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Abstract

Observed Engel curves are non-monotonic, hence consumption goods may be regarded as luxuries only for ranges of consumer income. This paper rationalises this evidence by postulating that quality of consumption governs the distribution of spending across goods. We argue that quality upgrading as income increases not only implies that virtually every variety of each good eventually becomes inferior. But also amends the notion of luxury good: a change in income, producing different quality variations across goods, causes heterogeneous spending responses. The resulting Engel curves shapes depend on the rise in quality of each good relative to the average consumption quality improvement. An illustrative simulation shows that the model captures the essential features of the observed Engel curves.

Keywords: Engel Curves, Nonhomothetic Preferences, Quality Ladders

JEL Classification: D11

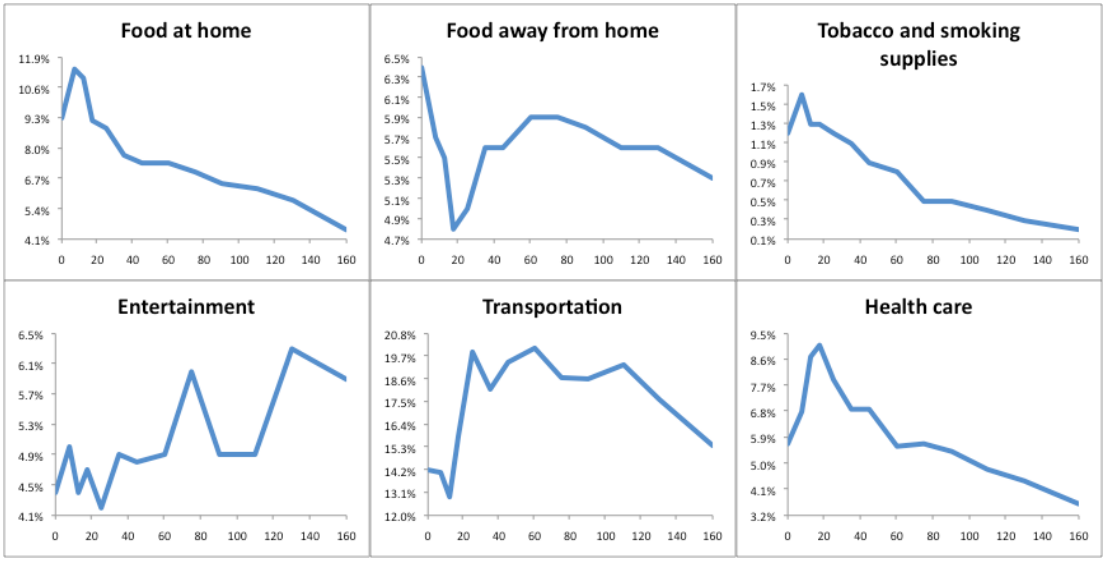
1 Introduction

> Of all the empirical regularities observed in economic data, Engel's [(1895)] Law is probably the best established. (Houthakker, 1987, p.142)

An Engel curve is a function that describes the relationship between demand for a good and consumer income, holding prices fixed. The curve illustrates the change in the consumption bundle composition occurring when consumers, as their income rises, devote a smaller fraction of resources to less desired goods (necessities), and a larger fraction to more desired ones (luxuries).

Empirical evidence based on observed Engel curves suggests that no good should be intrinsically considered a luxury or a necessity. Several studies document observed Engel curve non-monotonicities, including quadratic- or S-shapes, or non-monotonicities of higher order. (For a

Figure 1. Observed Engel curves.



Source: U.S. Bureau of Labor Statistics, year 2005.

review of this literature, see Lewbel, 2006; figure 1 exemplifies the Engel curves for a sample of six large aggregates in the U.S. for the year 2005, across different income clusters.) This evidence is suggestive of irregular patterns, which imply that goods should be regarded as luxury only for ranges of income. Notwithstanding, little effort in the economic literature is devoted to investigating the determinants for the complex relationship between consumer spending and consumer income.[1]

This paper aims to fill this gap in the literature. Section 2 constructs a model where quality levels set the different goods' relative appeal to consumers. In particular, we relate the evolution of consumer demand as income rises at an intra-industry level (*e.g.*, substitution of canned vegetables and salt-cured meat with fresh produce; of B&W TVs with HD flat screens) to that at an inter-industry level (*e.g.*, the demand shift from produce to visual entertainment), using the concept of 'quality upgrading' (Grossman and Helpman, 1991) as the common factor that determines consumer spending reallocations. For a given income level, each good is consumed in a particular quality, and the distribution of consumer spending across goods mirrors that of the consumed qualities. A rise in income implies, at an intra-industry level, a process of quality upgrading of each good. Depending on sectoral production characteristics, quality upgrading occurs at different pace across goods. At an inter-industry level, therefore, this mechanism

[1] An important exception is the notion of hierarchical preferences, postulating a priority ordering over the set of consumption goods (*e.g.*, Matsuyama, 2002; Foellmi and Zweimuller, 2006). This representation of consumers demand implies that every good is a luxury at sufficiently low-income levels, while gradually becoming a necessity as income rises. While S-shaped Engel curves can be justified by this assumption, more complex non-monotonicities cannot.

implies adjustments in the expenditure shares in response to the modified goods' relative appeal.

The model is consistent with the complex irregularities in the shapes of the observed Engel curves. While a direct testing is impossible due to the lack of relevant data, to illustrate this property I present in section 3 a simulation of the model.[2] Parameterisation is kept as simple as possible. First, I introduce good-specific upper-bounds to quality, to represent the technological state-of-the-art in the supply-side of the economy. Second, I let the cost of quality upgrading differ across goods. Even using this simple parameterisation, the simulation captures the fundamental patterns of the observed Engel curves.

2 The Model

Goods are organised along two dimensions: horizontal and vertical. (Figure 2 illustrates the commodity space.) The horizontal dimension (x-axis) designates the type of good, indexed by $v \in \mathbb{V} \equiv [0,1]$. The vertical dimension (y-axis) refers to the quality of the good, indicated by $q \in \mathbb{Q} \equiv \mathbb{R}_+$.[3] The commodity space is then given by the set $\mathbb{V} \times \mathbb{Q} = [0,1] \times [0,\infty)$, with each commodity identified by the pair $(v,q) \in \mathbb{V} \times \mathbb{Q}$.

The economy is inhabited by a continuum of individuals with identical preferences. I assume that the representative individual consumes strictly positive amounts of only one quality per good. Denote this quality by q_v, and the consumed quantity by $x_v \in \mathbb{R}_+$. The utility function then reads:

$$\begin{aligned} U = \ln C = \int_{\mathbb{V}} \ln c_v dv; \\ \text{with} \quad c_v = \begin{cases} x_v & \text{if } \ x_v < 1; \\ (x_v)^{q_v} & \text{if } \ x_v \geq 1; \end{cases} \end{aligned} \tag{1}$$

where C is an index of total consumption, and c_v represents the quality-adjusted consumption index for good v.

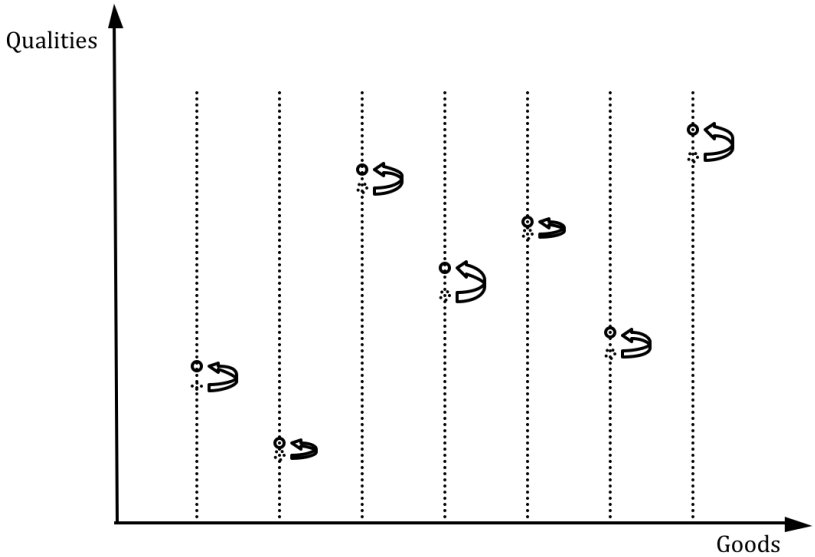
Utility (1) formalises the notions that quality is a desirable feature and turns increasingly desirable as the consumed quantity rises. Quality magnifies utility of (quantitative) consumption only when $x_v > 1$, capturing the idea that individuals first seek to satisfy their basic consumption needs, and only after these are met do they pay attention to the quality dimension of consumption.

The representative individual maximises utility (1) facing prices $\{p_v\}_{v \in \mathbb{V}}$ and the limit on

[2] The effects of quality on household decisions are empirically investigated mainly in the literature of international trade, where custom data at the product level permit constructing proxies for quality based on import prices (*e.g.*, Jaimovich and Merella, 2012). The lack of such figures for domestic trade disallows analogous proxies for aggregate consumption.

[3] Along with the proofs of all formal results, the appendix contains a definition of quality based on the theory of characteristics (Lancaster, 1966).

Figure 2. Quality upgrading.



Dashed and solid circles represent consumed qualities before and after wealth variation, respectively. Arrow sizes indicate the different intensities in quality upgrading across some representative goods.

spending $S \in \mathbb{R}_{++}$. The budget constraint is:

$$\int_{\mathbb{V}} p_v x_v dv = S. \tag{2}$$

I define $\beta_v \equiv p_v x_v / S$ as the *demand intensity* for good v, which measures the fractions of spending devoted to that good. In the optimum, the budget constraint (2) naturally binds, thus demand intensities sum up to one across goods (*i.e.*, $\int_{\mathbb{V}} \beta_v dv = 1$).

Regarding prices, I assume that higher qualities are more expensive, and that subsequent quality improvements become increasingly costly. Furthermore, to ease tractability, I assume that the price elasticity of quality upgrading, denoted by η_v, is constant along the quality space, although it may take different values across goods. Formally, price and quality are related by the function:[4]

$$p_v = (q_v)^{\eta_v} . \tag{3}$$

Before solving the consumer problem, it proves convenient to state the following preliminary result.

[4]Notice from (3) that the price elasticity of quality upgrading, formally $(dp_v/dq_v)(q_v/p_v)$, equals η_v since $dp_v/dq_v = \eta_v q_v^{\eta_v - 1}$ and $q_v/p_v = q_v^{1-\eta_v}$.

Lemma 1 *$x_v \geq 1$, $\forall v \in \mathbb{V}$ and $\forall S \in \mathbb{R}_{++}$.*

Consumed quantities are greater than one, regardless the amount of spending. Hence, in this specification of the model, individuals pay attention to the quality dimension of consumption at any level of income.

Problem 1 *The representative individual solves:*

$$\begin{aligned} \max_{\{q_v, x_v\}_{v \in \mathbb{V}}} \quad & \int_{\mathbb{V}} \ln c_v dv; \\ \textit{subject to:} \quad & \int_{\mathbb{V}} p_v x_v dv = S; \\ & c_v = \begin{cases} x_v & \textit{if} \;\; x_v < 1; \\ (x_v)^{q_v} & \textit{if} \;\; x_v \geq 1; \end{cases} \\ & p_v = (q_v)^{\eta_v}; \quad q_v, x_v \geq 0. \end{aligned}$$

Solution The **necessary and sufficient conditions** for a maximum are:

$$\ln \left[\beta_v S / (q_v)^{\eta_v}\right] = \eta_v; \tag{4}$$

$$q_v / \beta_v = \mu; \tag{5}$$

$$\int_{\mathbb{V}} \beta_v dv = 1; \tag{6}$$

where μ is a Lagrange multiplier associated to the budget constraint.

Consumer choice can be thought of as a two-stage decision. First, given the qualities distribution across goods, the individual chooses the fraction of spending devoted to consumption of each good. Formally:

$$\beta_v = \frac{q_v}{Q}, \;\; \forall v \in \mathbb{V}; \tag{7}$$

where $Q \equiv \int_{\mathbb{V}} q_z dz$ is an aggregate index measuring average consumption quality. The fraction of income spent on one good is thus determined by its quality relative to the average consumption quality. Second, for a given fraction of spending devoted to consumption of good v, the individual picks quality q_v to solve the 'trade-off' between quality and quantity by maximising c_v.

Proposition 1 *Given prices $\{p_v\}_{v \in \mathbb{V}}$ and income S, the consumer chooses, $\forall v \in \mathbb{V}$:*

$$x_v = e^{\eta_v} > 1; \tag{8}$$

$$q_v = e^{-\frac{\eta_v}{\eta_v - 1}} (S/Q)^{\frac{1}{\eta_v - 1}} > 0; \tag{9}$$

$$\beta_v = (eQ)^{-\frac{\eta_v}{\eta_v - 1}} (S)^{\frac{1}{\eta_v - 1}} > 0. \tag{10}$$

Consumed quantities differ across goods: the higher the price elasticity of quality upgrading, the larger the quantity consumed. This quantitative compensation for a higher cost of quality upgrading reflects the 'trade-off' between quality and quantity of consumption.

Note from (8) that a change in S has no effect on the consumed quantities. Hence, the analysis of the effects of a variation in total spending is limited to qualities and demand intensities.

Lemma 2 *Let $v', v'' \in \mathbb{V}$ be two goods such that $\eta_{v'} < \eta_{v''}$. Then:*

$$\frac{dq_v}{dS} = \frac{q_v}{\eta_v - 1}\frac{Q}{S}\left(\int_{\mathbb{V}} \frac{\eta_z q_z}{\eta_z - 1} dz\right)^{-1} > 0, \quad \forall v \in \mathbb{V}. \tag{11}$$

An increase in spending implies a rise in the quality consumed of each good. That is, the model endogenously generates a process of quality upgrading. The rationale is that consumers are mostly concerned with the quantitative aspects of consumption when their income is low, while they pay increasing attention to qualitative aspects as their income rises.

Proposition 2 *Denote $\tilde{\eta} \equiv \int_{\mathbb{V}} \eta_z q_z / (\eta_z - 1)\, dz / \int_{\mathbb{V}} q_z / (\eta_z - 1)\, dz$. Then:*

$$\frac{d\beta_v}{dS} = \frac{1}{S}\frac{\beta_v}{\eta_v - 1}\frac{\tilde{\eta} - \eta_v}{\tilde{\eta}}, \quad \forall v \in \mathbb{V}; \tag{12}$$

$$\frac{d\beta_v}{dS} \gtreqless 0 \quad \Leftrightarrow \quad \eta_v \lesseqgtr \tilde{\eta}. \tag{13}$$

Proposition 2 formalises the central result of the paper. The sign of the income effect on demand intensity is positive (negative) for sufficiently low (high) levels of price elasticity of quality upgrading. The sign switch is due to demand intensity not exclusively depending on the quality variation of the relevant good, but rather on that quality level change relative to the variation in the average consumption quality.

3 Simulation of the model

A mild relaxation of the assumptions adopted in section 2 allows capturing the fundamental patterns of the observed Engel curves: I introduce a change from finite to infinite magnitude of η_v at some level of quality. This variation implies an upper-bound to quality levels, which I normalise to one for all goods. The natural interpretation of this bound is that, at each point in time, the production of higher qualities may not (yet) be technologically feasible.

Formally, I modify the basic model by introducing two changes. First, my simulation takes only six goods into account. The commodity set is re-defined $\mathbb{V} \subset \mathbb{N} : v = 1, 2, ..., 6$. Second, for

each good, price relates to quality according to the function:

$$p_v = \begin{cases} a_v (q_v)^{\eta_v} & \text{if } \; q_v \leq 1 \\ \infty & \text{if } \; q_v > 1 \end{cases} \tag{14}$$

where a_v is a good-specific parameter introduced to re-scale price (3) following quality normalisation.

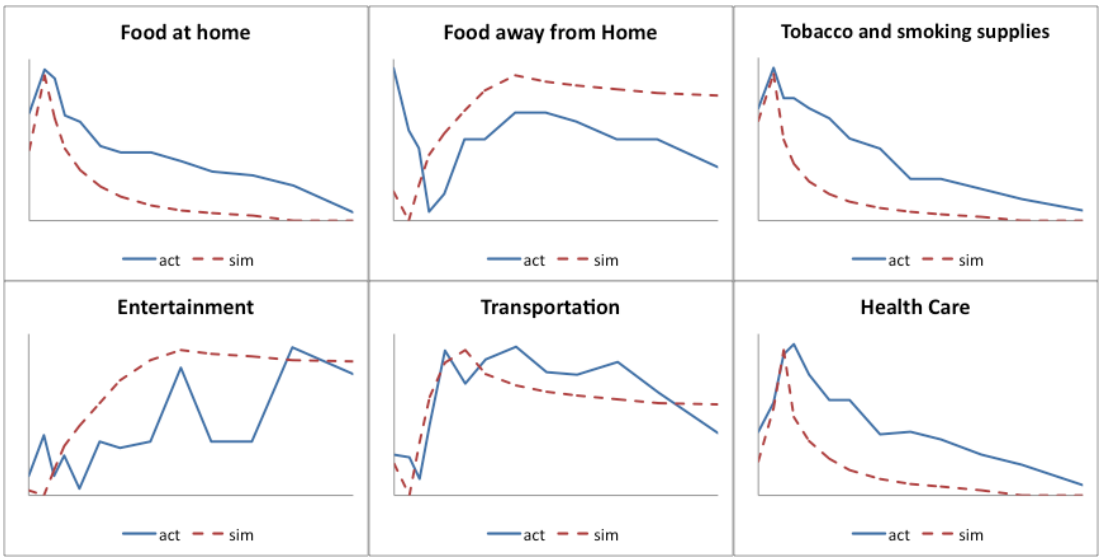
Proposition 3 *Let* $Q \equiv \sum_{v=1}^{6} q_v$. *Then:*

$$\beta_v = \max \left\{ (S/a_v)^{\frac{1}{\eta_v - 1}} (eQ)^{-\frac{\eta_v}{\eta_v - 1}}, 1/Q \right\}, \quad \forall v \in \mathbb{V}. \tag{15}$$

Equation (15) is used to generate the results of the simulation, illustrated in Figure 3. The parameters values $\{a_v, \eta_v\}_{v \in \mathbb{V}}$ are chosen such that the observed peak in the relevant expenditure share is replicated at the quality upper-bound.[5]

Figure 3 offers a simple comparison with actual data by reporting the observed curves form Figure 1. The fact that more than one change in the Engel curves slopes is replicated illustrates the central feature of my model. This generalises the representations of preferences found in the literature, which do not allow for other Engel curve patterns than monotonic or hump-shaped.

Figure 3. Simulated vs. observed Engel curves.



Source: U.S. Bureau of Labor Statistics, year 2005; and my calculations.

[5] The complete set of parameters is: "Food at home", $\{0.032, 1.3\}$; "Food away from home", $\{0.07, 1.7\}$; "Tobacco and smoking supplies", $\{0.03, 1.3\}$; "Entertainment", $\{0.09, 1.5\}$; "Transportation", $\{0.055, 1.65\}$; "Health care", $\{0.034, 1.3\}$.

4 Concluding Remarks

This paper proposes a model generating a process of quality upgrading that governs two important features of consumer demand. First, every good is bound to become inferior at some level of wealth. Second, demand shifts towards the most attractive goods are not necessarily monotonic, in line with the observed Engel curves. The central result is that increasing shares of spending are devoted to those goods whose quality can be more cheaply upgraded. A simulation exercise shows that the observed patterns of U.S. consumer spending across different levels of income can be replicated under a sensibly chosen parameterisation.

Appendix

The Appendix is organised as follows. Section A illustrates an extension of the model that allows to study the income effects on labour supply in the presence of varying willingness to pay for quality. Section B proposes a formal definition of quality based on elements of the theory of characteristics. Section C collects proofs of lemmas and propositions, along with the analytical derivations of other equations, that were omitted in the main text. Section D formalises some additional theoretical results.

A Extension: Wealth effects and labour supply

An important consequence of the analysis produced in the main article is that the consumer decision regarding the composition of the consumption bundle are not neutral to variations produced by a rise in income. In previous studies —see, *e.g.*, Grossman and Helpman (1991) and Aghion and Howitt (1992)— the choice of which levels of quality to consume not only depends merely on price-quality considerations and is not influenced by income, but also does not affect marginal utility of (quantitative) consumption. As a result, the presence of quality ladders only influences consumer welfare. In this appendix, I show that, under a preference representation such as that illustrated in Section 2, consumer taste for quality may also influence key economic variables other than welfare measures, such as labour supply.

The starting point is to construct a quantitative indicator of total consumption (obtained as an aggregator of the optimal allocations across the different goods) and the associated price index, using the sector-specific results of the model, the definition of average quality and standard aggregation techniques.

Corollary 1 *Let* $C \equiv X^Q$ *be the aggregate consumption index, and* $P \equiv \exp\left(\int_{\mathbb{V}} \beta_v \ln p_v dv\right) /$

$\exp\left(\int_{\mathbb{V}} \beta_v \ln \beta_v dv\right)$ *the price index. Furthermore, denote* $\bar{\eta} \equiv \int \eta_v \beta_v dv$. *Then:*

$$X = e^{\bar{\eta}}. \tag{16}$$

Proof. The optimal quantity consumed for each good v as a function of price can be obtained by replacing (7) into the definition of demand intensity, and rearranging to get: $x_v = (S/Q)(q_v/p_v)$. Replacing x_v in (1) with this expression yields the indirect utility: $\ln C = \int_{\mathbb{V}} q_v \ln\left[(S/Q)(q_v/p_v)\right] dv$. Isolating S, and using the definition of average quality and (7) again, I find the expenditure function in logarithmic terms: $\ln S = (1/Q)\ln C + \int_{\mathbb{V}} \beta_v \ln p_v dv - \int_{\mathbb{V}} \beta_v \ln \beta_v dv$. Taking exponentials:

$$S = \frac{\exp\left(\int_{\mathbb{V}} \beta_v \ln p_v dv\right)}{\exp\left(\int_{\mathbb{V}} \beta_v \ln \beta_v dv\right)} C^{1/Q}.$$

Imposing: $S = PX$; I can define: $X \equiv C^{1/Q}$; and: $P \equiv \exp\left(\int_{\mathbb{V}} \beta_v \ln p_v dv\right) / \exp\left(\int_{\mathbb{V}} \beta_v \ln \beta_v dv\right)$. Note that the price index can be rewritten as: $P = \exp\left(\int_{\mathbb{V}} \beta_v \ln(\beta_v/p_v)\, dv\right)$. From the definition of demand intensity: $\beta_v/p_v = x_v/S$; hence: $P = \exp\left(\int_{\mathbb{V}} \beta_v \ln(x_v/S)\, dv\right)$. Using (8) yields: $P = \exp\left(\int_{\mathbb{V}} \eta_v \beta_v dv\right)/S$. Hence: $X = S/P = S/\left[\exp\left(\int_{\mathbb{V}} \eta_v \beta_v dv\right)/S\right]$. Simplifying, and using the definition of $\bar{\eta}$, (16) obtains. ■

Corollary 1 points out that, although the good-specific optimally consumed quantities hold constant as income increases, the index of total consumption (16) decreases with income. The reason is that, for each good, the consumed quantity is proportional to the cost of quality upgrading. As Proposition 2 states, richer consumers tend to spend rising fractions of resources on goods that can be more cheaply upgraded. These goods are precisely those whose constant allocations are smaller. As a result, a rise in income shifts consumer demand away from goods consumed in larger amounts and towards goods whose consumed quantities are smaller, thereby generating a fall in total number of units consumed. This feature is formally captured by the index of average price elasticity of quality upgrading, $\bar{\eta}$, which for the same reason just stated falls as consumer spending rises.

With the aggregation indicators introduced above, it is possible to illustrate how taste for quality affects other consumer choices than the composition of the consumption bundle. In particular, here I focus on labour supply. In order to take labour market structure into account, I need to modify the utility function to account for leisure. Accordingly, I assume that the representative consumer devote a fraction L of the unit labour endowment to work, and the remaining fraction $(1-L)$ to leisure. The latter yields utility according to the function:

$$U = Q \ln X + \gamma \ln(1-L); \tag{17}$$

where γ is a preference parameter that measures the weight attached to leisure relative to consumption. In addition, the consumer's budget constraint must be redefined to account for the endogenous nature of total resources available for spending, which now depends on the amount of time devoted to work:

$$PX = wL, \tag{18}$$

where $w \in \mathbb{R}_{++}$ is the wage obtained by the representative consumer in return to labour services supplied to firms.

Note that, given the static nature of the model and the additive separable specification of preferences, the introduction of L implies a mere substitution of spending S with earning wL in the construction of the individual's maximisation problem. This means that only condition (4) for the solution of Problem 1 is affected, and now reads:

$$\ln\left[\beta_v wL/\left(q_v\right)^{\eta_v}\right] = \eta_v. \tag{19}$$

Therefore, changes in the fraction of labour devoted to work produce analogous effects, with regard to consumption goods choice, as an increase of spending S. As shown in Section 2, optimal qualities and optimal demand intensities can be seen as functions of S and, thereby, also the aggregation indicators constructed above. In this section, all these variables can thus be seen as functions of L. I follow this lead in the next proposition, which illustrates how the representative consumers chooses the optimal amount of labour supply by maximising utility (17) subject to the constraint (18). The result is expressed as a function of wages, whose variations are used to explore the wealth effects on labour supply.

Proposition 4 $L(w) = Q(w)/[Q(w)+\gamma]$; *with* $\partial L(w)/\partial w > 0$.

Proof. The representative consumer solves:

$$\max_{\{X,L\}} \quad U = Q\ln X + \gamma\ln(1-L);$$

$$\textit{subject to:} \quad X = e^{\bar{\eta}}, \quad Q = Q(L), \quad \bar{\eta} = \bar{\eta}(L);$$

where $Q = Q(L)$ and $\bar{\eta} = \bar{\eta}(L)$ indicate that the relevant indicators are, directly or indirectly, functions of one of the choice variables. By substituting the constraint into the objective function, the problem can be rewritten as: $\max_L U = Q(L)\,\bar{\eta}(L) + \gamma\ln(1-L)$. Differentiating with respect to L, and equating the resulting expression to zero yields:

$$\left(\frac{dQ}{dL}\bar{\eta} + Q\frac{d\bar{\eta}}{dL}\right) - \frac{\gamma}{1-L} = 0; \tag{20}$$

where, by analogy of dQ/dL with dQ/dS in (26) and of $d\beta_v/dL$ with $d\beta_v/dS$ in (12):

$$\frac{dQ}{dL} = \frac{Q}{L}\left(\int_{\mathbb{V}} \frac{q_z}{\eta_z - 1} dz\right) / \left(\int_{\mathbb{V}} \frac{\eta_z q_z}{\eta_z - 1} dz\right) = \frac{Q}{L}\frac{1}{\tilde{\eta}};$$

and:

$$\frac{d\bar{\eta}}{dL} = \int_{\mathbb{V}} \eta_v \frac{d\beta_v}{dL} dv = \frac{1}{L}\int_{\mathbb{V}} \frac{\eta_v}{\eta_v - 1}\frac{\tilde{\eta} - \eta_v}{\tilde{\eta}} \beta_v dv.$$

Using these two equations in (20), and recalling that $(\tilde{\eta} - 1)/\tilde{\eta} = 1/\left(\int_{\mathbb{V}} \eta_v \beta_v / (\eta_v - 1)\, dv\right)$:

$$\frac{\gamma}{1 - L} = \frac{Q}{L}\frac{\tilde{\eta} - 1}{\tilde{\eta}} \int_{\mathbb{V}} \frac{\eta_v}{\eta_v - 1} \beta_v dv = \frac{Q}{L}.$$

Rearranging, the first claim in Proposition 4 straightforwardly obtains.

The fact that optimal labour supply is increasing with wages immediately follows from considering that, according to Lemma 2, each good-specific quality and thereby average quality Q increase with w, and that:

$$dL/dQ = \gamma / (Q + \gamma) > 0;$$

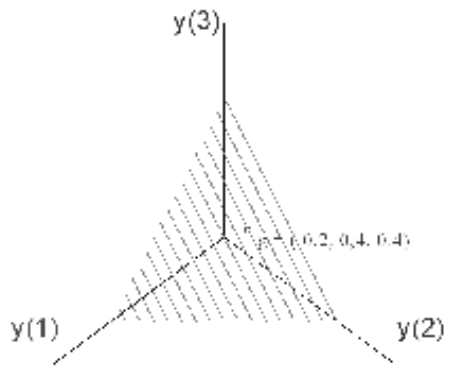
since $Q \geq 0$ and, by definition $\alpha, \gamma > 0$. ■

Proposition 4 shows that consumer taste for quality not only has a welfare impact, but also affects the consumer decision on the labour market. Labour supply increases with the level of average quality in consumption. Furthermore, the optimal amount of time spent on working rises as wages w increase. This prediction seems quite intuitive. On the one hand, if the level of quality in consumption is low, then the value of consumption in utility terms is also low: consumers tend to minimise their working time, because the resulting spending capability yields relatively little utility compared to leisure. On the other hand, if quality is high, then the value of consumption in utility terms is also high, and consumers tend to increase their working time. The equilibrium condition that follows from a preferences representation such as that adopted by Grossman and Helpman (1991) —formally, $L = 1/(1 + \gamma)$— neglects this phenomenon, predicting that the level of labour supply is set irrespective of the level of quality in consumption.

B Formal definition of quality

This section proposes a formal specification for the quality index. Alternative definitions can be found, among many others, in Stokey (1988) and Merella (2006). Although not essential —several contributions making explicit use of quality provide no formalization of the quality index— defining quality helps in characterising the theoretical difference between the vertical and the horizontal aspect of consumption goods differentiation.

Figure 4: A commodity in the characteristics simplex



When qualitative differentiation is introduced by assigning each commodity a value in a quality ladder, a strict separation between quantitative and qualitative aspects is required. For this reason, a commodity is referred to as a unit object, and its components as proportions of this object. Following the Lancastrian tradition, a commodity is outlined by its underlying *characteristics*, and it is thus depicted by the *allocations* of characteristics it contains.[6] In keeping quantitative and qualitative aspect of consumption separated, however, I depart from Lancaster's theory in assuming that the characteristic allocations lie in a unit interval, and that a commodity is properly identified only if the (fractional) allocations of the different characteristics it contains sum up to one.

Formally, I assume that there exists a *characteristic set* $\mathbb{J} \subset \mathbb{N} : j \leq J$, where j stands for a generic characteristic and J is the number of elements in $\mathbb{J}$. Each j identifies one dimension in the *characteristic allocation space* $\mathbb{Y} \subset \mathbb{R}_+^J : \{y_j \in [0,1], \forall j \in \mathbb{J}\}$, where y_j measures the jth characteristic allocation. A commodity is defined as a $(J \times 1)$ vector $\mathbf{g} \equiv [y_1, y_2, ..., y_J]'$ in $\mathbb{Y}$, such that $\mathbf{g}'\mathbf{I} = 1$, where $\mathbf{I}$ is the unit $(J \times 1)$ vector. The resulting commodity set, denoted by $\mathbb{S} \subset \mathbb{Y}$, can be geometrically represented by a $(J-1)$ dimensional *simplex*, an object often used in studies involving probabilities. In Figure 4, the simplex is portrayed by the gray wired triangle for a $J = 3$ case. Each point in the simplex represents a commodity (in the figure, one example is given by point $\mathbf{g}$), qualitatively different from all others.

The commodity set so obtained can be sorted by defining a set of proper *datum* points. Given the nature of the problem, a sensible benchmark is provided by human *needs*. These are assumed to define predetermined and objectively identifiable *ideal* objects, whose set is denoted by $\mathbb{V} \subset \mathbb{R} : v \in [0,1]$, where v indexes needs. I assume each ideal object finds concrete expression in a commodity equivalent (hereafter called *bliss*) in $\mathbb{S}$, denoted by $\mathbf{g}_v$. By comparing a generic

[6] For a review of the theory of characterisitcs, see Lancaster (1971).

$\mathbf{g}$ commodity to the bliss, I obtain a need-specific measure of qualitative differentiation. I name this measure *quality*, and denote it by $q(\mathbf{g}, \mathbf{g}_v) \in \mathbb{R}_+$ for some commodity $\mathbf{g}$ with reference to need v.

In this framework, it seems natural to imagine an inverse relation of quality to the Euclidean distance between the reference commodity and the bliss, denoted by $d(\mathbf{g}, \mathbf{g}_v) \equiv |\mathbf{g} - \mathbf{g}_v|$. Quality can thus be formally represented by the function $q(\mathbf{g}, \mathbf{g}_v) = f[d(\mathbf{g}, \mathbf{g}_v)]$, with $f'(\cdot) < 0$. Function f can be given a number of possible formalizations. A convenient way to define it is to allow quality to range in all positive real numbers as stated above. Other intervals may, however, be more appropriate in some contexts.

According to the framework developed so far, each commodity is able to satisfy, at least to some extent, all human needs. Although it possibly reflects reality in greater details, representing preferences in such a setting is extremely complex. The analysis is thus eased by assuming that one commodity can only satisfy one need. This univocal relation greatly simplifies preference representation, and the resulting demand functions take a more convenient form. A "qualitative priority" condition seems a sensible way to link each good to a single need: that is, a commodity satisfies the need for which that commodity is associated to the highest level of quality.

Denoting by $\mathbb{S}_v \subseteq \mathbb{S}$ the subset of commodities satisfying the vth need, I formally express the above condition as $\mathbf{g} \in \mathbb{S}_v \Leftrightarrow q(\mathbf{g}, \mathbf{g}_v) = \max_{z \in \mathbb{V}} \{q(\mathbf{g}, \mathbf{g}_z)\}$. If the level of quality of a commodity is the same with regard to two or more needs, then it might be assumed, for instance, that commodity is in the subset with the lowest v. Under these assumptions, the subsets of commodities satisfying different needs are disjoint, i.e. $\mathbb{S}_v \cap \mathbb{S}_z = \emptyset$, $\forall v, z \in \mathbb{V}$. I name these subsets *goods*. The definition of quality also provides a criterion for ordering the elements in each good set $\mathbb{S}_v$. The quality space is denoted by $\mathbb{Q} \subseteq \mathbb{R}_+ : q \equiv q(\mathbf{g}, \mathbf{g}_v)$, where q is the level of quality associated to commodity $\mathbf{g}$. Using the definitions of quality and good, hereafter each commodity $\mathbf{g} \in \mathbb{S}_v$ is identified by the pair $(v, q) \in \mathbb{V} \times \mathbb{Q}$.[7]

C Omitted proofs

Derivation of (1). Start from an aggregator of Dixit and Stiglitz (1977) type (in logarithmic terms): $\ln\left(\int_{\mathbb{V}} (c_v)^{(\zeta-1)/\zeta}\, dv\right)^{\zeta/(\zeta-1)}$. Computing the limit for $\zeta \to 1$ yields:

$$\lim_{\zeta \to 1} \ln\left(\int_{\mathbb{V}} (c_v)^{(\zeta-1)/\zeta}\, dv\right) / \left[(\zeta - 1)/\zeta\right] = 0/0.$$

[7] Notice that several varieties of the same good may provide the same level of quality. Among them, however, only one will be actively produced. Since individuals will be completely indifferent in puchasing any of such commodities, demand will be set according to minimum-price criteria.

This indeterminacy can be solved by applying the De l'Hopîtal rule, which gives:

$$\lim_{\zeta \to 1} \frac{\left(\int_{\mathbb{V}} \left(1/\zeta^2\right) (c_v)^{(\zeta-1)/\zeta} \ln c_v dv\right) / \left(\int_{\mathbb{V}} (c_v)^{(\zeta-1)/\zeta} dv\right)}{1/\zeta^2} = \int_{\mathbb{V}} \ln c_v dv.$$

Hence, the first expression in (1) obtains. ■

Proof of Lemma 1. Suppose the individual chooses $x_v < 1$, for some $v \in \mathbb{V}$. From the definition of good-specific quality-adjusted consumption index in (1), it follows that: $c_v = x_v$; so the individual will optimally choose the cheaper good, regardless of the level of quality attached to it. From (3), it follows that the consumption good in the baseline quality, $q = 0$, is costless, regardless the good considered. Therefore, the individual will optimally choose to consume the good in the baseline quality, $q_v = 0$. But this cannot be optimal, since the individual can achieve a higher level of c_v by increasing the quantity consumed x_v without any additional cost, leading to a contradiction. ■

Complete Solution of Problem 1. First note that, using Lemma 1 and the definition of demand intensities, solving Problem 1 is equivalent to solve the following problem:

$$\begin{aligned} \max_{\{q_v, \beta_v\}_{v \in \mathbb{V}}} \quad & \textstyle\int_{\mathbb{V}} \ln c_v dv, \\ \textit{subject to:} \quad & \textstyle\int_{\mathbb{V}} \beta_v dv = 1; \\ & c_v = (x_v)^{q_v}, \quad x_v = \beta_v S / p_v, \\ & p_v = (q_v)^{\eta_v}, \quad q_v, \beta_v \geq 0. \end{aligned} \tag{21}$$

Lemma 1 ensures that, under $q_v, x_v \geq 0$, there cannot be candidate maxima in the subset $\{x_v\}_{v \in \mathbb{V}} \in [0, 1)$, hence using the good-specific quality-adjusted consumption index $c_v = (x_v)^{q_v}$ is equivalent to use that specified in (1). The definition of demand intensity $\beta_v = p_v x_v / S$ allows for the maximisation problem to be formalised in terms of the choice variables $\{q_v, \beta_v\}_{v \in \mathbb{V}}$. This is useful for two reasons. Firstly, demand intensities are my main object of interest, hence discussing the optimality conditions taken with respect to these variables provides a deeper insight on how the consumer composes the consumption bundle. Secondly, it eases tractability, especially with regard to the computation of the sufficient conditions for a maximum.

I write the Lagrangian for the problem 21:

$$\mathcal{L} = \int_{\mathbb{V}} q_v \ln \left(\beta_v S / (q_v)^{\eta_v}\right) dv + \mu \left(1 - \int_{\mathbb{V}} \beta_v dv\right).$$

The first-order conditions for a maximum are:

$$\begin{gathered} d\mathcal{L}/dq_v = \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right) - \eta_v \leq 0; \quad q_v \geq 0; \\ q_v\left[\ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right) - \eta_v\right] = 0; \end{gathered} \tag{22}$$

$$\begin{gathered} d\mathcal{L}/d\beta_v = q_v/\beta_v - \mu \leq 0; \quad \beta_v \geq 0; \\ \beta_v\left(q_v/\beta_v - \mu\right) = 0; \end{gathered} \tag{23}$$

$$d\mathcal{L}/d\mu = 1 - \int_{\mathbb{V}} \beta_v dv = 0. \tag{24}$$

The Kuhn-Tucker Sufficiency Theorem —see Chiang and Wainwright (2005, pp. 424-5)— implies that the first-order conditions are necessary *and* sufficient for a *global* maximum in the nonnegative orthant, since: i) The objective function:

$$\int_{\mathbb{V}} q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right) dv$$

is strictly concave in the nonnegative orthant. This condition holds since the objective function is a sum over the strictly concave functions:

$$\left\{q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right)\right\}_{v\in\mathbb{V}}$$

in $(q_v, \beta_v) \in \mathbb{R}^2_+$. To prove this point, note that:

$$\begin{gathered} \partial^2 q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right)/\left(\partial q_v\right)^2 = -\eta_v/q_v; \\ \partial^2 q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right)/\left(\partial \beta_v\right)^2 = -q_v/\left(\beta_v\right)^2; \\ \partial q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right)/\left(\partial q_v \partial \beta_v\right) = 1/\beta_v = \partial q_v \ln\left(\beta_v S/\left(q_v\right)^{\eta_v}\right)/\left(\partial \beta_v \partial q_v\right). \end{gathered}$$

Hence, the Hessian matrix is negative definite, since:

$$\begin{aligned} |H_1| &= -\eta_v/q_v < 0; \\ |H_2| &= |H| = \left(\eta_v - 1\right)/\left(\beta_v\right)^2 > 0. \end{aligned}$$

ii) Each constraint is convex in the nonnegative orthant. This is trivial, since both the nonnegativity constraints and the budget constraint are linear, both in qualities and demand intensities. As a result: if there exists a candidate (interior) solution $\{q_v, \beta_v\}_{v\in\mathbb{V}}$ satisfying (4)-(6), then this solution also satisfies (22)-(24), and is the only possible solution to Problem 1. ■

Derivation of (7). From (5), multiply both sides by β_v, integrate across goods, and use the definition of average quality to obtain: $Q = \int_{\mathbb{V}} q_v dv = \mu \int_{\mathbb{V}} \beta_v dv = \mu$. Replace this result into (5) and rearrange to get (7). ■

Proof of Proposition 1. First, the result in (8) trivially follows from (4): (i) recalling that, by the definitions of price and demand intensity, $x_v = \beta_v S/\left(q_v\right)^{\eta_v}$, hence: $\ln x_v = \eta_v$; and (ii) taking exponentials on both sides of the resulting equation.

Second, replace (7) into (4) to obtain: $\ln\left(\left(q_v/Q\right)S/\left(q_v\right)^{\eta_v}\right) = \eta_v$. Take exponentials of both sides in the last equation and rearrange to get: $\left(q_v\right)^{1-\eta_v} S/Q = e^{\eta_v}$. Isolate q_v to have (9).

Third, divide both sides by Q to get: $\beta_v = e^{-\frac{\eta_v}{\eta_v-1}}\left(S\right)^{\frac{1}{\eta_v-1}}\left(Q\right)^{-\frac{1}{\eta_v-1}-1}$. Then, rearrange to obtain (10).

Finally, the inequalities follow from considering that: $\eta_v > 1$, $\forall v \in \mathbb{V}$; and that: $S, Q > 0$. ■

Proof of Lemma 2. Differentiate (9) with respect to S, and use (9) again to get:

$$\frac{dq_v}{dS} = \frac{q_v}{\eta_v - 1}\left(\frac{1}{S} - \frac{1}{Q}\frac{dQ}{dS}\right). \tag{25}$$

Use the definition of average quality and (25) to obtain: $dQ/dS = \int_{\mathbb{V}}\left(dq_z/dS\right)dz = \left(1/S\right)\int_{\mathbb{V}} q_z/\left(\eta_z - 1\right)dz - \left(1/Q\right)\left(dQ/dS\right)\int_{\mathbb{V}} q_z/\left(\eta_z - 1\right)dz$. Isolate dQ/dS and rearrange:

$$\frac{dQ}{dS} = \frac{Q}{S}\left(\int_{\mathbb{V}}\frac{q_z}{\eta_z - 1}dz\right)/\left(\int_{\mathbb{V}}\frac{\eta_z q_z}{\eta_z - 1}dz\right) > 0. \tag{26}$$

Use (26) into (25) to get:

$$\frac{dq_v}{dS} = \frac{1}{S}\frac{q_v}{\eta_v - 1}\left[1 - \left(\int_{\mathbb{V}} q_z/\left(\eta_z - 1\right)dz\right)/\left(\int_{\mathbb{V}}\eta_z q_z/\left(\eta_z - 1\right)dz\right)\right].$$

Rearranging, (11) obtains, where the inequality holds since: $q_v, q_z > 0$; and $\eta_v, \eta_z > 1$, for all $v, z \in \mathbb{V}$. ■

Proof of Proposition 2. First, from (7), differentiate with respect to S:

$$\frac{d\beta_v}{dS} = \frac{d\left(q_v/Q\right)}{dS} = \frac{1}{Q}\frac{dq_v}{dS} - \frac{\beta_v}{Q}\frac{dQ}{dS}; \tag{27}$$

use (25) to get: $d\beta_v/dS = \left(\beta_v/S\right)/\left(\eta_v - 1\right) - \left[1 + 1/\left(\eta_v - 1\right)\right]\left(\beta_v/Q\right)\left(dQ/dS\right)$. Rearrange:

$$\frac{d\beta_v}{dS} = \frac{\beta_v}{\eta_v - 1}\left(\frac{1}{S} - \frac{\eta_v}{Q}\frac{dQ}{dS}\right). \tag{28}$$

Use (26) to obtain:

$$\frac{d\beta_v}{dS} = \frac{\beta_v}{\eta_v - 1}\left\{\frac{1}{S} - \frac{\eta_v}{Q}\left[\frac{Q}{S}\left(\int_{\mathbb{V}}\frac{q_z}{\eta_z - 1}dz\right)/\left(\int_{\mathbb{V}}\frac{\eta_z q_z}{\eta_z - 1}dz\right)\right]\right\}.$$

Use the definition of $\tilde{\eta}$ to have: $d\beta_v/dS = \left[\beta_v/\left(\eta_v - 1\right)\right]\left[\left(1/S\right) - \left(\eta_v/S\right)/\tilde{\eta}\right]$. Rearranging, (12) obtains.

Second, from (12) it follows:

$$\frac{d\beta_v}{dS} \gtreqless 0 \quad \Leftrightarrow \quad \tilde{\eta} - \eta_v \gtreqless 0;$$

from which (13) straightforwardly obtains. ■

Proof of Proposition 3. The optimisation problem can be stated as:

$$\begin{aligned} \max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad & \textstyle\sum_{v=1}^{6} q_v \ln\left(\beta_v S/p_v\right); \\ \textit{subject to:} \quad & \textstyle\sum_{v=1}^{6} \beta_v = 1; \\ & \{p_v = a_v\left(q_v\right)^{\eta_v}\}_{v\in[1,6]}; \quad \{q_v \leq 1\}_{v\in[1,6]}. \end{aligned}$$

Write the Lagrangian:

$$L = \sum\nolimits_{v=1}^{6} q_v \ln\left(\left(S/a_v\right)\beta_v/\left(q_v\right)^{\eta_v}\right) + \mu\left(1 - \sum\nolimits_{v=1}^{6}\beta_v\right) + \sum\nolimits_{v=1}^{6}\lambda_v\left(1-q_v\right).$$

The first-order conditions for a maximum are:

$$dL/dq_v = \ln\left(\left(S/a_v\right)\beta_v/\left(q\right)^{\eta_v}\right) - \eta_v - \lambda_v = 0; \tag{29}$$

$$\frac{dL}{d\beta_v} = q_v/\beta_v - \mu = 0; \tag{30}$$

$$\frac{dL}{d\mu} = 1 - \sum\nolimits_{v=1}^{6}\beta_v = 0; \tag{31}$$

$$\begin{aligned} \frac{dL}{d\mu_v} = 1 - q_v \leq 0; \quad \mu_v \geq 0; \\ \lambda_v\left(1-q_v\right) = 0. \end{aligned} \tag{32}$$

From (30) and (31), it follows that: $\sum_{v=1}^{6} q_v = \mu\sum_{v=1}^{6}\beta_v = \mu = Q$. Hence: $\beta_v = q_v/\mu = q_v/Q$. Replacing this expression into (29) yields:

$$\ln\left(\left(S/a_v\right)\left(1/Q\right)/\left(q_v\right)^{\eta_v-1}\right) - \eta_v - \lambda_v = 0.$$

Hence: $q_v = \left(a_v Q/S\right)^{-1/(\eta_v-1)} e^{-(\eta_v+\lambda_v)/(\eta_v-1)}$. Finally, considering this expression together with (32):

$$q_v = \max\left\{\left[S/\left(a_v Q\right)\right]^{\frac{1}{\eta_v-1}} e^{-\frac{\eta_v}{\eta_v-1}}, 1\right\}. \tag{33}$$

Dividing both sides by Q, and rearranging, (15) obtains. ■

D Additional theoretical results

Corollary 2 *Let $v', v'' \in \mathbb{V}$ be two goods such that $\eta_{v'} < \eta_{v''}$. Then:*

$$x_{v'} < x_{v''}, \ \forall S \in \mathbb{R}_{++}; \tag{34}$$

and:

$$\beta_{v'} \lesseqqgtr \beta_{v''} \quad \Leftrightarrow \quad S \lesseqqgtr 1. \tag{35}$$

Proof. First, the result in (34) straightforwardly follows by considering (8) in conjunction with the inequality $\eta_{v'} < \eta_{v''}$.
Second, rewrite (10) as: $\beta_v = (S/eQ)^{\frac{\eta_v}{\eta_v - 1}} / S$; it follows that:

$$\beta_{v'} \lesseqqgtr \beta_{v''} \ \Leftrightarrow \ (S/eQ)^{\eta_{v'}/(\eta_{v'}-1)} \lesseqqgtr (S/eQ)^{\eta_{v''}/(\eta_{v''}-1)}.$$

Rearrange the second inequality to get: $(S/eQ)^{(\eta_{v''}-\eta_{v'})/[(\eta_{v'}-1)(\eta_{v''}-1)]} \lesseqqgtr 1$. Raise both sides to the power $(\eta_{v'} - 1)(\eta_{v''} - 1) / (\eta_{v''} - \eta_{v'})$, and rearrange:

$$\beta_{v'} \lesseqqgtr \beta_{v''} \quad \Leftrightarrow \quad S/Q \lesseqqgtr e. \tag{36}$$

Raise both sides of the second inequality to the power $\eta_v / (\eta_v - 1)$, and divide by S to get: $(S/eQ)^{\frac{\eta_v}{\eta_v - 1}} / S \lesseqqgtr 1/S$. Use (10) to get: $\beta_v \lesseqqgtr 1/S$. Integrating over goods yields:

$$\beta_{v'} \lesseqqgtr \beta_{v''} \quad \Leftrightarrow \quad 1 = \int_{\mathbb{V}} \beta_v dv \lesseqqgtr (1/S) \int_{\mathbb{V}} dv = 1/S; \tag{37}$$

from which (35) straightforwardly obtains. ■

Corollary 3 *Let $v', v'' \in \mathbb{V}$ be two goods such that $\eta_{v'} < \eta_{v''}$. Then:*

$$\frac{dq_{v'}}{dS} \lesseqqgtr \frac{dq_{v''}}{dS} \quad \Leftrightarrow \quad S \lesseqqgtr \hat{S}_{v',v''}; \tag{38}$$

where $\hat{S}_{v',v''} \equiv \int_{\mathbb{V}} \left\{ [(\eta_{v'} - 1) / (\eta_{v''} - 1)]^{(\eta_{v'}-1)(\eta_{v''}-1)/(\eta_{v''}-\eta_{v'})} \right\}^{\eta_v/(\eta_v - 1)} dv < 1.$

Proof. From (11) it follows:

$$\frac{dq_{v'}}{dS} \lesseqqgtr \frac{dq_{v''}}{dS} \quad \Leftrightarrow \quad \frac{q_{v'}}{\eta_{v'} - 1} \lesseqqgtr \frac{q_{v''}}{\eta_{v''} - 1}.$$

Use (10), multiply both sides of the second inequality by S/Q, and rearrange to have:

$$[S/(eQ)]^{(\eta_{v''}-\eta_{v'})/[(\eta_{v'}-1)(\eta_{v''}-1)]} \lesseqqgtr (\eta_{v'} - 1) / (\eta_{v''} - 1).$$

Raise both sides to the power $(\eta_{v'}-1)(\eta_{v''}-1)/(\eta_{v''}-\eta_{v'})$ to obtain:

$$S/(eQ) \lesseqgtr [(\eta_{v'}-1)/(\eta_{v''}-1)]^{(\eta_{v'}-1)(\eta_{v''}-1)/(\eta_{v''}-\eta_{v'})}.$$

Rearrange, raise both sides to the power $\eta_v/(\eta_v-1)$, and divide by S to get:

$$(1/S)[S/(eQ)]^{\eta_v/(\eta_v-1)} \lesseqgtr (1/S)[(\eta_{v'}-1)/(\eta_{v''}-1)]^{[(\eta_{v'}-1)(\eta_{v''}-1)/(\eta_{v''}-\eta_{v'})][\eta_v/(\eta_v-1)]}.$$

Using again (10), this inequality implies:

$$\frac{dq_{v'}}{dS} \lesseqgtr \frac{dq_{v''}}{dS} \quad \Leftrightarrow \quad \beta_v \lesseqgtr \frac{1}{S}\left(\frac{\eta_{v'}-1}{\eta_{v''}-1}\right)^{\frac{(\eta_{v'}-1)(\eta_{v''}-1)}{\eta_{v''}-\eta_{v'}}\frac{\eta_v}{\eta_v-1}};$$

integrating over goods on both sides of the second inequality yields:

$$1 = \int_{\mathbb{V}} \beta_v dv \gtreqless (1/S)\int_{\mathbb{V}} [(\eta_{v'}-1)/(\eta_{v''}-1)]^{[(\eta_{v'}-1)(\eta_{v''}-1)/(\eta_{v''}-\eta_{v'})][\eta_v/(\eta_v-1)]}\, dv;$$

from which (38) straightforwardly obtains. ■

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