High quality timetables for Italian schools

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Abstract

This work introduces a complex variant of the timetabling problem, which is motivated by the case of Italian schools. The new requirements enforce to (i) provide teachers with the same idle times, (ii) avoid consecutive days with heavy workload, (iii) limit multiple daily lessons for each class, (iv) introduce shorter time units to differentiate entry and exit times. We present an integer programming model for this problem, which is denoted by Italian High School Timetabling Problem (IHSTP). However, requirements (i), (ii), (iii) and (iv) cannot be expressed according to the current XHSTT standard. Since the IHSTP model is very hard to solve by an off-the-shelf solver, we present a two-step optimization method: the first step optimally assigns teachers to lesson times and the second step assigns classes to teachers. An extensive experimentation is performed on the model by realistic and real instances from Italian schools, as well as benchmark instances from the literature. Finally, the experiments show that the method is effective in solving both this new problem and the simplified problem without the new requirements.

Keywords*: Integer Programming, Timetabling, High School Timetabling.*

1 Introduction

The High School Timetabling problem (HST) is a relevant research area, which aims to schedule lectures to time slots. Its characteristics are country-dependent [16] and several solution approaches were proposed [18]. The introduction of the XHSTT (XML High School Time-Table) format for the HST problem has provided a uniform way to support a variety of possible constraints. However, new requirements have emerged in the case of Italian high schools and some of them do not fit with this format. This research is motivated by such a case.

The recent reforms in 2008 and 2015 deeply changed the educational structure of Italian high schools to make a service more oriented to students and decrease system costs [17]. The reduction in the weekly extension of lessons has led to an irregular distribution of lectures. Moreover, when two classes of the same year (or level) have few students, schools are requested to merge them into the so-called *articulated class*, even if students have different curricula. Therefore, the students of articulated classes may have few subjects in common and must be split when the different characterizing subjects are taught. In addition, full-time teachers must give lessons for 18 hours a week and, if this workload is not complete, they must be enrolled in other schools. Yet, some teachers may have additional days-off to account for possible additional duties.

These reforms increased the number of idle times for teachers, who claim that this number must be the same for all of them for the sake of equity. The new rules may also result in the planning of timetables with heavy workloads in consecutive days and lead to the burn-out of teachers. Moreover, it is recommended to schedule school days of same duration for a class to plan the transportation of students smoothly, even if this situation leads to an increase in the number of lessons. In addition, it is important to diversify the entrance and exit times of classes to limit crowds, as emphasized by the recent pandemic event.

Although relevant research exists in the HST problem for Italian schools [28, 1], it dates back in time and the recent changes in requirements were not taken into account. In this paper they are investigated and added to well-recognized requirements for the HST problem (e.g. assign teachers to classes, full-time and part-time teachers with one or more days off, surveillance in each class at any time slot.). The complete list of requirements is provided in Section 2. All in all, the new problem is denoted by the *Italian High-School Timetabling Problem* [IHSTP].

This paper presents an Integer Programming formulation for the [IHSTP], which is denoted by IHSTT. Since large-scale instances cannot be solved efficiently by an off-the-shelf optimization solver, we present a two-stage decomposition. In the first stage, we assign teachers to time slots in the so-called *Teacher Profile Problem* [TPP] through a MIP formulation denoted by TP. In the second stage, we solve a restricted version of IHSTT from the solution of TP. The overall method is denoted by TP-IHSTT. Since some of the new requirements are not supported by the XHSTT format, all models are implemented by a general-purpose modeling language and solved by a MIP (Mixed-Integer Programming) solver.

The two-step method is extensively tested in several instances, in order to assess to what extent it can be adopted. More precisely, in the first part of the experimentation we consider all requirements and compare the solutions of the MIP solver for the IHSTT and those provided by the two-step method, in which each sub-problem is solved by the same MIP solver. In the second part we focus on a simpler problem without the new features of [IHSTP] and compare the solutions of the MIP solver running IHSTT and the formulation proposed by [15], which is denoted by KSS. The KHE heuristic [12] [14] is also adopted to enrich the comparison. All variants are run without and with the TP step. In the second case, the methods are denoted by TP-KHE, TP-KSS and TP-IHSTT.

The experiments show the effectiveness of the two-step method, because it determines high-quality solutions for the problem at hand in terms of CPU times, costs and optimality gaps. Moreover, it is also effective for a simplified problem devoid of the new requirements: the method can be successfully applied both to KSS and IHSTT, but the results are far better in the latter case. Finally, IHSTT can effectively be used to solve some well-known benchmarks in the literature.

This paper is organized as follows. In section 2 the specific requirements for Italian high schools are presented. In section 3 the related work is critically discussed, to compare our problem with the case of other countries. In section 4 a complete Integer Programming formulation for Italian high schools is defined. In section 5 the two-step method is presented and the [TPP] is described and formulated. In section 6 experimental results are presented. In section 7 the conclusion is reported. Finally, appendices A, B and C report a table with slack variables, a glossary and complementary tables on the experimentation, respectively.

2 Italian High School Timetabling Problem

In Italian schools each student belongs to a class (or group of students) sharing the same lessons according to a *curriculum*. All students in a class must follow the same set of subjects for a fixed number of weekly hours. Lessons are daily organized in time slots (e.g. 1 hour, but fractional lesson units are also possible) and must be placed in a time horizon, which normally spans over a week and is repeated periodically for the entire school year. Lessons may span over multiple consecutive time slots, to accommodate special needs as in-class works or lab activities. These lessons are called multiple lessons (e.g. double and triple lessons).

Each subject is taught by a teacher or, more rarely, by a teacher and a co-presence teacher (or co-teacher), who has to teach always together with another colleague supervising the activity. From now on, for the sake of simplicity, teachers and co-teachers will be denoted as teachers, unless one refers explicitly to co-teachers and non-co-teachers.

Teachers may give lessons on more than one subject in one or more classes. Schools open from Monday to Saturday (very seldom until Friday) and teachers must have a day off for rest. They are classified as full-time and part-time teachers. Full-time teachers have to teach for a fixed number of hours a week (typically 18 hours, but some reductions are possible to do some management tasks) and must work in others schools to complete their workload. Part-time teachers have a shorter workload according to their annual contract and may work for several schools. As a result, some teachers must receive more than one day-off from each school. For the same reasons, some teachers may not be available to teach in some specific times.

Clearly, a teacher should not be employed for very few time slots a day. Conversely, a workload spanning all time slots in a day is not recommended, to prevent burn-out. Time-slot breaks are possible between lessons, even if they are not always required or appreciated.

The objective is to build a timetable, i.e. assign each lesson to a specific time slot of each day, such that a number of requirements are satisfied. They are divided into mandatory (or hard) and desirable (or soft). For the sake of clarity, in the following we enumerate all requirements and denote if they are hard or soft.

- R1 (hard) - Each class has to attend lessons for a given set of weekly days and a consecutive set of hours a day, as established by the school. For example, in a school all classes of the fifth year have to attend 32 hours a week and 5 hours a day, except on Tuesday and on Thursday, in which lessons are given for six hours. Every class of the second year has to attend lessons for 33 hours a week, in which the additional hour w.r.t. fifth year classes is given on Saturday.

- R2 (hard) - Every teacher has to teach for a fixed number of hours as established by national laws or school rules.

- R3 (hard) - Every teacher must have at least one day off a week. It can be determined according to two school-dependent policies: the day off can be *a priori* selected by the school or its decision is left *a posteriori* during timetable planning. Therefore, any methodological proposal must be able to deal with both policies.

- R4 (hard/soft) - A subset of teachers must/may receive additional days off according to specific conditions (e.g. employment in several schools, special contracts, additional administrative tasks, etc.). Unlike R3, these conditions affect whole days instead of specific daily parts.

- R5 (hard) - Since classes spend different time periods at school (on a daily and weekly basis), a lesson must be scheduled for a class only when the class is at school. For example, a fifth-year class cannot attend any lesson in the sixth hour on Saturday, if only five hours of lessons are scheduled for that class.

- R6 (hard) - A lesson must not be scheduled for a teacher in the case of specific commitments in specific periods of a day (e.g. employment in another school, special contracts, additional administrative tasks, etc.).

- R7 (hard) - Each class has to be taught by a given teacher for a fixed number of weekly hours. This number is called *week requirement* and is established by laws or school rules.

- R8 (hard) - A teacher-clash must be avoided: a teacher cannot teach simultaneously in two classes, unless they form an articulated class.

- R9 (hard) - A class-clash must be avoided: two teachers cannot teach the same class at the same time; the only exception is represented by the so-called co-teaching lessons (e.g. in some lab lessons).

- R10 (hard/soft) - The multiple lessons of a teacher in a class should be consecutive. It is important for multiple lessons of the same teacher in a class to be consecutive in a day. Clearly, a hard requirement for not splitting lessons could prevent the determination of a feasible timetable. As a result, both hard and soft options are possible. Moreover, consecutive lessons are welcome to have in-class works or written exams.

- R11 (hard) - For a limited number of hours, an articulated class must be divided into two or more groups attending different lessons with dedicated teachers. For example, a class could attend the lessons on the second foreign language with two different teachers at the same time: one for French and one for Spanish. The problem doubles in the case of co-teachers in articulated class: for example, if this class has two groups of students and the split groups must attend a lab lesson in co-teaching, four teachers must be involved with the class at the same time.

- R12 (hard) - Block lessons must be scheduled. These lessons take place at the same time for two or more classes, in order to share possible resources (e.g., gym or specialized language teachers). Blocks could also support the ordering of lessons by an optional offset, to enforce one lesson to precede another one in a class by a given number of periods.
- R13 (hard) - Preassigned lessons must be scheduled. In these lessons a teacher is already assigned to a class in a given period of a given day. They are often adopted when a teacher gives lessons for a short number of hours in a school.
- R14 (soft) - This requirement enforces a balanced distribution of the lessons among the workdays for a teacher in a class. This requirement can be denoted by horizontal distribution. For example, it holds for a teacher working for one hour on Monday, Tuesday, Thursday and Friday and for two hours on Wednesday (on Saturday no lessons are possible because the teacher must have a day-off).
- R15 (soft) - This requirement guarantees a balanced distribution among daily periods for a teacher in a class. This requirement can be referred to as vertical distribution. For example, it holds when a teacher gives lessons in a class no more than once in the first daily period, no more than once in the second daily period and so on.
- R16 (hard/soft) - Every teacher must/may give lessons in between a minimum and maximum number of *additional* days off. These numbers can be conveniently set to zero, if appropriate.
- R17 (soft) - Every teacher is willing to have a weekly timetable with no idle times between consecutive lessons. However, this requirement is often difficult to achieve in practice for every teacher.
- R18 (hard/soft) - The duration of each multiple lesson in a week must/may be limited between a minimum and a maximum number of periods.
- R19 (hard/soft) - Each teacher must/should not reach the maximum workload in two consecutive days.
- R20 (hard/soft) - A minimum and maximum number of double lessons must/should be scheduled in the week for some pairs of classes and teachers.
- R21 (hard/soft) - A minimum and maximum number of triple lessons must/should be scheduled in the week for some pairs of classes and teachers.
- R22 (hard/soft) - The timetable must/should avoid the occurrence of too many multiple lessons for a class in a particular day.
- R23 (hard/soft) - The daily number of periods of a teacher in a class must/should be in between a minimum and a maximum value.
- R24 (hard) - Fractional periods must be introduced to differentiate the times to start and end lessons for groups of classes. As a result, the duration of all lessons must be multiple quantities of this fractional time unit. This requirement could be hard and enforced for all classes, but it could also be ignored for all of them for the sake of equity.
- R25 (hard) - All teachers must have the same number of idle times. This requirement is set to be hard, because it must be enforced for all teachers or ignored for all of them for the sake of equity.

3 Related work

Several studies investigated the HST problem by Integer Programming. The problem characteristics are country-dependent and depend on the organizational model, which could be class-teacher (e.g. Australia, Bosnia, Brazil, Greece, Italy and South-Africa), course-based (e.g. USA) or a mix of them (e.g. Denmark, England, Finland and Netherlands). In the class-teacher model lessons are given to all students of a class, whereas in the course-based variant students attend lessons according to their individual plan, as in university course timetabling [2]. In the first case compact timetables are built from classes, which do not have idle times, whereas teachers typically have. In the second case, the timetable of teachers has no idle times, which can take place for students. This paper is in the area of class-teacher models and, to our knowledge, this is the first study investigating requirements R19, R22, R24 and R25.

Several constraints were defined in [23] on Bosnia and Herzegovina, but computational experiments are not provided. Moreover, it also neglects requirements on days off, obligation to take lessons (R5), balance of lessons spread in the week (R14, R15), teachers' idle times (R17) and limits on multiple daily lessons for classes (R18).

A lot of research was carried in Brazil on the so-called Class-Teacher Timetabling Problem with Compactness Constraints (CTTPC) ([24], [7], [8], [9], [25], [26], [27]).

Owing to the specific characteristics of Brazilian schools, these papers do not consider requirements on irregular weekly class layout (R5), articulated class (R11), block lessons (R12), balance of lessons spread in the week (R14, R15), multiple daily lessons limit for classes (R18) and restrictions on triple lessons (R21).

The Danish HST problem was described in [30] and [29]. However, they do not take into account requirements on weekly workload of teachers (R2), additional days off (R4), split lessons (R10), articulated classes (R11), balance of lessons spread in the week (R14, R15), multiple daily lessons limit for classes (R18), limits for number of double (R20) and triple lessons (R21), and restrictions on daily class-teacher workload (R23).

The French schools are investigated only in [19]. However, it ignores the requirements on articulated classes (R11), block lessons (R12), balance of lessons spread in the week (R14, R15), limited idle times (R17), multiple daily lessons limit for classes (R18), limits on number of double (R20) and triple lessons (R21), and class-teacher workload (R23). Experimental results were provided only for one instance and presented very synthetically.

The Greek schools are investigated in [3], [20], [4], [33] and [32]. Unlike in the Italian case, there are no requirements on lessons spread in the week with respect to daily periods (R15), limits for number of double (R20) and triple lessons (R21).

The HST problem was investigated in Italy by [28] and [1], who did not take into account the recent scholastic reforms. As a result, they could not consider the requirements on lessons spread in the week with respect to daily periods (R15), limits on the number of double (R20) and triple lessons (R21) and restrictions on the class-teacher workload (R23).

The HST problem was also generalized by [15] and [10] to support the XHSTT format and adopt Integer Programming formulations. Although the set of requirements is wide, it is not exhaustive for the Italian case.

Table 1 reports which problem requirements are faced in the most recent literature on HST problem. Column R13 reports only methods dealing with this requirement explicitly. Nevertheless, every heuristic or MIP formulation can deal with R13 by fixing proper decision variables, even if it is not explicitly stated.

Therefore, one can notice that requirements R19, R22, R24 and R25 have not been investigated so far. This paper covers this gap.

Year	Ref	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25
1997	[3]	X	X	X	X		X	X							X		X	X								
1999	[28]		X	X		X	X	X	X			X	X	X			X		X							
2003	[20]	X	X	X	X			X	X	X		X			X		X	X								
2007	[1]	X	X	X	X	X	X	X	X	X	X			X	X		X	X								
2009	[4]	X	X	X	X		X	X	X	X	X		X	X				X	X							
2012	[7]			X	X		X	X	X	X	X						X	X			X			X		
2012	[24]				X				X	X	X							X			X			X		
2012	[33]	X		X	X	X	X	X	X	X			X		X		X	X								
2014	[8]			X	X		X	X	X	X	X						X	X			X			X		
2014	[29]	X		X		X	X	X	X	X			X	X			X									
2015	[15]	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X		X		
2015	[23]	X	X				X	X	X	X	X	X	X	X			X				X	X		X		
2016	[9]			X	X		X	X	X	X	X						X	X			X	X		X		
2016	[19]	X	X	X		X	X	X	X	X	X	X					X									
2017	[10]	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X		X		
2017	[25]	X			X		X	X	X	X	X							X			X			X		
2018	[26]	X			X		X	X	X	X	X							X			X			X		
2020	[32]	X		X	X	X	X		X	X							X	X								
2020	[27]	X			X		X	X	X	X	X							X			X			X		

Table 1: Occurrence of the [IHSTP] requirements in the high school timetabling literature

Moreover, the current version of XHSTT (XHSTT-2014 [34]) is described in [22] and does not support the new requirements R22, R24 and R25. The implementation of requirement R19 is possible, but it requires much effort and has some limitations. More precisely, at the moment this requirement must be implemented with a different value for any pair of consecutive days and any teacher. As a result, it would be simpler and more effective to *a priori* enforce the maximum workload between consecutive days, in order to simplify the implementation and decrease the memory issues.

The timetabling XHSTT logic is based on *a priori* enumeration of variable length sub-events covering an event (week requirement). However, another logic is possible: the division of an event into sub-events with duration equal to 1 period. Since the second logic is expected to be less memory-consuming, it is of interest to develop a time-slot-based model and make a comparison to an event-based model. According to the example in [10], in an event with 4 periods, the KSS model can have 8 sub-events of different duration: 1, 1, 1, 1, 2, 2, 3 and 4. Conversely, in our model there are only 4 sub-events of 1 period each. Clearly this comparison is possible only in the problem without the new requirements. This paper investigates this comparison, as well.

Finally, high school timetabling is a challenging research area and different communities of researchers and practitioners have worked on benchmark instances of this problem. This is also shown by the organization of the Third International Timetabling Competition (ITC2011) [21]. We compare IHSTT to the Round 2 finalists, i.e. the evolutionary algorithm of team HFT [6] (Germany), the Simulated Annealing with Iterated Local Search of team GOAL [5], the hyper-heuristic [13] (UK), and the Adaptive Large neighborhood Search of team LECTIO [31]. Their programs were run for 10 times on each instance with different random number seeds and the solutions were ranked from the highest to the lowest.

4 Mathematical model for the [IHSTP]

In this section we present the mathematical formulation for the [IHSTP] for a single school. The formulation is based on the following sets: let C be the set of classes (or groups of students), T the set of teachers, $F \subseteq T$ the set of co-teachers, D the set of days, H the set of daily periods. N is the set of possible periods in multiple lessons, for example $N = \{2, 3\}$ implies that only 2-period or 3-period multiple lessons are allowed. Note that the duration of a single period is shorter than the length of a lesson in the case of multiple lessons and/or fractional time units.

The following parameters are defined. Let χ_{ct} be the number of weekly lessons for class $c \in C$ with teacher $t \in T$ (this number is typically called *timetable requirement*). Preassigned lessons are denoted by parameter π_{ctdh}, which takes value 1 if a lesson has to be scheduled on day $d \in D$ at period $h \in H$ for class $c \in C$ with teacher $t \in T$, 0 otherwise.

In order to handle block lessons, consider any two classes $c', c'' \in C$ and any two teachers $t', t'' \in T$, and define the quantity $\phi_{c't'c''t''}$, which denotes the number of lessons that must be located in the same time slot for teacher $t' \in T$ in class $c' \in C$ and teacher $t'' \in T$ and class $c'' \in C$. Let μ_{ctf} be the number of weekly lessons of both teacher $t \in T \backslash F$ and co-teacher $f \in F$ in class $c \in C$.

Let $\underline{\epsilon}_{ct}$ and $\overline{\epsilon}_{ct}$ the minimum and the maximum duration of a multiple lesson for class $c \in C$ with teacher $t \in T$, whereas $\underline{\zeta}_{ct}$ and $\overline{\zeta}_{ct}$ are its minimum and maximum occurrence of multiple lessons of class $c \in C$ with teacher $t \in T$ in the week. We also denote by θ_{ct} the penalty for the violation of multiple lessons of class $c \in C$ with teacher $t \in T$, denoting either the duration or the occurrence of the lesson on the basis of the instance requirement.

The following parameters are defined to link classes, days and periods. Let δ_{cdh} be a coefficient which takes value 1 if class $c \in C$ has to have a lesson on day $d \in D$ at period $h \in H$, 0 otherwise (note that, if $\delta_{cdh} = 0$, it is also possible for class $c \in C$ to have a lesson on day $d \in D$ at period $h \in H$). If a class $c \in C$ must not have a lesson on day $d \in D$ at period $h \in H$, the parameter β_{cdh} has value 0, and 1 otherwise.

The following parameters are defined to link teachers, days and periods. Let γ_{tdh} a boolean parameter with value 1 if teacher $t \in T$ is available to give a lesson on day $d \in D$ at period $h \in H$, 0 otherwise. According to the values of γ_{tdh}, one can easily detect the last assignable duty period ν_{td} for teacher $t \in T$ on day $d \in D$. In an assignable period a teacher must be available to teach even if his real activity depends on the timetabling. Moreover, teacher $t \in T$ must or could have in between $\underline{\alpha}_{td}$ and $\overline{\alpha}_{td}$ lessons on day $d \in D$.

The days-off of any teacher $t \in T$ are controlled by parameter τ_{ti}, in which index i takes integer values from 0 to 3. If $i = 0$, teacher $t \in T$ must have a day off on day $\tau_{t0} \in D \cup \{0\}$ (where 0 indicates a day off selected in the model solution); if $i = 1$, τ_{ti} represents the minimum number of additional days off of teacher $t \in T$ (since one day off must be guaranteed, the number of days off a week is at least $\tau_{t1} + 1$); if $i = 2$, τ_{ti} is the maximum number of additional days off for teacher $t \in T$ (hence, the number of days off a week is at most $\tau_{t2} + 1$); if $i = 3$, index τ_{ti} represents the (high) cost of violation of days off. Let $\tilde{D}_t$ be the singleton of the day off for teacher $t \in T$: $\tilde{D}_t = \{\tau_{t0}\}$.

The following parameters are defined to link teachers and classes. Teacher $t \in T$ must or could have in between $\underline{\rho}_{ct}$ and $\overline{\rho}_{ct}$ lessons with class $c \in C$.

In order to introduce a possible fractional time duration for all classes of a school, consider an integer positive parameter η, which represents the number of daily periods in a single lesson. For example, if the lesson takes 1 hour and the daily periods of set H represent 30 minutes, η takes value 2. Since some lessons cannot have a duration multiple of η, they need to be removed from the planning of fractional time units. As a result, define the set of incompatible periods $\tilde{N}_\eta = \{n \in N | (n \bmod \eta) \neq 0\}$.

The first decision variable is denoted by x_{ctdh}. It takes value 1 if class $c \in C$ is assigned to teacher $t \in T$ on day $d \in D$ at period $h \in H$, 0 otherwise. Note that $x_{ctdh} = 0$ if $d \in D \cap \tilde{D}_t$, $t \in T$, $c \in C$, $h \in H$. Clearly, this is the main decision variable, because its entries with value 1 define the timetable. The following auxiliary variables are also defined:

- a'_{td} is equal to 1 if at least one lesson of teacher $t \in T$ is scheduled on day $d \in D$, 0 otherwise;
- a''_{ctd} is equal to 1 if at least one lesson of teacher $t \in T$ and class $c \in C$ is scheduled on day $d \in D$, 0 otherwise;
- $b_{c't'c''t''dh}$ takes value 1 if teacher $t' \in T$ has a lesson on class $c' \in C$ and teacher $t'' \in T$ has a lesson on class $c'' \in C$ in the same period $h \in H$ of the same day $d \in D$, 0 otherwise;
- e_{ctfdh} is equal to 1 if teachers $t \in T$ and $f \in F$ have a lesson in class c on day $d \in D$ at period $h \in H$, 0 otherwise;
- m_{nctdh} is equal to 1 if a multiple lesson with duration $n \in N$ of teacher $t \in T$ starts at period $h \in H$ of day $d \in D$ in class $c \in C$, 0 otherwise;
- s^{min} are the minimum idle times for all teachers;
- s^{max} are the maximum idle times for all teachers;
- u'_{td} is the ordinal number of the first activity period of teacher $t \in T$ on day $d \in D$;
- v'_{td} is the ordinal number of the last activity period of teacher $t \in T$ on day $d \in D$;
- u''_{ctd} is the ordinal number of the first activity period of teacher $t \in T$ in class $c \in C$ on day $d \in D$;
- v''_{ctd} is the ordinal number of the last activity period of teacher $t \in T$ in class $c \in C$ on day $d \in D$.

For the sake of clarity, constraints are clustered in types depending on the requirements presented in Section 2. The link between constraints and requirements is reported in Table 2, which also reports a brief description of the types of constraints. The model will adopt slack variables also for hard constraints in order to make a comparison to the model by [15], where every constraint type could be hard or soft according to the specific instance at hand.

All constraints are described hereafter.

C_0 - Service constraints.

(1) $$a'_{td} \geq x_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$$

(2) $$\nu_{td} a''_{ctd} \geq \sum_{h \in H} x_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D$$

According to (1), any teacher $t \in T$ cannot be assigned to any class $c \in C$ in any period $h \in H$ of day $d \in D$ if he/she is not scheduled on this day. Constraint (2) enforces that any teacher $t \in T$ cannot be assigned to any period in class $c \in C$ on day $d \in D$ if he/she is not scheduled in this class on this day.

Requirements	Constraint	Description
-	C_0	Service constraints (required for the implementation of each requirement)
R1,R2,R7	C_1	Weekly requirement
R1	C_2	Class presence
R5,R9	C_3	Class unavailability
R6,R8	C_4	Teacher unavailability
R10	C_5	Split lessons
R3,R4	C_6	Days off
R9	C_7	Co-teaching
R11,R12	C_8	Block
R13	C_9	Pre-assigned lessons
R25	C_{10}	Equity in idle times
R17	C_{11}	Idle times
R18,R20,R21	C_{12}	Multiple lessons
R14	C_{13}	Horizontal distribution
R15	C_{14}	Vertical distribution
R16	C_{15}	Teacher workload restrictions
R23	C_{16}	Class/teacher workload restrictions
R22	C_{17}	Excessive multiple lessons
R19	C_{18}	Maximum workload
R24	C_{19}	Fractional time unit

Table 2: Grouping of requirements in types of constraints

C_1 **- Weekly requirement (R1, R2, R7, hard).** The sum of all lessons of teacher $t \in T$ in class $c \in C$ cannot differ from those required (χ_{ct}). Since the satisfaction of this hard constraint could not be guaranteed, a non-negative integer variable $s_{ct}^{C_1}$ is introduced. More formally,

$$(3) \quad \sum_{d\in D}\sum_{h\in H} x_{ctdh} - s_{ct}^{C_1} \leq \chi_{ct} \qquad \forall c \in C, \forall t \in T$$

$$(4) \quad \sum_{d\in D}\sum_{h\in H} x_{ctdh} + s_{ct}^{C_1} \geq \chi_{ct} \qquad \forall c \in C, \forall t \in T$$

(3) and (4) is similar to the analogous constraint introduced in [28].
Before introducing constraint types C_2 and C_3, it is worth noting that in each class there is at most a teacher $t \in T \setminus F$ and, if there is no teacher, the class cannot attend a lesson. These requirements can be directly enforced by the boolean parameters δ_{cdh} on class presence and β_{cdh} on class availability, in fact
$\delta_{cdh} \leq \sum_{t\in T\setminus F} x_{ctdh} \leq \beta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$
However, these constraints are not implemented as reported above, because we need to penalize their violation. Therefore, in what follows, we consider the constraints separately and introduce suitable auxiliary variables.
C_2 **- Class presence (R1, hard).** The following constraint enforces that each class must attend lessons in some periods and days of the weekly timetable:

$$(5) \quad \sum_{t\in T\setminus F} x_{ctdh} + s_{cdh}^{C_2} \geq \delta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$$

Note that the constraint holds despite the non-negative integer variable, because a teacher could be assigned to a class in a daily period, even if the class does not have to attend a lesson in that period. Clearly, this situation must not be penalized unlike in the converse case.
C_3 - **Class unavailability (R5, R9, hard).** The following constraint enforces that a class could attend lessons in some periods and days only if it is available in these periods and days of the weekly timetable:

$$(6) \quad \sum_{t\in T\setminus F} x_{ctdh} - s_{cdh}^{C_3} \leq \beta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$$

Note that the constraint holds despite the non-negative integer variable, because it is possible to have the availability of a class in a period of a day, but no teacher is assigned to the class. Clearly, this situation must not be penalized unlike in the converse case.

C_4 **- Teacher unavailability (R6, R8, hard).** Excluding the case of articulated classes, teacher $t \in T$ cannot be assigned to more than one class in each period of each day, i.e. $\sum_{c\in C} x_{ctdh} \leq 1$. The (un)availability of teachers is controlled by the boolean parameter γ_{tdh} and we must penalize the assignment of teachers when they are not available. Therefore, we introduce a boolean variable $s^{C_4}_{tdh}$, which takes value 1 of this critical situation occurs, 0 otherwise. Therefore, this constraint can be formulated as follows:

$$(7) \quad \sum_{c\in C} x_{ctdh} - s^{C_4}_{tdh} \leq \gamma_{tdh} a'_{td} \qquad \forall t \in T, \forall d \in D, \forall h \in H$$

C_5 **- Split lessons (R10, hard/soft).** Multiple lessons of any teacher $t \in T$ in class $c \in C$ must be consecutive on any day $d \in D$ (or without splits). This constraint can be enforced in period $h \in H$ by an upper bound of value h on the period of the first lesson and a lower bound of value h on the period of the last lesson for teacher $t \in T$ in class $c \in C$ on day $d \in D$, if this teacher is on duty in this class on this day. If $x_{ctdh} = 0$, these bounds must not be effective. More formally,

$$(8) \quad u''_{ctd} \leq (|H|+1)-(|H|+1-h)x_{ctdh} \qquad \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in H$$

$$(9) \quad v''_{ctd} \geq h x_{ctdh} \qquad \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in H$$

However, one must still link the time interval between the first and last teaching period to the number of lessons of a teacher in a day. The boolean variable $s^{C_5}_{ctd}$ is introduced to detect the split lessons of teacher $t \in T$ in class $c \in C$ on day $d \in D$ when it takes value 1, 0 otherwise. More formally,

$$(10) \quad a''_{ctd}+v''_{ctd}-u''_{ctd} \leq \sum_{h\in H} x_{ctdh}+s^{C_5}_{ctd}(|H|-2) \qquad \forall c\in C, \forall t\in T, \forall d\in D$$

A minor change in these constraints will be reported later to handle idle times.

C_6 **- Days off (R3, R4, hard/soft).** The overall number of days off must be in between the minimum and the maximum values, which are $1 + \tau_{t1}$ and $1 + \tau_{t2}$ for teacher $t \in T$, respectively. A non-negative integer variable $s^{C_6}_t$ is introduced to report how many times these constraints are not satisfied for teacher $t \in T$. Therefore,

$$(11) \quad 1 + \tau_{t1} - s^{C_6}_t \leq |D| - \sum_{d\in D} a'_{td} \qquad \forall t \in T$$

$$(12) \quad |D| - \sum_{d\in D} a'_{td} \leq 1 + \tau_{t2} + s^{C_6}_t \qquad \forall t \in T$$

C_7 **- Co-teaching (R9, hard).** Co-teaching cannot be performed either when the class or the teacher or the co-teacher are not available in a daily period.

$$(13) \quad e_{ctfdh} \leq \beta_{cdh}\cdot\gamma_{tdh}\cdot\gamma_{fdh}\cdot x_{ctdh} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F, \forall d\in D, \forall h\in H$$

$$(14) \quad e_{ctfdh} \leq \beta_{cdh}\cdot\gamma_{tdh}\cdot\gamma_{fdh}\cdot x_{cfdh} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F, \forall d\in D, \forall h\in H$$

Moreover, one must guarantee exactly μ_{ctf} co-teaching lessons in a week and a possible violation must be taken into account. Therefore, we introduce a non-negative integer variable $s^{C_7}_{ctf}$, which is an excess or lack of lessons for class $c \in C$ with teacher $t \in T$ and co-teacher $f \in F$:

$$(15) \quad \sum_{d\in D}\sum_{h\in H} e_{ctfdh}+s^{C_7}_{ctf} \geq \mu_{ctf} \qquad \forall c\in C, \forall t\in T\setminus F, \forall f\in F$$

$$(16) \quad \sum_{d\in D}\sum_{h\in H} e_{ctfdh}-s^{C_7}_{ctf} \leq \mu_{ctf} \qquad \forall c\in C, \forall t\in T\setminus F, \forall f\in F$$

C_8 **- Block lessons (R11, R12, hard).** Block lessons cannot be performed either when the first class or the second class or their teachers are not available in a daily period:

$$(17) \quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot x_{c't'dh} \qquad \forall c', c''\in C, \forall t', t''\in T, \forall d\in D, \forall h\in H$$

$$(18) \quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot x_{c''t''dh} \qquad \forall c', c''\in C, \forall t', t''\in T, \forall d\in D, \forall h\in H$$

Moreover, one must guarantee exactly $\phi_{c't'c''t''}$ block lessons in a week and a possible violation must be taken into account. Therefore, we introduce a non-negative integer variable $s^{C_8}_{c't'c''t''}$, which is an excess or lack of block lessons for classes $c', c'' \in C$ with teachers $t', t'' \in T$:

$$(19)\quad \sum_{d\in D}\sum_{h\in H} b_{c't'c''t''dh} + s^{C_8}_{c't'c''t''} \geq \phi_{c't'c''t''} \qquad \forall c', c'' \in C, \forall t', t'' \in T$$

$$(20)\quad \sum_{d\in D}\sum_{h\in H} b_{c't'c''t''dh} - s^{C_8}_{c't'c''t''} \leq \phi_{c't'c''t''} \qquad \forall c', c'' \in C, \forall t', t'' \in T$$

In the case of articulated classes, one could represent a teacher by an alias (i.e. the pair of teachers t' and t'' represent the same person).

C_9 **- Preassigned lessons (R13, hard).** The lessons of teacher $t \in T$ in class $c \in C$ have to be scheduled in period $h \in H$ of day $d \in D$ when the boolean parameter π_{ctdh} takes value 1. Since lessons could also be scheduled when π_{ctdh} is 0, the satisfaction of preassigned lessons can be enforced by

$$(21)\quad x_{ctdh} \geq \pi_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$$

However, the sake of consistency with the other constraints, the former expression is presented by a boolean variable $s^{C_9}_{ctdh}$, which takes value 1 only if the compulsory lesson of class $c \in C$ with teacher $t \in T$ in period $h \in H$ of day $d \in D$ is not scheduled:

$$(22)\quad x_{ctdh} + s^{C_9}_{ctdh} \geq \pi_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$$

C_{10} **- Equity in idle times (R25, hard).** The same number of idle times among all teachers can be pursued by the minimization of the difference between the maximum and the minimum idle times among all teachers. Therefore, one needs to introduce a new non-negative integer variable $s^{C_{10}} = s^{max} - s^{min}$ and minimize its value. Clearly, the number of idle times of each teacher must be in between s^{min} and s^{max} in the weekly planning horizon. More formally,

$$(23)\quad s^{min} + s^{C_{10}} = s^{max}$$

$$(24)\quad \sum_{d\in D}(v'_{td} + a'_{td} - u'_{td} - \sum_{c\in C}\sum_{h\in H} x_{ctdh}) \leq s^{max} \qquad \forall t \in T$$

$$(25)\quad \sum_{d\in D}(v'_{td} + a'_{td} - u'_{td} - \sum_{c\in C}\sum_{h\in H} x_{ctdh}) \geq s^{min} \qquad \forall t \in T$$

C_{11} **- Idle times (R17, soft).** The idle times of teacher $t \in T$ on day $d \in D$ can be derived from the first and the last activity daily period in a way similar to constraints C_5 on split lessons. More precisely, we compute for each period daily and teacher a lower bound on the last activity period and an upper bound on the first activity period. Their difference must be limited above by the number of lessons given by teacher $t \in T$ over all classes in a day. In order to guarantee the satisfaction of this constraint, an additional non-negative integer variable $s^{C_{11}}_{td}$ is introduced to report how many times a idle time occurs for teacher $t \in T$ over all periods $h \in H$ of day $d \in D$. Clearly, this variable will be minimized in this formulation. Therefore:

$$(26)\quad u'_{td} \leq (|H|+1) - (|H|+1-h)\sum_{c\in C} x_{ctdh} \qquad \forall t \in T, \forall d \in D, \forall h \in H$$

$$(27)\quad v'_{td} \geq h \sum_{c\in C} x_{ctdh} \qquad \forall t \in T, \forall d \in D, \forall h \in H$$

$$(28)\quad a'_{td} + v'_{td} - u'_{td} \leq \sum_{c\in C}\sum_{h\in H} x_{ctdh} + s^{C_{11}}_{td} \qquad \forall t \in T, \forall d \in D$$

C_{12} **- Multiple lessons (R18, R20, R21, hard/soft).** Consider a multiple lesson of length n starting in period 1 for teacher $t \in T$ in class $c \in C$ on day $d \in D$. As a consequence, in period $n+1$, $x_{ctd(n+1)}$ must take value 0. More formally:

$$(29)\quad \sum_{i=1}^{n} x_{ctdi} + 1 - x_{ctd(n+1)} \leq n + m_{nctd1} \qquad \forall n \in N \setminus \{|H|\}, \forall c \in C, \forall t \in T, \forall d \in D$$

Hence, m_{nctd1} must take value 1 when n consecutive lessons are followed by a period with no lesson between teacher t and class c.

If the multiple lesson of length n is scheduled in the last periods of a day, the former constraint is modified as follows:

$$(30)\quad 1-x_{ctd(\nu_{td}-n)}+\sum_{i=1}^{n} x_{ctd(\nu_{td}-n+i)} \leq n+m_{nctd(\nu_{td}-n+1)} \qquad \forall n\in N\setminus\{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D$$

The following constraint introduces for the special case in which multiple lessons span over all periods in a day:

$$(31)\quad \sum_{h\in H} x_{ctdh} \leq n-1+m_{nctd1} \qquad \forall n \in \{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D$$

We still need to introduce the case of multiple lessons starting after the first period and ending before the last one:

$$(32)\quad 1-x_{ctd(h-1)}+\sum_{i=1}^{n} x_{ctd(h+i-1)}+1-x_{ctd(h+n)} \leq n+1+m_{nctdh} \; \forall n\in N\setminus\{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in\{2..(\nu_{td}-n)\}$$

Sometimes the minimum ($\underline{\zeta}_{ct}$) and the maximum ($\overline{\zeta}_{ct}$) number of multiple lessons of predefined length (ranging between $\underline{\epsilon}_{ct}$ and $\overline{\epsilon}_{ct}$) must be considered for some teachers in some classes. The weekly number of multiple lessons can be computed by variable m_{nctdh}, but an additional non-negative integer variable $s_{ct}^{C_{12}}$ must be adopted to compute the number of violations for teacher $t \in T$ in class $c \in C$:

$$(33)\quad \sum_{n=\underline{\epsilon}_{ct}}^{\overline{\epsilon}_{ct}} \sum_{d\in D} \sum_{h=1}^{|H|+1-n} m_{nctdh} + s_{ct}^{C_{12}} \geq \underline{\zeta}_{ct} \qquad \forall c\in C, \forall t\in T$$

$$(34)\quad \sum_{n=\underline{\epsilon}_{ct}}^{\overline{\epsilon}_{ct}} \sum_{d\in D} \sum_{h=1}^{|H|+1-n} m_{nctdh} - s_{ct}^{C_{12}} \leq \overline{\zeta}_{ct} \qquad \forall c\in C, \forall t\in T$$

A congruence check of m_{nctdh} is needed: the sum of all lessons (multiple or single) must be equal to the week requirement

$$(35)\quad \sum_{n\in N} \sum_{d\in D} \sum_{h=1}^{|H|+1-n} (n\cdot m_{nctdh}) = \chi_{ct} \qquad \forall c\in C, \forall t\in T$$

C_{13} **- Horizontal distribution (R14, soft).** The lessons of a teacher in a class should not be clustered either in the first part or in the second part of a week. If the weekly number of lessons χ_{ct} of teacher $t \in T$ in class $c \in C$ is even, we enforce to have the same number of lessons in the two parts of the week; if χ_{ct} is odd, their difference should be one. Since this ideal balance could not be guaranteed in both previous cases, a non-negative integer variable $s_{ct}^{C_{13}}$ is introduced to report the difference between the periods of teacher $t \in T$ in class $c \in C$ in the two parts of the week. More formally:

$$(36)\quad \sum_{d=1}^{\lfloor |D|/2 \rfloor} \sum_{h\in H} x_{ctdh} - \sum_{d=\lceil |D|/2 \rceil+1}^{|D|} \sum_{h\in H} x_{ctdh} - s_{ct}^{C_{13}} \leq \lceil \frac{\chi_{ct}}{2} \rceil - \lfloor \frac{\chi_{ct}}{2} \rfloor \qquad \forall c \in C, \forall t \in T$$

$$(37)\quad \sum_{d=\lceil |D|/2 \rceil+1}^{|D|} \sum_{h\in H} x_{ctdh} - \sum_{d=1}^{\lfloor |D|/2 \rfloor} \sum_{h\in H} x_{ctdh} - s_{ct}^{C_{13}} \leq \lceil \frac{\chi_{ct}}{2} \rceil - \lfloor \frac{\chi_{ct}}{2} \rfloor \qquad \forall c \in C, \forall t \in T$$

Note that both of these constraints hold when $|D|$ is even or odd.

C_{14} **- Vertical distribution (R15, soft).** The number of lessons of teacher $t \in T$ in class $c \in C$ should not be clustered in a specific period $h \in H$ over all days of the weekly planning horizon. Therefore, we enforce an upper bound $\lceil \frac{\chi_{ct}}{|H|} \rceil$ and a lower bound $\lfloor \frac{\chi_{ct}}{|H|} \rfloor$ on the number of lessons scheduled for any teacher in any class in a given period. Since these bounds could be violated, a non-negative integer variable $s^{C_{14}}_{cth}$ is defined to report how many times they are not met for teacher $t \in T$ in class $c \in C$ in period $h \in H$. Therefore,

$$(38) \quad \sum_{d \in D} x_{ctdh} - s^{C_{14}}_{cth} \leq \lceil \frac{\chi_{ct}}{|H|} \rceil \qquad \forall c \in C, \forall t \in T, \forall h \in H$$

$$(39) \quad \sum_{d \in D} x_{ctdh} + s^{C_{14}}_{cth} \geq \lfloor \frac{\chi_{ct}}{|H|} \rfloor \qquad \forall c \in C, \forall t \in T, \forall h \in H$$

Due to (38) and (39) the lessons must have a balanced distribution over all daily periods.

C_{15} **- Teacher workload restrictions (R16, hard/soft).** The number of activity periods of each teacher $t \in T$ in any day $d \in D$ must be in between the lower bound $\underline{\alpha}_{td}$ and the upper bound $\overline{\alpha}_{td}$, if at least a lesson is scheduled for teacher $t \in T$ on day $d \in D$ (this is checked by the values of variable a'_{td}). Since this situation should not occur, the non-negative integer variable $s^{C_{15}}_{td}$ is defined to report how many times the violation occurs.

$$(40) \quad \sum_{c \in C} \sum_{h \in H} x_{ctdh} - \eta s^{C_{15}}_{td} \leq a'_{td} \overline{\alpha}_{td} \qquad \forall t \in T, d \in D$$

$$(41) \quad \sum_{c \in C} \sum_{h \in H} x_{ctdh} + \eta s^{C_{15}}_{td} \geq a'_{td} \underline{\alpha}_{td} \qquad \forall t \in T, \forall d \in D$$

Note that the parameter η accounts for the possible use of fractional periods.

C_{16} **- Class/teacher workload restrictions (R23, hard/soft).** The number of daily activity periods of each teacher $t \in T$ with class $c \in C$ must be in between the lower bound $\underline{\rho}_{ct}$ and the upper bound $\overline{\rho}_{ct}$, if at least a lesson is scheduled for teacher $t \in T$ with class $c \in C$ on day $d \in D$ (this is checked by the values of variable a''_{ctd}). Since this situation should not occur, the non-negative integer variable $s^{C_{16}}_{ctd}$ is defined to report how many times the violation occurs.

$$(42) \quad \sum_{h \in H} x_{ctdh} - \eta s^{C_{16}}_{ctd} \leq a''_{ctd} \overline{\rho}_{ct} \qquad \forall c \in C, \forall t \in T, \forall d \in D$$

$$(43) \quad \sum_{h \in H} x_{ctdh} + \eta s^{C_{16}}_{ctd} \geq a''_{ctd} \underline{\rho}_{ct} \qquad \forall c \in C, \forall t \in T, \forall d \in D$$

Note that the parameter η accounts for the possible use of fractional periods.

C_{17} **- Excessive multiple lessons (R22, hard/soft).** The number of periods with multiple daily lessons for class $c \in C$ on day $d \in D$ cannot be larger that a threshold value, which can be reasonably set to $\lceil \frac{|H|-1}{2} \rceil$ (e.g. half of the periods in a day, when $|H|$ is even). Since this situation should not occur, the non-negative integer variable $s^{C_{17}}_{cd}$ is defined to report how often it occurs.

$$(44) \quad \sum_{n \in N} \sum_{t \in T \setminus F} \sum_{h=1}^{|H|-(n-1)} n \cdot m_{nctdh} - s^{C_{17}}_{cd} \leq \lceil \frac{|H|-1}{2} \rceil \qquad \forall c \in C, \forall d \in D$$

Note that n allows to consider duration of multiple lessons, which cannot take the trivial length of one period. Note that co-teachers are not involved in (44) because they always work with other teachers.

Yet, multiple lessons with duration n cannot start after period $|H| - (n-1)$.

C_{18} **- Teacher maximum workload (R19, hard/soft).** The workload of teacher $t \in T$ in any two consecutive days $d \in D$ and $(d+1) \in D$ must take value one period less than the sum of maximum workload in these days (i.e. $\overline{\alpha}_{td} + \overline{\alpha}_{t(d+1)}$). The non-negative integer variable $s^{C_{18}}_{td}$ is introduced to quantify the violation of this requirement.

$$(45) \quad \sum_{c \in C} \sum_{h \in H} (x_{ctdh} + x_{ct(d+1)h}) - s^{C_{18}}_{td} \leq \overline{\alpha}_{td} + \overline{\alpha}_{t(d+1)} - 1 \qquad \forall t \in T, \forall d \in D \setminus \{|D|\}$$

Although the implementation of this constraint is possible in the standard XHSTT format, in common real cases we should replace it by several hundreds of constraints *Limit Busy Times*, when teachers and days are 100 and 6 respectively.

C_{19} **- Fractional time unit (R24, hard).** When fractional time units are possible, the duration of lessons must be multiple of parameter η. Since this condition may not always be met, a non-negative integer variable $s_{nct}^{C_{19}}$ is defined to report how often it is not satisfied.

$$(46)\quad \sum_{d\in D}\sum_{h=1}^{|H|+1-n} m_{nctdh} - s_{nct}^{C_{19}} = 0 \qquad \forall n\in\tilde{N}_\eta, \forall c\in C, \forall t\in T$$

The objective function is a linear combination of the violation of all types of constraints. Let o_i the overall violation of i-th constraint type and ω_i its weight. More formally, the violation of each constraint type is reported below:

$$o_1 = \sum_{c\in C}\sum_{t\in T} s_{ct}^{C_1} \quad o_2 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s_{cdh}^{C_2} \quad o_3 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s_{cdh}^{C_3} \quad o_4 = \sum_{t\in T}\sum_{d\in D}\sum_{h\in H} s_{tdh}^{C_4}$$

$$o_5 = \sum_{c\in C}\sum_{t\in T}\sum_{d\in D} s_{ctd}^{C_5} \quad o_6 = \sum_{t\in T} \tau_{t3} s_t^{C_6} \quad o_7 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s_{cdh}^{C_7}$$

$$o_8 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s_{cdh}^{C_8} \quad o_9 = \sum_{c\in C}\sum_{t\in T}\sum_{d\in D}\sum_{h\in H} s_{ctdh}^{C_9} \quad o_{10} = s^{C_{10}} \quad o_{11} = \sum_{t\in T}\sum_{d\in D} s_{td}^{C_{11}}$$

$$o_{12} = \sum_{c\in C}\sum_{t\in T} \theta_{ct} s_{ct}^{C_{12}} \quad o_{13} = \sum_{c\in C}\sum_{t\in T} s_{ct}^{C_{13}} \quad o_{14} = \sum_{c\in C}\sum_{t\in T}\sum_{h\in H} s_{cth}^{C_{14}} \quad o_{15} = \sum_{t\in T}\sum_{d\in D} s_{td}^{C_{15}}$$

$$o_{16} = \sum_{c\in C}\sum_{t\in T}\sum_{d\in D} s_{ctd}^{C_{16}} \quad o_{17} = \sum_{c\in C}\sum_{d\in D} s_{cd}^{C_{17}} \quad o_{18} = \sum_{t\in T}\sum_{d\in D\setminus\{|D|\}} s_{td}^{C_{18}} \quad o_{19} = \sum_{n\in N}\sum_{c\in C}\sum_{t\in T} s_{nct}^{C_{19}}$$

Hence, the objective function of IHSTT is:

$$(47)\quad f = \sum_{i=1}^{19} \omega_i o_i$$

The complete MIP model consists in minimizing f, subject to constraints (1)-(46). This model is very complex to solve as it is and, according to preliminary experiments, no significant gain is obtained by the removal of slack variables for hard constraints. Moreover, these variables help guarantee a full compatibility with [15], where every constraint type could be hard or soft according to the specific instance at hand. A two-step method is proposed in the following section to solve the problem.

5 A two-step method for the [IHSTP]

Since the model for [IHSTP] is expected to be very hard to solve, we present a two-step method to determine high-quality solutions within a reasonable time interval. The method is motivated by many possibilities for selecting the activity periods of each teacher, who gives lessons in a class for a limited number of periods w.r.t the the overall number of periods spent by students in the same class (e.g. a teachers must stay in a class for 4 hours a week and the same class attends lessons for 32 hours a week). Therefore, the [IHSTP] would be simplified if one *a priori* knows the schedule of teachers without details on the classes taught in each period.

Therefore, the proposed method decomposes [IHSTP] into two problems:

- The first problem assigns teaching periods to teachers to determine the so-called teacher profile. This problem is called Teacher Profile Problem [TPP].
- The second problem assign classes to teachers according to the solution of the [TPP] and results in a simplified version of the [IHSTP], which is called restricted [IHSTP] and denoted by [RIHSTP].

The details about the mathematical formulations of these problems are provided in Section 5.1 and Section 5.2. Figure 1 shows the connection between [TPP] and [RIHSTP].

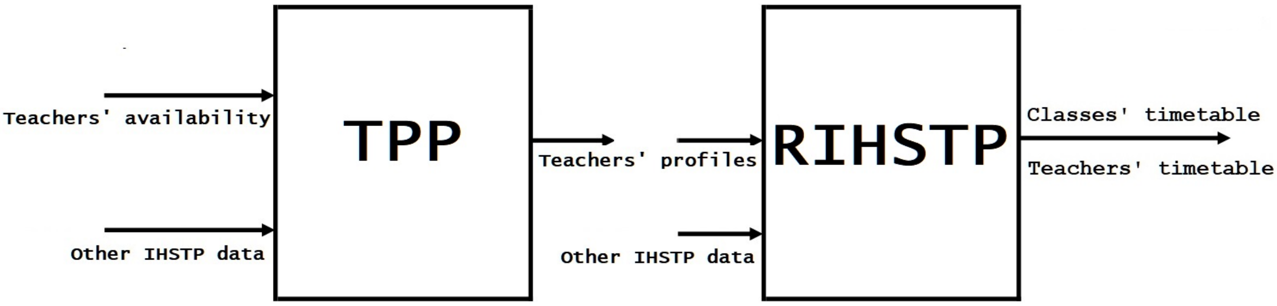

Figure 1: The connection between [TPP] and [RIHSTP]

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5.1 The teacher profile problem [TPP]

5.1.1 Problem description

Relevant data for the [TPP] are the periods in which teachers are available and lessons are given for each class in each day. Classes may not spend the same number of periods at school, because usually the number of school days in a week is not an exact divider of the overall week requirements (e.g. 33 hours over 6 days from Monday to Saturday). Schools have two choices to face this situation: fixing in advance which days have extra periods or letting this decision to the optimization phase. In the first case, parameters β_{cdh} and δ_{cdh} take the same value for all the weekly periods; in the second case they differ when the extra daily periods occur. Generally speaking, Italian schools prefer the first choice, because it results in a greater management control. Moreover, it would not be possible to determine the work shifts of the teachers if the attendance periods of classes at school are not known. Since the teacher profile is determined before the final timetable, in what follows the values of β_{cdh} and δ_{cdh} are supposed to be identical.

We aim to obtain a subset of the profiles for each teacher who is not a co-teacher (or teacher profile), while taking into account some requirements of the [IHSTP], but their determination must be computationally viable. Clearly, the periods in a (non-co-)teacher profile must be consecutive in a day, in order to a priori minimize idle times. In the [TPP] we consider daily profiles in which all teachers either start in the first period or end in the last period. This assumption decreases the number of possible profiles and is also motivated by equity issues. In fact, teachers starting in the second period have an edge over those starting in the first one, because they wake up later and come across less congested roads in their trips. Similarly, teachers ending in the last hour are more tired than those ending before and can go home later. Therefore, two possible shifts are considered: the first shift starts in the first period, the last shift ends in the last period. Note that the profiles of co-teachers are not determined in the [TPP], because they may end up working with teachers with different profiles and it may be impossible to satisfy all the requirements at the same time.

Figure 2 shows an example on the construction of a profile. The teacher has a day off on Wednesday and is available to teach from period 1 to period 6 in the other days. Assume to select in the first shift 3 periods on Monday, 4 periods on Tuesday and 3 periods on Saturday (a). In the last shift assume to select 2 periods on Monday, 3 periods on Thursday and 3 periods on Friday (b). The shifts can be merged and result in the final teacher profile (c). Although the profile of Monday has one idle period, it is acceptable owing to the relevant workload in this day. Note that this profile also satisfies the horizontal and vertical distribution, as defined in requirements R15 and R16.

In what follows, we enumerate all requirements of the profiles (or shifts).

1. **R26** (Shift selection). For non-co-teachers, the first and/or the second shift could be selected in a day.

2. **R27** (Duration of shifts). The length of shifts cannot be larger than the daily availability and teachers cannot be on duty in days which are not selected.

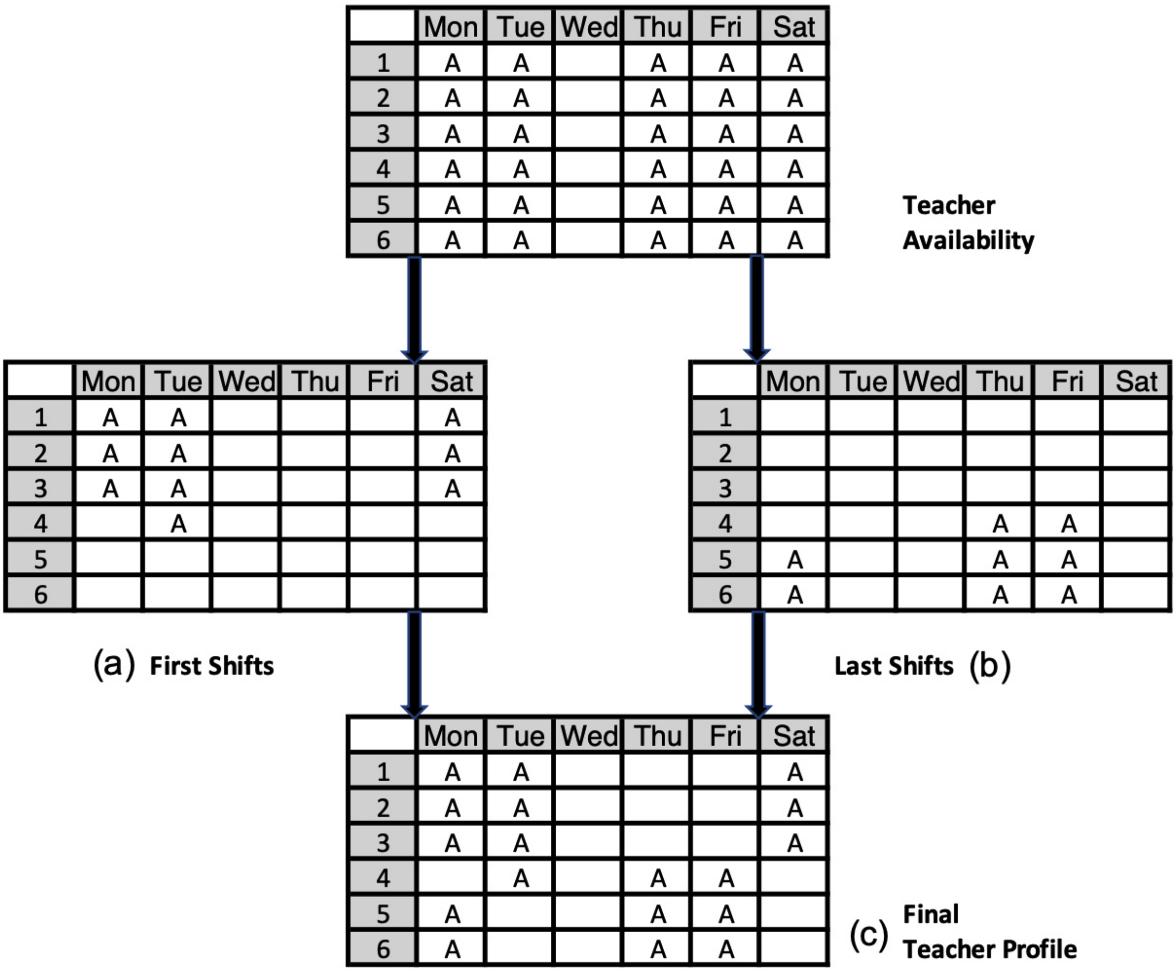

Figure 2: An example of optimized Teacher Profile

3. **R28** (Allocation of periods to shifts): Non-co-teachers must be on duty for all periods in a shift, if it is selected.

4. **R29** (Teacher profile definition). A period is part of a teacher profile if and only if the first shift or the second shift are selected.

5. **R30** (Profile consistency). Teachers cannot be assigned to profiles with periods in which they are not available. Moreover, the daily profiles cannot be selected in days off.

6. **R31** (Class surveillance). The profile of teachers must guarantee that each class is monitored by one of its non-co-teachers in each daily period.

7. **R32** (Alternated shifts). The profiles of teachers should encourage the alternation between the first and the last shift between any pair of consecutive days to incentivize a good vertical distribution possibly.

Finally, we need to restate a number of requirements of the [IHSTP] in terms of teacher profiles. More precisely, these requirements concern day off (**R33**, soft), additional days off (**R34**, soft), pre-assigned lessons (**R35**, hard), horizontal distribution (**R36**, hard), vertical distribution (**R37**, hard), block (**R38**, hard), fractional time unit (**R39**, hard), teacher workload restrictions (**R40**, soft), idle times (**R41**, soft), equity in idle times (**R42**, soft).

5.1.2 Optimization model

The [TPP] formulation is based on the notation already presented for the [IHSTP]. However, some additional notation needs to be introduced. Let ψ^c be a $|T\backslash F|$-column-vector, in which ψ^c_t takes value 1 if teacher $t \in T\backslash F$ teaches in class $c \in C$, and 0 otherwise. Let $\psi^{c\top}$ be the transpose of ψ^c. Moreover, let L_c be the set of classes with some teachers in common with class $c \in C$, including class c itself. It is possible to compute L_c from ψ^c as follows:
$L_c = \{c' \in C | \psi^{c\top}\psi^{c'} > 0\}$

Let y_{tdh} be a decision variable, which takes value 1 if the teacher $t \in T$ is on duty in day $d \in D$ at period $h \in H$, 0 otherwise. Clearly, y_{tdh} is the main decision variable of the [TPP], because all entries with value 1 represent the profile of teacher $t \in T$. The following variables of the IHSTT are also used with the same meaning in [TPP] model: $a'_{td}, b_{c't'c''t''dh}, u'_{td}, v'_{td}, s^{min}, s^{max}$. In addition, the following auxiliary variables are defined:

f_{tdh} 1 if $h \in H$ is the last period of the first shift of teacher $t \in T$ on day $d \in D$, 0 otherwise;
l_{tdh} 1 if $h \in H$ is the first period in last shift of teacher $t \in T$ on day $d \in D$, 0 otherwise;
n'_{td} length of the first shift of teacher $t \in T$ on day $d \in D$;
n''_{td} length of the last shift of teacher $t \in T$ on day $d \in D$;
$\tilde{m}_{ntdh}$ is equal to 1 if a block of duration $n \in N$ of teacher $t \in T$ starts at period $h \in H$ of day $d \in D$ in one of the shifts, 0 otherwise.

HC_1 - **Shift selection (R26, hard)**. The following constraint states that the first shift could be selected for any teacher in each day:

$$(48) \quad \sum_{h=1}^{\nu_{td}} \gamma_{tdh} f_{tdh} \leq 1 \qquad \forall t \in T\backslash F, \forall d \in D$$

Note that the first shift includes all periods between the first one and time slot such that f_{tdh} has value 1. A similar constraint is formulated for the last shift:

$$(49) \quad \sum_{h=2}^{\nu_{td}} \gamma_{tdh} l_{tdh} \leq 1 \qquad \forall t \in T\backslash F, \forall d \in D$$

Clearly, the last shift includes all periods between the time slot for which l_{tdh} has value 1 and the last one.

HC_2 - **Duration of shifts (R27, hard)**. The following constraints determine the duration of shifts for each teacher in each day from the values of variables f_{tdh} and l_{tdh} :

$$(50) \quad n'_{td} = \sum_{h \in H} h \gamma_{tdh} f_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D$$

$$(51) \quad n''_{td} = \sum_{h \in H} (\nu_{td} + 1 - h) \gamma_{tdh} l_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D$$

Moreover, (52) ensures that a'_{td} takes value 1 when teacher $t \in T$ works on day $d \in D$ and, in this case, the duration of duty shifts are bounded by suitable values.

$$(52) \quad n'_{td} + n''_{td} \leq a'_{td} \sum_{h \in H} \gamma_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D$$

In a workday at least one lesson has to be given by a teacher:

$$(53) \quad n'_{td} + n''_{td} \geq a'_{td} \qquad \forall t \in T\backslash F, \forall d \in D\backslash \tilde{D}_t$$

HC_3 - **Allocation of periods to shifts (R28, hard)**. The following constraints link variable y_{tdj} to f_{tdh} and l_{tdh}:

$$(54) \quad y_{tdj} \geq f_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D, \forall h \in H, j \in \{1, .., h\}$$

$$(55) \quad y_{tdj} \geq l_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D, \forall h \in H, j \in \{h, .., \nu_{td}\}$$

HC_4 - **Teacher profile definition (R29, hard)**.
The values of y_{tdh} are computed in the following constraint:

$$(56)\quad y_{tdh} = \sum_{i=h}^{\nu_{td}} f_{tdi} + \sum_{i=2}^{h} l_{tdi} \qquad \forall t \in T \setminus F, \forall d \in D, \forall h \in H$$

Note that three cases may occur: period $h \in H$ does not belong to any shift, or it is part of the first shift or part of the second shift.

HC_5 - **Profile consistency (R30, hard)**. Teachers cannot be assigned to profiles with periods in which they are not available. Moreover, profiles cannot be assigned to days which are not selected to give lessons:

$$(57)\quad y_{tdh} \le a'_{td}\gamma_{tdh} \qquad \forall t \in T \setminus F, \forall d \in D, \forall h \in H$$

HC_6 - **Class surveillance (R31, hard)**. Since no class should be left unattended, for each daily period the number of classes must be equal to the number of teachers:

$$(58)\quad \sum_{t \in T \setminus F} y_{tdh} = \sum_{c \in C} \delta_{cdh} \qquad \forall d \in D, \forall h \in H$$

The previous constraint does not guarantee that each class is attended by one of its teachers (for the sake of simplicity, we do not consider co-teachers). This is possible only if the sets of classes and teachers represent one partition or can be decomposed in several partitions (i.e when the subset of teachers gives lessons only in a subset of classes in a partition and vice versa). Therefore, we need to recall the definition of L_c from the values of ψ_t^c, to report the partition associated with class $c \in C$. If $L_c \equiv C$, there is one partition, else there are at least two partitions. Therefore, for each period of any day and class, a balance must be guaranteed between the number of teachers of the class and the number of classes in the same partition, provided that the class is available:

$$\sum_{t \in T \setminus F : \psi_t^c = 1} y_{tdh} = \delta_{cdh} |L_c|$$

However, some classes may not be available in the same daily periods. As a result, the former formula is modified as follows:

$$(59)\quad \sum_{t \in T \setminus F : \psi_t^c = 1} y_{tdh} = \sum_{c' \in L_c} \delta_{c'dh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$$

HC_7 - **Day off selection (R33, hard)**. Day off must be guaranteed for each teacher:

$$(60)\quad a'_{td} = 0 \qquad \forall t \in T \setminus F, \forall d \in D \cap \tilde{D}_t$$

HC_8 - **Preassigned lessons (R35, hard)**. Preassigned lessons must be scheduled

$$(61)\quad y_{tdh} \ge \pi_{ctdh} \qquad \forall c \in C, \forall t \in T \setminus F, \forall d \in D, \forall h \in H$$

(61) is very similar to (21) in constraint C_9 of the IHSTT.

HC_9 - **Horizontal distribution (R36, hard)**. Unlike in the [IHSTP], in the [TPP] the horizontal distribution of lessons is enforced on the overall activity of each teacher without paying attention to classes:

$$(62)\quad \sum_{d=1}^{|D|/2} \sum_{h \in H} y_{tdh} + \left\lceil \frac{\sum_{c \in C} \chi_{ct}}{2} \right\rceil - \left\lfloor \frac{\sum_{c \in C} \chi_{ct}}{2} \right\rfloor \ge \sum_{d=|D|/2+1}^{|D|} \sum_{h \in H} y_{tdh} \qquad \forall t \in T \setminus F$$

$$(63)\quad \sum_{d=1}^{|D|/2} \sum_{h \in H} y_{tdh} - \left\lceil \frac{\sum_{c \in C} \chi_{ct}}{2} \right\rceil + \left\lfloor \frac{\sum_{c \in C} \chi_{ct}}{2} \right\rfloor \le \sum_{d=|D|/2+1}^{|D|} \sum_{h \in H} y_{tdh} \qquad \forall t \in T \setminus F$$

Note that (62)-(63) are similar to (36)-(37) in C_{13} of the IHSTT.

HC_{10} - **Vertical distribution (R37, hard)**. The same logic holds for the vertical distribution:

$$(64)\quad \sum_{d \in D} y_{tdh} \le \left\lceil \frac{\sum_{c \in C} \chi_{ct}}{|H|} \right\rceil \qquad \forall t \in T \setminus F, \forall h \in H$$

$$(65)\quad \sum_{d \in D} y_{tdh} \ge \left\lfloor \frac{\sum_{c \in C} \chi_{ct}}{|H|} \right\rfloor \qquad \forall t \in T \setminus F, \forall h \in H$$

Clearly, (64)-(65) are similar to (38)-(39) in C_{13} of the IHSTT.
HC_{11} - **Block (R38, hard)**. Constrains on block lessons are enforced.

(66) $b_{c't'c''t''dh} \leq \beta_{c'dh} \cdot \beta_{c''dh} \cdot \gamma_{t'dh} \cdot \gamma_{t''dh} \cdot y_{t'dh}$ $\qquad \forall c', c'' \in C, \forall t', t'' \in T \backslash F, \forall d \in D, \forall h \in H$

(67) $b_{c't'c''t''dh} \leq \beta_{c'dh} \cdot \beta_{c''dh} \cdot \gamma_{t'dh} \cdot \gamma_{t''dh} \cdot y_{t''dh}$ $\qquad \forall c', c'' \in C, \forall t', t'' \in T \backslash F, \forall d \in D, \forall h \in H$

(68) $\sum_{d \in D} \sum_{h \in H} b_{c't'c''t''dh} = \phi_{c't'c''t''dh}$ $\qquad \forall c', c'' \in C, \forall t', t'' \in T \backslash F$

Note that (66)-(68) exhibit minor changes w.r.t. (17)-(20) in constraint C_8 of the IHSTT.

HC_{12} - **Fractional time unit (R39, hard)**. The duration of both shifts of every teacher must be a multiple quantity of the fractional time unit η.

(69) $\sum_{h \in H} y_{tdh} \leq n-1+\tilde{m}_{ntd1}$ $\qquad \forall n \in \{|H|\}, \forall t \in T \backslash F, \forall d \in D$

(70) $\sum_{i=1}^{n} y_{tdi} + (1-y_{td(n+1)}) \leq n+\tilde{m}_{ntd1}$ $\qquad \forall n \in N, \forall t \in T \backslash F, \forall d \in D$

(71) $1-y_{td(h-1)} + \sum_{i=1}^{n} y_{td(h+i-1)} + 1 - y_{td(h+n)} \leq n+1+\tilde{m}_{ntdh}$ $\qquad \forall n \in N, \forall t \in T \backslash F, \forall d \in D, \forall h \in \{2,..,(\nu_{td}-n)\}$

(72) $1-y_{td(\nu_{td}-n)} + \sum_{i=1}^{n} y_{td(\nu_{td}-n+i)} \leq n+\tilde{m}_{ntd(\nu_{td}-n+1)}$ $\qquad \forall n \in N, \forall t \in T \backslash F, \forall d \in D$

(73) $\sum_{d \in D} \sum_{h=1}^{\nu_{td}+1-n} \tilde{m}_{ntdh} = 0$ $\qquad \forall n \in \tilde{N}_{\eta}, \forall t \in T \backslash F$

Moreover, (69)-(72) compute the length of every shift, while (73) guarantees that each shift must have a length multiple of η. Clearly, these constraints can be skipped if $\eta = 1$.

SC_1 - **Alternated shifts (R32, soft)**. The first shift and last shift are recommended to be alternate in consecutive days.

(74) $1 - s_{tdh}^{SC_1} \leq y_{tdh} + y_{t(d+1)h}$ $\qquad \forall t \in T \backslash F, \forall d \in D \backslash \{|D|\}, \forall h \in \{1, \nu_{td}\}$

(75) $1 - s_{tdh}^{SC_1} \geq y_{tdh} - y_{t(d+1)h}$ $\qquad \forall t \in T \backslash F, \forall d \in D \backslash \{|D|\}, \forall h \in \{1, \nu_{td}\}$

(76) $1 - s_{tdh}^{SC_1} \geq y_{t(d+1)h} - y_{tdh}$ $\qquad \forall t \in T \backslash F, \forall d \in D \backslash \{|D|\}, \forall h \in \{1, \nu_{td}\}$

(77) $1 - s_{tdh}^{SC_1} \leq 2 - y_{tdh} - y_{t(d+1)h}$ $\qquad \forall t \in T \backslash F, \forall d \in D \backslash \{|D|\}, \forall h \in \{1, \nu_{td}\}$

SC_2 - **Days off placement (R34, soft)**. It is recommended to provide additional days off to each teacher:

(78) $\sum_{d \in D} a'_{td} + 1 + \tau_{t1} - s_t^{SC_2} \leq |D|$ $\qquad \forall t \in T \backslash F$

(79) $\sum_{d \in D} a'_{td} + 1 + \tau_{t2} + s_t^{SC_2} \geq |D|$ $\qquad \forall t \in T \backslash F$

Note that these constraints enforce the assignment of days off, when they were not indicated by teachers.
SC_3 - **Teacher workload restrictions (R40, soft)**. The following constraints play the same role of those in C_{15}, where $\sum_{c \in C} x_{ctdh}$ is replaced by y_{tdh}:

(80) $\sum_{h \in H} y_{tdh} + \eta s_{td}^{SC_3} \geq a'_{td} \underline{\alpha}_{td}$ $\qquad \forall t \in T \backslash F, \forall d \in D$

(81) $\sum_{h \in H} y_{tdh} \leq a'_{td} \overline{\alpha}_{td} + \eta s_{td}^{SC_3}$ $\qquad \forall t \in T \backslash F, d \in D$

SC_4 - **Idle times (R41, soft)**. The following constraints play the same role of those in C_{11}, where $\sum_{c \in C} x_{ctdh}$ is replaced by y_{tdh}:

(82) $u'_{td} \leq (\nu_{td}+1) - (\nu_{td}+1-h) y_{tdh}$ $\qquad \forall t \in T \backslash F, \forall d \in D, \forall h \in H$

$$(83)\quad v'_{td} \geq h{\cdot}y_{tdh} \qquad \forall t \in T\backslash F, \forall d \in D, \forall h \in H$$

$$(84)\quad a'_{td} + v'_{td} - u'_{td} \leq \sum_{h \in H} y_{tdh} + s^{SC_4}_{td} \qquad \forall t \in T\backslash F, \forall d \in D$$

SC_5 - **Equity in idle times (R42, soft)**. It is recommended for the teachers to have the same minimum idle times:

$$(85)\quad \sum_{d \in D} s^{SC_4}_{td} \leq s^{max} \qquad \forall t \in T\backslash F$$

$$(86)\quad \sum_{d \in D} s^{SC_4}_{td} \geq s^{min} \qquad \forall t \in T\backslash F$$

$$(87)\quad s^{min} + s^{SC_5} \geq s^{max}$$

Teacher Profile Problem objective function

The objective function of the [TPP] is the sum of all constraint deviation multiplied by a proper weight:

$$(88)\quad f = \omega_1 \sum_{t \in T\backslash F} \sum_{d \in D} \sum_{h \in \{1,\nu_{td}\}} s^{SC_1}_{tdh} + \omega_3 \sum_{t \in T\backslash F} s^{SC_3}_{t} + \omega_4 \sum_{t \in T\backslash F} \sum_{d \in D} s^{SC_4}_{td} + \omega_4 \sum_{t \in T\backslash F} \sum_{d \in D} s^{SC_4}_{td} + \omega_5 s^{SC_5}$$

The complete formulation of the [TPP] consists in minimizing f, subject to constraints (48)-(87)

5.2 The restricted [IHSTP]

This problem is obtained by replacing γ_{tdh} with values of y_{tdh} in IHSTT, as determined in the solution of the [TPP]. This substitution occurs in constraints (7), (13)-(14), (17)-(18). All in all, the two-step method is supposed to be effective owing to the larger number of null entries of y_{tdh} as opposed to γ_{tdh}. The real effectiveness of the method will be evaluated in the following experimentation.

Figure 3 shows how the solution of the [TPP] can be adopted to obtain a possible solution of the restricted [IHSTP] for teacher A, who must give lessons in classes denoted by 3C, 4C and 5C. For example, according to the TPP, teacher A must give lessons on Monday from period 1 to period 3 and from period 5 to period 6. The restricted [IHSTP] assigns the selected work periods of teacher A to each class. In Figure 3, teacher A is assigned to class 3C from period 1 to period 2, class 5C in period 3, class 4C from period 5 to period 6.

	Mon	Tue	Wed	Thu	Fri	Sat
1	A	A				A
2	A	A				A
3	A	A				A
4		A		A	A	
5	A			A	A	
6	A			A	A	

	Mon	Tue	Wed	Thu	Fri	Sat
1	3C	5C				4C
2	3C	5C				4C
3	5C	4C				3C
4		3C		5C	4C	
5	4C			5C	3C	
6	4C			3C	5C	

Figure 3: A teacher timetable obtained as the solution of [RIHSTP] program from the Teacher Profile

6 Experimentation

6.1 Experimental settings and results

This experimentation has several objectives. First, we aim to show to what extent the IHSTT can be solved to tackle specific and realistic instances of Italian high schools, where requirements R19, R22, R24 and R25 are taken into account. Second, we want to assess how much the two-step method is effective both in terms of running time and objective function, and how long the computation takes in each step. Third, we face a simplified setting without R19, R22, R24 and R25, and compare the results of a MIP solver to solve the IHSTT, the KSS model ([15], [10]), as well as the outcomes from the KHE heuristic[1]. Fourth, in the simplified setting, we extend and evaluate the teacher profile phase to the KSS model and the KHE heuristic, to have a deeper understanding on the effectiveness of the two-step method. Fifth, we perform a "stress test" on the viability of the IHSTT model by complex instances coming from 4 real Italian schools. Moreover, IHSTT is also tested on some well-known benchmark instances taken from the ITC2011 competition. Last but not least, we run IHSTT on some well-known benchmark instances taken from XHSTT datasets with short and long running times to evaluate solution quality and speed, respectively.

The KHE is a well-known freeware open-source C program, that implements an advanced heuristic described in [14] and supports the XHSTT format. Although KHE does not have any time limit for optimization, the option of multiple separate threads can be introduced to obtain better solutions. Clearly, the KSS model supports XHSTT format and can be solved by any MIP solver, but additional implementations were performed to process the main decision variables in order to scale to larger problems instances.

The experimentation is organized according to two experimental settings, which differ for which constraints are hard, soft or disabled. These settings are called *Setting1* and *Setting2*. For the sake of clarity we denote by *Setting1* the experimentation with requirements R19, R22, R24 and R25, whereas in *Setting2* they are ignored. Therefore, *Setting1* represents the current case of Italian schools and *Setting2* is the simplified problem. Table 3 reports the types of constraints and the methods run in these settings.

Constraints	Requirements	*Setting1* IHSTT/TP-IHSTT	*Setting2* KHE/KSS/IHSTT/TP-KHE/TP-KSS/TP-IHSTT
C1	R1,R2,R7	Hard	Hard
C2	R1	Hard	Hard
C3	R5,R9	Hard	Hard
C4	R6,R8	Hard	Hard
C5	R10	Soft/Hard/Disabled	Soft/Hard/Disabled
C6	R3,R4	Hard	Hard
C7	R9	Hard/Disabled	Hard/Disabled
C8	R11,R12	Hard/Disabled	Hard/Disabled
C9	R13	Hard/Disabled	Hard/Disabled
C10	**R25**	**Hard**	**Disabled**
C11	R17	Soft	Soft
C12	R18,R20,R21	Soft/Hard	Soft/Hard
C13	R14	Soft	Disabled
C14	R15	Soft	Disabled
C15	R16	Hard	Hard
C16	R23	Hard	Hard
C17	**R22**	**Soft**	**Disabled**
C18	**R19**	**Soft**	**Disabled**
C19	**R24**	**Disabled/Hard**	**Disabled**

Table 3: Types of constraints in the two experimental settings.

The values of coefficients ω_i in the objective function of IHSTT indicate whether the i-th constraint is hard, soft or disabled. They can be set by schools according to their policies. When two options are

[1] *http://jeffreykingston.id.au/khe/*

reported (e.g. hard and disabled) in Table 3, some instances consider one option and other instances the other option. In this experimentation, hard constraints have values of ω_i equal to $100,000$. Soft-constraints have values of ω_i much lower than $100,000$ and typically range between 1 and 100. If i-th constraint is not used (or disabled), the corresponding value of ω_i is 0. The details about the values are reported for each instance in tables 14 and 15 of Appendix C.

In *Setting1* we compare the solutions provided by a MIP solver on IHSTT and TP-IHSTT. The outcomes of *Setting1* are reported in Table 5; in addition, the value of slack variables are reported in Table 16 and Table 17. In *Setting2* we compare the KHE heuristic, the same MIP solver running KSS and IHSTT, as well as TP-KHE, TP-KSS, TP-IHSTT. These outcomes are reported in Table 6.

Although XHSTT benchmark instances exist in the literature, 24 new specific instances are built for this experimentation in order to capture all novelties arising in this problem setting. They describe real situations or realistic conditions according to expert-based opinions. The first 20 instances are grouped according to their size: in the first group, instances are denoted from 1 to 9 and their size ranges from small to medium; in the second group, instances are denoted from 10 to 20 and their size ranges from medium to huge. Finally the last 4 instances are grouped according to their complexity.

Instance 1 comes from the timetable data of a *real* middle school with 5 daily periods. Instance 2 is more realistic for high-schools, because it has 6 daily periods. Instance 3 adds to Instance 2 co-teaching with one full-time teacher and one part-time teacher. Instance 4 is more complex than the previous ones, because it features 3 articulated class (one teacher must teach two class in the same time) and blocks. Instance 5 represents a school in which classes have 5 hours and half every day, to have uniform entrance and exit times among all school classes. Instance 6 and Instance 8 introduce fractional time units (of 30 and 15 minutes, respectively), to allow splitting the entrance of students (in two or four groups, respectively), reducing overcrowding. This is an emerging issue owing to the COVID-19 pandemic. Instance 7 is quite realistic, because every class has 12 teachers in a week. Instance 9 is more complex and larger than Instance 7: it features 18 classes and teachers with variable week requirements ranging from 2 to 4 time slots. The second group (instances between 10 and 20) is generated using a common block with 12 teachers and 6 classes with fixed week requirement (3 periods). Instances 21 and 22 are provided by mid-schools and have up to 24 classes. Instance 23 has 42 classes and come from a high-school with scientific, classic and linguistic curriculum. Instance 24 describes a technical school with 40 classes with several curricula, articulated classes and labs.

The most important problem data of each instance are reported in Table 4, where *Requir. (1)* indicates the number of timetable requirements, Δ the duration of a single period (in minutes), *CoTea (2)* number of lessons in co-teaching, *Artic. (3)* number of lessons in articulated classes, *Blocks (4)* number of block lessons. The data files of instances in *Setting2* are also available in XHSTT format[2], whereas this is not possible for *Setting1*, because some constraints of [IHSTP] are not supported in XHSTT standard. In this experimentation, we adopt the modeling language IBM OPL to call the MIP solver CPLEX 20.1 for implementing and solving all models. All the experiments are performed on a computer with an Intel I5-4460 3.20 GHz 4-core CPU equipped with 32 GBytes of DDR3 RAM and 1 TBytes SSD drive running Ubuntu 20.04 LTS. The time limit is 3 hours.

Table 5 focuses on *Setting1* and is organized into three groups of columns. The first column lists the instances, the second group reports the outcomes of the IHSTT, the third group shows the results of TP-IHSTT. For example, in instance 5 the TP is solved in 81.4 seconds and the overall two-step method in 304.7 seconds. All the instances cannot be solved by the IHSTT within the time limit.

Two quality measures are reported in the results: the objective value (denoted by *Obj*) and the mean of the idle times (denoted by *Idle times*). For example, according to the solution of the IHSTT, in instance 16 this value is 5.7 hours, but it is obtained at the time limit, when constraint C10 (*Equity Idle Times*) is not satisfied. The same instance is effectively solved by the TP-IHSTT, which returns a much lower value of idle times for all teachers.

Two types of gaps are reported. The column *Gap* indicates the relative difference between *Obj* and the lower bound L_B returned by the MIP solver. It is computed as:

$$(89)\quad Gap = \left[100\frac{Obj - L_B}{L_B}\right]$$

When $L_B = 0$, no value of *Gap* is reported. Moreover, the reported value of *Gap* is ∞ when hard

[2] *https://github.com/ClaudioCrobu/IHSTP*

constraints are not satisfied. This gap is not reported in the group of columns TP-IHSTT, because it takes always value zero.

The column *LBGap* indicates the relative difference between *Obj* and the lower bound $\overline{L}_B$ at the root node after CPLEX cuts:

$$(90) \quad LBGap = \left\lceil 100 \frac{Obj - \overline{L}_B}{Obj} \right\rceil$$

The best outcomes are emphasized in bold.

Instance	$\lvert C\rvert$	$\lvert T\rvert$	*Requir. (1)*	$\lvert D\rvert$	$\lvert H\rvert$	$\lvert D\rvert \cdot \lvert H\rvert$	$\lvert C\rvert \cdot \lvert D\rvert \cdot \lvert H\rvert$	η	Δ	$\underline{\rho}$	$\overline{\rho}$	$\underline{\alpha}$	$\overline{\alpha}$	*CoTea (2)*	*Artic (3)*	*Blocks (4)*
1	5	13	50	6	5	30	150	1	60'	1	2	2	5	-	-	-
2	3	6	18	6	6	36	108	1	60'	1	2	3	5	-	-	-
3	3	8	36	6	6	36	108	1	60'	1	2	3	5	21	-	3
4	6	6	36	6	6	36	216	1	60'	1	2	3	5	-	18	90
5	3	6	18	6	11	66	198	1	30'	2	7	3	10	-	-	-
6	3	6	18	6	12	72	216	2	30'	2	4	6	10	-	-	-
7	6	12	72	6	6	36	216	1	60'	1	2	3	5	-	-	-
8	3	6	18	6	24	144	432	4	15'	4	8	4	12	-	-	-
9	18	36	216	6	6	36	648	1	60'	1	1	3	5	-	-	648
10	18	36	216	6	6	36	648	1	60'	1	1	3	5	-	-	-
11	24	48	288	6	6	36	864	1	60'	1	1	3	5	-	-	-
12	30	60	360	6	6	36	1080	1	60'	1	1	3	5	-	-	-
13	36	72	432	6	6	36	1296	1	60'	1	1	3	5	-	-	-
14	42	84	504	6	6	36	1512	1	60'	1	1	3	5	-	-	-
15	42	85	504	6	6	36	1512	1	60'	1	1	3	5	18	-	-
16	48	96	576	6	6	36	1728	1	60'	1	1	3	5	-	-	-
17	54	108	648	6	6	36	1944	1	60'	1	1	3	5	-	-	-
18	60	120	720	6	6	36	2160	1	60'	1	1	3	5	-	-	-
19	78	156	936	6	6	36	2808	1	60'	1	1	3	5	-	-	-
20	156	312	1872	6	6	36	5616	1	60'	1	1	3	5	-	-	-
21	23	44	230	6	5	30	690	1	60'	1	2	1	5	-	-	-
22	24	44	288	5	6	30	720	1	60'	1	2	1	5	-	-	-
23	42	79	491	6	6	36	1512	1	60'	1	3	1	5	-	-	-
24	40	103	697	6	6	36	1440	1	60	1	4	1	6	270	30	30

Table 4: Description of the Italian schools' instances

Table 6 pertains to *Setting2* and is organized into eight groups of columns. The first group lists the instances, the following six groups report the outcomes of KHE, KSS, IHSTT, TP-KHE, TP-KSS and TP-IHSTT, the last group shows the common time of the first phase of the 2-phase methods.

In order to make a fairer comparison on KHE, it was used with the option of parallel threads. We have run KHE with different values of threads and reported the best one in column *Threads*. For the first group of instances 1, 10, 100 and 1000 threads number were used; the option with 1000 threads was not used for the second group of instances because of a memory problem. We remark that the concept of threads in KHE is different from the one adopted in CPLEX which corresponds to the CPU cores. Since KHE does not compute a lower bound, in the computation of *Gap*, this is replaced by the best upper bound computed by the other methods. The string MEM means that the computer's available memory was insufficient for building the instance.

In the last row of Table 6 the average rank of each method is reported. It is computed by assigning value 1 to the minimum objective function in the group, value 2 to the second and so on, but in case of equality the method with the minimum time is considered. According to this logic, the best results are emphasized in bold.

Setting1	*IHSTT*					*TP-IHSTT*				
Instance	*Time*	*Obj*	*Gap*	*Idle times*	*LBGap*	*Time*	*Obj*	*Idle times*	*LBGap*	*TP time*
1	TL	2625	2917	0.9	100	**40.5**	**87**	**0.0**	**0**	19.5
2	TL	1970	228	2.3	100	**4.1**	**600**	**1.0**	**0**	1.8
3	TL	2364	224	2.1	100	**160.9**	**730**	**0.9**	**4**	1.4
4	TL	2436	121	1.8	100	**60.0**	**1100**	**1.0**	**0**	45.6
5	TL	2610	226	3.8	100	**304.7**	**800**	**1.0**	**25**	81.4
6	TL	3904	225	5.7	100	**8.8**	**1200**	**2.0**	**0**	7.0
7	TL	3615	198	2.5	100	**42.1**	**1215**	**1.0**	**0**	0.1
8	TL	8688	262	11.3	100	**234.9**	**2400**	**4.0**	**0**	145.8
9	TL	21600	454	4.7	100	**270.2**	**3840**	**1.0**	**0**	22.1
10	TL	21600	500	4.8	100	**21.7**	**3600**	**1.0**	**0**	6.4
11	TL	33600	600	5.0	100	**27.0**	**4800**	**1.0**	**0**	8.7
12	TL	42000	600	5.4	100	**45.9**	**6000**	**1.0**	**0**	11.3
13	TL	64800	800	5.6	100	**38.3**	**7200**	**1.0**	**0**	13.6
14	TL	75600	800	5.4	100	**46.2**	**8400**	**1.0**	**0**	16.3
15	TL	75600	800	5.3	100	**86.7**	**8400**	**1.0**	**0**	16.2
16	TL	86409	800	5.7	100	**50.9**	**9600**	**1.0**	**0**	21.9
17	TL	108012	900	5.3	100	**57.7**	**10800**	**1.0**	**0**	29.0
18	TL	146189	1118	5.5	100	**272.8**	**12000**	**1.0**	**0**	42.3
19	TL	175146	1023	5.4	100	**75.2**	**15600**	**1.0**	**0**	27.6
20	TL	379861	1118	5.1	100	**2499.1**	**31203**	**1.0**	**0**	183.0

Table 5: Results of *Setting1* (idle times are expressed in hours, all remaining times in seconds; TL = Time Limit = 10800 seconds)

Setting2	*KHE*			*KSS*			*IHSTT*			*TP-KHE*			*TP-KSS*			*TP-IHSTT*		*TP*
Instance	*Time*	*Obj*	*Threads*	*Time*	*Obj*	*Gap*	*Time*	*Obj*	*Gap*	*Time*	*Obj*	*Threads*	*Time*	*Obj*	*Gap*	*Time*	*Obj*	*Time*
1	350.2	4000	1000	TL	200	-	593.1	**0**	0	195.2	12000	1000	6.9	0	0	**5.6**	**0**	3.6
2	388.8	(5,1200)	1000	TL	**600**	0	TL	**600**	0	408.1	(6,600)	1000	**2.7**	**600**	0	2.8	600	1.6
3	339.2	(3,900)	1000	6550.1	**600**	0	TL	700	17	214.1	(4,600)	1000	2.8	600	0	**2.5**	**600**	1.4
4	784.4	(11,2500)	1000	**2335.6**	**1100**	0	TL	1100	0	447.7	(15,1100)	1000	**67.2**	**1100**	0	78.3	1100	64.7
5	476.1	3600	1000	TL	(1,3600)	∞	TL	**800**	33	569.1	(20,1300)	1000	**75.0**	**600**	0	82.6	600	64.8
6	1059.0	(4,1800)	1000	TL	**1200**	0	TL	**1200**	0	622.1	(5,1200)	1000	9.3	1200	0	**7.6**	**1200**	6.6
7	1195.3	1300	1000	TL	1400	17	7645.1	**1200**	0	406.6	1200	1000	**5.5**	**1200**	0	15.5	1200	0.1
8	2821.1	(4,5600)	1000		MEM		TL	**2400**	0	2164.3	(6,2400)	1000		MEM		170.6	**2400**	165.3
9	2951.6	**3900**	1000	TL	4800	33	TL	4200	17	104.1	3600	100	20.6	3600	0	**16.5**	**3600**	14.5
10	342.1	4000	100	TL	7000	94	TL	**3900**	8	107.0	3600	100	20.3	3600	0	**16.9**	**3600**	14.2
11	406.7	5400	100	TL	(3,13500)	∞	TL	**5300**	10	136.7	4800	100	21.4	4800	0	**17.3**	**4800**	12.7
12	490.1	6700	100	TL	(5,17500)	∞	TL	**6500**	8	181.5	6000	100	27.8	6000	0	**21.2**	**6000**	16.1
13	693.2	8300	100		MEM		TL	**7900**	10	190.3	7200	100		MEM		**25.6**	**7200**	19.5
14	871.9	**9600**	100		MEM		TL	9800	17	245.9	8400	100		MEM		**30.0**	**8400**	22.2
15	830.5	**9400**	100		MEM		TL	9700	15	247.0	8400	100		MEM		**32.4**	**8400**	22.4
16	1094.6	**10900**	100		MEM		TL	11100	16	480.2	9600	100		MEM		**196.1**	**9600**	183.4
17	1286.4	**12400**	100		MEM		TL	12400	15	456.5	10800	100		MEM		**87.3**	**10800**	73.3
18	1302.8	**13400**	100		MEM		TL	14100	18	602.4	12000	100		MEM		203.0	**12000**	183.5
19	2053.6	17900	100		MEM		TL	**17800**	14	855.9	15600	100		MEM		**269.6**	**15600**	232.9
20	6557.6	**37200**	100		MEM		TL	39500	27	624.7	31200	10		MEM		388.4	31200	303.2
Average rank		*1.90*			*2.55*			***1.45***			*2.20*			*2.15*			***1.20***	

Table 6: Results of *Setting2* (All times are expressed in seconds; TL = Time Limit = 10800 seconds; ∞ = Infeasible; MEM = memory exhausted)

Statistics	*Setting1*								
	IHSTT			*TP-IHSTT*			*TP*		
Instance	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*
1	2668	6068	26816	1268	2472	8481	1312	1990	8090
2	1053	2834	12296	720	1758	5886	647	734	3600
3	1605	3828	15884	1083	2558	8849	647	734	3600
4	2595	6516	26352	1734	4059	13077	1817	1918	9608
5	1593	4904	22286	990	2664	9456	1127	1192	7920
6	1521	6002	26120	918	2670	9126	827	1114	5922
7	3136	6922	32086	2290	4345	15224	251	242	906
8	2457	18608	112868	1580	8401	37561	2248	9922	95587
9	9682	7152	79146	6402	4288	27596	3877	4356	22752
10	9769	7212	80616	6480	4302	27756	3877	4320	21672
11	13025	9616	107488	8640	5760	37296	5169	5760	28896
12	16281	12020	134360	10800	7170	46260	6461	7200	36120
13	19537	14424	161232	12960	8640	55872	7753	8640	43344
14	22793	16828	188104	15120	10080	65352	9045	10080	50568
15	22793	16832	188156	15120	10080	65388	9045	10080	50568
16	26049	19232	214976	17280	11520	74496	10337	11520	57792
17	29305	21636	241848	19440	12852	82728	11629	16200	71496
18	32561	24040	268729	21600	14328	92304	12921	18000	79440
19	42329	31252	349336	28080	18642	120276	16797	18720	93912
20	84657	62504	698672	56160	37290	240504	33593	46800	206544

Statistics	*Setting2*														
	KSS			*IHSTT*			*TP-KSS*			*TP-IHSTT*			*TP*		
Instance	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*	*#Var.*	*#Con.*	*#NZ*
1	11119	14274	88779	2134	5425	21022	4444	2022	27104	1070	2194	7130	1144	1879	6906
2	7836	9768	65460	873	1841	8096	3474	1362	22242	594	1506	4662	640	718	4402
3	8097	10295	64721	1321	2750	11726	3424	1599	21341	932	2259	7209	640	718	3402
4	14424	18020	121436	2294	4817	16938	6391	2555	41447	1482	3555	10629	1385	1615	6711
5	16800	28146	269184	1350	4446	17700	6111	3278	79656	834	2364	7584	1120	1186	7878
6	18672	30720	273216	1260	5508	21510	4827	3090	44955	684	2202	6948	827	1114	5922
7	16460	20182	123918	2715	6457	26880	7228	3500	43350	1923	3932	13174	264	253	983
8		MEM		1980	17682	104130		MEM		1053	6637	29061	610	910	3894
9	39126	48450	217794	8070	7530	59955	16848	7956	58392	4938	2766	20274	4020	4164	20160
10	39348	48888	218880	8100	5544	61020	16956	8064	58608	4968	2772	20520	4020	4164	20160
11	53904	66624	293280	10800	7392	81360	22608	10752	78144	6624	3696	27360	5360	5552	26880
12	18978	10261	84576	13500	9240	101700	28260	13440	97680	8280	4620	34200	6700	6940	33600
13		MEM		16200	11088	122040		MEM		9936	5544	41040	8040	8328	40320
14		MEM		18900	12936	142380		MEM		11592	6468	47880	9380	9716	47040
15		MEM		18900	12936	142380		MEM		11592	6468	47880	9380	9716	47040
16		MEM		21600	14784	162720		MEM		13248	7392	54720	10720	13984	59520
17		MEM		24300	16632	183060		MEM		14904	8316	61560	12060	15732	66960
18		MEM		27000	18480	203400		MEM		16560	9240	68400	13400	17480	74400
19		MEM		35100	24024	264420		MEM		21528	12012	88920	17420	22724	96720
20		MEM		70200	48048	528840		MEM		43056	24024	177840	34840	45448	193440

Table 7: Statistics of the number of variables, constraints and non-zeros in IHSTT,TP-IHSTT programs for *Setting1* (Table 5) and in KSS,IHSTT,TP-KSS and TP-IHSTT programs for *Setting2* (Table 6)

Setting2	*KHE TL1*		*KHE TL2*		*KSS model size*		*KSS TL1*		*KSS TL2*		*IHSTT model size*		*IHSTT TL1*		*IHSTT TL2*	
Instance	*Obj*	*Threads*	*Obj*	*Threads*	*#var*	*#con*	*Obj*	*Gap*	*Obj*	*Gap*	*#var*	*#con*	*Obj*	*Gap*	*Obj*	*Gap*
21	25900	10	25900	10	39330	17908	**1600**	-	1200	-	11766	26621	2500	-	**400**	-
22	**4300**	100	**4300**	100	42228	29261	7000	-	6100	-	12668	28679	6200	-	**4300**	-
23	**1100**	10	**1100**	10	MEM		MEM		MEM		26628	59637	1410700	-	30100	-
24	(22,1300)	10	(22,1300)	10	MEM		MEM		MEM		29290	52158	**638700**	2400	**48500**	79
Average rank	*1.75 = (3+1+1+2)/4*		*1.75 = (3+1+1+2)/4*				*2.50 = =(1+3+3+3)/4*		*2.75 = =(2+3+3+3)/4*				*1.75 = (2+2+2+1)/4*		*1.25 = (1+1+2+1)/4*	

Table 8: Results of *Setting2* for instances 21-24 (All times are expressed in seconds; TL1 = Time Limit 1 = 10800 seconds; TL2 = Time Limit 2 = 43200 seconds; ∞ = Infeasible; MEM = memory exhausted)

Without time limits			
Instance	*Best known*	*KSS*	*IHSTT*
Brazil 1	41*	41	41
Brazil 2	5*	5	5
Brazil 3	24*	26	26
Brazil 4	51*	61	59
Brazil 5	19*	30	41
Brazil 6	35*	60	98
Brazil 7	53	122	113

Table 9: Brazilian instances without any time limits (* means *optimum*)

Table 7 reports the number of variables (*#Var.*), constraints (*#Con.*) and non-zero coefficients (*#NZ*) in the constraints of both *Setting1* and *Setting2*. This table clearly shows the decrease in the size of instances when one switches from IHSTT to TP-IHSTT (*Setting1*), as well as from KSS to IHSTT with or without the TP step (*Setting2*).

Table 8 focuses on the last four instances. They are solved by KHE, KSS and IHSTT within two time limits TL1 and TL2, which amount to 10800 seconds and 43200 seconds, respectively. As for KHE, we report the objective function Obj and the optimal number of threads. Next, data and outcomes of the KSS are shown. More precisely, we report the model size in terms of number of variables (*#Var.*) and constraints (*#Con.*). The outcomes are the objective (*Obj*) and the gap (*Gap*) computed by (89). The same organization of results is adopted for IHSTT. The average rank is computed as in Table 6, but it is separately computed for TL1 and TL2.

Although IHSTT was motivated by the case of Italian schools, we aim to show its compatibility with respect to some benchmark instances and its capability in obtaining good solutions even in this case. The experimentation is divided into two parts. In the first part, some XHSTT instances are solved without any time limits, according to the policy of round 1 of the ITC2011 competition. This experimentation is reported in Table 9 on instances from *Brazil 1* to *Brazil 7*, for which the best known solutions are taken from [34]. The solutions of IHSTT are compared to those for KSS, as reported in [15]. In the second part, four instances of the Round 2 of the ITC2011 competition are solved to evaluate the IHSTT formulation with respect to some well-known benchmarks. The rules of Round 2 are rigorously followed. The time limit is set by the ITC2011 benchmark utility and only one thread is used. The results of IHSTT are reported in Table 10 and compared to all participants of Round 2, as reported in [11]. Moreover, Table 11 compares the size of these instances for IHSTT and KSS according to [15].

Finally, since this problem is naturally affected by symmetry, we rerun all tests while disabling the symmetry control options in CPLEX. However, these results are never better than those presented so far.

6.2 Analysis of results

6.2.1 *Setting1*

Consider the columns denoted by IHSTT in Table 5. They show that all instances use the overall available time to determine low quality upper-bounds. Moreover, the lower bounds at the root node after CPLEX

ITC2011		*GOAL*		*HySST*		*Lectio*		*HFT*		*IHSTT*	
Instance	*Seed*	*Obj*	*Rank*	*Obj*	*Rank*	*Obj*	*Rank*	*Obj*	*Rank*	*Obj*	*Rank*
BrazilInstance2	102545520	1.00063	3	1.00078	4	0.00046	1	7.00189	5	0.00103	2
BrazilInstance2	109328591	1.00054	3	1.00075	4	0.00057	1	5.00183	5	0.00103	2
BrazilInstance2	234546972	1.00087	4	1.00081	3	0.00028	1	7.00180	5	0.00103	2
BrazilInstance2	317604170	1.00051	3	1.00078	4	0.00019	1	6.00186	5	0.00103	2
BrazilInstance2	584363925	1.00054	3	1.00069	4	0.00047	1	5.00198	5	0.00103	2
BrazilInstance2	65843198	1.00063	3	1.00082	4	0.00038	1	6.00207	5	0.00103	2
BrazilInstance2	792992094	1.00063	3	1.00087	4	0.00025	1	7.00195	5	0.00103	2
BrazilInstance2	802033156	1.00066	3	1.00072	4	0.00034	1	6.00165	5	0.00103	2
BrazilInstance2	856676505	1.00066	3	1.00072	4	0.00034	1	7.00210	5	0.00103	2
BrazilInstance2	96247109	1.00051	3	1.00078	4	0.00053	1	7.00189	5	0.00103	2
BrazilInstance3	102545520	0.00132	3	0.00096	1	0.00159	4	29.00264	5	0.00126	2
BrazilInstance3	109328591	0.00134	3	0.00126	1	0.00175	4	31.00288	5	0.00126	1
BrazilInstance3	234546972	0.00138	3	0.00123	1	0.00153	4	28.00285	5	0.00126	2
BrazilInstance3	317604170	0.00087	1	0.00111	2	0.00112	3	30.00306	5	0.00126	4
BrazilInstance3	584363925	0.00117	2	0.00096	1	0.00150	4	26.00264	5	0.00126	3
BrazilInstance3	65843198	0.00135	3	0.00123	1	0.00171	4	32.00276	5	0.00126	2
BrazilInstance3	792992094	0.00129	2	0.00132	3	0.00136	4	29.00273	5	0.00126	1
BrazilInstance3	802033156	0.00137	3	0.00135	2	0.00167	4	29.00303	5	0.00126	1
BrazilInstance3	856676505	0.00120	1	0.00133	3	0.00149	4	29.00288	5	0.00126	2
BrazilInstance3	96247109	0.00111	2	0.00102	1	0.00149	4	32.00288	5	0.00126	3
BrazilInstance4	102545520	17.00099	4	5.00221	3	1.00188	2	64.00258	5	0.00165	1
BrazilInstance4	109328591	18.00090	4	3.00241	3	2.00202	2	67.00243	5	0.00165	1
BrazilInstance4	234546972	18.00093	4	2.00238	3	1.00172	2	66.00246	5	0.00165	1
BrazilInstance4	317604170	18.00093	4	4.00242	3	2.00185	2	66.00234	5	0.00165	1
BrazilInstance4	584363925	17.00111	4	3.00233	2	4.00265	3	68.00243	5	0.00165	1
BrazilInstance4	65843198	17.00102	4	3.00210	3	3.00201	2	63.00225	5	0.00165	1
BrazilInstance4	792992094	17.00102	4	4.00223	3	2.00215	2	67.00243	5	0.00165	1
BrazilInstance4	802033156	18.00083	4	5.00227	3	3.00212	2	68.00195	5	0.00165	1
BrazilInstance4	856676505	16.00107	4	3.00239	3	3.00200	2	68.00222	5	0.00165	1
BrazilInstance4	96247109	16.00104	4	3.00235	3	2.00150	2	68.00258	5	0.00165	1
BrazilInstance6	102545520	4.00234	4	3.00273	3	0.00250	1	22.00438	5	0.00703	2
BrazilInstance6	109328591	4.00225	4	2.00270	3	0.00192	1	23.00363	5	0.00703	2
BrazilInstance6	234546972	4.00236	4	3.00281	3	0.00204	1	24.00369	5	0.00703	2
BrazilInstance6	317604170	4.00222	4	3.00240	3	0.00218	1	22.00360	5	0.00703	2
BrazilInstance6	584363925	4.00230	4	3.00284	3	0.00323	1	23.00438	5	0.00703	2
BrazilInstance6	65843198	4.00228	4	2.00229	3	0.00183	1	25.00387	5	0.00703	2
BrazilInstance6	792992094	4.00246	4	3.00298	3	0.00241	1	21.00423	5	0.00703	2
BrazilInstance6	802033156	4.00210	4	3.00256	3	0.00191	1	24.00372	5	0.00703	2
BrazilInstance6	856676505	4.00207	3	4.00270	4	0.00261	1	22.00384	5	0.00703	2
BrazilInstance6	96247109	4.00228	4	3.00291	3	0.00239	1	23.00369	5	0.00703	2
Final ranking		*GOAL*	***3.33***	*HySST*	***2.88***	*Lectio*	***2.00***	*HFT*	***5.00***	*IHSTT*	***1.78***

Table 10: Results of simulated *ITC2011* - Round 2 (Time limit used for IHSTT = 556 seconds - CPLEX parameter threads was set to 1 - *Obj* corresponds to *Cost* in ITC2011 tables)

	IHSTT				**KSS**			
Instance	*#Var*	*#Con*	*#NZ*	*#Var×#Con*	*#Var*	*#Con*	*#NZ*	*#Var×#Con*
Brazil Instance 2	1E+04	5E+03	5E+04	5E+07	3E+04	1E+04	1E+05	3E+08
Brazil Instance 3	1E+04	6E+03	6E+04	9E+07	3E+04	2E+04	2E+05	6E+08
Brazil Instance 4	3E+04	1E+04	1E+05	4E+08	5E+04	2E+04	2E+05	1E+09
Brazil Instance 6	4E+04	2E+04	2E+05	6E+08	6E+04	3E+04	3E+05	2E+09
Mean relative model size	**100%**				**446%**			
Mean relative non-zeros size	**100%**				**232%**			

Table 11: Model size comparison on *ITC2011*

cuts are always zero. In addition, the average idle times are not acceptable.

Consider the groups columns denoted by TP-IHSTT in Table 5. In this case, all instances are optimally solved within the time limit. The column "*TP time*" shows that an acceptable time is spent for solving the TP. The time spent in the TP step is on average 68% of the total running time for first group of instances and 80% for second group, if the default parameters are used for the configuration of CPLEX. The lower bounds at the root node after CPLEX cuts are often equal to the final integer solution. Generally speaking, the two-step method returns lower values of the average idle times, i.e.

higher-quality timetables from the viewpoint of teachers in real applications. Moreover, the left side of Table 7 shows the decrease in the number of variables, constraints and non-zeroes entries if one switches from IHSTT to TP-IHSTT. Therefore, the two-step method looks a promising approach for solving [HST] problems and it is worth investigating its viability also in *Setting2*.

6.2.2 *Setting2* without TP step

Consider the columns denoted by KHE, KSS and IHSTT in Table 6. In the first group of instances (1-9), IHSTT outperforms KSS: KSS performs better for 4 times out of 9, while IHSTT determines the optimal solutions for 6 instances out of 9. Furthermore KSS does not get the first feasible solution within the time limit for two instances; such a situation never occurs to IHSTT. Although KHE software does not give guarantees of optimality, the comparison to IHSTT shows that it performs better only for one time and obtains infeasible solutions only five instances out of nine.

In the second group of instances IHSTT works better in five instances out of eleven. The other solutions have an optimality gap ranging from 15% to 27%. KSS gets a feasible solution one time out of eleven, in two instances it does not return the first feasible solution within the time limit. Therefore, IHSTT is always superior to KSS in all instances. KHE always obtains feasible solutions. Hence, it looks better than KSS and slightly worse than IHSTT.

Moreover, Table 7 shows the decrease in the number of variables, constraints and non-zeroes entries if one switches from KSS to IHSTT.

6.2.3 *Setting2* with TP step

Consider the columns denoted by prefix **TP-** in Table 6. In the first group of instances (1-9), TP-IHSTT proves to be superior to TP-KHE and TP-KSS in 5 instances out of 9. KHE does not take advantage of the TP step and in seven cases it worsens w.r.t. the case without the TP step. The benefits of the TP step are instead very clear for both KSS and IHSTT, as they show significant improvements in gaps and optimization times.

In the second group of instances (10-20) TP-KHE improves all solutions owing to the TP step. The comparison between TP-KSS and TP-IHSTT indicates a much better effectiveness of TP-IHSTT in terms of running times. Furthermore, TP-KSS is more demanding from the point of view of memory use, as the 8 largest instances cannot be solved and compared to TP-IHSTT.

Yet, Table 7 shows the decrease in the number of variables, constraints and non-zeroes entries if one switches from TP-KSS to TP-IHSTT.

According to the former results, it is of interest to run IHSTT for a larger time limit, to possibly obtain optimal solutions for all instances in Table 4. A final experimentation is carried out with a time limit of 24 hours and the optimal solutions are eventually obtained for all instances. These solutions are equal to those of the two-step method, i.e. the proposed method returns the optimal solutions for all instances in Table 4. Moreover, the method exhibits a considerable speed-up in running times.

6.2.4 Real instances from 4 Italian schools

According to Table 8, KSS can determine feasible solutions for two instances out of four within the usual time limit of 3 hours = 10800 seconds. In both cases, the value of Gap cannot be computed by (formula), because $L_B = 0$. If the running time is increased to 12 hours = 43200 seconds, the improvement is marginal in the first two instances, whereas it is still not possible to build the model for the last two instances. On the other hand, IHSTT can provide feasible solutions for all instances after 3 hours and a significant improvement is obtained after 12 hours. This is an experimental confirmation on its better use of memory. Finally, KHE can provide either very good and very poor solutions and no improvement can be obtained by larger running times.

6.2.5 Comparison to benchmarks

Table 9 shows that IHSTT can obtain two best known solutions. All in all, it seems to have equivalent performances as opposed to KSS. Table 10 shows that this formulation leads to good-quality results in the time available with respect to the meta-heuristics taking part in the round 2 of this competition. Moreover, Table 11 compares the model dimensions of these instances according to the data reported in [15]. According to Table 11, in these instances the average increases in memory size and number of nonzeros from IHSTT to KSS are 4.46 and 2.32, respectively. The memory size is computed by the product between the number of constraints and the number of decision variables.

7 Conclusion

This paper has investigated the Italian High School Timetabling Problem. It has well-established characteristics like co-teachers, articulated classes, multiple lessons, additional days-off, as well as quality indicators, such as the horizontal and vertical distributions of lessons. However, it exhibits new features which have not been investigated so far: fractional time units, equity in idle times, avoidance of consecutive heavy days and excessive workload for classes. All in all, this problem is more complex than those in the literature on the class-teacher paradigm. Moreover, the generalized HST problem based on KSS [15] [10] does not incorporate all requirements in the [IHSTP].

A mixed integer programming model (denoted by IHSTT) has been proposed for [IHSTP], in order to pursue the maximum compatibility with KSS. Since KSS timetabling is based on the decomposition into sub-events and the IHSTT is built on equally-sized sub-events, the larger cardinality of the sets of sub-events makes the IHSTT more suitable to solve also realistic size instances of a simplified problem, in which the new requirements are omitted.

In order to obtain fast solutions for both the complete and the simplified [IHSTP], a two-step method is proposed. In the first step, the TP model is solved to cleverly decrease the initial solution space of IHSTT and determine the profiles of teachers. Next, the IHSTT model with restricted data is solved very effectively in the second step. The two-step method results in good-shaped timetables and suitable computing times even for the most complex problem instances. Although the method does not guarantee the optimality, it returns the optimal solutions for all instances motivating this research. In our opinion, the two-step method is quite general and could be applied to other class-teacher problems for countries with a similar problem setting.

Some possible research developments are listed below:

- investigating the applicability of the TPP also to the XHSTT standard instances;
- allowing the IHSTT model to have a better compatibility with XHSTT on-line database [34];
- facing the multiple school timetabling problem with some shared teachers among two or more schools;
- planning temporary timetables in which some teachers may not be available or they have to be substituted;
- planning timetables according to teachers' preferences;
- investigating the class-teacher assignment problem by a full MIP model, and evaluating its impact on the final timetabling.

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A Appendix A - Summary of [IHSTP]-[TPP] slack variables

Var.	Type	Description
$s_{ct}^{C_1}$	Integer non-negative	Not assigned weekly lessons for class c and teacher t
$s_{cdh}^{C_2}$	Boolean	Not assigned required lesson for class c on day d at period h
$s_{cdh}^{C_3}$	Boolean	Violated availability periods of class c on day d at period h
$s_{tdh}^{C_4}$	Boolean	Violated availability periods of teacher t on day d at period h
$s_{ctd}^{C_5}$	Boolean	Split lessons for class $c \in C$ and teacher t on day d
$s_{t}^{C_6}$	Integer non-negative	Lack/excess of days off for teacher t
$s_{ctf}^{C_7}$	Integer non-negative	Lab lessons for class c, teacher t and co-teacher f in excess or in lack
$s_{c't'c''t''}^{C_8}$	Integer non-negative	Block lessons for classes c', c'' with teachers t', t'' in excess or in lack
$s_{cdh}^{C_9}$	Boolean	Not assigned preassigned lesson for class c on day d at period h
$s^{C_{10}}$	Integer non-negative	Difference between maximum and minimum idle times for teachers
$s_{td}^{C_{11}}$	Integer non-negative	Idle times for teacher t on day d
$s_{l}^{C_{12}}$	Integer non-negative	Violation for multiple lessons limit $l \in L$
$s_{ct}^{C_{13}}$	Integer non-negative	Violation of ideal weekly lessons' distribution for class c and teacher t
$s_{cth}^{C_{14}}$	Integer non-negative	Violation of ideal daily lessons' distribution for class c and teacher t for period h
$s_{td}^{C_{15}}$	Integer non-negative	Violation of under-load/over-load limits for teacher t on day d
$s_{ctd}^{C_{16}}$	Integer non-negative	Violation of under-load/over-load limits for class c/teacher t on day d
$s_{cd}^{C_{17}}$	Integer non-negative	Presence of multiple lessons overload for class c on day d
$s_{td}^{C_{18}}$	Integer non-negative	Presence of two consecutive heavy days $d, d+1$ for teacher t
$s_{td}^{C_{19}}$	Integer non-negative	Violation of fractional time units for teacher t on day d

Table 12: IHSTT Slack variables summary

Var.	Type	Description
s_{tdh}^{SC1}	Boolean	1 if teacher t teaches in the same period h in two consecutive days $d, d+1$, 0 otherwise
s_{t}^{SC2}	Integer non-negative	Violation of minimum/maximum days off required
s_{td}^{SC3}	Integer non-negative	Violation of under-load/over-load limits for teacher t on day d
s_{td}^{SC4}	Integer non-negative	Idle times for teacher t on day d
s^{SC5}	Integer non-negative	0 if all teachers have the same minimum idle times, positive otherwise

Table 13: TP Slack variables summary

B Appendix B - Glossary

Articulated class
A class made with the union of two or more *classes* with a small number of students.

Block
Two lessons for two pairs of *classes* and teachers who have to work together or separately in the same time slot.

Class (or group)
Group of students taking lessons from the same *curriculum* at the same time.

Co-presence teacher (co-teacher)
A teacher who always works together with another colleague.

Curriculum
It is the set of subjects in a class and the number of lessons for each subject.

Daily period
A time interval with a constant duration equal to the minimum lesson unit.

Day off
A day when the teacher does not teach.

Double lesson
A lesson with length of two periods which must be consecutive for the same class and teacher.

Fractional time unit
A period with a duration of a fraction of an hour.

Full-time teacher
A teacher with a weekly workload equal to a fixed number of hours.

Idle time
A pause between two lessons non-consecutive of a teacher.

Lesson unit
The minimum interval of time of a lesson (normally it is equal to one hour).

Multiple lesson
A lesson with length of some periods which must be consecutive for the same class and teacher.

Part-time teacher
A teacher with a reduced weekly workload compared to a *full-time teacher*.

Split lesson
A lesson which is not given in consecutive periods by a teacher in a class.

Triple lesson
A lesson with length of three periods which must be consecutive for the same class and teacher.

C Appendix C - Weights of instances used in experimentation

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	100,000	100,000	100,000	100,000	1,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	0	1,000	0
2	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
3	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	0	10	3	100,000	100,000	1	1,000	0
4	100,000	100,000	100,000	100,000	100,000	100,000	0	100,000	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
5	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
6	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	1	10	3	100,000	100,000	1	1,000	1
7	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
8	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	1	10	3	100,000	100,000	1	1,000	1
9	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
10	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
11	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
12	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
13	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
14	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
15	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
16	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
17	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
18	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
19	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
20	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0

Table 14: Weights of constraints C1-C19 used in *Setting1* (Table 5)

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	100,000	100,000	100,000	100,000	1,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
2	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
3	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	0	0	0	100,000	100,000	0	0	0
4	100,000	100,000	100,000	100,000	100,000	100,000	0	100,000	0	0	100	0	0	0	100,000	100,000	0	0	0
5	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
6	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	1	0	0	100,000	100,000	0	0	0
7	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
8	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	1	0	0	100,000	100,000	0	0	0
9	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
10	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
11	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
12	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
13	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
14	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
15	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
16	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
17	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
18	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
19	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
20	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0

Table 15: Weights of constraints C1-C19 used in *Setting2* (Table 6)

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	0	0	0	0	0	0	0	0	0	0	26	0	8	1	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	18	0	8	30	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	21	0	12	48	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	22	0	8	52	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	24	0	0	70	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	36	0	16	48	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	36	0	0	5	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	72	0	96	176	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	216	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	216	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	336	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	420	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	648	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	756	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	756	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	864	0	0	3	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	1,080	0	0	4	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	1,440	0	128	303	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	1,716	0	252	342	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	3,744	0	370	587	0	0	0	0	0

Table 16: Constraints C1-C19 slacks' values in IHSTT (*Setting1* Table 5)

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	0	0	0	0	0	0	0	0	0	0	0	0	28	3	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	7	0	0	10	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	11	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	6	0	20	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	12	0	0	5	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	24	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	36	0	24	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	36	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	48	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	72	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	84	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	84	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	108	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	120	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	156	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	312	0	0	1	0	0	0	0	0

Table 17: Constraints C1-C19 slacks' values in TP-IHSTT (*Setting1* Table 5)