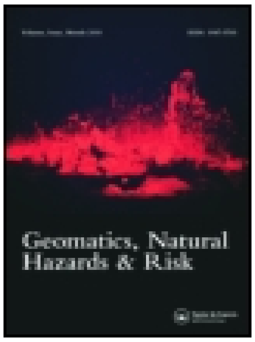

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Tornadoes and related damage costs: statistical modelling with a semi-Markov approach

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ABSTRACT
We propose a statistical approach to modelling for predicting and simulating occurrences of tornadoes and accumulated cost distributions over a time interval. This is achieved by modelling the tornado intensity, measured with the Fujita scale, as a stochastic process. Since the Fujita scale divides tornado intensity into six states, it is possible to model the tornado intensity by using Markov and semi-Markov models. We demonstrate that the semi-Markov approach is able to reproduce the duration effect that is detected in tornado occurrence. The superiority of the semi-Markov model as compared to the Markov chain model is also affirmed by means of a statistical test of hypothesis. As an application, we compute the expected value and the variance of the costs generated by the tornadoes over a given time interval in a given area. The paper contributes to the literature by demonstrating that semi-Markov models represent an effective tool for physical analysis of tornadoes as well as for the estimation of the economic damages to human things.

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Tornadoes modelling; Markov process; semi-Markov process; reward process

1. Introduction

Every year tornadoes cause deaths and several damages to people and things. Only in the USA, tornadoes killed, on average, more than 100 people per year from 2004 to 2013 (Simmons et al. 2013). Just to give an example of the monetary damages of tornadoes in the USA, in 2013, the estimated cost was about 200 millions of dollars (Simmons et al. 2013). In this scenario, the development of techniques to estimate and model the probabilities of these events is needed and can be of great benefit for the society. Many researchers are working on this subject (see e.g. Boswell et al. 1999; Sisson et al. 2006; de Melo Mendes & Pericchi 2009; Obeysekera et al. 2011; Bashan et al. 2013; Delphin et al. 2013). The approaches used can be typically divided into two main groups, one analytical and another statistical (see e.g. Bryan & Rotunno 2009; Dotzek et al. 2003, respectively).

Here we propose a statistical approach based on semi-Markov model, the approach already used in many disciplines, as finance or insurance, to model and quantify reward processes (see e.g. Howard 1971; Balcer & Sahin 1986; De Dominicis & Manca 1986; Masuda & Sumita 1991; Masuda 1993; Soltani & Khorshidian 1998; McClean et al. 2004; Papadopoulou 2004; Stenberg et al. 2006; Papadopoulou & Tsaklidis 2007; Stenberg et al. 2007; Limnios & Oprisan 2012; Papadopoulou et al. 2012; Papadopoulou 2013; D'Amico et al. 2014b). This kind of models generalize the more common Markov chain models, and their main feature is the possibility to reproduce the duration effect of

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the considered random phenomenon. This is made possible by considering sojourn times in the states of the process that are distributed according to any type of probability distribution functions, non-memoryless distributions included. In this work, we choose to model the tornado's intensity as a stochastic process. The tornado's intensity is measured by the Fujita scale (F-scale) which is an empirical scale related to the gravity of the damages produced by the tornado. Since the F-scale divides tornado intensity into six states, it is possible to model the tornado intensity by using semi-Markov models. The database used in this work is made available from the National Oceanic and Atmospheric Administration (NOAA) (USA) that counts of more than 60,000 tornadoes from 1950 until 2013. The proposal of a semi-Markov model for modelling tornadoes allows the estimation of probability of an occurrence of a tornadoes with a certain intensity at each time in a given location. This also gives the possibility to compute the total costs of damages caused by the tornadoes which is a relevant indicator of environmental hazards. The paper is organized as follows. In Section 2, we introduce the database and the object of investigation. In Section 3, we present the semi-Markov model and the related reward (cost) process. Section 4 shows the main application of the model to the tornado process. At last, in Section 5, we give some concluding remarks.

2. Database

The data used in this work come from NOAA's National Weather Service and they are freely available on the website www.spc.noaa.gov/wcm/#data. Almost 60,000 events from 1950 to 2013 are collected in the database, all of them geographically distributed in the USA (as it is possible to see in Figure 1). For each event, date, time, state, F-scale, injuries, fatalities, starting latitude and longitude, ending latitude and longitude are recorded. The physical quantity of our interest is the F-scale. This is an empirical scale that measures tornado intensity based on the damage produced to man-made structures. It can be also related to the wind speed, for example, for a tornado classified F0, the wind speed can go from 64 to 116 m/s, instead for an F5 tornado from 419 to 512 m/s (Fujita 1973). As it is well known, the F-scale admits six values of tornado intensity that goes from F0 to F5. Given that it is a discrete scale, the tornado intensities, measured by the F-scale, can be naturally modelled

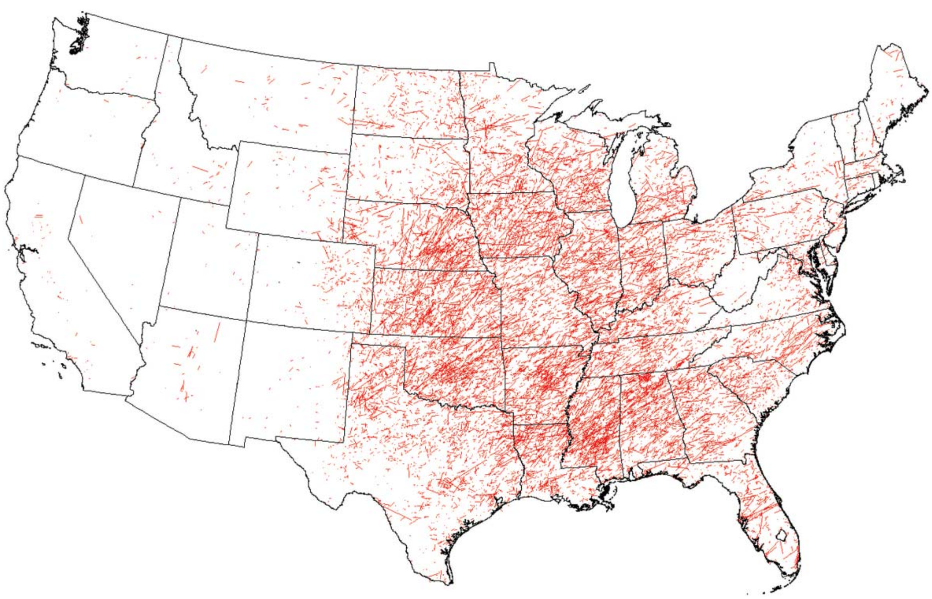

Figure 1. Geographical distribution of the database's events, extrapolated from http://www.spc.noaa.gov/gis/svrgis/images/tornado.png.

through a discrete-state stochastic model. For the present work, we choose to use a semi-Markov model.

3. Semi-Markov process

We define an homogeneous semi-Markov process with values in a finite state space $E = \{1, 2, ..., m\}$ (see e.g. Limnios & Oprisan 2001; Janssen & Manca 2006). Let $(\Omega, \mathbf{F}, P)$ be a probability space; we consider two sequences of random variables $J = \{J_n\}_{n \in \mathrm{IN}}$ and $T = \{T_n\}_{n \in \mathrm{IN}}$, where

$$J_n : \Omega \to E; \quad T_n : \Omega \to \mathrm{IN}.$$

They denote the state and time of the nth transition of the system, respectively. In our application, J_n is the intensity of the nth tornado and T_n the time of its occurrence.

We assume that (J, T) is a Markov renewal process on the state space $E \times \mathrm{IN}$ with kernel $Q_{ij}(t)$, $i, j \in E$, $t \in \mathrm{IN}$. The kernel has the following probabilistic interpretation:

$$\begin{aligned} &P\left[J_{n+1} = j, T_{n+1} - T_n \le t | \sigma(J_h, T_h), h \le n, J_n = i\right] = \\ &P\left[J_{n+1} = j, T_{n+1} - T_n \le t | J_n = i\right] = Q_{ij}(t), \end{aligned} \tag{1}$$

where $(\sigma(J_h, T_h), h \le n)$ represents the set of past values of the Markov renewal process (J, T). Relation (1) asserts that the knowledge of the last tornado's intensity suffices to give the conditional distribution of the couple $(J_{n+1}, T_{n+1} - T_n)$, whatever the past values of the variables might be.

It is simple to realize that $p_{ij} := P\left[J_{n+1} = j | J_n = i\right] = \lim_{t \to \infty} Q_{ij}(t);\ i, j \in E,\ t \in \mathrm{IN}$, where $\mathbf{P} = (p_{ij})$ is the transition probability matrix of the embedded Markov chain J_n.

Simple probabilistic reasoning allows the computation of the conditional probability distribution of the sojourn time $T_{n+1} - T_n$ in the state J_n given that the next visited state is J_{n+1}. In formula,

$$\begin{aligned} G_{ij}(t) &:= P\left\{T_{n+1} - T_n \le t | J_n = i, J_{n+1} = j\right\} = \\ &\begin{cases} \dfrac{Q_{ij}(t)}{p_{ij}} & \text{if } p_{ij} \ne 0 \\ 1 & \text{if } p_{ij} = 0. \end{cases} \end{aligned} \tag{2}$$

The $G_{ij}(\cdot)$ denotes the waiting time distribution function in state i given that, with the next transition, the process will be in the state j. The sojourn time distribution $G_{ij}(\cdot)$ can be any distribution function. We recover the discrete-time Markov chain when the $G_{ij}(\cdot)$ are all geometrically distributed. Therefore, we should find out whether the inter-arrival times between two tornadoes of given intensities follow a geometric distribution or not. This is a primary question to which we will respond in the next section.

Now it is possible to define the time homogeneous semi-Markov chain $Z(t)$ as

$$Z(t) = J_{N(t)}, \quad \forall t \in \mathrm{IN}, \tag{3}$$

where $N(t) = \sup\{n \in \mathrm{IN} : T_n \le t\}$. Then, $Z(t)$ represents the state of the system for each waiting time.

At this point, we introduce the discrete backward recurrence time process linked to the semi-Markov chain. For each time $t \in \mathrm{IN}$, we define the following stochastic process:

$$B(t) = t - T_{N(t)}. \tag{4}$$

We call it discrete backward recurrence time process. It denotes the time elapsed from the occurrence of the last tornado to the current time t.

The joint stochastic process ($Z(t)$, $B(t)$, $t \in$ IN) with values in $E \times$ IN is a Markov process. That is,

$$\begin{aligned} P[Z(T) = j, B(T) = v' | \sigma(Z(h), B(h)), h \le t, Z(t) = i, B(t) = v] \\ = P[Z(T) = j, B(T) = v' | Z(t) = i, B(t) = v] =: {}^{b}\phi_{ij}^{b}(v; v', t). \end{aligned}$$

with the following evolution equation (see e.g. D'Amico & Petroni 2012):

$$\begin{aligned} {}^{b}\phi_{ij}^{b}(v; v', t) = \delta_{ij} \frac{[1 - \sum_{a \in E} Q_{ia}(t+v)]}{[1 - \sum_{a \in E} Q_{ia}(v)]} 1_{\{v'=t+v\}} \\ + \sum_{k \in E} \sum_{s=1}^{t} \frac{Q_{ik}(s+v) - Q_{ik}(s+v-1)}{[1 - \sum_{a \in E} Q_{ia}(v)]} {}^{b}\phi_{kj}^{b}(0; v', t-s). \end{aligned} \quad (5)$$

Expression (5) provides the probability of having a tornado of intensity j after $t - v'$ periods and no additional tornado within the times $\{t - v' + 1, t - v' + 2, \ldots, t\}$ given that the last tornado occurred v periods before the present time and was of intensity i.

We can now define the accumulated discounted reward (cost), $\xi(t)$, during the time interval $(0, t]$, by the following relation:

$$\xi(t) = \sum_{n=1}^{N(t)} \psi_{J_n} e^{-\delta T_n}, \quad (6)$$

where ψ_{J_n} is the cost caused by the nth tornado that had an intensity J_n. This cost has to be discounted using a deterministic force of interest δ and the time T_n of occurrence of the event. The total damage over the time interval $[0, t]$ is obtained by summation over the random number of tornadoes $N(t)$ up to time t.

In the application section, we will compute the expected value $E[\xi(t)]$ and the second-order moment $E[\xi^2(t)]$. For an extended treatment of the semi-Markov reward process (see e.g. Stenberg et al. 2006).

4. Application to real data

4.1. Test

The first step of our application is to test the validity of the Markov hypothesis by means of a statistical test of hypothesis proposed by Stenberg et al. (2006) and shortly described in this article. The geometric distributions for the waiting times are of specific parameters for Markov models. Other distributions for the sojourn times show that the Markov modelling is inappropriate. The probability distribution function of the sojourn time in state i before making a transition in state j has been denoted by $G_{ij}(\cdot)$. Define the corresponding probability mass function by

$$g_{ij}(t) = P\{T_{n+1} - T_n = t | J_n = i, J_{n+1} = j\} = \begin{cases} G_{ij}(t) - G_{ij}(t-1) & \text{if } t > 1 \\ G_{ij}(1) & \text{if } t = 1. \end{cases} \quad (7)$$

Under the geometrical hypothesis, the equality $g_{ij}(1)(1 - g_{ij}(1)) - g_{ij}(2) = 0$ must hold, and then a sufficiently strong deviation from this equality has to be interpreted as an evidence against the

Table 1. Results of the test.

State	State	Score	Decision
$i = 1$	$j = 2$	9.79	H_0 rejected
$i = 1$	$j = 3$	4.43	H_0 rejected
$i = 3$	$j = 1$	4.24	H_0 rejected
$i = 4$	$j = 1$	5.50	H_0 rejected

Markovian hypothesis and in favour of the semi-Markov model. The test statistic is as follows:

$$\widehat{S}_{ij} = \frac{\sqrt{N(i,j)}(\widehat{g}_{ij}(1)(1-\widehat{g}_{ij}(1)) - \widehat{g}_{ij}(2))}{\sqrt{\widehat{g}_{ij}(1)(1-\widehat{g}_{ij}(1))^2(2-\widehat{g}_{ij}(1))}}, \quad (8)$$

where $N(i, j)$ denotes the number of transitions from state i to state j observed in the sample and $\widehat{g}_{ij}(x)$ is the empirical estimator of the probability $g_{ij}(x)$ which is given by the ratio between the number of transition from i to j occurring exactly after x unit of time and $N(i, j)$. This statistic, under the geometrical hypothesis H_0 (or Markovian hypothesis), has approximately the standard normal distribution (see Stenberg et al. 2006).

We applied this procedure to our data to execute tests at a significance level of 95%. Because we have six states, we estimated the $6 \times (6 - 1)$ waiting time distribution functions and for each of them we computed the value of the test statistic (8). The geometric hypothesis is rejected for 17 of the 30 distributions. In Table 1, we show the results of the test applied to the waiting time distribution functions for few states.The large values of the test statistic suggest the rejection of the Markovian hypothesis in favour of the more general semi-Markov one.

4.2. Probability transition matrices

To set the Markov model and the semi-Markov one, described in the previous section, we use the Matlab Application Semi-Markov Toolbox (D'Amico et al. 2014a). This application allows to estimate the transition matrices for Markov and semi-Markov models starting from real discrete data of a given phenomenon. This application is also able to produce synthetic time series, of the same length as the real one, by means of Monte Carlo simulation. The Monte Carlo algorithm consists of repeated random sampling to compute successive visited states of the random variables $\{J_0, J_1, ...\}$ up to the horizon time L. The difference between semi-Markov and Markov models is that, in the first case, the jump times $\{T_0, T_1, ...\}$ between successive transitions are considered as a random variable. The algorithm for semi-Markov model consists of four steps:

(1) Set $n = 0, J_0 = i, T_0 = 0$, horizon time $= L$;
(2) Sample J from $\widehat{p}_{J_n}$ and set $J_{n+1} = J(\omega)$;
(3) Sample W from $\widehat{G}_{J_n,J_{n+1}}$ and set $T_{n+1} = T_n + W(\omega)$;
(4) If $T_{n+1} \geq L$ stop

else set $n = n + 1$ and go to (2).

We show the results of the application in terms of transition probability matrices of the two considered models later. Particularly, in Figure 2, we show graphically the transition probability matrix of the Markov model.

In Figures 3 and 4 instead, we show the transition probability matrices of the semi-Markov model. The different matrices are plotted by varying the time t, by fixing $v = 1$ (Figure 3), while the different matrices are plotted by varying the backward v, by fixing $t = 1$ (Figure 4). As it is possible to note the dependence of the tornado process by the backward is stronger with respect to time dependence. This is evident in Figure 4, where for little variations of the backward we have great

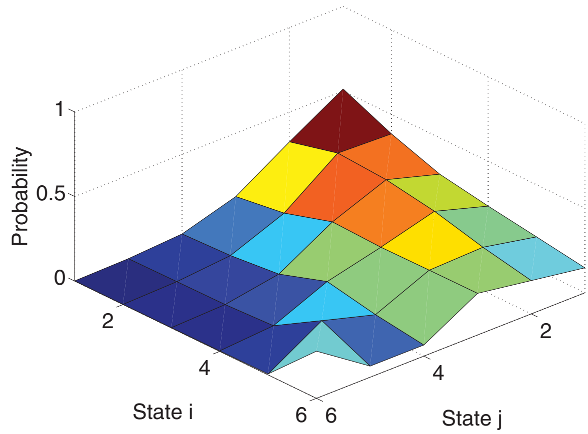

Figure 2. Transition probability matrix of the embedded Markov model.

variation on the probability transition matrices. From Figure 4, we highlight the strong dependence of the process from backward by observing extreme states. For example, if we have an F5 tornado (state 6), we can observe that the probability to have, in the next step, a tornado with the same intensity increases with the increasing of backward, going from 0.27 for $v = 1$ to 0.62 for $v = 6$. A similar

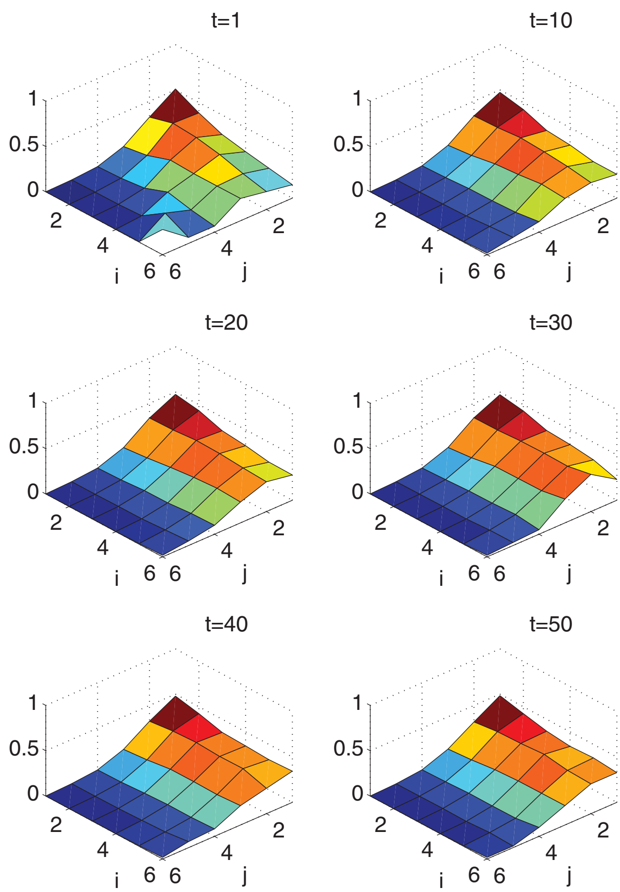

Figure 3. Transition probability matrix of the semi-Markov model varying the time *t*.

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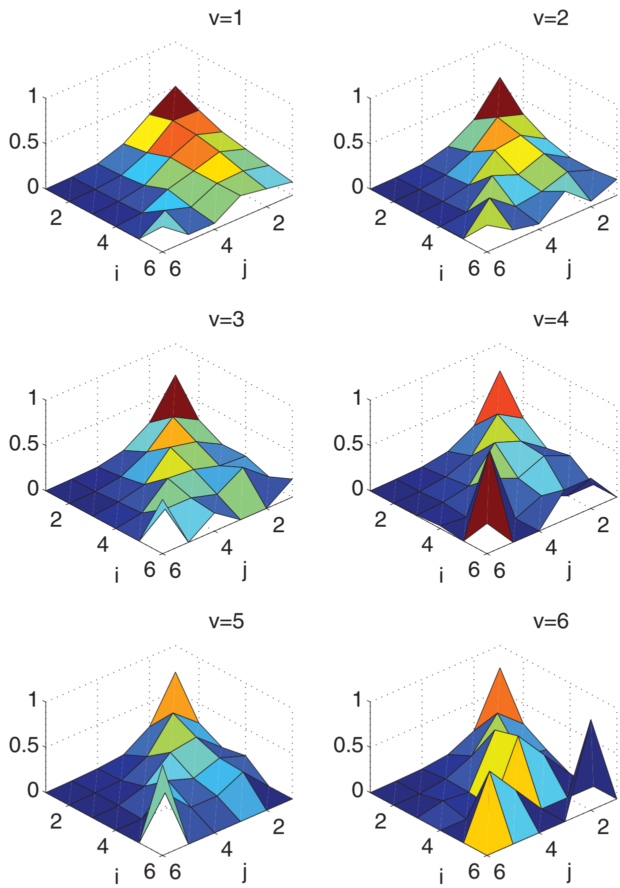

Figure 4. Transition probability matrix of the semi-Markov model varying the backward v.

observation can be made for the virtual transition on the state 1, which corresponds to F0 intensity where we have a variation from 0.51 to 0.75, respectively, for v equal to 1 and 6. More generally, we can note, at the increasing of backward, a movement of probability mass on the main diagonal of the transition probability matrices.

4.3. Cost application

As a further application we apply the cost model to the tornado time series. Particularly, we transform the original process into cost that a State has to pay due to tornado damages. To do this, we apply the results of Simmons et al. (2013). The F-scale is then transformed into costs, by associating to each grade a cost in the following way: 8689$, 62,440$, 121,141$, 146,564$, 177,824$ and 89,192$ that are, respectively, the mean costs of tornado degrees 0, 1, 2, 3, 4 and 5. As previously said, we compute the expected value and the variance of the accumulated discounted cost (see Figures 5 and 6, respectively). In both figures, the continuous lines are referred to real data while the dashed lines to the synthetic one. In these figures, we show the quantities as a function of the number of tornadoes and we highlight the dependencies with the actual state i and the backward process v by varying them. It is possible to affirm that the semi-Markov model well caught the behaviour of real data. It is possible to note that the results, in term of goodness of fit, for the mean of the accumulated discounted cost remain almost constant for large number of successive tornadoes, instead the variance starts to diverge near 40 successive tornadoes in both examples plotted.

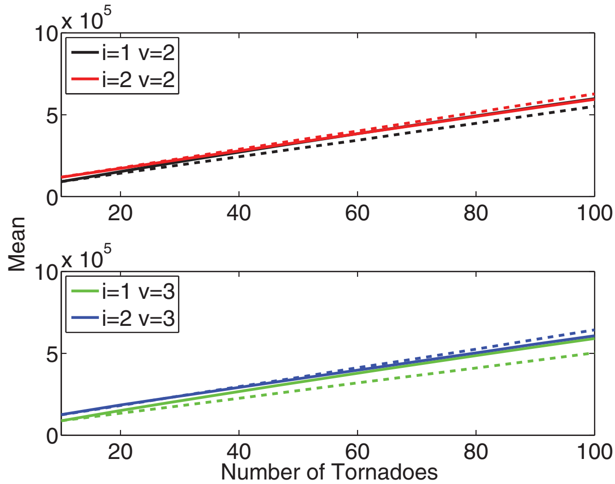

Figure 5. Expected value of the accumulated discounted cost. Comparison between real (continuous line) and synthetic data (dashed line).

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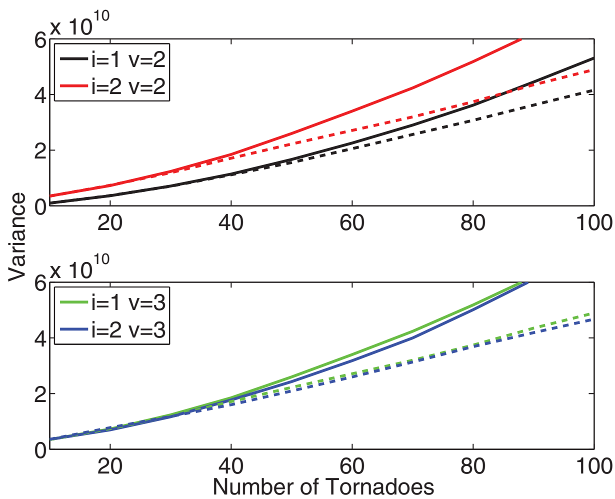

Figure 6. Variance of the accumulated discounted cost. Comparison between real (continuous line) and synthetic data (dashed line).

5. Conclusion

In this paper, we model the statistical behaviour of tornadoes in a vast region of the USA. To do this, we use a first-order semi-Markov model that is more general of the Markov chain model. We show, through a statistical test, that the latter one is not able to capture the duration effect of the tornadoes. The more general semi-Markov model in fact, by considering the time of permanence in a given state as generated by non-memoryless distribution, is able to reproduce the duration effect. Moreover, since we believe that the costs of the tornado damages are a serious problem related to this natural phenomenon, as an economic application, we compute the expected value and the variance of the accumulated discounted cost and we show its dependency by the intensity and the duration of the initial tornado. We have shown that the model is able to capture statistical feature of tornado occurrence, intensity and damages costs. It can then be used to make statistical prediction of those quantities.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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