normal distribution, so characterized by excess kurtosis and fat tails compared with the normal. In other terms, spot prices will expectedly remain unchanged more often than in the case of a normal distribution, and, at the same time, extreme variations are more often observed. The author also shows how this pricing feeds into the volatility smile.

Most papers have focused on the predictive power of the VIX Index (or CBOE Volatility Index) for predicting future stock market returns. Giot (2005) proves that high (low) levels of the VIX correspond to positive (negative) future returns. Also, Chow et al. (2014) show the existence of a positive relationship between market returns and the VIX Index. Regarding the VIX features, Fleming et al. (1995) for the first time showed the persistence (long-memory behavior) of this index, not considered in the previous literature on the main European volatility indices (i.e. VStoxx and VDax). Stanescu and Tunaru (2013) focused the attention on the linkages between Eurostoxx50, S&P500, VSTOXX, VIX, and VSTOXX futures series, and showed that these linkages can be used by equity investors to generate alpha and protect their investments during turbulent times. Fahling et al. (2019) show that the best forecasting model for the one-month VDAX is a GARCHX(1, 1) model and an ARX(1) model for the one-year VDAX, while for the VSTOXX, an ARX model is the best fit under each scenario for both strategies.

3. Model and Data Description

To conduct the empirical analysis, we use daily VStoxx close prices for the interval 2000-2019 (5423 observations) transformed in log returns (5422 observations). Logarithmic transformation allows time-series stationarity and a closer-to-normal distribution. Data come from *Bloomberg*.

Figure 1 reports the Vstoxx log returns distribution over the considered time span, evidencing that the central bars and the tails of the histogram are higher than the normal distribution expected ones, while the side bars report a lower frequency, in the typical fat-tailed leptokurtic distribution.

Table 1 reports the descriptive statistics of the VStoxx log returns series for the considered time span.

Figure 2 shows the graphical comparison among the Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness, and Hurst exponent. **Table 2** reports the descriptive statistics of the same parameters data derived from VStoxx

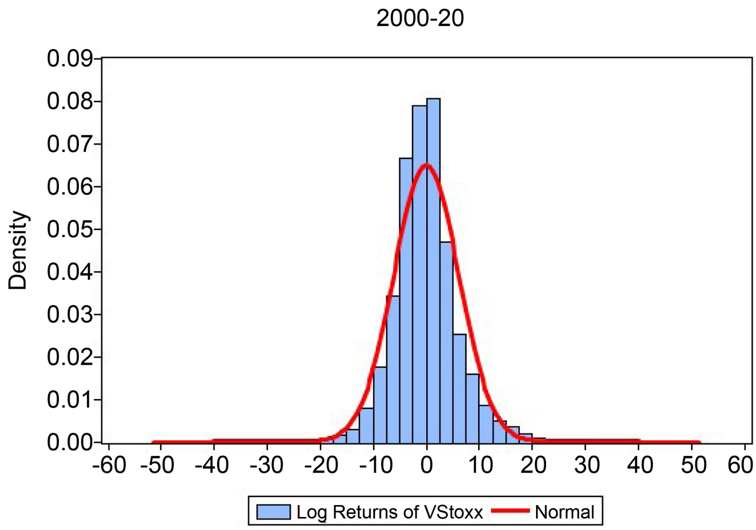

Figure 1. Example of VStoxx log returns density distribution series from January 2000 to December 2019. Source: authors' calculations in Eviews 10 based on Bloomberg data.

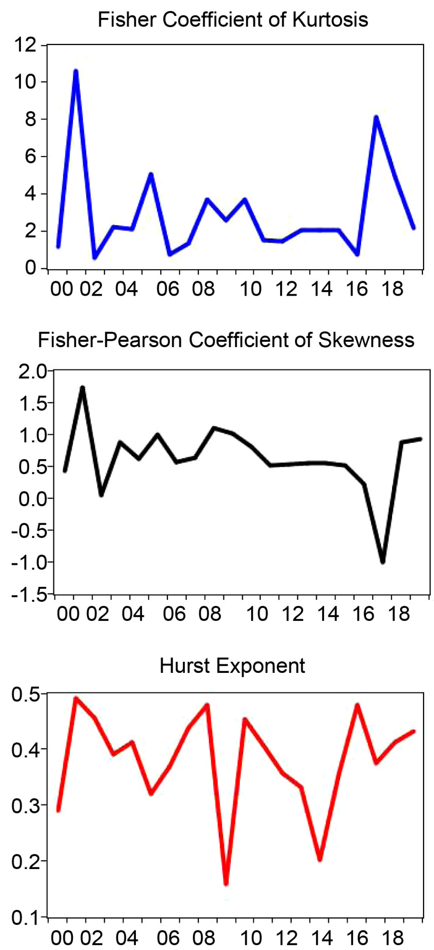

Figure 2. Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness, and Hurst exponent data derived from VStoxx log returns distribution from January 2000 to December 2019 (20 annual obs.) Source: authors' calculations in Eviews 10 based on Bloomberg data.

Table 1. Descriptive statistics of data from January 2000 to December 2019 (5422 daily obs.)

Statistics	VStoxx Log Returns (%)
Mean	−0.005
Median	−0.331
Maximum	47.031
Minimum	−43.472
Std. Dev.	6.148
Skewness	0.758
Kurtosis	7.529

Source: Authors' calculations based on Bloomberg data.

Table 2. Descriptive statistics of Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness, and Hurst exponent from January 2000 to December 2019 (20 annual obs.)

Statistics	Fisher Coefficient of Kurtosis	Fisher-Pearson Coefficient of Skewness	Hurst Exponent
Mean	2.941	0.619	0.381
Median	2.089	0.582	0.398
Maximum	10.599	1.730	0.492
Minimum	0.569	−1.007	0.158
Std. Dev.	2.560	0.524	0.088
Skewness	1.775	−1.088	−1.036
Kurtosis	5.544	6.416	3.596

Source: Authors' calculations based on Bloomberg data.

log returns distribution.

Let FK_t (Fisher coefficient of kurtosis at time *t*) and FS_t (Fisher-Pearson coefficient of skewness at time *t*) define as (Joanes & Gill, 1998):

$$FK_t = \frac{m_{4_t}}{\left(m_2\right)_t^2} - 3 \tag{1}$$

$$FS_t = \frac{m_{3_t}}{\left(m_2\right)_t^{3/2}} \tag{2}$$

where m_{2_t}, m_{3_t}, and m_{4_t} are respectively the second, the third, and the fourth central moment computed for the time interval *t*.

The econometric model implemented develops in two sequential parts. The first part is based on the methodology suggested by Gonzalo and Granger (1995), while the second part is based on an OLS regression.

Firstly, through a cointegration analysis, employing the Augmented Dickey-Fuller test, we verify whether the short-term deviations of these two series converge towards the long-term equilibrium. The existence of a linear combination between these two series, indeed, supports the presence of a long-term equilibrium adjustment process, even if the series deviates one from the other in the short term.

If the series are cointegrated[1], employing a bivariate Vector Error Correction Model (VECM), as suggested by Engle and Granger (1987), we verify which variable moves more rapidly than the other one, to evidence the role of the leader and follower variables.

To provide some information about the statistical properties of European stocks' volatility index time series, we estimate the Hurst Exponent using re-scaled range analysis (R/S), following the intuition and the structure suggested by Sang et al. (2001). Re-scaled range analysis approach is robust to heavy tails (Barunik & Kristoufek, 2010). Starting from a VStoxx log return series Rt_n, and posing τ as the time span of the entire discrete series, the cumulative sum of the difference between the return series and their mean, $X(n,\tau)$, is defined as:

$$X(n,\tau)=\sum_{i=1}^{n}\left(Rt_i-\frac{1}{\tau}\sum_{n=1}^{\tau}Rt_n\right) \tag{3}$$

where R is the difference between the maximum and minimum of $X(n,\tau)$, and S is the series standard deviation. Therefore, R and S are defined as:

$$R(\tau)=X(n,\tau)-\min X(n,\tau) \quad \text{for} \quad n\in[1,\tau] \tag{4}$$

$$S(\tau)=\sqrt{\frac{1}{\tau}\sum_{n=1}^{\tau}\left(Rt_n-\frac{1}{\tau}\sum_{n=1}^{\tau}Rt_n\right)^2} \tag{5}$$

R and S are functions of τ. The R/S ratio can then be represented by the following empirical equation:

$$\frac{R}{S}=(c\tau)^H \tag{6}$$

where c is a constant and H is the Hurst exponent. The Hurst exponent is a classical self-similarity parameter that measures the long-range dependence in a time series and provides a measure of long-term nonlinearity (Millen & Beard, 2003). **Figure 3** reports its distribution in the considered time span.

The formal specification of the model is defined by the following equations:

$$\Delta H_t=\beta_{10}+\sum_{t=1}^{l}\beta_{1t}\Delta H_{t-1}+\sum_{t=1}^{l}\alpha_{1t}\Delta FK_{t-1}+\lambda_1 ECT_{t-1}+\varepsilon_{1t} \tag{7}$$

$$\Delta FK_t=\beta_{20}+\sum_{t=1}^{l}\beta_{2t}\Delta H_{t-1}+\sum_{t=1}^{l}\alpha_{2t}\Delta FK_{t-1}+\lambda_2 ECT_{t-1}+\varepsilon_{2t} \tag{8}$$

where:

- ΔH_t and ΔFK_t are the first differences for the Hurst exponent and the Fisher coefficient of the kurtosis series;
- β_{10} and β_{20} are the constant terms of the Equations (7) and (8);
- ΔH_{t-1} and ΔFK_{t-1} are the delayed first differences for the Hurst exponent and the Fisher coefficient of the kurtosis series;
- l is the number of lags;

[1]If the two series are not cointegrated then the VECM cannot be implemented.

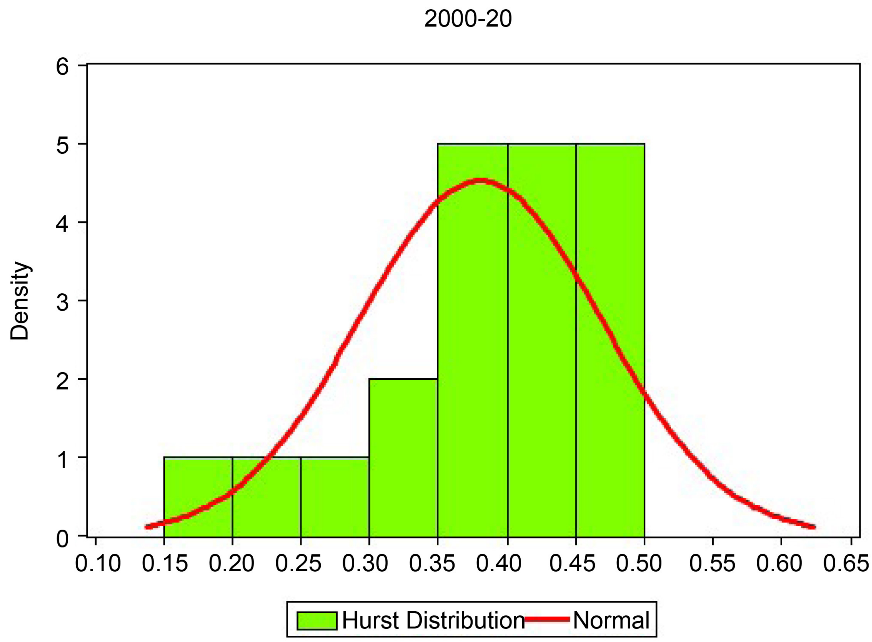

Figure 3. Hurst exponent distribution from January 2000 to December 2019 (20 annual obs.). Source: authors' calculations in Eviews 10 based on Bloomberg data.

- ECT_{t-1} is the Error Correction Term (ECT). It is defined as $ECT_{t-1} = H_{t-1} - \alpha - \gamma FK_{t-1}$. In simple terms, it measures the deviations between the Hurst exponent and Fisher coefficient of kurtosis at the time $(t-1)$ with respect to the theoretical long-period equilibrium. γ is the cointegrating coefficient and α is the intercept of the cointegrating term;
- λ_1 and λ_2 are the adjustment coefficients. They describe the speed of adjustment back to the long-period equilibrium, that is they measure the proportion of correction of the series deviations from the long-run relationship;
- ε_{1t} and ε_{2t} are the error terms of Equations (7) and (8).

For the aim of the analysis, the sign[2] and statistical significance of the adjustment coefficients (λ_1 and λ_2) determine which market contributes to the adjustment process toward the long-period equilibrium, and which variable can move more rapidly than the other one. Hence, we should distinguish four cases:

1) If λ_1 is statistically significant and negative, then the Fisher coefficient of kurtosis adjusts more rapidly than the Hurst exponent. This means that the Hurst exponent is trying to restore the long-run equilibrium;

2) If λ_2 is statistically significant and positive, then the Hurst exponent moves more rapidly than the Fisher coefficient of kurtosis adjustment. This means that the Fisher coefficient of kurtosis adjustment is trying to restore the long-run equilibrium;

3) If λ_1 is statistically significant and negative and λ_2 is statistically significant and positive then both variables contribute to the adjustment process toward the

[2]We should expect the negative sign for λ_1 and the positive sign for λ_2 to favor the process of adjustment.

long-run equilibrium. In this case, by following Gonzalo and Granger (1995), to evaluate the effective contribution of each market in the adjustment process, we follow the concept of Market Share (MS)[3]. We distinguish three sub-cases:

a) If $MS \approx 1$ then the Hurst exponent is the leading variable, and the Fisher coefficient of kurtosis is the lagging variable;

b) If $MS \approx 0$ then the Fisher coefficient of kurtosis is the leading variable, and the Hurst exponent is the lagging variable;

c) If $MS \approx 0.5$ then both variables contribute in the same way;

4) If only one of the adjustment coefficients is statistically significant and it presents the correct sign then only that variable contributes to the adjustment process toward the equilibrium.

The log returns of spot prices usually show a leptokurtic distribution rather than a normal distribution (Iqbal, 2018), meaning that it exhibits excess kurtosis and more weight in the tails compared with the normal distribution. The evolution of the excess of kurtosis variations (which represent a stylized feature of the volatility distribution), therefore, should be evaluated as the main determinant of the other fundamental aspect of the volatility distribution, the skewness progression. In this way, we can roughly estimate the "gamma effect"[4] of the excess of kurtosis on the distribution skewness.

Thus, the following OLS regression allows verifying if the kurtosis variations can be an explanatory variable for the skewness variations:

$$\Delta FS_t = c + \beta \Delta FK_t + \varepsilon_t \tag{9}$$

where:

- ΔFS_t is the first difference of the log variation of the Fisher-Pearson coefficient of skewness at time t;
- ΔFK_t is the first difference of the log variation of the Fisher coefficient of kurtosis at time t;
- c is the constant term;
- β is the regressor's coefficient;
- ε_t is the error term.

The above OLS regression is run on annual values so that we have 16 annual observations (after adjustment[5]) for regression (9). The number of observations matters for inference, particularly in presence of non-normal distributed residuals. Jenkins and Quintana-Ascencio (2020) investigate the process to identify a minimum N (number of observations) needed for a study. Authors recommend a minimum $N = 8$ for a tight data pattern (i.e. very low variance) and a minimum $N \approx 25$ to match a model to the data pattern with high variance. Their findings support our OLS model performance.

[3]The formula suggested by Gonzalo and Granger (1995) is the following: $MS = \frac{\lambda_2}{\lambda_2 - \lambda_1}$.

[4]This expression is just an analogy with the options instruments terminology. "Gamma", indeed, is the rate of change in an option's delta per 1-point move in the underlying asset's price.

[5]The Fisher-Pearson coefficient of skewness is negative in 2017.

4. Results

All preliminary and complementary tests on time series and further statistical tests validating the acceptance of OLS assumptions are not reported here. These latter tests, confirm the presence of heteroskedasticity, no serial correlation, and non-normal distributed residuals. To limit the problem of heteroskedasticity, we calculate robust estimates by using the *Huber-White* procedure. To deal with non-normal distributed residuals, we follow Jenkins and Quintana-Ascencio (2020).

As suggested by Liew (2004), we use Akaike's Information Criterion (AIC) as lag length selection criteria in determining the autoregressive lag length. The author shows its superiority over the other criteria under study in the case of a small sample (60 observations and below).

4.1. VECM: Hurst Exponent and Fisher Coefficient of Kurtosis

Firstly, we tested for the cointegration between the two series through the Augmented Dickey-Fuller test, reported in **Table 3**.

Results show that the two series are cointegrated. Therefore, we can perform the VECM to assess which market contributes to the adjustment process toward the long-term equilibrium. Akaike's information criterion suggests three lags as the optimal lag length structure. The VECM estimation outputs are reported in the following **Table 4(a)** and **Table 4(b)**.

Table 4(a) and **Table 4(b)** show that only λ_1 is statistically significant and negative while λ_2 is positive but not statistically significant. This means that the Fisher coefficient of kurtosis adjusts more rapidly than the Hurst exponent. This latter moves toward a restoration of the long-run equilibrium relationship, proving that the Hurst exponent is a proxy measure of the degree of volatility mean reversion.

4.2. OLS Regression

Table 5 reports the augmented Dickey-Fuller test.

The augmented Dickey-Fuller tests show that the series is stationary. **Table 6** shows the estimated coefficients for Equation (9).

The estimated determination coefficient R^2 for the considered time span is 0.78 for Equation (9). The relationship between the Fisher-Pearson coefficient of skewness elasticity (*dependent variable*) and the Fisher coefficient of kurtosis elasticity (*independent variable*) is positive, as expected, reporting a coefficient of about 0.98. This means that an increase/decrease of 1 percent of the regressor

Table 3. Augmented Dickey-Fuller test: period 2000-20 (20 annual obs.).

Augmented Dickey-Fuller Test	**Residuals**
Period	2000-20
*Prob.**	0.0056

Source: Authors' calculations based on Bloomberg data.

Table 4. (a) Results of VECM estimation: dependent variable ΔH 2000-20 (20 annual obs.); (b) Results of VECM estimation: dependent variable ΔFK 2000-20 (20 annual obs.).

(a)

Variable	Coefficient	Std. Error	Prob.
β_{10}	0.025	0.022	0.300
β_{11}	1.144	0.882	0.231
β_{12}	0.633	0.582	0.309
β_{13}	0.124	0.261	0.646
α_{11}	−0.089	0.041	0.060
α_{12}	−0.052	0.028	0.107
α_{13}	−0.012	0.015	0.430
λ_1	−2.685*	1.216	0.058

Note: *** signals parameter significance at 1%, ** signals parameter significance at 5%, and * signals parameter significance at 10%. Source: Authors' calculations in Eviews 10 based on Bloomberg data.

(b)

Variable	Coefficient	Std. Error	Prob.
β_{20}	0.070	0.705	0.923
β_{21}	−3.355	20.287	0.873
β_{22}	4.119	14.911	0.789
β_{23}	0.727	4.974	0.887
α_{21}	−0.459	1.034	0.669
α_{22}	−0.359	0.765	0.651
α_{23}	−0.274	0.391	0.503
λ_2	9.620	29.542	0.753

Note: *** signals parameter significance at 1%, ** signals parameter significance at 5%, and * signals parameter significance at 10%. Source: Authors' calculations in Eviews 10 based on Bloomberg data.

Table 5. Augmented dickey-fuller test: period 2000-20 (20 annual obs.)

Augmented Dickey-Fuller Test	Residuals
Period	2000-20
Prob.*	0.0000

Source: Authors' calculations based on Bloomberg data.

Table 6. OLS regression: dependent variable ΔFS during period 2000-20 (16 annual obs.)[6].

Variable	Coefficient	Std. Error	Prob.
c	0.010	0.305	0.9755
β	0.983***	0.199	0.0002

Note: *** signals parameter significance at 1%, ** signals parameter significance at 5%, and * signals parameter significance at 10%. Source: Authors' calculations in Eviews 10 based on Bloomberg data.

[6]The Fisher-Pearson coefficient of skewness is negative in 2017.

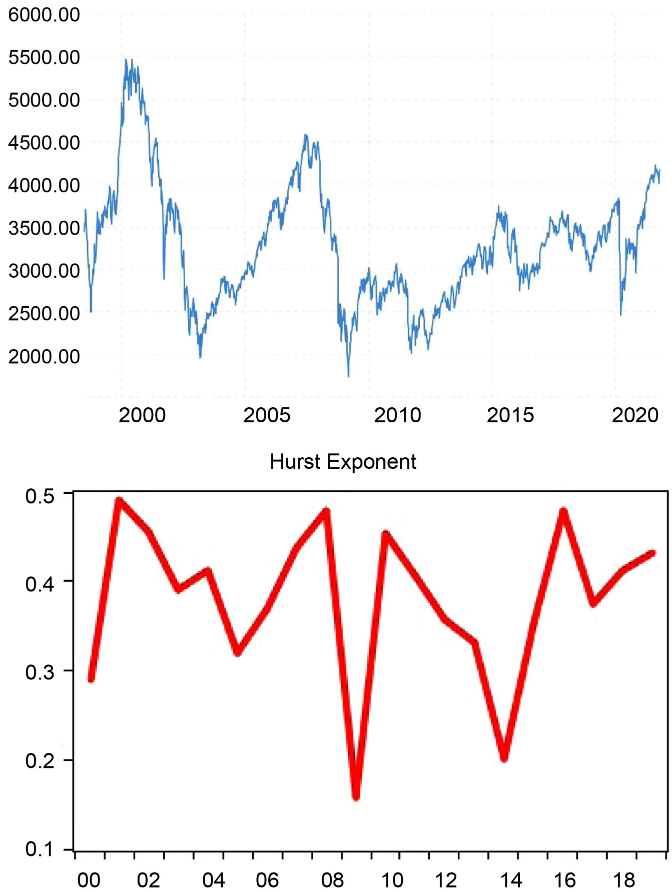

Figure 4. Eurostoxx 50 and Hurst Exponent: historical chart since the 2000s Source: authors' calculations in Eviews 10 based on Bloomberg data.

corresponds to an increase/decrease of approximately 98 basis points of the distribution skewness elasticity. This last result would enhance the underlying idea that, by making different hypotheses on the Hurst exponent within its habitat, it could further improve the simulation scenarios for the optimization of the volatility option pricing procedure. More, these dynamics further confirm how the Hurst exponent information can be used to evaluate potential near-term market excess in combination with the classical equity metrics (such as P/E, P/S, Dividend Yield, etc.).

5. Economic Discussion and Conclusions

From an empirical point of view, this paper suggests some interesting effects regarding the anti-persistence and mean-reversion characteristics of the historical price series of the volatility indices. This also suggests the existence of relationships between the variables relating to persistence and the moments of the distribution of the historical series of the VStoxx index, and more specifically the links between the trend of anti-persistence and kurtosis on the one hand and the excess of kurtosis and skewness elasticity on the other hand. In short, our results suggest that the Hurst Exponent can have an active role in signaling potential near-term market inversion points (see **Figure 4**).

More detailed results can possibly be obtained by making a hypothesis on the Hurst Exponent within its "habitat" for the option pricing procedure, and by developing the analysis on a rolling window, to have a more focused calibration of the time shift between the signals obtained from the Hurst exponent and the market trend inversion.

As it is possible to observe from **Figure 4**, high values of the Hurst exponent anticipated some important inversion points in the early 2000s, mid-2000s, and the local major top in 2017. Of course, it does not mean it represents the Holy Grail for anticipating major market tops, but it can be useful to exploit information from implied volatility structure and put it together with the classical fundamental and technical metrics to have further confirmation about the market trend progression.

The theoretical and empirical structure of this work does not have the ambition to be exhaustive but rather opens the door to various insights and themes of investigation in various directions. The evidence of the structural anti-persistence of the historical series of the returns of the VStoxx (as the main European volatility index) and the links between the Hurst exponent and the moments of distribution (in particular, skewness and kurtosis) leaves room for further study and fields of investigation.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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